# Computer algebra independent integration tests

Summer 2022 edition

## 4-Trig-functions/4.1-Sine/66-4.1.10-c+d-x-^m-a+b-sin-^n

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# Contents

# Chapter 1

# Introduction

## Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 348 ]. This is test number [ 66 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 348 ) | 0.00 ( 0 ) |
| Mathematica | 100.00 ( 348 ) | 0.00 ( 0 ) |
| Fricas | 92.53 ( 322 ) | 7.47 ( 26 ) |
| Maple | 75.86 ( 264 ) | 24.14 ( 84 ) |
| Maxima | 58.33 ( 203 ) | 41.67 ( 145 ) |
| Giac | 51.44 ( 179 ) | 48.56 ( 169 ) |
| Mupad | 41.09 ( 143 ) | 58.91 ( 205 ) |
| Sympy | 33.33 ( 116 ) | 66.67 ( 232 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

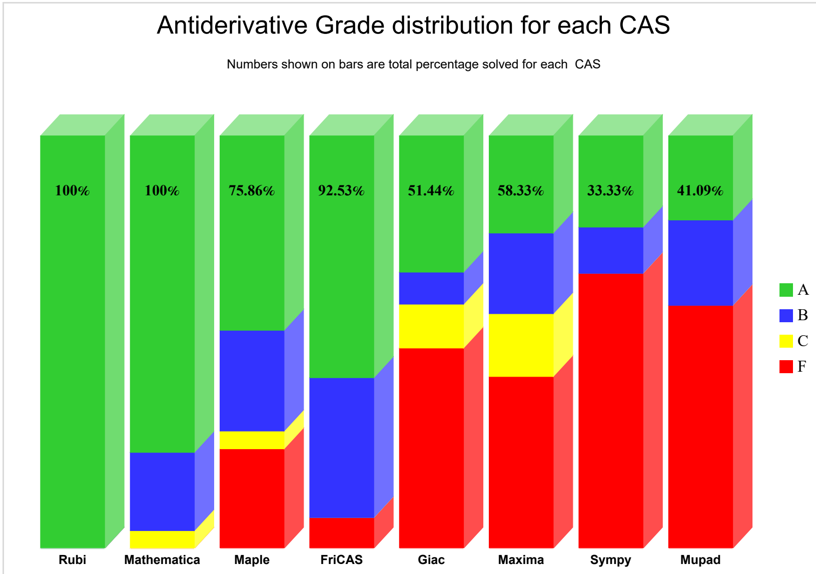
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

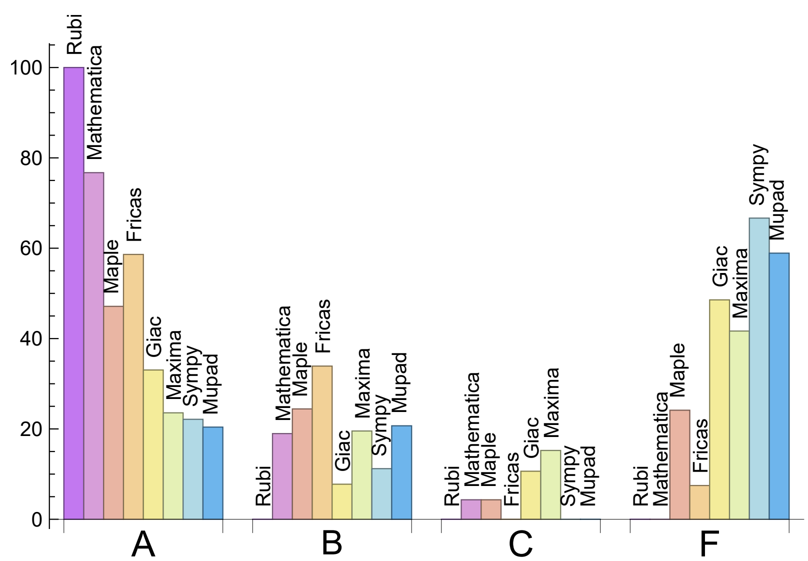
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100.00 | 0.00 | 0.00 | 0.00 |
| Mathematica | 76.72 | 18.97 | 4.31 | 0.00 |
| Fricas | 58.62 | 33.91 | 0.00 | 7.47 |
| Maple | 47.13 | 24.43 | 4.31 | 24.14 |
| Giac | 33.05 | 7.76 | 10.63 | 48.56 |
| Maxima | 23.56 | 19.54 | 15.23 | 41.67 |
| Sympy | 22.13 | 11.21 | 0.00 | 66.67 |
| Mupad | N/A | 20.69 | 0.00 | 58.91 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 % | 0.00 % | 0.00 % |
| Mathematica | 0 | 0.00 % | 0.00 % | 0.00 % |
| Maple | 84 | 100.00 % | 0.00 % | 0.00 % |
| Fricas | 26 | 23.08 % | 0.00 % | 76.92 % |
| Giac | 169 | 78.70 % | 21.30 % | 0.00 % |
| Maxima | 145 | 35.86 % | 0.00 % | 64.14 % |
| Sympy | 232 | 80.60 % | 17.67 % | 1.72 % |
| Mupad | 205 | 62.44 % | 37.56 % | 0.00 % |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

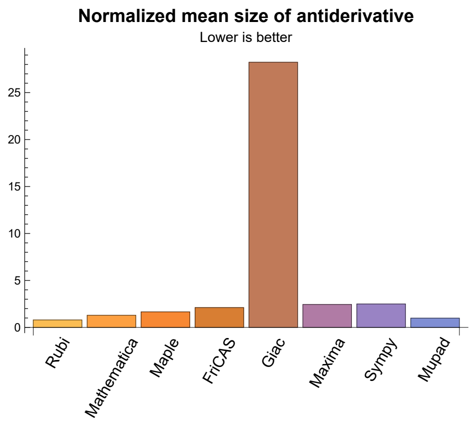
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

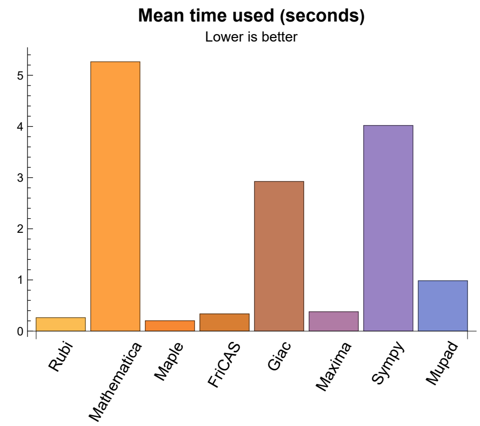
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.26 | 218.88 | 0.80 | 117.50 | 1.00 |
| Mathematica | 5.26 | 504.68 | 1.29 | 122.50 | 0.95 |
| Maple | 0.20 | 314.63 | 1.66 | 149.00 | 1.35 |
| Maxima | 0.38 | 466.57 | 2.44 | 157.00 | 1.46 |
| Fricas | 0.34 | 772.12 | 2.11 | 190.50 | 1.47 |
| Sympy | 4.02 | 257.02 | 2.49 | 0.00 | 0.00 |
| Giac | 2.92 | 4344.71 | 28.24 | 95.00 | 1.26 |
| Mupad | 0.98 | 94.38 | 0.98 | 16.00 | 0.65 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
5
10
15
20
25
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

## 1.4 list of integrals that has no closed form antiderivative

{26, 27, 31, 32, 36, 37, 65, 66, 71, 75, 76, 110, 111, 115, 116, 120, 121, 137, 138, 142, 143, 144, 145, 149, 150, 166, 167, 171, 172, 173, 177, 178, 183, 184, 189, 190, 195, 196, 201, 202, 207, 208, 213, 214, 215, 216, 217, 218, 219, 240, 241, 242, 243, 244, 255, 256, 273, 274, 279, 280, 285, 286, 290, 291, 292, 293, 314, 315, 316, 317, 318}

## 1.5 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {28, 29, 203, 204, 209, 210, 226, 230, 234, 236, 237, 238, 245, 246, 247, 248, 249, 250, 269, 270, 275, 281, 282, 300, 304, 308, 310, 311, 312, 324, 327, 329, 330, 331, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

### 1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

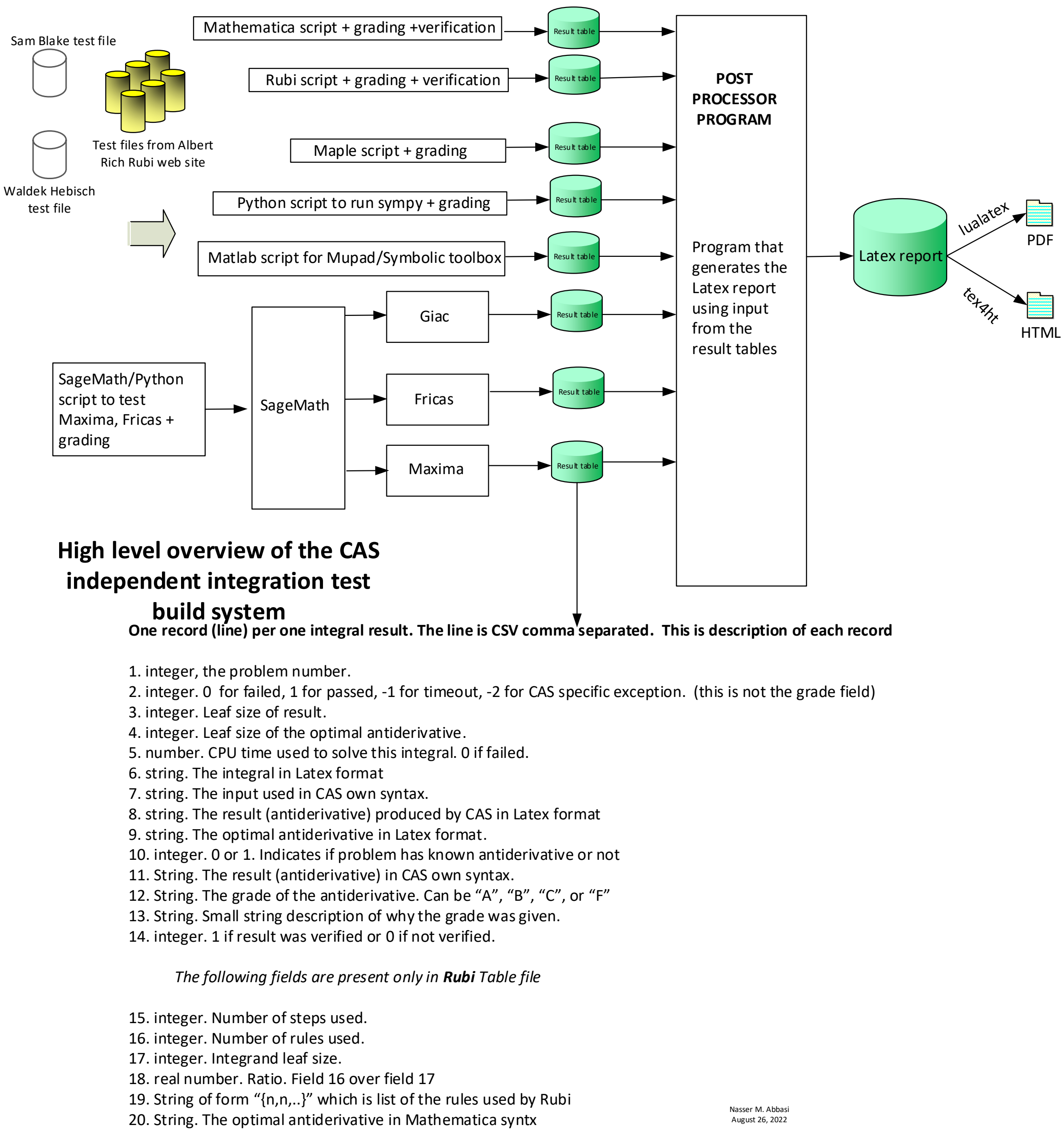
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



# Chapter 2

# detailed summary tables of results

## Local contents

## 2.1 List of integrals sorted by grade for each CAS

### Local contents

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 36, 37, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 183, 184, 186, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 200, 201, 202, 204, 206, 207, 208, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 225, 227, 229, 230, 231, 233, 235, 237, 239, 240, 241, 242, 243, 244, 245, 248, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 276, 277, 278, 279, 280, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 305, 307, 309, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 326, 328, 332, 334, 335, 336, 340, 343, 344, 348 }

B grade: { 28, 29, 34, 35, 52, 59, 175, 181, 182, 185, 187, 192, 199, 203, 205, 209, 210, 211, 220, 224, 226, 228, 232, 234, 236, 238, 246, 247, 249, 250, 253, 260, 269, 270, 271, 275, 281, 282, 283, 298, 300, 301, 302, 303, 304, 306, 308, 310, 311, 312, 323, 324, 325, 327, 329, 330, 331, 333, 337, 338, 339, 341, 342, 345, 346, 347 }

C grade: { 38, 39, 40, 41, 42, 43, 44, 60, 61, 62, 63, 64, 68, 132, 133 }

F grade: { }

### 2.1.3 Maple

A grade: { 4, 5, 6, 7, 12, 13, 14, 15, 19, 20, 21, 22, 26, 27, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 71, 75, 76, 98, 99, 100, 104, 105, 106, 110, 111, 114, 115, 116, 120, 121, 137, 138, 142, 143, 144, 145, 149, 150, 154, 155, 156, 160, 161, 162, 166, 167, 171, 172, 173, 177, 178, 182, 183, 184, 187, 188, 189, 190, 192, 194, 195, 196, 200, 201, 202, 206, 207, 208, 212, 213, 214, 215, 216, 217, 218, 219, 223, 227, 231, 235, 239, 240, 241, 242, 243, 244, 245, 248, 254, 255, 256, 259, 260, 261, 262, 265, 266, 267, 268, 272, 273, 274, 276, 277, 278, 279, 280, 284, 285, 286, 290, 291, 292, 293, 297, 301, 305, 309, 313, 314, 315, 316, 317, 318, 328, 332, 336, 340, 344, 348 }

B grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 23, 24, 25, 28, 29, 33, 34, 35, 95, 96, 97, 101, 102, 103, 107, 108, 112, 113, 117, 118, 151, 152, 153, 157, 158, 159, 165, 170, 179, 180, 185, 186, 191, 193, 197, 198, 199, 203, 204, 205, 209, 210, 211, 222, 226, 230, 234, 238, 251, 252, 253, 257, 258, 263, 264, 269, 270, 271, 275, 281, 282, 283, 296, 300, 304, 308, 312, 320, 323, 327, 331, 335, 339, 343, 347 }

C grade: { 77, 78, 79, 80, 81, 82, 83, 109, 119, 122, 123, 124, 181, 319, 322 }

F grade: { 67, 68, 69, 70, 72, 73, 74, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 163, 164, 168, 169, 174, 175, 176, 220, 221, 224, 225, 228, 229, 232, 233, 236, 237, 246, 247, 249, 250, 287, 288, 289, 294, 295, 298, 299, 302, 303, 306, 307, 310, 311, 321, 324, 325, 326, 329, 330, 333, 334, 337, 338, 341, 342, 345, 346 }

### 2.1.4 Maxima

A grade: { 4, 19, 26, 27, 31, 32, 36, 37, 65, 66, 71, 75, 76, 103, 110, 111, 115, 116, 120, 121, 137, 138, 142, 143, 144, 145, 149, 150, 159, 166, 167, 171, 172, 173, 177, 178, 182, 183, 184, 189, 190, 200, 201, 202, 213, 214, 215, 216, 217, 218, 219, 240, 241, 242, 243, 244, 253, 254, 255, 256, 264, 265, 266, 272, 273, 274, 279, 280, 284, 290, 291, 297, 305, 309, 314, 315, 316, 317, 318, 332, 340, 348 }

B grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 23, 24, 25, 28, 29, 30, 33, 34, 35, 95, 96, 97, 101, 102, 107, 108, 109, 112, 113, 114, 117, 118, 119, 151, 152, 153, 157, 158, 179, 180, 181, 185, 186, 187, 188, 194, 197, 198, 199, 206, 209, 210, 211, 212, 251, 252, 257, 258, 259, 260, 263, 269, 270, 271, 275, 276, 277, 278 }

C grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 98, 99, 100, 104, 105, 106, 154, 155, 156, 160, 161, 162, 261, 262, 267, 268 }

F grade: { 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 163, 164, 165, 168, 169, 170, 174, 175, 176, 191, 192, 193, 195, 196, 203, 204, 205, 207, 208, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 245, 246, 247, 248, 249, 250, 281, 282, 283, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 310, 311, 312, 313, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 26, 27, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 114, 115, 116, 120, 121, 137, 138, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 171, 172, 173, 174, 175, 176, 177, 178, 182, 183, 184, 187, 188, 189, 190, 193, 194, 195, 196, 201, 202, 207, 208, 213, 214, 215, 216, 217, 218, 219, 223, 227, 231, 235, 240, 241, 242, 243, 244, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 277, 278, 279, 280, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 297, 301, 305, 309, 313, 314, 315, 316, 317, 318, 319, 328, 332, 336, 340, 344, 348 }

B grade: { 7, 14, 15, 22, 23, 24, 25, 28, 29, 33, 34, 35, 52, 106, 107, 108, 109, 112, 113, 117, 118, 119, 163, 164, 165, 168, 169, 170, 179, 180, 181, 185, 186, 191, 192, 197, 198, 199, 200, 203, 204, 205, 206, 209, 210, 211, 212, 220, 221, 222, 224, 225, 226, 228, 229, 230, 232, 233, 234, 236, 237, 238, 239, 245, 246, 247, 248, 249, 250, 251, 252, 253, 269, 270, 271, 275, 276, 281, 282, 283, 294, 295, 296, 298, 299, 300, 302, 303, 304, 306, 307, 308, 310, 311, 312, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347 }

C grade: { }

F grade: { 67, 68, 70, 91, 92, 93, 94, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 144 }

### 2.1.6 Sympy

A grade: { 4, 19, 26, 27, 31, 32, 36, 37, 60, 61, 62, 63, 64, 65, 66, 71, 75, 76, 97, 103, 110, 111, 115, 116, 120, 121, 137, 138, 142, 143, 144, 149, 150, 153, 159, 166, 167, 177, 178, 183, 184, 189, 190, 201, 202, 207, 208, 213, 214, 215, 216, 217, 218, 219, 240, 241, 242, 243, 244, 254, 255, 256, 273, 274, 279, 280, 285, 286, 290, 291, 292, 293, 314, 315, 316, 317, 318 }

B grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 95, 96, 101, 102, 109, 114, 119, 151, 152, 157, 158, 181, 182, 187, 188, 193, 194, 223, 227, 257, 258, 259, 260, 263, 264, 265, 266, 297, 301 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 104, 105, 106, 107, 108, 112, 113, 117, 118, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 145, 146, 147, 148, 154, 155, 156, 160, 161, 162, 163, 164, 165, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 185, 186, 191, 192, 195, 196, 197, 198, 199, 200, 203, 204, 205, 206, 209, 210, 211, 212, 220, 221, 222, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 245, 246, 247, 248, 249, 250, 251, 252, 253, 261, 262, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 281, 282, 283, 284, 287, 288, 289, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 26, 27, 31, 32, 36, 65, 66, 71, 75, 76, 95, 96, 97, 101, 102, 103, 110, 111, 115, 116, 120, 121, 122, 123, 124, 131, 137, 138, 144, 145, 149, 150, 151, 152, 153, 157, 158, 159, 166, 167, 171, 172, 173, 177, 178, 182, 183, 184, 188, 189, 190, 194, 195, 196, 200, 201, 206, 212, 215, 216, 217, 218, 219, 223, 227, 231, 235, 239, 240, 241, 242, 243, 244, 254, 255, 256, 260, 266, 272, 273, 274, 278, 279, 284, 290, 291, 292, 293, 297, 301, 305, 309, 313, 314, 315, 316, 317, 318, 328, 332, 336, 340, 348 }

B grade: { 6, 13, 21, 30, 99, 105, 109, 114, 119, 128, 129, 130, 132, 133, 155, 161, 181, 187, 193, 257, 258, 259, 263, 264, 265, 277, 344 }

C grade: { 5, 7, 12, 14, 15, 20, 22, 38, 39, 40, 41, 45, 46, 47, 48, 53, 54, 55, 56, 60, 61, 62, 98, 100, 104, 106, 125, 126, 127, 154, 156, 160, 162, 261, 262, 267, 268 }

F grade: { 23, 24, 25, 28, 29, 33, 34, 35, 37, 42, 43, 44, 49, 50, 51, 52, 57, 58, 59, 63, 64, 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 107, 108, 112, 113, 117, 118, 134, 135, 136, 139, 140, 141, 142, 143, 146, 147, 148, 163, 164, 165, 168, 169, 170, 174, 175, 176, 179, 180, 185, 186, 191, 192, 197, 198, 199, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 220, 221, 222, 224, 225, 226, 228, 229, 230, 232, 233, 234, 236, 237, 238, 245, 246, 247, 248, 249, 250, 251, 252, 253, 269, 270, 271, 275, 276, 280, 281, 282, 283, 285, 286, 287, 288, 289, 294, 295, 296, 298, 299, 300, 302, 303, 304, 306, 307, 308, 310, 311, 312, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347 }

### 2.1.8 Mupad

A grade: { 26, 27, 31, 32, 36, 37, 65, 66, 71, 75, 76, 110, 111, 115, 116, 120, 121, 137, 138, 142, 143, 144, 145, 149, 150, 166, 167, 171, 172, 173, 177, 178, 183, 184, 189, 190, 195, 196, 201, 202, 207, 208, 213, 214, 215, 216, 217, 218, 219, 240, 241, 242, 243, 244, 255, 256, 273, 274, 279, 280, 285, 286, 290, 291, 292, 293, 314, 315, 316, 317, 318 }

B grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 30, 67, 69, 70, 95, 96, 97, 101, 102, 103, 109, 114, 119, 122, 123, 124, 151, 152, 153, 157, 158, 159, 181, 182, 187, 188, 193, 194, 200, 206, 212, 223, 227, 231, 235, 239, 254, 257, 258, 259, 260, 263, 264, 265, 266, 272, 277, 278, 284, 297, 301, 305, 309, 313, 328, 332, 336, 340, 344, 348 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 68, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 104, 105, 106, 107, 108, 112, 113, 117, 118, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 154, 155, 156, 160, 161, 162, 163, 164, 165, 168, 169, 170, 174, 175, 176, 179, 180, 185, 186, 191, 192, 197, 198, 199, 203, 204, 205, 209, 210, 211, 220, 221, 222, 224, 225, 226, 228, 229, 230, 232, 233, 234, 236, 237, 238, 245, 246, 247, 248, 249, 250, 251, 252, 253, 261, 262, 267, 268, 269, 270, 271, 275, 276, 281, 282, 283, 287, 288, 289, 294, 295, 296, 298, 299, 300, 302, 303, 304, 306, 307, 308, 310, 311, 312, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 77 | 551 | 490 | 170 | 311 | 171 | 221 |
| N.S. | 1 | 1.00 | 0.84 | 5.99 | 5.33 | 1.85 | 3.38 | 1.86 | 2.40 |
| time (sec) | N/A | 0.068 | 0.187 | 0.083 | 0.303 | 0.438 | 0.367 | 2.980 | 0.791 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 62 | 308 | 285 | 110 | 202 | 111 | 147 |
| N.S. | 1 | 1.00 | 0.87 | 4.34 | 4.01 | 1.55 | 2.85 | 1.56 | 2.07 |
| time (sec) | N/A | 0.048 | 0.109 | 0.056 | 0.315 | 0.345 | 0.227 | 3.771 | 0.621 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 45 | 148 | 141 | 63 | 112 | 65 | 84 |
| N.S. | 1 | 1.00 | 0.90 | 2.96 | 2.82 | 1.26 | 2.24 | 1.30 | 1.68 |
| time (sec) | N/A | 0.034 | 0.097 | 0.050 | 0.310 | 0.342 | 0.138 | 4.225 | 0.547 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 27 | 52 | 53 | 30 | 46 | 31 | 35 |
| N.S. | 1 | 1.00 | 0.96 | 1.86 | 1.89 | 1.07 | 1.64 | 1.11 | 1.25 |
| time (sec) | N/A | 0.012 | 0.053 | 0.027 | 0.294 | 0.408 | 0.080 | 2.884 | 0.527 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 49 | 78 | 141 | 78 | 0 | 597 | -1 |
| N.S. | 1 | 1.00 | 0.96 | 1.53 | 2.76 | 1.53 | 0.00 | 11.71 | -0.02 |
| time (sec) | N/A | 0.070 | 0.056 | 0.053 | 0.384 | 0.340 | 0.000 | 4.294 | 0.000 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 66 | 112 | 164 | 124 | 0 | 521 | -1 |
| N.S. | 1 | 1.00 | 0.92 | 1.56 | 2.28 | 1.72 | 0.00 | 7.24 | -0.01 |
| time (sec) | N/A | 0.085 | 0.149 | 0.063 | 0.359 | 0.345 | 0.000 | 4.262 | 0.000 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 87 | 150 | 199 | 209 | 0 | 5727 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 1.44 | 1.91 | 2.01 | 0.00 | 55.07 | -0.01 |
| time (sec) | N/A | 0.103 | 0.465 | 0.102 | 0.422 | 0.432 | 0.000 | 3.779 | 0.000 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 132 | 1030 | 735 | 286 | 660 | 222 | 349 |
| N.S. | 1 | 1.00 | 0.82 | 6.40 | 4.57 | 1.78 | 4.10 | 1.38 | 2.17 |
| time (sec) | N/A | 0.078 | 0.357 | 0.128 | 0.385 | 0.393 | 0.554 | 3.689 | 1.089 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 106 | 587 | 442 | 189 | 456 | 153 | 229 |
| N.S. | 1 | 1.00 | 0.79 | 4.38 | 3.30 | 1.41 | 3.40 | 1.14 | 1.71 |
| time (sec) | N/A | 0.051 | 0.267 | 0.086 | 0.330 | 0.483 | 0.457 | 3.746 | 0.847 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 77 | 289 | 232 | 112 | 264 | 94 | 179 |
| N.S. | 1 | 1.00 | 0.81 | 3.04 | 2.44 | 1.18 | 2.78 | 0.99 | 1.88 |
| time (sec) | N/A | 0.037 | 0.193 | 0.059 | 0.308 | 0.456 | 0.249 | 3.998 | 0.202 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 52 | 112 | 96 | 54 | 126 | 48 | 57 |
| N.S. | 1 | 1.00 | 0.95 | 2.04 | 1.75 | 0.98 | 2.29 | 0.87 | 1.04 |
| time (sec) | N/A | 0.018 | 0.117 | 0.035 | 0.289 | 0.406 | 0.132 | 3.388 | 0.095 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 65 | 114 | 162 | 88 | 0 | 612 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 1.46 | 2.08 | 1.13 | 0.00 | 7.85 | -0.01 |
| time (sec) | N/A | 0.128 | 0.072 | 0.052 | 0.347 | 0.390 | 0.000 | 3.242 | 0.000 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 75 | 156 | 171 | 130 | 0 | 535 | -1 |
| N.S. | 1 | 1.00 | 0.93 | 1.93 | 2.11 | 1.60 | 0.00 | 6.60 | -0.01 |
| time (sec) | N/A | 0.099 | 0.276 | 0.076 | 0.358 | 0.442 | 0.000 | 2.865 | 0.000 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 101 | 193 | 206 | 223 | 0 | 5141 | -1 |
| N.S. | 1 | 1.00 | 0.89 | 1.71 | 1.82 | 1.97 | 0.00 | 45.50 | -0.01 |
| time (sec) | N/A | 0.131 | 0.785 | 0.121 | 0.415 | 0.499 | 0.000 | 3.290 | 0.000 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 122 | 229 | 258 | 341 | 0 | 7832 | -1 |
| N.S. | 1 | 1.00 | 0.75 | 1.41 | 1.59 | 2.10 | 0.00 | 48.35 | -0.01 |
| time (sec) | N/A | 0.127 | 0.803 | 0.188 | 0.526 | 0.488 | 0.000 | 3.489 | 0.000 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 150 | 1023 | 934 | 351 | 772 | 351 | 533 |
| N.S. | 1 | 1.00 | 0.67 | 4.55 | 4.15 | 1.56 | 3.43 | 1.56 | 2.37 |
| time (sec) | N/A | 0.174 | 0.613 | 0.109 | 0.344 | 0.448 | 0.830 | 3.241 | 1.547 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 127 | 560 | 541 | 227 | 495 | 231 | 365 |
| N.S. | 1 | 1.00 | 0.73 | 3.20 | 3.09 | 1.30 | 2.83 | 1.32 | 2.09 |
| time (sec) | N/A | 0.111 | 0.600 | 0.093 | 0.318 | 0.370 | 0.536 | 3.122 | 1.109 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 86 | 265 | 270 | 131 | 284 | 137 | 174 |
| N.S. | 1 | 1.00 | 0.70 | 2.15 | 2.20 | 1.07 | 2.31 | 1.11 | 1.41 |
| time (sec) | N/A | 0.068 | 0.266 | 0.073 | 0.297 | 0.427 | 0.344 | 3.824 | 0.980 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 59 | 95 | 104 | 62 | 126 | 69 | 79 |
| N.S. | 1 | 1.00 | 0.79 | 1.27 | 1.39 | 0.83 | 1.68 | 0.92 | 1.05 |
| time (sec) | N/A | 0.029 | 0.129 | 0.052 | 0.328 | 0.444 | 0.189 | 4.313 | 0.630 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 102 | 172 | 279 | 154 | 0 | 6296 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 1.42 | 2.31 | 1.27 | 0.00 | 52.03 | -0.01 |
| time (sec) | N/A | 0.174 | 0.157 | 0.065 | 0.355 | 0.399 | 0.000 | 3.763 | 0.000 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 175 | 245 | 306 | 238 | 0 | 1000 | -1 |
| N.S. | 1 | 1.00 | 1.21 | 1.69 | 2.11 | 1.64 | 0.00 | 6.90 | -0.01 |
| time (sec) | N/A | 0.171 | 0.688 | 0.105 | 0.412 | 0.397 | 0.000 | 4.160 | 0.000 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 221 | 318 | 341 | 401 | 0 | 116534 | -1 |
| N.S. | 1 | 1.00 | 1.20 | 1.73 | 1.85 | 2.18 | 0.00 | 633.34 | -0.01 |
| time (sec) | N/A | 0.243 | 0.520 | 0.167 | 0.516 | 0.382 | 0.000 | 6.718 | 0.000 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 364 | 633 | 716 | 820 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.97 | 3.42 | 3.87 | 4.43 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.095 | 0.345 | 0.071 | 0.390 | 0.396 | 0.000 | 0.000 | 0.000 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 210 | 361 | 398 | 504 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.71 | 2.93 | 3.24 | 4.10 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.063 | 0.194 | 0.052 | 0.362 | 0.368 | 0.000 | 0.000 | 0.000 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 134 | 124 | 175 | 252 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.00 | 1.85 | 2.61 | 3.76 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.027 | 0.055 | 0.013 | 0.344 | 0.380 | 0.000 | 0.000 | 0.000 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.015 | 4.024 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.014 | 4.793 | 0.030 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 384 | 541 | 1654 | 676 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.40 | 4.79 | 14.64 | 5.98 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.147 | 6.454 | 0.069 | 0.429 | 0.403 | 0.000 | 0.000 | 0.000 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 181 | 276 | 552 | 379 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.18 | 3.33 | 6.65 | 4.57 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.097 | 2.541 | 0.061 | 0.376 | 0.379 | 0.000 | 0.000 | 0.000 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 52 | 53 | 217 | 46 | 0 | 1251 | 55 |
| N.S. | 1 | 1.00 | 1.79 | 1.83 | 7.48 | 1.59 | 0.00 | 43.14 | 1.90 |
| time (sec) | N/A | 0.019 | 0.083 | 0.023 | 0.290 | 0.370 | 0.000 | 4.418 | 1.176 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.025 | 3.981 | 0.029 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.026 | 3.997 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 309 | 309 | 478 | 1056 | 3886 | 1744 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.55 | 3.42 | 12.58 | 5.64 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.161 | 2.519 | 0.133 | 1.608 | 0.423 | 0.000 | 0.000 | 0.000 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 423 | 548 | 1938 | 972 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.35 | 3.04 | 10.77 | 5.40 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.104 | 6.586 | 0.079 | 0.617 | 0.402 | 0.000 | 0.000 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 292 | 246 | 763 | 452 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.68 | 2.26 | 7.00 | 4.15 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.049 | 1.233 | 0.049 | 0.443 | 0.377 | 0.000 | 0.000 | 0.000 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.027 | 21.020 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.025 | 23.179 | 0.043 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 124 | 233 | 263 | 190 | 0 | 1246 | -1 |
| N.S. | 1 | 1.00 | 0.64 | 1.19 | 1.35 | 0.97 | 0.00 | 6.39 | -0.01 |
| time (sec) | N/A | 0.312 | 0.078 | 0.042 | 0.314 | 0.374 | 0.000 | 3.144 | 0.000 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 125 | 188 | 242 | 156 | 0 | 779 | -1 |
| N.S. | 1 | 1.00 | 0.74 | 1.11 | 1.42 | 0.92 | 0.00 | 4.58 | -0.01 |
| time (sec) | N/A | 0.178 | 0.066 | 0.041 | 0.317 | 0.363 | 0.000 | 4.271 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 123 | 145 | 196 | 127 | 0 | 426 | -1 |
| N.S. | 1 | 1.00 | 0.87 | 1.02 | 1.38 | 0.89 | 0.00 | 3.00 | -0.01 |
| time (sec) | N/A | 0.127 | 0.064 | 0.037 | 0.332 | 0.353 | 0.000 | 5.066 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 121 | 99 | 159 | 107 | 0 | 168 | -1 |
| N.S. | 1 | 1.00 | 1.03 | 0.85 | 1.36 | 0.91 | 0.00 | 1.44 | -0.01 |
| time (sec) | N/A | 0.096 | 0.038 | 0.032 | 0.302 | 0.367 | 0.000 | 2.441 | 0.000 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 148 | 140 | 129 | 146 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.06 | 1.01 | 0.93 | 1.05 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.149 | 0.214 | 0.024 | 0.600 | 0.351 | 0.000 | 0.000 | 0.000 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 162 | 180 | 129 | 208 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.96 | 1.07 | 0.77 | 1.24 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.172 | 0.410 | 0.037 | 0.626 | 0.377 | 0.000 | 0.000 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 208 | 220 | 129 | 297 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.08 | 1.14 | 0.67 | 1.54 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.203 | 0.327 | 0.023 | 0.595 | 0.395 | 0.000 | 0.000 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 194 | 242 | 295 | 258 | 0 | 1324 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 1.05 | 1.28 | 1.12 | 0.00 | 5.73 | -0.00 |
| time (sec) | N/A | 0.303 | 1.411 | 0.039 | 0.552 | 0.383 | 0.000 | 3.870 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 175 | 197 | 274 | 195 | 0 | 816 | -1 |
| N.S. | 1 | 1.00 | 0.86 | 0.97 | 1.35 | 0.96 | 0.00 | 4.02 | -0.00 |
| time (sec) | N/A | 0.247 | 1.073 | 0.034 | 0.531 | 0.371 | 0.000 | 4.938 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 149 | 150 | 229 | 148 | 0 | 434 | -1 |
| N.S. | 1 | 1.00 | 0.94 | 0.95 | 1.45 | 0.94 | 0.00 | 2.75 | -0.01 |
| time (sec) | N/A | 0.192 | 0.313 | 0.035 | 0.524 | 0.368 | 0.000 | 4.289 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 126 | 108 | 187 | 114 | 0 | 165 | -1 |
| N.S. | 1 | 1.00 | 0.97 | 0.83 | 1.44 | 0.88 | 0.00 | 1.27 | -0.01 |
| time (sec) | N/A | 0.160 | 0.151 | 0.037 | 0.499 | 0.364 | 0.000 | 4.469 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 149 | 145 | 135 | 138 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.10 | 1.07 | 1.00 | 1.02 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.175 | 0.237 | 0.054 | 0.598 | 0.360 | 0.000 | 0.000 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 158 | 189 | 136 | 209 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.93 | 1.11 | 0.80 | 1.23 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.219 | 0.923 | 0.054 | 0.607 | 0.365 | 0.000 | 0.000 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 216 | 244 | 230 | 136 | 328 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.13 | 1.06 | 0.63 | 1.52 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.232 | 1.315 | 0.053 | 0.618 | 0.420 | 0.000 | 0.000 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | C | B | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 661 | 273 | 136 | 422 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.68 | 1.11 | 0.55 | 1.71 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.285 | 2.929 | 0.066 | 0.674 | 0.419 | 0.000 | 0.000 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 410 | 410 | 542 | 476 | 547 | 371 | 0 | 2479 | -1 |
| N.S. | 1 | 1.00 | 1.32 | 1.16 | 1.33 | 0.90 | 0.00 | 6.05 | -0.00 |
| time (sec) | N/A | 0.800 | 2.059 | 0.033 | 0.547 | 0.393 | 0.000 | 3.949 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 354 | 354 | 389 | 384 | 499 | 300 | 0 | 1548 | -1 |
| N.S. | 1 | 1.00 | 1.10 | 1.08 | 1.41 | 0.85 | 0.00 | 4.37 | -0.00 |
| time (sec) | N/A | 0.682 | 1.043 | 0.030 | 0.570 | 0.387 | 0.000 | 5.095 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 266 | 296 | 424 | 246 | 0 | 848 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 0.97 | 1.39 | 0.81 | 0.00 | 2.79 | -0.00 |
| time (sec) | N/A | 0.331 | 0.473 | 0.027 | 0.534 | 0.377 | 0.000 | 4.733 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 257 | 202 | 210 | 377 | 212 | 0 | 332 | -1 |
| N.S. | 1 | 1.00 | 0.79 | 0.82 | 1.47 | 0.82 | 0.00 | 1.29 | -0.00 |
| time (sec) | N/A | 0.269 | 0.364 | 0.029 | 0.541 | 0.359 | 0.000 | 4.610 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 300 | 288 | 253 | 274 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.11 | 1.07 | 0.94 | 1.01 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.372 | 0.616 | 0.029 | 0.674 | 0.377 | 0.000 | 0.000 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 292 | 496 | 368 | 253 | 388 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.70 | 1.26 | 0.87 | 1.33 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.492 | 1.529 | 0.029 | 0.679 | 0.415 | 0.000 | 0.000 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | C | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 356 | 356 | 1429 | 450 | 254 | 549 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.01 | 1.26 | 0.71 | 1.54 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.537 | 6.219 | 0.032 | 0.677 | 0.422 | 0.000 | 0.000 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | A | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 60 | 87 | 106 | 72 | 117 | 220 | -1 |
| N.S. | 1 | 1.00 | 0.69 | 1.00 | 1.22 | 0.83 | 1.34 | 2.53 | -0.01 |
| time (sec) | N/A | 0.073 | 0.011 | 0.040 | 0.289 | 0.352 | 16.082 | 3.283 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | A | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 69 | 65 | 84 | 54 | 85 | 176 | -1 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.29 | 0.83 | 1.31 | 2.71 | -0.02 |
| time (sec) | N/A | 0.037 | 0.009 | 0.019 | 0.334 | 0.350 | 1.080 | 3.549 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | A | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 59 | 42 | 67 | 38 | 54 | 136 | -1 |
| N.S. | 1 | 1.00 | 1.28 | 0.91 | 1.46 | 0.83 | 1.17 | 2.96 | -0.02 |
| time (sec) | N/A | 0.022 | 0.007 | 0.013 | 0.289 | 0.351 | 0.578 | 4.740 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 64 | 60 | 38 | 57 | 80 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.59 | 0.89 | 1.25 | 0.00 | -0.02 |
| time (sec) | N/A | 0.040 | 0.017 | 0.014 | 0.557 | 0.342 | 2.305 | 0.000 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 111 | 79 | 38 | 69 | 114 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.28 | 0.91 | 0.44 | 0.79 | 1.31 | 0.00 | -0.01 |
| time (sec) | N/A | 0.059 | 0.060 | 0.019 | 0.590 | 0.354 | 17.746 | 0.000 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.022 | 10.477 | 0.037 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.022 | 10.392 | 0.040 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 33 | 0 | 0 | 0 | 0 | 0 | 36 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.95 |
| time (sec) | N/A | 0.043 | 0.244 | 0.128 | 0.000 | 0.000 | 0.000 | 0.000 | 1.052 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 185 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.073 | 2.866 | 0.122 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 35 | 0 | 0 | 52 | 0 | 0 | 140 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 1.24 | 0.00 | 0.00 | 3.33 |
| time (sec) | N/A | 0.050 | 0.186 | 0.118 | 0.000 | 0.358 | 0.000 | 0.000 | 3.107 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 58 | 0 | 0 | 0 | 0 | 0 | 253 |
| N.S. | 1 | 1.00 | 0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.05 |
| time (sec) | N/A | 0.063 | 0.376 | 0.151 | 0.000 | 0.000 | 0.000 | 0.000 | 4.486 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.028 | 0.522 | 0.024 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 267 | 267 | 251 | 0 | 0 | 188 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.70 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.223 | 9.422 | 0.133 | 0.000 | 0.153 | 0.000 | 0.000 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 211 | 0 | 0 | 136 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.30 | 0.00 | 0.00 | 0.84 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.154 | 0.420 | 0.050 | 0.000 | 0.100 | 0.000 | 0.000 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 121 | 0 | 0 | 94 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 0.74 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.057 | 0.031 | 0.023 | 0.000 | 0.097 | 0.000 | 0.000 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.013 | 5.382 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.025 | 0.552 | 0.016 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 79 | 454 | 0 | 52 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 5.75 | 0.00 | 0.66 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.052 | 0.014 | 0.051 | 0.000 | 0.089 | 0.000 | 0.000 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 75 | 353 | 0 | 52 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 4.71 | 0.00 | 0.69 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.049 | 0.012 | 0.041 | 0.000 | 0.087 | 0.000 | 0.000 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 79 | 290 | 0 | 52 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 3.67 | 0.00 | 0.66 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.051 | 0.012 | 0.045 | 0.000 | 0.107 | 0.000 | 0.000 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 75 | 378 | 0 | 48 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 5.04 | 0.00 | 0.64 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.047 | 0.011 | 0.044 | 0.000 | 0.084 | 0.000 | 0.000 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 63 | 426 | 0 | 48 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.91 | 6.17 | 0.00 | 0.70 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.049 | 0.016 | 0.046 | 0.000 | 0.091 | 0.000 | 0.000 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 65 | 529 | 0 | 52 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.92 | 7.45 | 0.00 | 0.73 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.046 | 0.015 | 0.053 | 0.000 | 0.098 | 0.000 | 0.000 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 79 | 599 | 0 | 52 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 7.58 | 0.00 | 0.66 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.051 | 0.012 | 0.051 | 0.000 | 0.110 | 0.000 | 0.000 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 118 | 0 | 0 | 77 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.22 | 0.00 | 0.00 | 0.79 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.112 | 0.232 | 0.034 | 0.000 | 0.126 | 0.000 | 0.000 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 120 | 0 | 0 | 77 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.17 | 0.00 | 0.00 | 0.75 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.104 | 0.223 | 0.036 | 0.000 | 0.104 | 0.000 | 0.000 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 116 | 0 | 0 | 77 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.17 | 0.00 | 0.00 | 0.78 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.105 | 0.219 | 0.043 | 0.000 | 0.100 | 0.000 | 0.000 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 120 | 0 | 0 | 69 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.17 | 0.00 | 0.00 | 0.67 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.096 | 0.186 | 0.036 | 0.000 | 0.101 | 0.000 | 0.000 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 99 | 0 | 0 | 64 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.19 | 0.00 | 0.00 | 0.77 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.091 | 0.163 | 0.051 | 0.000 | 0.116 | 0.000 | 0.000 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 117 | 0 | 0 | 77 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.16 | 0.00 | 0.00 | 0.76 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.099 | 0.218 | 0.037 | 0.000 | 0.096 | 0.000 | 0.000 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 121 | 0 | 0 | 77 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.25 | 0.00 | 0.00 | 0.79 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.106 | 0.264 | 0.035 | 0.000 | 0.093 | 0.000 | 0.000 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 29 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.086 | 0.257 | 0.078 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 87 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.141 | 0.341 | 0.078 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 29 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.071 | 0.270 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 57 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.090 | 1.371 | 0.092 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 123 | 482 | 498 | 170 | 264 | 157 | 191 |
| N.S. | 1 | 1.00 | 1.37 | 5.36 | 5.53 | 1.89 | 2.93 | 1.74 | 2.12 |
| time (sec) | N/A | 0.084 | 0.314 | 0.076 | 0.307 | 0.344 | 0.247 | 2.351 | 0.259 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 81 | 241 | 260 | 104 | 151 | 95 | 112 |
| N.S. | 1 | 1.00 | 1.19 | 3.54 | 3.82 | 1.53 | 2.22 | 1.40 | 1.65 |
| time (sec) | N/A | 0.059 | 0.244 | 0.050 | 0.298 | 0.364 | 0.148 | 2.633 | 0.153 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 51 | 90 | 103 | 53 | 68 | 47 | 54 |
| N.S. | 1 | 1.00 | 1.13 | 2.00 | 2.29 | 1.18 | 1.51 | 1.04 | 1.20 |
| time (sec) | N/A | 0.028 | 0.186 | 0.033 | 0.312 | 0.380 | 0.088 | 1.513 | 0.097 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 54 | 103 | 181 | 94 | 0 | 712 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 1.61 | 2.83 | 1.47 | 0.00 | 11.12 | -0.02 |
| time (sec) | N/A | 0.101 | 0.140 | 0.053 | 0.346 | 0.353 | 0.000 | 1.865 | 0.000 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 110 | 141 | 208 | 138 | 0 | 578 | -1 |
| N.S. | 1 | 1.00 | 1.25 | 1.60 | 2.36 | 1.57 | 0.00 | 6.57 | -0.01 |
| time (sec) | N/A | 0.118 | 0.295 | 0.075 | 0.390 | 0.352 | 0.000 | 1.828 | 0.000 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 104 | 177 | 279 | 231 | 0 | 6157 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 1.44 | 2.27 | 1.88 | 0.00 | 50.06 | -0.01 |
| time (sec) | N/A | 0.141 | 0.373 | 0.102 | 0.407 | 0.422 | 0.000 | 1.983 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 216 | 1135 | 1041 | 372 | 779 | 339 | 452 |
| N.S. | 1 | 1.00 | 0.91 | 4.79 | 4.39 | 1.57 | 3.29 | 1.43 | 1.91 |
| time (sec) | N/A | 0.173 | 0.731 | 0.128 | 0.325 | 0.353 | 0.442 | 1.363 | 1.319 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 182 | 567 | 549 | 216 | 456 | 207 | 255 |
| N.S. | 1 | 1.00 | 1.08 | 3.38 | 3.27 | 1.29 | 2.71 | 1.23 | 1.52 |
| time (sec) | N/A | 0.124 | 0.392 | 0.096 | 0.309 | 0.386 | 0.287 | 1.747 | 0.970 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 80 | 219 | 224 | 105 | 219 | 107 | 127 |
| N.S. | 1 | 1.00 | 0.68 | 1.86 | 1.90 | 0.89 | 1.86 | 0.91 | 1.08 |
| time (sec) | N/A | 0.066 | 0.472 | 0.061 | 0.292 | 0.351 | 0.159 | 1.684 | 0.727 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 114 | 198 | 359 | 191 | 0 | 7049 | -1 |
| N.S. | 1 | 1.00 | 0.79 | 1.37 | 2.48 | 1.32 | 0.00 | 48.61 | -0.01 |
| time (sec) | N/A | 0.237 | 0.169 | 0.078 | 0.451 | 0.349 | 0.000 | 2.019 | 0.000 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 206 | 274 | 396 | 289 | 0 | 1134 | -1 |
| N.S. | 1 | 1.00 | 1.27 | 1.69 | 2.44 | 1.78 | 0.00 | 7.00 | -0.01 |
| time (sec) | N/A | 0.204 | 0.402 | 0.104 | 0.408 | 0.370 | 0.000 | 2.925 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 353 | 347 | 504 | 484 | 0 | 124086 | -1 |
| N.S. | 1 | 1.00 | 1.57 | 1.54 | 2.24 | 2.15 | 0.00 | 551.49 | -0.00 |
| time (sec) | N/A | 0.324 | 0.596 | 0.150 | 0.593 | 0.452 | 0.000 | 3.593 | 0.000 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 148 | 126 | 484 | 1054 | 951 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 3.27 | 7.12 | 6.43 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.199 | 0.738 | 0.125 | 0.388 | 0.362 | 0.000 | 0.000 | 0.000 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 94 | 254 | 329 | 533 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 2.25 | 2.91 | 4.72 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.140 | 0.489 | 0.070 | 0.354 | 0.374 | 0.000 | 0.000 | 0.000 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 51 | 73 | 185 | 107 | 272 | 696 | 66 |
| N.S. | 1 | 1.00 | 0.85 | 1.22 | 3.08 | 1.78 | 4.53 | 11.60 | 1.10 |
| time (sec) | N/A | 0.043 | 0.110 | 0.062 | 0.300 | 0.421 | 0.453 | 1.570 | 1.040 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.038 | 3.328 | 0.050 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.036 | 3.206 | 0.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 309 | 309 | 257 | 807 | 3779 | 1764 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 2.61 | 12.23 | 5.71 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.255 | 1.341 | 0.942 | 0.943 | 0.455 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 243 | 175 | 421 | 871 | 940 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.72 | 1.73 | 3.58 | 3.87 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.181 | 1.529 | 0.885 | 0.598 | 0.380 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 148 | 225 | 169 | 993 | 217 | 1336 | 3094 | 183 |
| N.S. | 1 | 1.00 | 1.52 | 1.14 | 6.71 | 1.47 | 9.03 | 20.91 | 1.24 |
| time (sec) | N/A | 0.062 | 0.618 | 0.393 | 0.333 | 0.339 | 0.855 | 2.433 | 4.810 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.037 | 9.397 | 0.247 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.036 | 9.800 | 0.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 124 | 484 | 1059 | 952 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 3.29 | 7.20 | 6.48 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.208 | 0.812 | 0.129 | 0.476 | 0.376 | 0.000 | 0.000 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 92 | 254 | 331 | 536 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 2.27 | 2.96 | 4.79 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.141 | 0.518 | 0.089 | 0.385 | 0.353 | 0.000 | 0.000 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 47 | 73 | 185 | 108 | 272 | 697 | 66 |
| N.S. | 1 | 1.00 | 0.80 | 1.24 | 3.14 | 1.83 | 4.61 | 11.81 | 1.12 |
| time (sec) | N/A | 0.045 | 0.111 | 0.079 | 0.362 | 0.339 | 0.448 | 2.030 | 0.890 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.043 | 3.480 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.040 | 3.203 | 0.056 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F(-2) | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 108 | 145 | 0 | 0 | 0 | 117 | 82 |
| N.S. | 1 | 1.00 | 0.90 | 1.21 | 0.00 | 0.00 | 0.00 | 0.98 | 0.68 |
| time (sec) | N/A | 0.091 | 0.160 | 0.077 | 0.000 | 0.000 | 0.000 | 1.736 | 0.958 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F(-2) | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 92 | 119 | 0 | 0 | 0 | 93 | 64 |
| N.S. | 1 | 1.00 | 0.94 | 1.21 | 0.00 | 0.00 | 0.00 | 0.95 | 0.65 |
| time (sec) | N/A | 0.072 | 0.118 | 0.037 | 0.000 | 0.000 | 0.000 | 1.715 | 0.841 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F(-2) | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 76 | 93 | 0 | 0 | 0 | 69 | 47 |
| N.S. | 1 | 1.00 | 1.31 | 1.60 | 0.00 | 0.00 | 0.00 | 1.19 | 0.81 |
| time (sec) | N/A | 0.038 | 0.096 | 0.035 | 0.000 | 0.000 | 0.000 | 1.524 | 0.234 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 83 | 0 | 0 | 0 | 0 | 383 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 3.79 | -0.01 |
| time (sec) | N/A | 0.096 | 0.088 | 0.025 | 0.000 | 0.000 | 0.000 | 2.538 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 117 | 0 | 0 | 0 | 0 | 1140 | -1 |
| N.S. | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 8.77 | -0.01 |
| time (sec) | N/A | 0.104 | 0.171 | 0.012 | 0.000 | 0.000 | 0.000 | 2.195 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 153 | 0 | 0 | 0 | 0 | 1487 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 8.55 | -0.01 |
| time (sec) | N/A | 0.119 | 0.177 | 0.011 | 0.000 | 0.000 | 0.000 | 1.639 | 0.000 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 337 | 337 | 231 | 0 | 0 | 0 | 0 | 1210 | -1 |
| N.S. | 1 | 1.00 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 3.59 | -0.00 |
| time (sec) | N/A | 0.154 | 0.843 | 0.017 | 0.000 | 0.000 | 0.000 | 3.075 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 271 | 271 | 191 | 0 | 0 | 0 | 0 | 633 | -1 |
| N.S. | 1 | 1.00 | 0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 2.34 | -0.00 |
| time (sec) | N/A | 0.114 | 0.622 | 0.010 | 0.000 | 0.000 | 0.000 | 3.170 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 165 | 113 | 0 | 0 | 0 | 0 | 274 | -1 |
| N.S. | 1 | 1.00 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 1.66 | -0.01 |
| time (sec) | N/A | 0.062 | 0.327 | 0.012 | 0.000 | 0.000 | 0.000 | 2.558 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 127 | 0 | 0 | 0 | 0 | 138 | -1 |
| N.S. | 1 | 1.00 | 0.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.62 | -0.00 |
| time (sec) | N/A | 0.184 | 0.312 | 0.010 | 0.000 | 0.000 | 0.000 | 2.973 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 263 | 226 | 0 | 0 | 0 | 0 | 541 | -1 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 2.06 | -0.00 |
| time (sec) | N/A | 0.185 | 0.545 | 0.010 | 0.000 | 0.000 | 0.000 | 3.995 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 332 | 332 | 295 | 0 | 0 | 0 | 0 | 1344 | -1 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 4.05 | -0.00 |
| time (sec) | N/A | 0.226 | 0.460 | 0.010 | 0.000 | 0.000 | 0.000 | 2.806 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 417 | 417 | 306 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.148 | 0.518 | 0.016 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 293 | 245 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.119 | 0.384 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 231 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.061 | 1.022 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.047 | 2.103 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.045 | 0.494 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 691 | 691 | 455 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.239 | 1.783 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 435 | 435 | 352 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.167 | 1.208 | 0.011 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 249 | 249 | 308 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.085 | 1.661 | 0.016 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.056 | 20.592 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.051 | 10.460 | 0.014 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.046 | 2.007 | 0.010 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.032 | 0.752 | 0.024 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 449 | 449 | 376 | 0 | 0 | 392 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 0.87 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.412 | 0.566 | 0.130 | 0.000 | 0.127 | 0.000 | 0.000 | 0.000 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 260 | 0 | 0 | 274 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.92 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.253 | 0.190 | 0.158 | 0.000 | 0.113 | 0.000 | 0.000 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 148 | 199 | 0 | 0 | 138 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.34 | 0.00 | 0.00 | 0.93 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.097 | 1.791 | 0.042 | 0.000 | 0.113 | 0.000 | 0.000 | 0.000 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.035 | 0.559 | 0.058 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.033 | 3.600 | 0.143 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 124 | 482 | 498 | 170 | 264 | 157 | 191 |
| N.S. | 1 | 1.00 | 1.38 | 5.36 | 5.53 | 1.89 | 2.93 | 1.74 | 2.12 |
| time (sec) | N/A | 0.082 | 0.245 | 0.065 | 0.295 | 0.347 | 0.250 | 5.801 | 0.804 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 84 | 241 | 260 | 104 | 151 | 95 | 112 |
| N.S. | 1 | 1.00 | 1.24 | 3.54 | 3.82 | 1.53 | 2.22 | 1.40 | 1.65 |
| time (sec) | N/A | 0.058 | 0.196 | 0.049 | 0.370 | 0.366 | 0.146 | 5.389 | 0.670 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 43 | 90 | 103 | 53 | 68 | 47 | 50 |
| N.S. | 1 | 1.00 | 0.96 | 2.00 | 2.29 | 1.18 | 1.51 | 1.04 | 1.11 |
| time (sec) | N/A | 0.028 | 0.084 | 0.030 | 0.291 | 0.341 | 0.091 | 4.540 | 0.634 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 57 | 103 | 181 | 94 | 0 | 712 | -1 |
| N.S. | 1 | 1.00 | 0.89 | 1.61 | 2.83 | 1.47 | 0.00 | 11.12 | -0.02 |
| time (sec) | N/A | 0.085 | 0.077 | 0.049 | 0.342 | 0.360 | 0.000 | 4.930 | 0.000 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 72 | 141 | 208 | 138 | 0 | 578 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 1.60 | 2.36 | 1.57 | 0.00 | 6.57 | -0.01 |
| time (sec) | N/A | 0.103 | 0.232 | 0.175 | 0.373 | 0.374 | 0.000 | 2.769 | 0.000 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 94 | 177 | 279 | 231 | 0 | 6157 | -1 |
| N.S. | 1 | 1.00 | 0.76 | 1.44 | 2.27 | 1.88 | 0.00 | 50.06 | -0.01 |
| time (sec) | N/A | 0.127 | 0.513 | 0.110 | 0.489 | 0.368 | 0.000 | 2.846 | 0.000 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 250 | 232 | 1125 | 1031 | 386 | 779 | 371 | 497 |
| N.S. | 1 | 1.00 | 0.93 | 4.50 | 4.12 | 1.54 | 3.12 | 1.48 | 1.99 |
| time (sec) | N/A | 0.175 | 0.709 | 0.122 | 0.391 | 0.362 | 0.446 | 6.013 | 2.643 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 182 | 249 | 561 | 543 | 230 | 456 | 229 | 281 |
| N.S. | 1 | 1.00 | 1.37 | 3.08 | 2.98 | 1.26 | 2.51 | 1.26 | 1.54 |
| time (sec) | N/A | 0.124 | 0.416 | 0.097 | 0.304 | 0.363 | 0.290 | 4.835 | 1.152 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 96 | 216 | 221 | 113 | 219 | 119 | 143 |
| N.S. | 1 | 1.00 | 0.83 | 1.86 | 1.91 | 0.97 | 1.89 | 1.03 | 1.23 |
| time (sec) | N/A | 0.066 | 0.371 | 0.062 | 0.344 | 0.369 | 0.160 | 4.863 | 0.743 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 134 | 221 | 358 | 194 | 0 | 7397 | -1 |
| N.S. | 1 | 1.00 | 0.86 | 1.42 | 2.29 | 1.24 | 0.00 | 47.42 | -0.01 |
| time (sec) | N/A | 0.209 | 0.156 | 0.065 | 0.403 | 0.371 | 0.000 | 3.900 | 0.000 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 232 | 301 | 395 | 286 | 0 | 1135 | -1 |
| N.S. | 1 | 1.00 | 1.27 | 1.64 | 2.16 | 1.56 | 0.00 | 6.20 | -0.01 |
| time (sec) | N/A | 0.230 | 0.374 | 0.103 | 0.503 | 0.367 | 0.000 | 4.294 | 0.000 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 245 | 395 | 374 | 503 | 476 | 0 | 123654 | -1 |
| N.S. | 1 | 1.00 | 1.61 | 1.53 | 2.05 | 1.94 | 0.00 | 504.71 | -0.00 |
| time (sec) | N/A | 0.290 | 0.756 | 0.148 | 0.513 | 0.394 | 0.000 | 9.919 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 495 | 495 | 401 | 0 | 0 | 2237 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.81 | 0.00 | 0.00 | 4.52 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.647 | 0.165 | 0.058 | 0.000 | 0.548 | 0.000 | 0.000 | 0.000 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 367 | 367 | 296 | 0 | 0 | 1599 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.81 | 0.00 | 0.00 | 4.36 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.529 | 0.121 | 0.040 | 0.000 | 0.535 | 0.000 | 0.000 | 0.000 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 182 | 492 | 0 | 1045 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.78 | 2.10 | 0.00 | 4.47 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.287 | 0.029 | 0.076 | 0.000 | 0.546 | 0.000 | 0.000 | 0.000 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.037 | 0.268 | 0.036 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.034 | 0.227 | 0.040 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 925 | 925 | 742 | 0 | 0 | 5206 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.80 | 0.00 | 0.00 | 5.63 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.024 | 2.136 | 0.609 | 0.000 | 0.728 | 0.000 | 0.000 | 0.000 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 671 | 671 | 530 | 0 | 0 | 3197 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 4.76 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.762 | 1.109 | 0.710 | 0.000 | 0.602 | 0.000 | 0.000 | 0.000 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 305 | 236 | 641 | 0 | 1586 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.77 | 2.10 | 0.00 | 5.20 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.338 | 0.688 | 1.445 | 0.000 | 0.573 | 0.000 | 0.000 | 0.000 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.037 | 23.013 | 0.260 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.035 | 69.619 | 0.373 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.032 | 0.716 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 607 | 607 | 552 | 0 | 0 | 442 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.542 | 11.854 | 0.169 | 0.000 | 0.123 | 0.000 | 0.000 | 0.000 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 318 | 707 | 0 | 0 | 282 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.22 | 0.00 | 0.00 | 0.89 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.262 | 9.483 | 0.157 | 0.000 | 0.103 | 0.000 | 0.000 | 0.000 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 148 | 166 | 0 | 0 | 138 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.12 | 0.00 | 0.00 | 0.93 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.096 | 2.233 | 0.031 | 0.000 | 0.101 | 0.000 | 0.000 | 0.000 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.036 | 0.247 | 0.042 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.034 | 1.577 | 0.176 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 240 | 537 | 1316 | 1051 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.46 | 3.27 | 8.02 | 6.41 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.223 | 1.191 | 0.168 | 0.631 | 0.386 | 0.000 | 0.000 | 0.000 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 181 | 293 | 409 | 587 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.40 | 2.27 | 3.17 | 4.55 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.160 | 0.988 | 0.112 | 0.599 | 0.389 | 0.000 | 0.000 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 199 | 88 | 274 | 152 | 456 | 772 | 80 |
| N.S. | 1 | 1.00 | 2.62 | 1.16 | 3.61 | 2.00 | 6.00 | 10.16 | 1.05 |
| time (sec) | N/A | 0.063 | 0.331 | 0.126 | 0.498 | 0.353 | 0.724 | 3.256 | 1.159 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 72 | 37 | 50 | 54 | 80 | 32 | 27 |
| N.S. | 1 | 1.00 | 2.57 | 1.32 | 1.79 | 1.93 | 2.86 | 1.14 | 0.96 |
| time (sec) | N/A | 0.025 | 0.084 | 0.058 | 0.486 | 0.339 | 0.616 | 3.428 | 0.736 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.029 | 5.841 | 0.062 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.028 | 5.791 | 0.082 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 1378 | 759 | 4616 | 1319 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.58 | 3.07 | 18.69 | 5.34 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.320 | 2.697 | 0.269 | 0.838 | 0.424 | 0.000 | 0.000 | 0.000 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 263 | 419 | 613 | 721 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.40 | 2.23 | 3.26 | 3.84 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.236 | 1.730 | 0.369 | 0.710 | 0.409 | 0.000 | 0.000 | 0.000 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 236 | 192 | 1763 | 200 | 1867 | 20204 | 164 |
| N.S. | 1 | 1.00 | 2.13 | 1.73 | 15.88 | 1.80 | 16.82 | 182.02 | 1.48 |
| time (sec) | N/A | 0.100 | 0.462 | 0.216 | 0.546 | 0.350 | 1.410 | 5.818 | 1.768 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 85 | 54 | 129 | 69 | 422 | 77 | 69 |
| N.S. | 1 | 1.00 | 1.89 | 1.20 | 2.87 | 1.53 | 9.38 | 1.71 | 1.53 |
| time (sec) | N/A | 0.055 | 0.101 | 0.072 | 0.504 | 0.351 | 1.251 | 5.187 | 1.185 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.047 | 5.825 | 0.249 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.046 | 6.504 | 0.311 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 382 | 382 | 391 | 881 | 0 | 1569 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.02 | 2.31 | 0.00 | 4.11 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.406 | 3.370 | 0.212 | 0.000 | 0.442 | 0.000 | 0.000 | 0.000 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 278 | 278 | 931 | 492 | 0 | 851 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.35 | 1.77 | 0.00 | 3.06 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.315 | 1.522 | 0.521 | 0.000 | 0.406 | 0.000 | 0.000 | 0.000 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 298 | 452 | 0 | 256 | 4653 | 44105 | 246 |
| N.S. | 1 | 1.00 | 1.89 | 2.86 | 0.00 | 1.62 | 29.45 | 279.15 | 1.56 |
| time (sec) | N/A | 0.139 | 0.834 | 0.271 | 0.000 | 0.367 | 2.921 | 12.406 | 1.940 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 117 | 91 | 212 | 92 | 1127 | 91 | 92 |
| N.S. | 1 | 1.00 | 1.56 | 1.21 | 2.83 | 1.23 | 15.03 | 1.21 | 1.23 |
| time (sec) | N/A | 0.043 | 0.135 | 0.092 | 0.541 | 0.353 | 2.484 | 5.174 | 3.286 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.046 | 4.189 | 0.645 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.045 | 3.844 | 0.885 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 352 | 352 | 565 | 1151 | 2842 | 2929 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.61 | 3.27 | 8.07 | 8.32 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.329 | 4.033 | 0.261 | 0.819 | 0.510 | 0.000 | 0.000 | 0.000 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 249 | 249 | 363 | 643 | 1443 | 1666 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.46 | 2.58 | 5.80 | 6.69 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.228 | 1.741 | 0.154 | 0.493 | 0.443 | 0.000 | 0.000 | 0.000 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 300 | 245 | 527 | 626 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.24 | 1.83 | 3.93 | 4.67 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.088 | 0.619 | 0.173 | 0.403 | 0.401 | 0.000 | 0.000 | 0.000 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 48 | 34 | 51 | 97 | 0 | 38 | 39 |
| N.S. | 1 | 1.00 | 1.26 | 0.89 | 1.34 | 2.55 | 0.00 | 1.00 | 1.03 |
| time (sec) | N/A | 0.034 | 0.047 | 0.080 | 0.286 | 0.344 | 0.000 | 6.062 | 1.214 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.030 | 7.067 | 0.085 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.031 | 8.150 | 0.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 463 | 463 | 1208 | 1705 | 0 | 4792 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.61 | 3.68 | 0.00 | 10.35 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.503 | 20.540 | 0.288 | 0.000 | 0.527 | 0.000 | 0.000 | 0.000 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 327 | 327 | 559 | 942 | 0 | 2577 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.71 | 2.88 | 0.00 | 7.88 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.343 | 9.847 | 0.200 | 0.000 | 0.452 | 0.000 | 0.000 | 0.000 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 396 | 351 | 0 | 881 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.34 | 2.08 | 0.00 | 5.21 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.137 | 1.051 | 0.178 | 0.000 | 0.416 | 0.000 | 0.000 | 0.000 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 57 | 59 | 112 | 156 | 0 | 88 | 83 |
| N.S. | 1 | 1.00 | 1.12 | 1.16 | 2.20 | 3.06 | 0.00 | 1.73 | 1.63 |
| time (sec) | N/A | 0.057 | 0.135 | 0.086 | 0.290 | 0.348 | 0.000 | 5.941 | 1.268 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.047 | 14.795 | 0.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.046 | 29.653 | 0.131 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 600 | 600 | 1370 | 2257 | 12815 | 7847 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.28 | 3.76 | 21.36 | 13.08 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.735 | 26.699 | 0.269 | 12.480 | 0.652 | 0.000 | 0.000 | 0.000 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 392 | 392 | 916 | 1215 | 6160 | 4060 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.34 | 3.10 | 15.71 | 10.36 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.498 | 11.526 | 0.252 | 2.629 | 0.515 | 0.000 | 0.000 | 0.000 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 216 | 484 | 468 | 2080 | 1388 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.24 | 2.17 | 9.63 | 6.43 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.200 | 2.163 | 0.231 | 0.900 | 0.439 | 0.000 | 0.000 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 85 | 87 | 157 | 232 | 0 | 112 | 116 |
| N.S. | 1 | 1.00 | 1.04 | 1.06 | 1.91 | 2.83 | 0.00 | 1.37 | 1.41 |
| time (sec) | N/A | 0.066 | 0.362 | 0.116 | 0.277 | 0.346 | 0.000 | 6.512 | 1.387 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.046 | 53.954 | 0.132 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.045 | 110.101 | 0.154 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.042 | 4.555 | 0.141 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.027 | 1.034 | 0.029 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.035 | 0.549 | 0.055 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.027 | 20.815 | 0.051 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.042 | 20.581 | 0.056 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 544 | 544 | 1528 | 0 | 0 | 2317 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.81 | 0.00 | 0.00 | 4.26 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.658 | 2.504 | 0.073 | 0.000 | 0.568 | 0.000 | 0.000 | 0.000 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 408 | 408 | 702 | 0 | 0 | 1654 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.72 | 0.00 | 0.00 | 4.05 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.564 | 1.979 | 0.063 | 0.000 | 0.506 | 0.000 | 0.000 | 0.000 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 267 | 267 | 343 | 548 | 0 | 1057 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.28 | 2.05 | 0.00 | 3.96 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.322 | 2.846 | 0.107 | 0.000 | 0.542 | 0.000 | 0.000 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 59 | 68 | 0 | 237 | 335 | 77 | 139 |
| N.S. | 1 | 1.00 | 1.04 | 1.19 | 0.00 | 4.16 | 5.88 | 1.35 | 2.44 |
| time (sec) | N/A | 0.046 | 0.080 | 0.064 | 0.000 | 0.381 | 38.048 | 5.114 | 1.996 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 643 | 643 | 1590 | 0 | 0 | 2665 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.47 | 0.00 | 0.00 | 4.14 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.781 | 2.695 | 0.148 | 0.000 | 0.623 | 0.000 | 0.000 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 479 | 479 | 786 | 0 | 0 | 1864 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.64 | 0.00 | 0.00 | 3.89 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.654 | 1.823 | 0.289 | 0.000 | 0.564 | 0.000 | 0.000 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 311 | 311 | 709 | 625 | 0 | 1160 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.28 | 2.01 | 0.00 | 3.73 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.380 | 4.596 | 0.411 | 0.000 | 0.552 | 0.000 | 0.000 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 71 | 90 | 0 | 283 | 1690 | 99 | 127 |
| N.S. | 1 | 1.00 | 0.95 | 1.20 | 0.00 | 3.77 | 22.53 | 1.32 | 1.69 |
| time (sec) | N/A | 0.079 | 0.125 | 0.103 | 0.000 | 0.360 | 175.767 | 3.142 | 2.499 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 802 | 802 | 1851 | 0 | 0 | 3002 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.31 | 0.00 | 0.00 | 3.74 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.911 | 3.081 | 0.112 | 0.000 | 0.685 | 0.000 | 0.000 | 0.000 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 592 | 592 | 909 | 0 | 0 | 2059 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.54 | 0.00 | 0.00 | 3.48 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.770 | 2.207 | 0.319 | 0.000 | 0.587 | 0.000 | 0.000 | 0.000 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 382 | 382 | 752 | 686 | 0 | 1254 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.97 | 1.80 | 0.00 | 3.28 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.438 | 4.916 | 0.689 | 0.000 | 0.562 | 0.000 | 0.000 | 0.000 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 97 | 146 | 0 | 359 | 0 | 151 | 199 |
| N.S. | 1 | 1.00 | 0.91 | 1.36 | 0.00 | 3.36 | 0.00 | 1.41 | 1.86 |
| time (sec) | N/A | 0.128 | 0.181 | 0.130 | 0.000 | 0.382 | 0.000 | 3.684 | 3.011 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 732 | 732 | 2189 | 0 | 0 | 3580 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.99 | 0.00 | 0.00 | 4.89 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.755 | 8.845 | 0.091 | 0.000 | 0.701 | 0.000 | 0.000 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 528 | 528 | 982 | 0 | 0 | 2426 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.86 | 0.00 | 0.00 | 4.59 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.626 | 2.494 | 0.082 | 0.000 | 0.607 | 0.000 | 0.000 | 0.000 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 325 | 764 | 660 | 0 | 1438 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.35 | 2.03 | 0.00 | 4.42 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.359 | 3.881 | 0.149 | 0.000 | 0.642 | 0.000 | 0.000 | 0.000 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 77 | 67 | 0 | 297 | 0 | 83 | 173 |
| N.S. | 1 | 1.00 | 1.15 | 1.00 | 0.00 | 4.43 | 0.00 | 1.24 | 2.58 |
| time (sec) | N/A | 0.053 | 0.061 | 0.142 | 0.000 | 0.409 | 0.000 | 5.618 | 2.677 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 882 | 882 | 2452 | 0 | 0 | 4553 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.78 | 0.00 | 0.00 | 5.16 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.971 | 41.169 | 0.106 | 0.000 | 0.772 | 0.000 | 0.000 | 0.000 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 639 | 639 | 1276 | 0 | 0 | 2984 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.00 | 0.00 | 0.00 | 4.67 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.784 | 21.170 | 0.093 | 0.000 | 0.685 | 0.000 | 0.000 | 0.000 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 370 | 370 | 822 | 766 | 0 | 1695 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.22 | 2.07 | 0.00 | 4.58 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.411 | 7.607 | 0.165 | 0.000 | 0.627 | 0.000 | 0.000 | 0.000 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 111 | 101 | 0 | 400 | 0 | 130 | 222 |
| N.S. | 1 | 1.00 | 1.34 | 1.22 | 0.00 | 4.82 | 0.00 | 1.57 | 2.67 |
| time (sec) | N/A | 0.089 | 0.316 | 0.141 | 0.000 | 0.410 | 0.000 | 4.533 | 2.960 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.045 | 4.586 | 0.125 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.027 | 0.445 | 0.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.039 | 0.247 | 0.032 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.029 | 11.937 | 0.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.046 | 22.990 | 0.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 574 | 574 | 833 | 750 | 0 | 1515 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.45 | 1.31 | 0.00 | 2.64 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.115 | 8.580 | 1.417 | 0.000 | 0.570 | 0.000 | 0.000 | 0.000 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1106 | 1106 | 2332 | 0 | 0 | 3142 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.11 | 0.00 | 0.00 | 2.84 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.675 | 21.431 | 0.829 | 0.000 | 0.615 | 0.000 | 0.000 | 0.000 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1512 | 1512 | 6310 | 0 | 0 | 5191 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.17 | 0.00 | 0.00 | 3.43 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.033 | 23.994 | 0.825 | 0.000 | 0.750 | 0.000 | 0.000 | 0.000 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 751 | 751 | 959 | 1084 | 0 | 2443 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.28 | 1.44 | 0.00 | 3.25 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.974 | 11.326 | 2.343 | 0.000 | 0.668 | 0.000 | 0.000 | 0.000 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1584 | 1584 | 13567 | 0 | 0 | 5793 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.57 | 0.00 | 0.00 | 3.66 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.936 | 21.932 | 1.655 | 0.000 | 0.833 | 0.000 | 0.000 | 0.000 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 2348 | 2348 | 12936 | 0 | 0 | 10616 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.51 | 0.00 | 0.00 | 4.52 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 5.097 | 20.836 | 0.736 | 0.000 | 1.212 | 0.000 | 0.000 | 0.000 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 275 | 691 | 519 | 492 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.82 | 4.58 | 3.44 | 3.26 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.151 | 1.051 | 0.184 | 0.353 | 0.379 | 0.000 | 0.000 | 0.000 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 198 | 433 | 297 | 308 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.74 | 3.80 | 2.61 | 2.70 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.133 | 0.611 | 0.133 | 0.337 | 0.347 | 0.000 | 0.000 | 0.000 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 246 | 203 | 116 | 160 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.11 | 2.57 | 1.47 | 2.03 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.077 | 0.287 | 0.150 | 0.338 | 0.364 | 0.000 | 0.000 | 0.000 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 19 | 18 | 16 | 24 | 19 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 1.19 | 1.12 | 1.00 | 1.50 | 1.19 | 1.00 |
| time (sec) | N/A | 0.017 | 0.009 | 0.050 | 0.275 | 0.360 | 0.230 | 4.070 | 0.054 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.028 | 2.026 | 0.082 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.027 | 2.608 | 0.096 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 102 | 436 | 534 | 155 | 984 | 24528 | 184 |
| N.S. | 1 | 1.00 | 1.03 | 4.40 | 5.39 | 1.57 | 9.94 | 247.76 | 1.86 |
| time (sec) | N/A | 0.090 | 0.289 | 0.181 | 0.503 | 0.352 | 2.836 | 7.276 | 3.020 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 74 | 215 | 309 | 97 | 605 | 33978 | 110 |
| N.S. | 1 | 1.00 | 0.99 | 2.87 | 4.12 | 1.29 | 8.07 | 453.04 | 1.47 |
| time (sec) | N/A | 0.072 | 0.229 | 0.140 | 0.506 | 0.335 | 2.929 | 7.827 | 2.964 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 53 | 78 | 151 | 51 | 326 | 13408 | 53 |
| N.S. | 1 | 1.00 | 1.04 | 1.53 | 2.96 | 1.00 | 6.39 | 262.90 | 1.04 |
| time (sec) | N/A | 0.039 | 0.292 | 0.111 | 0.494 | 0.362 | 1.620 | 4.406 | 2.936 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 97 | 35 | 52 | 17 | 88 | 34 | 29 |
| N.S. | 1 | 1.00 | 5.11 | 1.84 | 2.74 | 0.89 | 4.63 | 1.79 | 1.53 |
| time (sec) | N/A | 0.026 | 0.095 | 0.097 | 0.482 | 0.380 | 1.172 | 5.108 | 2.774 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 58 | 104 | 163 | 95 | 0 | 716 | -1 |
| N.S. | 1 | 1.00 | 0.81 | 1.44 | 2.26 | 1.32 | 0.00 | 9.94 | -0.01 |
| time (sec) | N/A | 0.124 | 0.155 | 0.145 | 0.332 | 0.333 | 0.000 | 5.523 | 0.000 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 80 | 137 | 172 | 137 | 0 | 3408 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 1.44 | 1.81 | 1.44 | 0.00 | 35.87 | -0.01 |
| time (sec) | N/A | 0.132 | 0.234 | 0.199 | 0.356 | 0.352 | 0.000 | 6.330 | 0.000 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 219 | 132 | 736 | 572 | 266 | 2725 | 55344 | 339 |
| N.S. | 1 | 1.00 | 0.60 | 3.36 | 2.61 | 1.21 | 12.44 | 252.71 | 1.55 |
| time (sec) | N/A | 0.152 | 0.612 | 0.131 | 0.320 | 0.359 | 5.324 | 9.112 | 3.494 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 95 | 339 | 289 | 152 | 1528 | 38869 | 187 |
| N.S. | 1 | 1.00 | 0.59 | 2.11 | 1.80 | 0.94 | 9.49 | 241.42 | 1.16 |
| time (sec) | N/A | 0.109 | 0.681 | 0.203 | 0.295 | 0.338 | 3.956 | 9.382 | 3.203 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 52 | 113 | 114 | 69 | 724 | 26480 | 84 |
| N.S. | 1 | 1.00 | 0.57 | 1.24 | 1.25 | 0.76 | 7.96 | 290.99 | 0.92 |
| time (sec) | N/A | 0.060 | 0.744 | 0.137 | 0.283 | 0.347 | 2.941 | 10.737 | 3.038 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 24 | 28 | 25 | 25 | 158 | 25 | 22 |
| N.S. | 1 | 1.00 | 0.75 | 0.88 | 0.78 | 0.78 | 4.94 | 0.78 | 0.69 |
| time (sec) | N/A | 0.029 | 0.043 | 0.075 | 0.279 | 0.372 | 2.194 | 4.617 | 2.642 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-2) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 105 | 163 | 282 | 166 | 0 | 4828 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 1.27 | 2.20 | 1.30 | 0.00 | 37.72 | -0.01 |
| time (sec) | N/A | 0.196 | 0.309 | 0.178 | 0.384 | 0.360 | 0.000 | 4.611 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 203 | 235 | 309 | 258 | 0 | 49990 | -1 |
| N.S. | 1 | 1.00 | 1.16 | 1.34 | 1.77 | 1.47 | 0.00 | 285.66 | -0.01 |
| time (sec) | N/A | 0.222 | 0.502 | 0.260 | 0.404 | 0.370 | 0.000 | 14.473 | 0.000 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 502 | 502 | 1438 | 1265 | 3854 | 1871 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.86 | 2.52 | 7.68 | 3.73 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.324 | 8.892 | 0.304 | 1.193 | 0.478 | 0.000 | 0.000 | 0.000 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 278 | 278 | 748 | 677 | 1932 | 1087 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.69 | 2.44 | 6.95 | 3.91 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.171 | 8.661 | 0.191 | 0.524 | 0.426 | 0.000 | 0.000 | 0.000 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 655 | 303 | 725 | 517 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.81 | 1.76 | 4.22 | 3.01 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.095 | 2.957 | 0.199 | 0.408 | 0.405 | 0.000 | 0.000 | 0.000 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 30 | 43 | 47 | 58 | 0 | 58 | 33 |
| N.S. | 1 | 1.00 | 0.81 | 1.16 | 1.27 | 1.57 | 0.00 | 1.57 | 0.89 |
| time (sec) | N/A | 0.036 | 0.043 | 0.090 | 0.279 | 0.413 | 0.000 | 6.414 | 0.084 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.030 | 13.933 | 0.122 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.028 | 22.479 | 0.149 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 475 | 475 | 1200 | 1124 | 5130 | 1548 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.53 | 2.37 | 10.80 | 3.26 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.381 | 9.454 | 0.354 | 1.299 | 0.481 | 0.000 | 0.000 | 0.000 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 343 | 343 | 564 | 573 | 1328 | 868 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.64 | 1.67 | 3.87 | 2.53 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.246 | 5.160 | 0.236 | 0.643 | 0.437 | 0.000 | 0.000 | 0.000 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 231 | 181 | 1115 | 159 | 0 | 6656 | 240 |
| N.S. | 1 | 1.00 | 1.52 | 1.19 | 7.34 | 1.05 | 0.00 | 43.79 | 1.58 |
| time (sec) | N/A | 0.097 | 0.988 | 0.180 | 0.313 | 0.377 | 0.000 | 6.407 | 7.667 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 45 | 70 | 129 | 49 | 0 | 67 | 71 |
| N.S. | 1 | 1.00 | 1.07 | 1.67 | 3.07 | 1.17 | 0.00 | 1.60 | 1.69 |
| time (sec) | N/A | 0.034 | 0.066 | 0.120 | 0.281 | 0.346 | 0.000 | 3.295 | 2.802 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.043 | 20.011 | 0.191 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.042 | 25.661 | 0.222 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 698 | 698 | 2368 | 2161 | 0 | 2559 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.39 | 3.10 | 0.00 | 3.67 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.476 | 8.484 | 0.370 | 0.000 | 0.620 | 0.000 | 0.000 | 0.000 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 431 | 431 | 1577 | 1119 | 0 | 1546 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.66 | 2.60 | 0.00 | 3.59 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.258 | 8.494 | 0.365 | 0.000 | 0.481 | 0.000 | 0.000 | 0.000 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 241 | 853 | 483 | 0 | 803 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.54 | 2.00 | 0.00 | 3.33 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.135 | 5.447 | 0.296 | 0.000 | 0.421 | 0.000 | 0.000 | 0.000 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 75 | 67 | 91 | 125 | 0 | 96 | 74 |
| N.S. | 1 | 1.00 | 0.97 | 0.87 | 1.18 | 1.62 | 0.00 | 1.25 | 0.96 |
| time (sec) | N/A | 0.058 | 0.068 | 0.152 | 0.297 | 0.360 | 0.000 | 4.642 | 0.096 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.045 | 23.854 | 0.240 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.046 | 34.792 | 0.424 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 449 | 449 | 411 | 0 | 0 | 354 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 0.79 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.424 | 2.610 | 0.112 | 0.000 | 0.124 | 0.000 | 0.000 | 0.000 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 277 | 253 | 0 | 0 | 197 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.71 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.212 | 1.212 | 0.122 | 0.000 | 0.114 | 0.000 | 0.000 | 0.000 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 154 | 187 | 0 | 0 | 136 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.21 | 0.00 | 0.00 | 0.88 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.109 | 1.900 | 0.072 | 0.000 | 0.114 | 0.000 | 0.000 | 0.000 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.028 | 5.462 | 0.036 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.035 | 0.655 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.027 | 77.344 | 0.068 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.042 | 10.520 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 432 | 432 | 410 | 0 | 0 | 1777 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 4.11 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.384 | 0.142 | 0.088 | 0.000 | 0.556 | 0.000 | 0.000 | 0.000 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 320 | 302 | 0 | 0 | 1243 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 3.88 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.333 | 0.115 | 0.069 | 0.000 | 0.526 | 0.000 | 0.000 | 0.000 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 212 | 197 | 1006 | 0 | 781 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.93 | 4.75 | 0.00 | 3.68 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.185 | 0.035 | 0.138 | 0.000 | 0.574 | 0.000 | 0.000 | 0.000 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 19 | 18 | 18 | 41 | 19 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.00 | 1.00 | 2.28 | 1.06 | 1.00 |
| time (sec) | N/A | 0.017 | 0.007 | 0.056 | 0.286 | 0.356 | 0.327 | 3.457 | 0.058 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 618 | 618 | 1588 | 0 | 0 | 2325 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.57 | 0.00 | 0.00 | 3.76 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.690 | 3.379 | 0.248 | 0.000 | 0.613 | 0.000 | 0.000 | 0.000 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 460 | 460 | 782 | 0 | 0 | 1645 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.70 | 0.00 | 0.00 | 3.58 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.611 | 2.651 | 0.254 | 0.000 | 0.548 | 0.000 | 0.000 | 0.000 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 716 | 1123 | 0 | 1040 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.40 | 3.77 | 0.00 | 3.49 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.342 | 5.088 | 0.402 | 0.000 | 0.563 | 0.000 | 0.000 | 0.000 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 361 | 96 | 0 | 214 | 1923 | 95 | 318 |
| N.S. | 1 | 1.00 | 5.16 | 1.37 | 0.00 | 3.06 | 27.47 | 1.36 | 4.54 |
| time (sec) | N/A | 0.080 | 0.911 | 0.124 | 0.000 | 0.361 | 173.132 | 5.331 | 3.917 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 737 | 737 | 2452 | 0 | 0 | 2676 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.33 | 0.00 | 0.00 | 3.63 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.570 | 6.581 | 0.374 | 0.000 | 0.625 | 0.000 | 0.000 | 0.000 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 548 | 548 | 2397 | 0 | 0 | 1790 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.37 | 0.00 | 0.00 | 3.27 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.468 | 3.135 | 0.615 | 0.000 | 0.563 | 0.000 | 0.000 | 0.000 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 2165 | 1750 | 0 | 1043 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 6.17 | 4.99 | 0.00 | 2.97 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.263 | 13.724 | 0.961 | 0.000 | 0.584 | 0.000 | 0.000 | 0.000 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 55 | 54 | 55 | 53 | 0 | 56 | 55 |
| N.S. | 1 | 1.00 | 0.90 | 0.89 | 0.90 | 0.87 | 0.00 | 0.92 | 0.90 |
| time (sec) | N/A | 0.044 | 0.073 | 0.083 | 0.291 | 0.375 | 0.000 | 5.572 | 0.090 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 937 | 937 | 1977 | 0 | 0 | 3065 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.11 | 0.00 | 0.00 | 3.27 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.069 | 4.643 | 0.126 | 0.000 | 0.688 | 0.000 | 0.000 | 0.000 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 667 | 667 | 1085 | 0 | 0 | 2051 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.63 | 0.00 | 0.00 | 3.07 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.766 | 3.421 | 0.109 | 0.000 | 0.655 | 0.000 | 0.000 | 0.000 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 413 | 413 | 2743 | 861 | 0 | 1193 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 6.64 | 2.08 | 0.00 | 2.89 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.432 | 14.860 | 0.209 | 0.000 | 0.632 | 0.000 | 0.000 | 0.000 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 64 | 71 | 64 | 62 | 0 | 71 | 69 |
| N.S. | 1 | 1.00 | 0.85 | 0.95 | 0.85 | 0.83 | 0.00 | 0.95 | 0.92 |
| time (sec) | N/A | 0.056 | 0.043 | 0.093 | 0.298 | 0.363 | 0.000 | 4.143 | 0.198 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 923 | 923 | 2241 | 0 | 0 | 4118 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.43 | 0.00 | 0.00 | 4.46 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.283 | 9.367 | 0.160 | 0.000 | 0.820 | 0.000 | 0.000 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 659 | 659 | 1368 | 0 | 0 | 2671 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.08 | 0.00 | 0.00 | 4.05 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.970 | 7.268 | 0.138 | 0.000 | 0.686 | 0.000 | 0.000 | 0.000 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 349 | 842 | 1542 | 0 | 1273 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.41 | 4.42 | 0.00 | 3.65 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.559 | 5.572 | 0.329 | 0.000 | 0.583 | 0.000 | 0.000 | 0.000 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 152 | 112 | 0 | 305 | 0 | 107 | 149 |
| N.S. | 1 | 1.00 | 1.81 | 1.33 | 0.00 | 3.63 | 0.00 | 1.27 | 1.77 |
| time (sec) | N/A | 0.065 | 0.189 | 0.119 | 0.000 | 0.375 | 0.000 | 5.836 | 4.228 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.042 | 3.422 | 0.089 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.029 | 1.884 | 0.038 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.036 | 0.243 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.027 | 99.364 | 0.056 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.041 | 8.684 | 0.052 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 73 | 194 | 0 | 341 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.95 | 2.52 | 0.00 | 4.43 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.047 | 0.250 | 1.384 | 0.000 | 0.368 | 0.000 | 0.000 | 0.000 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 280 | 280 | 355 | 606 | 0 | 1401 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.27 | 2.16 | 0.00 | 5.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.341 | 3.156 | 1.193 | 0.000 | 0.577 | 0.000 | 0.000 | 0.000 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 418 | 418 | 703 | 0 | 0 | 2299 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.68 | 0.00 | 0.00 | 5.50 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.565 | 1.051 | 0.978 | 0.000 | 0.561 | 0.000 | 0.000 | 0.000 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 112 | 349 | 0 | 627 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.97 | 3.01 | 0.00 | 5.41 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.067 | 0.735 | 2.375 | 0.000 | 0.375 | 0.000 | 0.000 | 0.000 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 357 | 357 | 1104 | 946 | 0 | 2389 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.09 | 2.65 | 0.00 | 6.69 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.398 | 11.462 | 2.306 | 0.000 | 0.662 | 0.000 | 0.000 | 0.000 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 753 | 753 | 2311 | 0 | 0 | 4952 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.07 | 0.00 | 0.00 | 6.58 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.842 | 15.784 | 0.770 | 0.000 | 0.741 | 0.000 | 0.000 | 0.000 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 765 | 765 | 1897 | 0 | 0 | 3082 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.48 | 0.00 | 0.00 | 4.03 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.921 | 1.544 | 0.138 | 0.000 | 0.680 | 0.000 | 0.000 | 0.000 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 557 | 557 | 917 | 0 | 0 | 2123 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.65 | 0.00 | 0.00 | 3.81 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.762 | 1.092 | 0.129 | 0.000 | 0.626 | 0.000 | 0.000 | 0.000 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 812 | 1207 | 0 | 1280 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.31 | 3.44 | 0.00 | 3.65 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.441 | 4.032 | 0.253 | 0.000 | 0.646 | 0.000 | 0.000 | 0.000 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 90 | 94 | 0 | 262 | 0 | 94 | 896 |
| N.S. | 1 | 1.00 | 1.20 | 1.25 | 0.00 | 3.49 | 0.00 | 1.25 | 11.95 |
| time (sec) | N/A | 0.121 | 0.085 | 0.158 | 0.000 | 0.405 | 0.000 | 5.236 | 5.394 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 763 | 763 | 3808 | 0 | 0 | 3448 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.99 | 0.00 | 0.00 | 4.52 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.913 | 8.560 | 0.801 | 0.000 | 0.759 | 0.000 | 0.000 | 0.000 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 566 | 566 | 1740 | 0 | 0 | 2275 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.07 | 0.00 | 0.00 | 4.02 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.720 | 7.989 | 0.547 | 0.000 | 0.680 | 0.000 | 0.000 | 0.000 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 379 | 379 | 2209 | 1721 | 0 | 1296 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.83 | 4.54 | 0.00 | 3.42 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.410 | 13.600 | 1.678 | 0.000 | 0.633 | 0.000 | 0.000 | 0.000 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 53 | 55 | 54 | 55 | 0 | 56 | 98 |
| N.S. | 1 | 1.00 | 0.90 | 0.93 | 0.92 | 0.93 | 0.00 | 0.95 | 1.66 |
| time (sec) | N/A | 0.073 | 0.053 | 0.183 | 0.269 | 0.379 | 0.000 | 7.948 | 4.686 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1138 | 1138 | 2489 | 0 | 0 | 4197 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.19 | 0.00 | 0.00 | 3.69 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.339 | 5.895 | 0.526 | 0.000 | 0.912 | 0.000 | 0.000 | 0.000 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 825 | 825 | 1221 | 0 | 0 | 2802 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.48 | 0.00 | 0.00 | 3.40 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.053 | 3.109 | 0.956 | 0.000 | 0.729 | 0.000 | 0.000 | 0.000 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 524 | 524 | 934 | 1877 | 0 | 1622 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.78 | 3.58 | 0.00 | 3.10 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.589 | 9.542 | 1.725 | 0.000 | 0.681 | 0.000 | 0.000 | 0.000 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 143 | 180 | 0 | 350 | 0 | 183 | 1320 |
| N.S. | 1 | 1.00 | 1.15 | 1.45 | 0.00 | 2.82 | 0.00 | 1.48 | 10.65 |
| time (sec) | N/A | 0.179 | 0.186 | 0.276 | 0.000 | 0.457 | 0.000 | 6.656 | 6.640 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 852 | 852 | 3114 | 0 | 0 | 3898 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.65 | 0.00 | 0.00 | 4.58 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.143 | 42.155 | 0.129 | 0.000 | 0.754 | 0.000 | 0.000 | 0.000 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 616 | 616 | 1905 | 0 | 0 | 2541 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.09 | 0.00 | 0.00 | 4.12 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.901 | 22.021 | 0.129 | 0.000 | 0.622 | 0.000 | 0.000 | 0.000 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 386 | 386 | 2314 | 1732 | 0 | 1432 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.99 | 4.49 | 0.00 | 3.71 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.506 | 13.633 | 0.330 | 0.000 | 0.635 | 0.000 | 0.000 | 0.000 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 54 | 59 | 57 | 69 | 0 | 72 | 118 |
| N.S. | 1 | 1.00 | 0.90 | 0.98 | 0.95 | 1.15 | 0.00 | 1.20 | 1.97 |
| time (sec) | N/A | 0.080 | 0.066 | 0.131 | 0.416 | 0.369 | 0.000 | 7.443 | 4.808 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1144 | 1144 | 4632 | 0 | 0 | 4695 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.05 | 0.00 | 0.00 | 4.10 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.674 | 40.579 | 0.619 | 0.000 | 0.959 | 0.000 | 0.000 | 0.000 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 840 | 840 | 2574 | 0 | 0 | 3089 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.06 | 0.00 | 0.00 | 3.68 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.379 | 21.135 | 0.608 | 0.000 | 0.729 | 0.000 | 0.000 | 0.000 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 517 | 517 | 1019 | 1890 | 0 | 1762 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.97 | 3.66 | 0.00 | 3.41 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.711 | 9.256 | 1.551 | 0.000 | 0.688 | 0.000 | 0.000 | 0.000 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 146 | 155 | 0 | 396 | 0 | 221 | 1167 |
| N.S. | 1 | 1.00 | 1.40 | 1.49 | 0.00 | 3.81 | 0.00 | 2.12 | 11.22 |
| time (sec) | N/A | 0.170 | 0.534 | 0.231 | 0.000 | 0.466 | 0.000 | 5.606 | 6.109 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1432 | 1432 | 4084 | 0 | 0 | 4903 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.85 | 0.00 | 0.00 | 3.42 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.899 | 30.101 | 1.004 | 0.000 | 1.033 | 0.000 | 0.000 | 0.000 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F(-2) | B | F | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1051 | 1051 | 5228 | 0 | 0 | 3146 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.97 | 0.00 | 0.00 | 2.99 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.398 | 10.381 | 1.361 | 0.000 | 0.770 | 0.000 | 0.000 | 0.000 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 641 | 641 | 2504 | 2485 | 0 | 1719 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.91 | 3.88 | 0.00 | 2.68 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.786 | 13.808 | 3.296 | 0.000 | 0.691 | 0.000 | 0.000 | 0.000 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 86 | 90 | 91 | 133 | 0 | 105 | 233 |
| N.S. | 1 | 1.00 | 0.90 | 0.94 | 0.95 | 1.39 | 0.00 | 1.09 | 2.43 |
| time (sec) | N/A | 0.104 | 0.130 | 0.147 | 0.286 | 0.429 | 0.000 | 4.711 | 5.054 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [341] had the largest ratio of [36]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 5 | 2 | 1.00 | 14 | 0.143 |
| 2 | A | 4 | 2 | 1.00 | 14 | 0.143 |
| 3 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 4 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 5 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 6 | A | 4 | 4 | 1.00 | 14 | 0.286 |
| 7 | A | 5 | 4 | 1.00 | 14 | 0.286 |
| 8 | A | 6 | 4 | 1.00 | 16 | 0.250 |
| 9 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 10 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 11 | A | 2 | 1 | 1.00 | 14 | 0.071 |
| 12 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 13 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 14 | A | 7 | 6 | 1.00 | 16 | 0.375 |
| 15 | A | 7 | 7 | 1.00 | 16 | 0.438 |
| 16 | A | 12 | 4 | 1.00 | 16 | 0.250 |
| 17 | A | 8 | 4 | 1.00 | 16 | 0.250 |
| 18 | A | 6 | 4 | 1.00 | 16 | 0.250 |
| 19 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 20 | A | 8 | 4 | 1.00 | 16 | 0.250 |
| 21 | A | 8 | 4 | 1.00 | 16 | 0.250 |
| 22 | A | 12 | 5 | 1.00 | 16 | 0.312 |
| 23 | A | 9 | 5 | 1.00 | 14 | 0.357 |
| 24 | A | 7 | 4 | 1.00 | 14 | 0.286 |
| 25 | A | 5 | 3 | 1.00 | 12 | 0.250 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 26 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 27 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 28 | A | 6 | 6 | 1.00 | 16 | 0.375 |
| 29 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 30 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 31 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 32 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 33 | A | 15 | 8 | 1.00 | 16 | 0.500 |
| 34 | A | 9 | 6 | 1.00 | 16 | 0.375 |
| 35 | A | 6 | 4 | 1.00 | 14 | 0.286 |
| 36 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 37 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 38 | A | 8 | 6 | 1.00 | 16 | 0.375 |
| 39 | A | 7 | 6 | 1.00 | 16 | 0.375 |
| 40 | A | 6 | 6 | 1.00 | 16 | 0.375 |
| 41 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 42 | A | 6 | 6 | 1.00 | 16 | 0.375 |
| 43 | A | 7 | 6 | 1.00 | 16 | 0.375 |
| 44 | A | 8 | 6 | 1.00 | 16 | 0.375 |
| 45 | A | 10 | 9 | 1.00 | 18 | 0.500 |
| 46 | A | 9 | 8 | 1.00 | 18 | 0.444 |
| 47 | A | 8 | 7 | 1.00 | 18 | 0.389 |
| 48 | A | 7 | 6 | 1.00 | 18 | 0.333 |
| 49 | A | 7 | 7 | 1.00 | 18 | 0.389 |
| 50 | A | 9 | 8 | 1.00 | 18 | 0.444 |
| 51 | A | 9 | 9 | 1.00 | 18 | 0.500 |
| 52 | A | 11 | 8 | 1.00 | 18 | 0.444 |
| 53 | A | 23 | 8 | 1.00 | 18 | 0.444 |
| 54 | A | 20 | 8 | 1.00 | 18 | 0.444 |
| 55 | A | 14 | 7 | 1.00 | 18 | 0.389 |
| 56 | A | 12 | 6 | 1.00 | 18 | 0.333 |
| 57 | A | 12 | 6 | 1.00 | 18 | 0.333 |
| 58 | A | 18 | 7 | 1.00 | 18 | 0.389 |
| 59 | A | 19 | 8 | 1.00 | 18 | 0.444 |
| 60 | A | 4 | 3 | 1.00 | 12 | 0.250 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 61 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 62 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 63 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 64 | A | 4 | 3 | 1.00 | 12 | 0.250 |
| 65 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 66 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 67 | A | 2 | 1 | 1.00 | 25 | 0.040 |
| 68 | A | 3 | 2 | 1.00 | 29 | 0.069 |
| 69 | A | 2 | 1 | 1.00 | 28 | 0.036 |
| 70 | A | 3 | 1 | 1.00 | 28 | 0.036 |
| 71 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 72 | A | 8 | 3 | 1.00 | 16 | 0.188 |
| 73 | A | 5 | 3 | 1.00 | 16 | 0.188 |
| 74 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 75 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 76 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 77 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 78 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 79 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 80 | A | 3 | 2 | 1.00 | 10 | 0.200 |
| 81 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 82 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 83 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 84 | A | 5 | 3 | 1.00 | 14 | 0.214 |
| 85 | A | 5 | 3 | 1.00 | 14 | 0.214 |
| 86 | A | 5 | 3 | 1.00 | 14 | 0.214 |
| 87 | A | 5 | 3 | 1.00 | 12 | 0.250 |
| 88 | A | 5 | 3 | 1.00 | 14 | 0.214 |
| 89 | A | 5 | 3 | 1.00 | 14 | 0.214 |
| 90 | A | 5 | 3 | 1.00 | 14 | 0.214 |
| 91 | A | 4 | 2 | 1.00 | 28 | 0.071 |
| 92 | A | 7 | 5 | 1.00 | 32 | 0.156 |
| 93 | A | 4 | 2 | 1.00 | 28 | 0.071 |
| 94 | A | 5 | 2 | 1.00 | 28 | 0.071 |
| 95 | A | 6 | 3 | 1.00 | 18 | 0.167 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 96 | A | 5 | 3 | 1.00 | 18 | 0.167 |
| 97 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 98 | A | 5 | 4 | 1.00 | 18 | 0.222 |
| 99 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 100 | A | 7 | 5 | 1.00 | 18 | 0.278 |
| 101 | A | 10 | 6 | 1.00 | 20 | 0.300 |
| 102 | A | 9 | 7 | 1.00 | 20 | 0.350 |
| 103 | A | 6 | 4 | 1.00 | 18 | 0.222 |
| 104 | A | 9 | 5 | 1.00 | 20 | 0.250 |
| 105 | A | 9 | 5 | 1.00 | 20 | 0.250 |
| 106 | A | 15 | 8 | 1.00 | 20 | 0.400 |
| 107 | A | 7 | 7 | 1.00 | 20 | 0.350 |
| 108 | A | 6 | 6 | 1.00 | 20 | 0.300 |
| 109 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 110 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 111 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 112 | A | 10 | 9 | 1.00 | 20 | 0.450 |
| 113 | A | 9 | 9 | 1.00 | 20 | 0.450 |
| 114 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 115 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 116 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 117 | A | 7 | 7 | 1.00 | 21 | 0.333 |
| 118 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 119 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 120 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 121 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 122 | A | 5 | 3 | 1.00 | 18 | 0.167 |
| 123 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 124 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 125 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 126 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 127 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 128 | A | 9 | 5 | 1.00 | 18 | 0.278 |
| 129 | A | 7 | 5 | 1.00 | 18 | 0.278 |
| 130 | A | 4 | 4 | 1.00 | 16 | 0.250 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 131 | A | 9 | 5 | 1.00 | 18 | 0.278 |
| 132 | A | 9 | 5 | 1.00 | 18 | 0.278 |
| 133 | A | 13 | 6 | 1.00 | 18 | 0.333 |
| 134 | A | 10 | 6 | 1.00 | 18 | 0.333 |
| 135 | A | 8 | 5 | 1.00 | 18 | 0.278 |
| 136 | A | 6 | 4 | 1.00 | 16 | 0.250 |
| 137 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 138 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 139 | A | 16 | 9 | 1.00 | 18 | 0.500 |
| 140 | A | 10 | 7 | 1.00 | 18 | 0.389 |
| 141 | A | 7 | 5 | 1.00 | 16 | 0.312 |
| 142 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 143 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 144 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 145 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 146 | A | 12 | 5 | 1.00 | 20 | 0.250 |
| 147 | A | 9 | 5 | 1.00 | 20 | 0.250 |
| 148 | A | 5 | 3 | 1.00 | 18 | 0.167 |
| 149 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 150 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 151 | A | 6 | 3 | 1.00 | 18 | 0.167 |
| 152 | A | 5 | 3 | 1.00 | 18 | 0.167 |
| 153 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 154 | A | 5 | 4 | 1.00 | 18 | 0.222 |
| 155 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 156 | A | 7 | 5 | 1.00 | 18 | 0.278 |
| 157 | A | 10 | 6 | 1.00 | 20 | 0.300 |
| 158 | A | 9 | 7 | 1.00 | 20 | 0.350 |
| 159 | A | 6 | 4 | 1.00 | 18 | 0.222 |
| 160 | A | 10 | 5 | 1.00 | 20 | 0.250 |
| 161 | A | 11 | 7 | 1.00 | 20 | 0.350 |
| 162 | A | 14 | 8 | 1.00 | 20 | 0.400 |
| 163 | A | 12 | 7 | 1.00 | 20 | 0.350 |
| 164 | A | 10 | 6 | 1.00 | 20 | 0.300 |
| 165 | A | 8 | 5 | 1.00 | 18 | 0.278 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 166 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 167 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 168 | A | 22 | 9 | 1.00 | 20 | 0.450 |
| 169 | A | 18 | 10 | 1.00 | 20 | 0.500 |
| 170 | A | 11 | 8 | 1.00 | 18 | 0.444 |
| 171 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 172 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 173 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 174 | A | 18 | 5 | 1.00 | 20 | 0.250 |
| 175 | A | 10 | 5 | 1.00 | 20 | 0.250 |
| 176 | A | 5 | 3 | 1.00 | 18 | 0.167 |
| 177 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 178 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 179 | A | 9 | 9 | 1.00 | 26 | 0.346 |
| 180 | A | 8 | 8 | 1.00 | 26 | 0.308 |
| 181 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 182 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 183 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 184 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 185 | A | 14 | 11 | 1.00 | 28 | 0.393 |
| 186 | A | 12 | 10 | 1.00 | 28 | 0.357 |
| 187 | A | 8 | 6 | 1.00 | 26 | 0.231 |
| 188 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 189 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 190 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 191 | A | 19 | 13 | 1.00 | 28 | 0.464 |
| 192 | A | 17 | 13 | 1.00 | 28 | 0.464 |
| 193 | A | 11 | 7 | 1.00 | 26 | 0.269 |
| 194 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 195 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 196 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 197 | A | 17 | 10 | 1.00 | 26 | 0.385 |
| 198 | A | 14 | 11 | 1.00 | 26 | 0.423 |
| 199 | A | 9 | 7 | 1.00 | 24 | 0.292 |
| 200 | A | 3 | 3 | 1.00 | 19 | 0.158 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 201 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 202 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 203 | A | 24 | 10 | 1.00 | 28 | 0.357 |
| 204 | A | 20 | 11 | 1.00 | 28 | 0.393 |
| 205 | A | 12 | 7 | 1.00 | 26 | 0.269 |
| 206 | A | 5 | 5 | 1.00 | 21 | 0.238 |
| 207 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 208 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 209 | A | 40 | 13 | 1.00 | 28 | 0.464 |
| 210 | A | 30 | 13 | 1.00 | 28 | 0.464 |
| 211 | A | 19 | 8 | 1.00 | 26 | 0.308 |
| 212 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 213 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 214 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 215 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 216 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 217 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 218 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 219 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 220 | A | 14 | 9 | 1.00 | 26 | 0.346 |
| 221 | A | 12 | 8 | 1.00 | 26 | 0.308 |
| 222 | A | 10 | 6 | 1.00 | 24 | 0.250 |
| 223 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 224 | A | 19 | 11 | 1.00 | 28 | 0.393 |
| 225 | A | 16 | 10 | 1.00 | 28 | 0.357 |
| 226 | A | 13 | 8 | 1.00 | 26 | 0.308 |
| 227 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 228 | A | 24 | 13 | 1.00 | 28 | 0.464 |
| 229 | A | 21 | 13 | 1.00 | 28 | 0.464 |
| 230 | A | 16 | 9 | 1.00 | 26 | 0.346 |
| 231 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 232 | A | 22 | 9 | 1.00 | 26 | 0.346 |
| 233 | A | 18 | 8 | 1.00 | 26 | 0.308 |
| 234 | A | 14 | 7 | 1.00 | 24 | 0.292 |
| 235 | A | 5 | 5 | 1.00 | 19 | 0.263 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 236 | A | 29 | 11 | 1.00 | 28 | 0.393 |
| 237 | A | 24 | 12 | 1.00 | 28 | 0.429 |
| 238 | A | 17 | 9 | 1.00 | 26 | 0.346 |
| 239 | A | 7 | 7 | 1.00 | 21 | 0.333 |
| 240 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 241 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 242 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 243 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 244 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 245 | A | 21 | 9 | 1.00 | 24 | 0.375 |
| 246 | A | 30 | 11 | 1.00 | 26 | 0.423 |
| 247 | A | 36 | 10 | 1.00 | 26 | 0.385 |
| 248 | A | 48 | 11 | 1.00 | 24 | 0.458 |
| 249 | A | 73 | 16 | 1.00 | 26 | 0.615 |
| 250 | A | 92 | 14 | 1.00 | 26 | 0.538 |
| 251 | A | 6 | 6 | 1.00 | 26 | 0.231 |
| 252 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 253 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 254 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 255 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 256 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 257 | A | 6 | 4 | 1.00 | 28 | 0.143 |
| 258 | A | 5 | 4 | 1.00 | 28 | 0.143 |
| 259 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 260 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 261 | A | 5 | 5 | 1.00 | 28 | 0.179 |
| 262 | A | 6 | 6 | 1.00 | 28 | 0.214 |
| 263 | A | 10 | 8 | 1.00 | 28 | 0.286 |
| 264 | A | 7 | 5 | 1.00 | 28 | 0.179 |
| 265 | A | 6 | 6 | 1.00 | 26 | 0.231 |
| 266 | A | 2 | 1 | 1.00 | 21 | 0.048 |
| 267 | A | 9 | 6 | 1.00 | 28 | 0.214 |
| 268 | A | 11 | 7 | 1.00 | 28 | 0.250 |
| 269 | A | 22 | 13 | 1.00 | 26 | 0.500 |
| 270 | A | 13 | 10 | 1.00 | 26 | 0.385 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 271 | A | 10 | 8 | 1.00 | 24 | 0.333 |
| 272 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 273 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 274 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 275 | A | 20 | 12 | 1.00 | 28 | 0.429 |
| 276 | A | 16 | 12 | 1.00 | 28 | 0.429 |
| 277 | A | 7 | 7 | 1.00 | 26 | 0.269 |
| 278 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 279 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 280 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 281 | A | 32 | 16 | 1.00 | 28 | 0.571 |
| 282 | A | 17 | 12 | 1.00 | 28 | 0.429 |
| 283 | A | 11 | 7 | 1.00 | 26 | 0.269 |
| 284 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 285 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 286 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 287 | A | 14 | 6 | 1.00 | 28 | 0.214 |
| 288 | A | 9 | 6 | 1.00 | 28 | 0.214 |
| 289 | A | 5 | 4 | 1.00 | 28 | 0.143 |
| 290 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 291 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 292 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 293 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 294 | A | 11 | 6 | 1.00 | 26 | 0.231 |
| 295 | A | 9 | 5 | 1.00 | 26 | 0.192 |
| 296 | A | 7 | 4 | 1.00 | 24 | 0.167 |
| 297 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 298 | A | 18 | 11 | 1.00 | 28 | 0.393 |
| 299 | A | 15 | 10 | 1.00 | 28 | 0.357 |
| 300 | A | 12 | 8 | 1.00 | 26 | 0.308 |
| 301 | A | 5 | 5 | 1.00 | 21 | 0.238 |
| 302 | A | 21 | 14 | 1.00 | 28 | 0.500 |
| 303 | A | 16 | 10 | 1.00 | 28 | 0.357 |
| 304 | A | 13 | 10 | 1.00 | 26 | 0.385 |
| 305 | A | 3 | 2 | 1.00 | 21 | 0.095 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 306 | A | 29 | 10 | 1.00 | 26 | 0.385 |
| 307 | A | 24 | 9 | 1.00 | 26 | 0.346 |
| 308 | A | 19 | 8 | 1.00 | 24 | 0.333 |
| 309 | A | 6 | 4 | 1.00 | 19 | 0.210 |
| 310 | A | 29 | 13 | 1.00 | 28 | 0.464 |
| 311 | A | 24 | 14 | 1.00 | 28 | 0.500 |
| 312 | A | 15 | 11 | 1.00 | 26 | 0.423 |
| 313 | A | 5 | 5 | 1.00 | 21 | 0.238 |
| 314 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 315 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 316 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 317 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 318 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 319 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 320 | A | 9 | 6 | 1.00 | 26 | 0.231 |
| 321 | A | 11 | 7 | 1.00 | 26 | 0.269 |
| 322 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 323 | A | 12 | 9 | 1.00 | 26 | 0.346 |
| 324 | A | 19 | 11 | 1.00 | 26 | 0.423 |
| 325 | A | 33 | 14 | 1.00 | 32 | 0.438 |
| 326 | A | 27 | 13 | 1.00 | 32 | 0.406 |
| 327 | A | 21 | 11 | 1.00 | 30 | 0.367 |
| 328 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 329 | A | 34 | 17 | 1.00 | 34 | 0.500 |
| 330 | A | 26 | 13 | 1.00 | 34 | 0.382 |
| 331 | A | 22 | 13 | 1.00 | 32 | 0.406 |
| 332 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 333 | A | 53 | 18 | 1.00 | 34 | 0.529 |
| 334 | A | 41 | 18 | 1.00 | 34 | 0.529 |
| 335 | A | 31 | 14 | 1.00 | 32 | 0.438 |
| 336 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 337 | A | 48 | 19 | 1.00 | 34 | 0.559 |
| 338 | A | 37 | 17 | 1.00 | 34 | 0.500 |
| 339 | A | 28 | 15 | 1.00 | 32 | 0.469 |
| 340 | A | 4 | 3 | 1.00 | 27 | 0.111 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 341 | A | 66 | 20 | 1.00 | 36 | 0.556 |
| 342 | A | 53 | 22 | 1.00 | 36 | 0.611 |
| 343 | A | 38 | 16 | 1.00 | 34 | 0.471 |
| 344 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 345 | A | 85 | 21 | 1.00 | 36 | 0.583 |
| 346 | A | 60 | 20 | 1.00 | 36 | 0.556 |
| 347 | A | 45 | 17 | 1.00 | 34 | 0.500 |
| 348 | A | 4 | 3 | 1.00 | 29 | 0.103 |

# Chapter 3

# Listing of integrals

## Local contents

# 3.1 $\int (c+dx)^4 \sin(a+bx)\,dx$

**Optimal**. Leaf size=92

$$-\frac{24d^4\cos(a+bx)}{b^5}+\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}-\frac{(c+dx)^4\cos(a+bx)}{b}-\frac{24d^3(c+dx)\sin(a+bx)}{b^4}+\frac{4d(c+dx)^3 \ldots}{}$$

```
[Out] -24*d^4*cos(b*x+a)/b^5+12*d^2*(d*x+c)^2*cos(b*x+a)/b^3-(d*x+c)^4*cos(b*x+a)
/b-24*d^3*(d*x+c)*sin(b*x+a)/b^4+4*d*(d*x+c)^3*sin(b*x+a)/b^2
```

**Rubi** [A]

time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2718}

$$-\frac{24d^4\cos(a+bx)}{b^5}-\frac{24d^3(c+dx)\sin(a+bx)}{b^4}+\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}+\frac{4d(c+dx)^3\sin(a+bx)}{b^2}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sin[a + b*x],x]
```

```
[Out] (-24*d^4*Cos[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 - ((c +
d*x)^4*Cos[a + b*x])/b - (24*d^3*(c + d*x)*Sin[a + b*x])/b^4 + (4*d*(c + d*
x)^3*Sin[a + b*x])/b^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4\sin(a+bx)\,dx &= -\frac{(c+dx)^4\cos(a+bx)}{b}+\frac{(4d)\int (c+dx)^3\cos(a+bx)\,dx}{b}\\
&= -\frac{(c+dx)^4\cos(a+bx)}{b}+\frac{4d(c+dx)^3\sin(a+bx)}{b^2}-\frac{(12d^2)\int (c+dx)^2\sin(a+ \ldots}{b^2}\\
&= \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}-\frac{(c+dx)^4\cos(a+bx)}{b}+\frac{4d(c+dx)^3\sin(a+bx)}{b^2}- \ldots\\
&= \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}-\frac{(c+dx)^4\cos(a+bx)}{b}-\frac{24d^3(c+dx)\sin(a+bx)}{b^4} \ldots\\
&= -\frac{24d^4\cos(a+bx)}{b^5}+\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}-\frac{(c+dx)^4\cos(a+bx)}{b}-24d \ldots
\end{aligned}$$

**Mathematica** [**A**]
time = 0.19, size = 77, normalized size = 0.84

$$\frac{-((24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cos(a+bx))+4bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\sin(a+bx)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sin[a + b*x],x]
```

```
[Out] (-((24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x]) + 4*b*
d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Sin[a + b*x])/b^5
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(92) = 184$.
time = 0.08, size = 551, normalized size = 5.99 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^4*a^4*d^4*cos(b*x+a)+4/b^3*a^3*c*d^3*cos(b*x+a)-4/b^4*a^3*d^4*(si
n(b*x+a)-(b*x+a)*cos(b*x+a))-6/b^2*a^2*c^2*d^2*cos(b*x+a)+12/b^3*a^2*c*d^3*
(sin(b*x+a)-(b*x+a)*cos(b*x+a))+6/b^4*a^2*d^4*(-(b*x+a)^2*cos(b*x+a)+2*cos(
b*x+a)+2*(b*x+a)*sin(b*x+a))+4/b*a*c^3*d*cos(b*x+a)-12/b^2*a*c^2*d^2*(sin(b
*x+a)-(b*x+a)*cos(b*x+a))-12/b^3*a*c*d^3*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a
)+2*(b*x+a)*sin(b*x+a))-4/b^4*a*d^4*(-(b*x+a)^3*cos(b*x+a)+3*(b*x+a)^2*sin(
b*x+a)-6*sin(b*x+a)+6*(b*x+a)*cos(b*x+a))-c^4*cos(b*x+a)+4/b*c^3*d*(sin(b*x
+a)-(b*x+a)*cos(b*x+a))+6/b^2*c^2*d^2*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2
*(b*x+a)*sin(b*x+a))+4/b^3*c*d^3*(-(b*x+a)^3*cos(b*x+a)+3*(b*x+a)^2*sin(b*x
+a)-6*sin(b*x+a)+6*(b*x+a)*cos(b*x+a))+1/b^4*d^4*(-(b*x+a)^4*cos(b*x+a)+4*(
b*x+a)^3*sin(b*x+a)+12*(b*x+a)^2*cos(b*x+a)-24*cos(b*x+a)-24*(b*x+a)*sin(b*
x+a)))
```

**Maxima** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(92) = 184$.
time = 0.30, size = 490, normalized size = 5.33

$$-\frac{c^4\cos(bx+a)-\frac{4ac^3d\cos(bx+a)}{b}+\frac{6a^2c^2d^2\cos(bx+a)}{b^2}-\frac{4a^3cd^3\cos(bx+a)}{b^3}+\frac{a^4d^4\cos(bx+a)}{b^4}+\frac{4((bx+a)\cos(bx+a)-\sin(bx+a))c^3d}{b}-\frac{12((bx+a)\cos(bx+a)-\sin(bx+a))ac^2d^2}{b^2}+\frac{12((bx+a)\cos(bx+a)-\sin(bx+a))a^2cd^3}{b^3}-\frac{4((bx+a)\cos(bx+a)-\sin(bx+a))a^3d^4}{b^4}+\frac{6\left(\left((bx+a)^2-2\right)\cos(bx+a)-2(bx+a)\sin(bx+a)\right)c^2d^2}{b^2}-\frac{12\left(\left((bx+a)^2-2\right)\cos(bx+a)-2(bx+a)\sin(bx+a)\right)acd^3}{b^3}+\frac{6\left(\left((bx+a)^2-2\right)\cos(bx+a)-2(bx+a)\sin(bx+a)\right)a^2d^4}{b^4}+\frac{4\left(\left((bx+a)^3-6bx-6a\right)\cos(bx+a)-3\left((bx+a)^2-2\right)\sin(bx+a)\right)cd^3}{b^3}-\frac{4\left(\left((bx+a)^3-6bx-6a\right)\cos(bx+a)-3\left((bx+a)^2-2\right)\sin(bx+a)\right)ad^4}{b^4}+\frac{\left(\left((bx+a)^4-12(bx+a)^2+24\right)\cos(bx+a)-4\left((bx+a)^3-6bx-6a\right)\sin(bx+a)\right)d^4}{b^4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(c^4*cos(b*x + a) - 4*a*c^3*d*cos(b*x + a)/b + 6*a^2*c^2*d^2*cos(b*x + a)/
b^2 - 4*a^3*c*d^3*cos(b*x + a)/b^3 + a^4*d^4*cos(b*x + a)/b^4 + 4*((b*x + a
)*cos(b*x + a) - sin(b*x + a))*c^3*d/b - 12*((b*x + a)*cos(b*x + a) - sin(b
*x + a))*a*c^2*d^2/b^2 + 12*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^2*c*d
^3/b^3 - 4*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^3*d^4/b^4 + 6*(((b*x +
```

```
 a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 12*(((b*x
 + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 6*(((b*
x + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 4*(((b
*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*x + a))*c
*d^3/b^3 - 4*(((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - 3*((b*x + a)^2 - 2
)*sin(b*x + a))*a*d^4/b^4 + (((b*x + a)^4 - 12*(b*x + a)^2 + 24)*cos(b*x +
a) - 4*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b
```

**Fricas** [**A**]
time = 0.44, size = 170, normalized size = 1.85

$$-\frac{\left(b^{4} d^{4} x^{4} + 4 \, b^{4} c d^{3} x^{3} + b^{4} c^{4} - 12 \, b^{2} c^{2} d^{2} + 24 \, d^{4} + 6 \, {\left(b^{4} c^{2} d^{2} - 2 \, b^{2} d^{4}\right)} x^{2} + 4 \, {\left(b^{4} c^{3} d - 6 \, b^{2} c d^{3}\right)} x\right)} \cos\left(b x + a\right) - 4 \, {\left(b^{3} d^{4} x^{3} + 3 \, b^{3} c d^{3} x^{2} + b^{3} c^{3} d - 6 \, b c d^{3} + 3 \, {\left(b^{3} c^{2} d^{2} - 2 \, b d^{4}\right)} x\right)} \sin\left(b x + a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -((b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b
^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a) -
 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2
- 2*b*d^4)*x)*sin(b*x + a))/b^5
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(92) = 184$.
time = 0.37, size = 311, normalized size = 3.38

$$\begin{cases} - \frac{c^{4} \cos{\left(a + b x \right)}}{b} - \frac{4 c^{3} d x \cos{\left(a + b x \right)}}{b} - \frac{6 c^{2} d^{2} x^{2} \cos{\left(a + b x \right)}}{b} - \frac{4 c d^{3} x^{3} \cos{\left(a + b x \right)}}{b} - \frac{d^{4} x^{4} \cos{\left(a + b x \right)}}{b} + \frac{4 c^{3} d \sin{\left(a + b x \right)}}{b^{2}} + \frac{12 c^{2} d^{2} x \sin{\left(a + b x \right)}}{b^{2}} + \frac{12 c d^{3} x^{2} \sin{\left(a + b x \right)}}{b^{2}} + \frac{4 d^{4} x^{3} \sin{\left(a + b x \right)}}{b^{2}} + \frac{12 c^{2} d^{2} \cos{\left(a + b x \right)}}{b^{3}} + \frac{24 c d^{3} x \cos{\left(a + b x \right)}}{b^{3}} + \frac{12 d^{4} x^{2} \cos{\left(a + b x \right)}}{b^{3}} - \frac{24 c d^{3} \sin{\left(a + b x \right)}}{b^{4}} - \frac{24 d^{4} x \sin{\left(a + b x \right)}}{b^{4}} - \frac{24 d^{4} \cos{\left(a + b x \right)}}{b^{5}} & \text{for}\: b \neq 0 \\\left(c^{4} x + 2 c^{3} d x^{2} + 2 c^{2} d^{2} x^{3} + c d^{3} x^{4} + \frac{d^{4} x^{5}}{5}\right) \sin{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)/b - 4*c**3*d*x*cos(a + b*x)/b - 6*c**2*d**2*x
**2*cos(a + b*x)/b - 4*c*d**3*x**3*cos(a + b*x)/b - d**4*x**4*cos(a + b*x)/
b + 4*c**3*d*sin(a + b*x)/b**2 + 12*c**2*d**2*x*sin(a + b*x)/b**2 + 12*c*d*
*3*x**2*sin(a + b*x)/b**2 + 4*d**4*x**3*sin(a + b*x)/b**2 + 12*c**2*d**2*co
s(a + b*x)/b**3 + 24*c*d**3*x*cos(a + b*x)/b**3 + 12*d**4*x**2*cos(a + b*x)
/b**3 - 24*c*d**3*sin(a + b*x)/b**4 - 24*d**4*x*sin(a + b*x)/b**4 - 24*d**4
*cos(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3
+ c*d**3*x**4 + d**4*x**5/5)*sin(a), True))
```

**Giac** [**A**]
time = 2.98, size = 171, normalized size = 1.86

$$-\frac{{\left(b^{4} d^{4} x^{4} + 4 \, b^{4} c d^{3} x^{3} + 6 \, b^{4} c^{2} d^{2} x^{2} + 4 \, b^{4} c^{3} d x + b^{4} c^{4} - 12 \, b^{2} d^{4} x^{2} - 24 \, b^{2} c d^{3} x - 12 \, b^{2} c^{2} d^{2} + 24 \, d^{4}\right)} \cos\left(b x + a\right)}{b^{5}} + \frac{4 \, {\left(b^{3} d^{4} x^{3} + 3 \, b^{3} c d^{3} x^{2} + 3 \, b^{3} c^{2} d^{2} x + b^{3} c^{3} d - 6 \, b d^{4} x - 6 \, b c d^{3}\right)} \sin\left(b x + a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a
)/b^5 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*
b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

**Mupad [B]**
time = 0.79, size = 221, normalized size = 2.40

$$\frac{4\,x\,\cos\left(a+b\,x\right)\,\left(6\,c\,d^3-b^2\,c^3\,d\right)}{b^3}-\frac{4\,\sin\left(a+b\,x\right)\,\left(6\,c\,d^3-b^2\,c^3\,d\right)}{b^4}-\frac{d^4\,x^4\,\cos\left(a+b\,x\right)}{b}-\frac{\cos\left(a+b\,x\right)\,\left(b^4\,c^4-12\,b^2\,c^2\,d^2+24\,d^4\right)}{b^5}+\frac{4\,d^4\,x^3\,\sin\left(a+b\,x\right)}{b^2}-\frac{12\,x\,\sin\left(a+b\,x\right)\,\left(2\,d^4-b^2\,c^2\,d^2\right)}{b^4}+\frac{6\,x^2\,\cos\left(a+b\,x\right)\,\left(2\,d^4-b^2\,c^2\,d^2\right)}{b^3}-\frac{4\,c\,d^3\,x^3\,\cos\left(a+b\,x\right)}{b}+\frac{12\,c\,d^3\,x^2\,\sin\left(a+b\,x\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (4*x*cos(a + b*x)*(6*c*d^3 - b^2*c^3*d))/b^3 - (4*sin(a + b*x)*(6*c*d^3 - b
^2*c^3*d))/b^4 - (d^4*x^4*cos(a + b*x))/b - (cos(a + b*x)*(24*d^4 + b^4*c^4
 - 12*b^2*c^2*d^2))/b^5 + (4*d^4*x^3*sin(a + b*x))/b^2 - (12*x*sin(a + b*x)
*(2*d^4 - b^2*c^2*d^2))/b^4 + (6*x^2*cos(a + b*x)*(2*d^4 - b^2*c^2*d^2))/b^
3 - (4*c*d^3*x^3*cos(a + b*x))/b + (12*c*d^3*x^2*sin(a + b*x))/b^2
```

# 3.2 $\int (c+dx)^3 \sin(a+bx)\,dx$

**Optimal**. Leaf size=71

$$\frac{6d^2(c+dx)\cos(a+bx)}{b^3}-\frac{(c+dx)^3\cos(a+bx)}{b}-\frac{6d^3\sin(a+bx)}{b^4}+\frac{3d(c+dx)^2\sin(a+bx)}{b^2}$$

```
[Out] 6*d^2*(d*x+c)*cos(b*x+a)/b^3-(d*x+c)^3*cos(b*x+a)/b-6*d^3*sin(b*x+a)/b^4+3*
d*(d*x+c)^2*sin(b*x+a)/b^2
```

**Rubi** [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2717}

$$-\frac{6d^3\sin(a+bx)}{b^4}+\frac{6d^2(c+dx)\cos(a+bx)}{b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{b^2}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x],x]
```

```
[Out] (6*d^2*(c + d*x)*Cos[a + b*x])/b^3 - ((c + d*x)^3*Cos[a + b*x])/b - (6*d^3*
Sin[a + b*x])/b^4 + (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\sin(a+bx)\,dx &= -\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\cos(a+bx)\,dx}{b}\\ &= -\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{3d(c+dx)^2\sin(a+bx)}{b^2}-\frac{(6d^2)\int (c+dx)\sin(a+bx}{b^2}\\ &= \frac{6d^2(c+dx)\cos(a+bx)}{b^3}-\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{3d(c+dx)^2\sin(a+bx)}{b^2}-\\ &= \frac{6d^2(c+dx)\cos(a+bx)}{b^3}-\frac{(c+dx)^3\cos(a+bx)}{b}-\frac{6d^3\sin(a+bx)}{b^4}+\frac{3d(c+dx}{}\end{aligned}$$

**Mathematica** [A]
time = 0.11, size = 62, normalized size = 0.87

$$\frac{-b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)+3d\left(-2d^2+b^2(c+dx)^2\right)\sin(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x],x]
```

```
[Out] (-(b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x]) + 3*d*(-2*d^2 + b^2
*(c + d*x)^2)*Sin[a + b*x])/b^4
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(71) = 142$.
time = 0.06, size = 308, normalized size = 4.34

| method | result |
|---|---|
| risch | $-\frac{\left(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2\right)\cos(bx+a)}{b^3}+\frac{3d\left(d^2x^2b^2+2b^2cdx+b^2c^2-2d^2\right)\sin(bx+a)}{b^4}$ |
| norman | $\frac{\frac{\left(2b^2c^3-12c\,d^2\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^3}+\frac{d^3x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{d^3x^3}{b}-\frac{3d\left(b^2c^2-2d^2\right)x}{b^3}-\frac{3c\,d^2x^2}{b}+\frac{6d\left(b^2c^2-2d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^4}+\frac{6d^3x}{}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$ |
| derivativedivides | $\frac{\frac{a^3d^3\cos(bx+a)}{b^3}-\frac{3a^2c\,d^2\cos(bx+a)}{b^2}+\frac{3a^2d^3(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^3}+\frac{3a\,c^2d\cos(bx+a)}{b}-\frac{6ac\,d^2(\sin(bx+a)-(bx+a)\cos(bx+}{b^2}}{}$ |
| default | $\frac{\frac{a^3d^3\cos(bx+a)}{b^3}-\frac{3a^2c\,d^2\cos(bx+a)}{b^2}+\frac{3a^2d^3(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^3}+\frac{3a\,c^2d\cos(bx+a)}{b}-\frac{6ac\,d^2(\sin(bx+a)-(bx+a)\cos(bx+}{b^2}}{}$ |
| meijerg | $\frac{8d^3\sqrt{\pi}\,\cos(a)\left(\frac{xb\left(-\frac{5x^2b^2}{2}+15\right)\cos(bx)}{20\sqrt{\pi}}-\frac{\left(-\frac{15x^2b^2}{2}+15\right)\sin(bx)}{20\sqrt{\pi}}\right)}{b^4}+\frac{8d^3\sqrt{\pi}\,\sin(a)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2b^2}{2}+3\right)\cos(b}{4\sqrt{\pi}}\right.}{b^4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^3*a^3*d^3*cos(b*x+a)-3/b^2*a^2*c*d^2*cos(b*x+a)+3/b^3*a^2*d^3*(sin
(b*x+a)-(b*x+a)*cos(b*x+a))+3/b*a*c^2*d*cos(b*x+a)-6/b^2*a*c*d^2*(sin(b*x+a
)-(b*x+a)*cos(b*x+a))-3/b^3*a*d^3*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2*(b*
x+a)*sin(b*x+a))-c^3*cos(b*x+a)+3/b*c^2*d*(sin(b*x+a)-(b*x+a)*cos(b*x+a))+3
/b^2*c*d^2*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2*(b*x+a)*sin(b*x+a))+1/b^3*
d^3*(-(b*x+a)^3*cos(b*x+a)+3*(b*x+a)^2*sin(b*x+a)-6*sin(b*x+a)+6*(b*x+a)*co
s(b*x+a)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(71) = 142$.
time = 0.32, size = 285, normalized size = 4.01

$$-\frac{c^3\cos(bx+a)-\frac{3ac^2d\cos(bx+a)}{b}+\frac{3a^2cd^2\cos(bx+a)}{b^2}-\frac{a^3d^3\cos(bx+a)}{b^3}+\frac{3((bx+a)\cos(bx+a)-\sin(bx+a))c^2d}{b}-\frac{6((bx+a)\cos(bx+a)-\sin(bx+a))acd^2}{b^2}+\frac{3((bx+a)\cos(bx+a)-\sin(bx+a))a^2d^3}{b^3}+\frac{3\left(\left((bx+a)^2-2\right)\cos(bx+a)-2(bx+a)\sin(bx+a)\right)cd^2}{b^2}-\frac{3\left(\left((bx+a)^2-2\right)\cos(bx+a)-2(bx+a)\sin(bx+a)\right)ad^3}{b^3}+\frac{\left(\left((bx+a)^3-6bx-6a\right)\cos(bx+a)-3\left((bx+a)^2-2\right)\sin(bx+a)\right)d^3}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(c^3*cos(b*x + a) - 3*a*c^2*d*cos(b*x + a)/b + 3*a^2*c*d^2*cos(b*x + a)/b^
2 - a^3*d^3*cos(b*x + a)/b^3 + 3*((b*x + a)*cos(b*x + a) - sin(b*x + a))*c^
2*d/b - 6*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a*c*d^2/b^2 + 3*((b*x + a
)*cos(b*x + a) - sin(b*x + a))*a^2*d^3/b^3 + 3*(((b*x + a)^2 - 2)*cos(b*x +
 a) - 2*(b*x + a)*sin(b*x + a))*c*d^2/b^2 - 3*(((b*x + a)^2 - 2)*cos(b*x +
a) - 2*(b*x + a)*sin(b*x + a))*a*d^3/b^3 + (((b*x + a)^3 - 6*b*x - 6*a)*cos
(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

**Fricas** [A]
time = 0.35, size = 110, normalized size = 1.55

$$-\frac{\left(b^{3} d^{3} x^{3}+3 \, b^{3} c d^{2} x^{2}+b^{3} c^{3}-6 \, b c d^{2}+3 \, \left(b^{3} c^{2} d-2 \, b d^{3}\right) x\right) \cos \left(b x+a\right)-3 \, \left(b^{2} d^{3} x^{2}+2 \, b^{2} c d^{2} x+b^{2} c^{2} d-2 \, d^{3}\right) \sin \left(b x+a\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b
*d^3)*x)*cos(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)
*sin(b*x + a))/b^4
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(70) = 140$.
time = 0.23, size = 202, normalized size = 2.85

$$\begin{cases} - \frac{c^{3} \cos{\left(a + b x \right)}}{b} - \frac{3 c^{2} d x \cos{\left(a + b x \right)}}{b} - \frac{3 c d^{2} x^{2} \cos{\left(a + b x \right)}}{b} - \frac{d^{3} x^{3} \cos{\left(a + b x \right)}}{b} + \frac{3 c^{2} d \sin{\left(a + b x \right)}}{b^{2}} + \frac{6 c d^{2} x \sin{\left(a + b x \right)}}{b^{2}} + \frac{3 d^{3} x^{2} \sin{\left(a + b x \right)}}{b^{2}} + \frac{6 c d^{2} \cos{\left(a + b x \right)}}{b^{3}} + \frac{6 d^{3} x \cos{\left(a + b x \right)}}{b^{3}} - \frac{6 d^{3} \sin{\left(a + b x \right)}}{b^{4}} & \text{for}\: b \neq 0 \\\left(c^{3} x + \frac{3 c^{2} d x^{2}}{2} + c d^{2} x^{3} + \frac{d^{3} x^{4}}{4}\right) \sin{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)/b - 3*c**2*d*x*cos(a + b*x)/b - 3*c*d**2*x**2
*cos(a + b*x)/b - d**3*x**3*cos(a + b*x)/b + 3*c**2*d*sin(a + b*x)/b**2 + 6
*c*d**2*x*sin(a + b*x)/b**2 + 3*d**3*x**2*sin(a + b*x)/b**2 + 6*c*d**2*cos(
a + b*x)/b**3 + 6*d**3*x*cos(a + b*x)/b**3 - 6*d**3*sin(a + b*x)/b**4, Ne(b
, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a), True
))
```

**Giac** [A]
time = 3.77, size = 111, normalized size = 1.56

$$-\frac{\left(b^{3} d^{3} x^{3}+3 \, b^{3} c d^{2} x^{2}+3 \, b^{3} c^{2} d x+b^{3} c^{3}-6 \, b d^{3} x-6 \, b c d^{2}\right) \cos \left(b x+a\right)}{b^{4}}+\frac{3 \, \left(b^{2} d^{3} x^{2}+2 \, b^{2} c d^{2} x+b^{2} c^{2} d-2 \, d^{3}\right) \sin \left(b x+a\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b
*c*d^2)*cos(b*x + a)/b^4 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d
^3)*sin(b*x + a)/b^4
```

**Mupad** [B]
time = 0.62, size = 147, normalized size = 2.07

$$\frac{\cos\left(a+b\,x\right)\,\left(6\,c\,d^2-b^2\,c^3\right)}{b^3}-\frac{3\,\sin\left(a+b\,x\right)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^4}-\frac{d^3\,x^3\,\cos\left(a+b\,x\right)}{b}+\frac{3\,d^3\,x^2\,\sin\left(a+b\,x\right)}{b^2}+\frac{3\,x\,\cos\left(a+b\,x\right)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^3}+\frac{6\,c\,d^2\,x\,\sin\left(a+b\,x\right)}{b^2}-\frac{3\,c\,d^2\,x^2\,\cos\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (cos(a + b*x)*(6*c*d^2 - b^2*c^3))/b^3 - (3*sin(a + b*x)*(2*d^3 - b^2*c^2*d
))/b^4 - (d^3*x^3*cos(a + b*x))/b + (3*d^3*x^2*sin(a + b*x))/b^2 + (3*x*cos
(a + b*x)*(2*d^3 - b^2*c^2*d))/b^3 + (6*c*d^2*x*sin(a + b*x))/b^2 - (3*c*d^
2*x^2*cos(a + b*x))/b
```

# 3.3 $\int (c+dx)^2 \sin(a+bx)\,dx$

**Optimal**. Leaf size=50

$$\frac{2d^2\cos(a+bx)}{b^3}-\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{2d(c+dx)\sin(a+bx)}{b^2}$$

```
[Out] 2*d^2*cos(b*x+a)/b^3-(d*x+c)^2*cos(b*x+a)/b+2*d*(d*x+c)*sin(b*x+a)/b^2
```

**Rubi** [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2718}

$$\frac{2d^2\cos(a+bx)}{b^3}+\frac{2d(c+dx)\sin(a+bx)}{b^2}-\frac{(c+dx)^2\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x],x]
```

```
[Out] (2*d^2*Cos[a + b*x])/b^3 - ((c + d*x)^2*Cos[a + b*x])/b + (2*d*(c + d*x)*Si
n[a + b*x])/b^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\sin(a+bx)\,dx &= -\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(2d)\int (c+dx)\cos(a+bx)\,dx}{b}\\
&= -\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{2d(c+dx)\sin(a+bx)}{b^2}-\frac{(2d^2)\int \sin(a+bx)\,dx}{b^2}\\
&= \frac{2d^2\cos(a+bx)}{b^3}-\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{2d(c+dx)\sin(a+bx)}{b^2}
\end{aligned}$$

**Mathematica** [**A**]
time = 0.10, size = 45, normalized size = 0.90

$$\frac{-((-2d^2+b^2(c+dx)^2)\cos(a+bx))+2bd(c+dx)\sin(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x],x]
```

```
[Out] (-((-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x]) + 2*b*d*(c + d*x)*Sin[a + b*x])
/b^3
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(50) = 100$.
time = 0.05, size = 148, normalized size = 2.96

| method | result |
|---|---|
| risch | $-\frac{(d^2x^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{b^3}+\frac{2d(dx+c)\sin(bx+a)}{b^2}$ |
| norman | $\frac{\frac{-2b^2c^2+4d^2}{b^3}+\frac{d^2x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{d^2x^2}{b}-\frac{2cdx}{b}+\frac{4cd\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{4d^2x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{2cdx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$ |
| derivativedivides | $\frac{-\frac{a^2d^2\cos(bx+a)}{b^2}+\frac{2acd\cos(bx+a)}{b}-\frac{2a\,d^2(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^2}-c^2\cos(bx+a)+\frac{2cd(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}+\frac{d}{}}{b}$ |
| default | $\frac{-\frac{a^2d^2\cos(bx+a)}{b^2}+\frac{2acd\cos(bx+a)}{b}-\frac{2a\,d^2(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^2}-c^2\cos(bx+a)+\frac{2cd(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}+\frac{d}{}}{b}$ |
| meijerg | $\frac{4d^2\sqrt{\pi}\,\cos(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2b^2}{2}+1\right)\cos(bx)}{2\sqrt{\pi}}+\frac{xb\sin(bx)}{2\sqrt{\pi}}\right)}{b^3}+\frac{4d^2\sqrt{\pi}\,\sin(a)\left(\frac{x\left(b^2\right)^{\frac{3}{2}}\cos(bx)}{2\sqrt{\pi}\,b^2}-\frac{\left(b^2\right)^{\frac{3}{2}}\left(-\frac{3x^2b^2}{2}+\right.}{6\sqrt{\pi}\,b}\right.}{b^2\sqrt{b^2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^2*a^2*d^2*cos(b*x+a)+2/b*a*c*d*cos(b*x+a)-2/b^2*a*d^2*(sin(b*x+a)
-(b*x+a)*cos(b*x+a))-c^2*cos(b*x+a)+2/b*c*d*(sin(b*x+a)-(b*x+a)*cos(b*x+a))
+1/b^2*d^2*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2*(b*x+a)*sin(b*x+a)))
```

**Maxima** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(50) = 100$.
time = 0.31, size = 141, normalized size = 2.82

$$-\frac{c^2\cos(bx+a)-\frac{2\,acd\cos(bx+a)}{b}+\frac{a^2d^2\cos(bx+a)}{b^2}+\frac{2\,((bx+a)\cos(bx+a)-\sin(bx+a))cd}{b}-\frac{2\,((bx+a)\cos(bx+a)-\sin(bx+a))ad^2}{b^2}+\frac{\left(\left((bx+a)^2-2\right)\cos(bx+a)-2\,(bx+a)\sin(bx+a)\right)d^2}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(c^2*cos(b*x + a) - 2*a*c*d*cos(b*x + a)/b + a^2*d^2*cos(b*x + a)/b^2 + 2*
((b*x + a)*cos(b*x + a) - sin(b*x + a))*c*d/b - 2*((b*x + a)*cos(b*x + a) -
 sin(b*x + a))*a*d^2/b^2 + (((b*x + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*si
n(b*x + a))*d^2/b^2)/b
```

**Fricas** [A]
time = 0.34, size = 63, normalized size = 1.26

$$-\frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,d^2)\cos{(bx + a)} - 2\,(bd^2x + bcd)\sin{(bx + a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*(b*d^2*x +
 b*c*d)*sin(b*x + a))/b^3
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.
time = 0.14, size = 112, normalized size = 2.24

$$\begin{cases} -\frac{c^2\cos{(a+bx)}}{b} - \frac{2cdx\cos{(a+bx)}}{b} - \frac{d^2x^2\cos{(a+bx)}}{b} + \frac{2cd\sin{(a+bx)}}{b^2} + \frac{2d^2x\sin{(a+bx)}}{b^2} + \frac{2d^2\cos{(a+bx)}}{b^3} & \text{for}\ b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)/b - 2*c*d*x*cos(a + b*x)/b - d**2*x**2*cos(a
+ b*x)/b + 2*c*d*sin(a + b*x)/b**2 + 2*d**2*x*sin(a + b*x)/b**2 + 2*d**2*co
s(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a), True
))
```

**Giac** [A]
time = 4.22, size = 65, normalized size = 1.30

$$-\frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,d^2)\cos{(bx + a)}}{b^3} + \frac{2\,(bd^2x + bcd)\sin{(bx + a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 + 2*(b*d^2*
x + b*c*d)*sin(b*x + a)/b^3
```

**Mupad** [B]
time = 0.55, size = 84, normalized size = 1.68

$$\frac{\cos{(a + b\,x)}\,\left(2\,d^2 - b^2\,c^2\right)}{b^3} - \frac{d^2\,x^2\,\cos{(a + b\,x)}}{b} + \frac{2\,c\,d\,\sin{(a + b\,x)}}{b^2} + \frac{2\,d^2\,x\,\sin{(a + b\,x)}}{b^2} - \frac{2\,c\,d\,x\,\cos{(a + b\,x)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (cos(a + b*x)*(2*d^2 - b^2*c^2))/b^3 - (d^2*x^2*cos(a + b*x))/b + (2*c*d*si
n(a + b*x))/b^2 + (2*d^2*x*sin(a + b*x))/b^2 - (2*c*d*x*cos(a + b*x))/b
```

# 3.4 $\int (c+dx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=28

$$-\frac{(c+dx)\cos(a+bx)}{b}+\frac{d\sin(a+bx)}{b^2}$$

```
[Out] -(d*x+c)*cos(b*x+a)/b+d*sin(b*x+a)/b^2
```

**Rubi** [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3377, 2717}

$$\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x],x]
```

```
[Out] -(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sin(a+bx)\,dx &= -\frac{(c+dx)\cos(a+bx)}{b}+\frac{d\int \cos(a+bx)\,dx}{b}\\ &= -\frac{(c+dx)\cos(a+bx)}{b}+\frac{d\sin(a+bx)}{b^2}\end{aligned}$$

**Mathematica** [A]
time = 0.05, size = 27, normalized size = 0.96

$$\frac{-b(c+dx)\cos(a+bx)+d\sin(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x],x]
```

```
[Out] (-(b*(c + d*x)*Cos[a + b*x]) + d*Sin[a + b*x])/b^2
```

**Maple [A]**
time = 0.03, size = 52, normalized size = 1.86

| method | result |
|---|---|
| risch | $-\frac{(dx+c)\cos(bx+a)}{b}+\frac{d\sin(bx+a)}{b^2}$ |
| derivativedivides | $\frac{\frac{da\cos(bx+a)}{b}-c\cos(bx+a)+\frac{d(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}}{b}$ |
| default | $\frac{\frac{da\cos(bx+a)}{b}-c\cos(bx+a)+\frac{d(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}}{b}$ |
| norman | $\frac{\frac{2c\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{2d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{dx}{b}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$ |
| meijerg | $\frac{2d\sqrt{\pi}\,\cos(a)\left(-\frac{xb\cos(bx)}{2\sqrt{\pi}}+\frac{\sin(bx)}{2\sqrt{\pi}}\right)}{b^2}+\frac{2d\sqrt{\pi}\,\sin(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(bx)}{2\sqrt{\pi}}+\frac{xb\sin(bx)}{2\sqrt{\pi}}\right)}{b^2}+\frac{c\sqrt{\pi}\,\cos(a)\left(\frac{1}{\sqrt{\pi}}\right.}{b}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b*d*a*cos(b*x+a)-c*cos(b*x+a)+1/b*d*(sin(b*x+a)-(b*x+a)*cos(b*x+a)))
```

**Maxima [A]**
time = 0.29, size = 53, normalized size = 1.89

$$-\frac{c\cos(bx+a)-\frac{ad\cos(bx+a)}{b}+\frac{((bx+a)\cos(bx+a)-\sin(bx+a))d}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(c*cos(b*x + a) - a*d*cos(b*x + a)/b + ((b*x + a)*cos(b*x + a) - sin(b*x +
 a))*d/b)/b
```

**Fricas [A]**
time = 0.41, size = 30, normalized size = 1.07

$$-\frac{(bdx+bc)\cos(bx+a)-d\sin(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -((b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))/b^2
```

**Sympy [A]**
time = 0.08, size = 46, normalized size = 1.64

$$\begin{cases} -\frac{c\cos(a+bx)}{b} - \frac{dx\cos(a+bx)}{b} + \frac{d\sin(a+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)/b - d*x*cos(a + b*x)/b + d*sin(a + b*x)/b**2, Ne
(b, 0)), ((c*x + d*x**2/2)*sin(a), True))
```

**Giac [A]**
time = 2.88, size = 31, normalized size = 1.11

$$-\frac{(bdx + bc)\cos(bx + a)}{b^2} + \frac{d\sin(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*d*x + b*c)*cos(b*x + a)/b^2 + d*sin(b*x + a)/b^2
```

**Mupad [B]**
time = 0.53, size = 35, normalized size = 1.25

$$\frac{d\,\sin(a + b\,x)}{b^2} - \frac{c\,\cos(a + b\,x) + d\,x\,\cos(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(c + d*x),x)
```

```
[Out] (d*sin(a + b*x))/b^2 - (c*cos(a + b*x) + d*x*cos(a + b*x))/b
```

# 3.5 $\int \frac{\sin(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=51

$$\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] cos(a-b*c/d)*Si(b*c/d+b*x)/d+Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

**Rubi** [A]
time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3384, 3380, 3383}

$$\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x),x]
```

```
[Out] (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (Cos[a - (b*c)/d]*SinInte
gral[(b*c)/d + b*x])/d
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{c+dx}\,dx &= \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\\ &= \frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\end{aligned}$$

**Mathematica** [A]
time = 0.06, size = 49, normalized size = 0.96

$$\frac{\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)+\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x),x]
[Out] (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral
[(b*c)/d + b*x])/d
```

**Maple** [A]
time = 0.05, size = 78, normalized size = 1.53

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}$ | 78 |
| default | $-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}$ | 78 |
| risch | $\frac{i\,\mathrm{e}^{\frac{i(da-cb)}{d}}\,\text{expIntegral}\left(1,-ibx-ia-\frac{-iad+ibc}{d}\right)}{2d}-\frac{i\,\mathrm{e}^{-\frac{i(da-cb)}{d}}\,\text{expIntegral}\left(1,ibx+ia-\frac{i(da-cb)}{d}\right)}{2d}$ | 98 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
[Out] -Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a
*d+b*c)/d)/d
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.38, size = 141, normalized size = 2.76

$$-\frac{b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] -1/2*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_int
egral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(ex
p_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -(I*
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/(b*d)
```

**Fricas** [A]
time = 0.34, size = 78, normalized size = 1.53

$$\frac{\left(\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+2\,\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*sin(-
(b*c - a*d)/d) + 2*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + bx\right)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x), x)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.29, size = 597, normalized size = 11.71

$$\frac{\Im\left(\operatorname{Ci}\left(bx + \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{bc}{2d}\right)^2 - \Im\left(\operatorname{Ci}\left(-bx - \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{bc}{2d}\right)^2 + 2\operatorname{Si}\left(\frac{bdx+bc}{d}\right) \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{bc}{2d}\right)^2 + 2\Re\left(\operatorname{Ci}\left(bx + \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{bc}{2d}\right) + 2\Re\left(\operatorname{Ci}\left(-bx - \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{bc}{2d}\right) - 2\Re\left(\operatorname{Ci}\left(bx + \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right) \tan\left(\frac{bc}{2d}\right)^2 - 2\Re\left(\operatorname{Ci}\left(-bx - \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right) \tan\left(\frac{bc}{2d}\right)^2 - \Im\left(\operatorname{Ci}\left(bx + \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right)^2 + \Im\left(\operatorname{Ci}\left(-bx - \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right)^2 - 2\operatorname{Si}\left(\frac{bdx+bc}{d}\right) \tan\left(\frac{1}{2}a\right)^2 + 4\Im\left(\operatorname{Ci}\left(bx + \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right) \tan\left(\frac{bc}{2d}\right) - 4\Im\left(\operatorname{Ci}\left(-bx - \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right) \tan\left(\frac{bc}{2d}\right) + 8\operatorname{Si}\left(\frac{bdx+bc}{d}\right) \tan\left(\frac{1}{2}a\right) \tan\left(\frac{bc}{2d}\right) - \Im\left(\operatorname{Ci}\left(bx + \frac{bc}{d}\right)\right) \tan\left(\frac{bc}{2d}\right)^2 + \Im\left(\operatorname{Ci}\left(-bx - \frac{bc}{d}\right)\right) \tan\left(\frac{bc}{2d}\right)^2 - 2\operatorname{Si}\left(\frac{bdx+bc}{d}\right) \tan\left(\frac{bc}{2d}\right)^2 + 2\Re\left(\operatorname{Ci}\left(bx + \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right) + 2\Re\left(\operatorname{Ci}\left(-bx - \frac{bc}{d}\right)\right) \tan\left(\frac{1}{2}a\right) - 2\Re\left(\operatorname{Ci}\left(bx + \frac{bc}{d}\right)\right) \tan\left(\frac{bc}{2d}\right) - 2\Re\left(\operatorname{Ci}\left(-bx - \frac{bc}{d}\right)\right) \tan\left(\frac{bc}{2d}\right) + \Im\left(\operatorname{Ci}\left(bx + \frac{bc}{d}\right)\right) - \Im\left(\operatorname{Ci}\left(-bx - \frac{bc}{d}\right)\right) + 2\operatorname{Si}\left(\frac{bdx+bc}{d}\right)}{2\left(d\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{bc}{2d}\right)^2 + d\tan\left(\frac{1}{2}a\right)^2 + d\tan\left(\frac{bc}{2d}\right)^2 + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_
integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integra
l(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*
c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*ta
n(1/2*a)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 2*sin_int
egral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*imag_part(cos_integral(b*x + b*c/d)
)*tan(1/2*a)*tan(1/2*b*c/d) - 4*imag_part(cos_integral(-b*x - b*c/d))*tan(1
/2*a)*tan(1/2*b*c/d) + 8*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b
*c/d) - imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 + imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 2*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x - b*c/d))*ta
n(1/2*b*c/d) + imag_part(cos_integral(b*x + b*c/d)) - imag_part(cos_integra
l(-b*x - b*c/d)) + 2*sin_integral((b*d*x + b*c)/d))/(d*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 + d*tan(1/2*a)^2 + d*tan(1/2*b*c/d)^2 + d)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x),x)
[Out] int(sin(a + b*x)/(c + d*x), x)
```

# 3.6 $\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=72

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sin(a+bx)}{d(c+dx)}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}$$

```
[Out] b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2-b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2-sin(b*x+a
)/d/(d*x+c)
```

**Rubi** [A]
time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {3378, 3384, 3380, 3383}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sin(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - Sin[a + b*x]/(d*(c +
d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx = -\frac{\sin(a+bx)}{d(c+dx)} + \frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}$$
$$= -\frac{\sin(a+bx)}{d(c+dx)} + \frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} - \frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}$$
$$= \frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{\sin(a+bx)}{d(c+dx)} - \frac{b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d^2}$$

**Mathematica [A]**
time = 0.15, size = 66, normalized size = 0.92

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right) - \frac{d\sin(a+bx)}{c+dx} - b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - (d*Sin[a + b*x])/(c + d*x) -
 b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)])/d^2
```

**Maple [A]**
time = 0.06, size = 112, normalized size = 1.56

| method | result |
|---|---|
| derivativedivides | $b\left(-\frac{\sin(bx+a)}{(-da+cb+d(bx+a))d} + \frac{-\frac{\mathrm{sinIntegral}\left(-bx-a-\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d} + \frac{\mathrm{cosineIntegral}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}}{d}\right.$ |
| default | $b\left(-\frac{\sin(bx+a)}{(-da+cb+d(bx+a))d} + \frac{-\frac{\mathrm{sinIntegral}\left(-bx-a-\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d} + \frac{\mathrm{cosineIntegral}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}}{d}\right.$ |
| risch | $-\frac{b\,\mathrm{e}^{\frac{i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,-ibx-ia-\frac{-iad+ibc}{d}\right)}{2d^2} - \frac{b\,\mathrm{e}^{-\frac{i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,ibx+ia-\frac{i(da-cb)}{d}\right)}{2d^2} - \frac{(-2dxb-2cb)}{2d(dx+c)(-}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-sin(b*x+a)/(-d*a+c*b+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b
*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.36, size = 164, normalized size = 2.28

$$-\frac{b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{2\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2
*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(2,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b*c*d + (b*x +
a)*d^2 - a*d^2)*b)
```

**Fricas** [A]
time = 0.34, size = 124, normalized size = 1.72

$$-\frac{2\,(bdx+bc)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-\left((bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+2\,d\sin(bx+a)}{2\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - (
(b*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(
b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 2*d*sin(b*x + a))/(d^3*x + c*d^2)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x)**2, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(72) = 144$.
time = 4.26, size = 521, normalized size = 7.24

$$\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)+b^3c\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)-ab^2d\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)+(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)+b^3c\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)-ab^2d\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)+b^2d\sin\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)\right)d^2}{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c - a*d)/d)*cos_
integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + b
^3*c*cos(-(b*c - a*d)/d)*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(
d*x + c)) + b*c - a*d)/d) - a*b^2*d*cos(-(b*c - a*d)/d)*cos_integral(((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + (d*x + c)*(b - b
*c/(d*x + c) + a*d/(d*x + c))*b^2*sin(-(b*c - a*d)/d)*sin_integral(-((d*x +
 c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + b^3*c*sin(-(b*c -
 a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
 - a*d)/d) - a*b^2*d*sin(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/
(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + b^2*d*sin(-(d*x + c)*(b - b*c/
(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*
x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x)^2,x)
```

```
[Out] int(sin(a + b*x)/(c + d*x)^2, x)
```

# 3.7 $\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=104

$$-\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3}-\frac{\sin(a+bx)}{2d(c+dx)^2}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

```
[Out] -1/2*b*cos(b*x+a)/d^2/(d*x+c)-1/2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-1/2*b^
2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^3-1/2*sin(b*x+a)/d/(d*x+c)^2
```

**Rubi** [A]
time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3378, 3384, 3380, 3383}

$$-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -1/2*(b*Cos[a + b*x])/(d^2*(c + d*x)) - (b^2*CosIntegral[(b*c)/d + b*x]*Sin
[a - (b*c)/d])/(2*d^3) - Sin[a + b*x]/(2*d*(c + d*x)^2) - (b^2*Cos[a - (b*c
)/d]*SinIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx = -\frac{\sin(a+bx)}{2d(c+dx)^2}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin(a+bx)}{2d(c+dx)^2}-\frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2}$$
$$= -\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin(a+bx)}{2d(c+dx)^2}-\frac{\left(b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2}-\frac{\left(b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\text{co}}{}$$
$$= -\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3}-\frac{\sin(a+bx)}{2d(c+dx)^2}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

**Mathematica [A]**
time = 0.46, size = 87, normalized size = 0.84

$$-\frac{b^2\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\frac{d(b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^2}+b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -1/2*(b^2*CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a
 + b*x] + d*Sin[a + b*x]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b
*(c/d + x)])/d^3
```

**Maple [A]**
time = 0.10, size = 150, normalized size = 1.44

| method | result |
|---|---|
| derivativedivides | $b^2\left(-\frac{\sin(bx+a)}{2(-da+cb+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-da+cb+d(bx+a))d}-\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(bx\right.}{}}{d}}{2d}\right.$ |
| default | $b^2\left(-\frac{\sin(bx+a)}{2(-da+cb+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-da+cb+d(bx+a))d}-\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(bx\right.}{}}{d}}{2d}\right.$ |

| | |
|---|---|
| risch | $-\frac{i b^2 e^{\frac{i(da-cb)}{d}} \operatorname{expIntegral}\left(1,-ibx-ia-\frac{-iad+ibc}{d}\right)}{4d^3}+\frac{i b^2 e^{-\frac{i(da-cb)}{d}} \operatorname{expIntegral}\left(1,ibx+ia-\frac{i(da-cb)}{d}\right)}{4d^3}+\frac{i(2ib^3d^3x$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*sin(b*x+a)/(-d*a+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-d*a+c*b+d*
(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c
)/d)*sin((-a*d+b*c)/d)/d)/d)/d)
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.42, size = 199, normalized size = 1.91

$$-\frac{b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{2\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\,(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3
*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(3,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^2*c^2*d - 2*a
*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(96) = 192.
time = 0.43, size = 209, normalized size = 2.01

$$-\frac{2\,d^2\sin(bx+a)+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)+2\,(bd^2x+bcd)\cos(bx+a)+\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{4\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*sin(b*x + a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*
c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2*(b*d^2*x + b*c*d)*cos(b*x + a
) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d) +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)/d))*sin(-
(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/(d*x+c)**3,x)

[Out] Integral(sin(a + b*x)/(c + d*x)**3, x)

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.78, size = 5727, normalized size = 55.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")

```
[Out] -1/4*(b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*
x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/
2*b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)
^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^2*d^2*x^2*real_part(cos_integral(b*x +
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 +
 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan
(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*imag_
part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^2*d^2*x^2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*d^
2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b^2*d^2
*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2
*b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^
2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*t
an(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 4*b^2*c*d*x*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c
/d) - b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1
/2*b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x
)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*
b*x)^2*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b^2*d^2*x^
2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*
d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + 2*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^
2 + b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^
```

```
2*tan(1/2*b*c/d)^2 - b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*
b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c^2*sin_integral((b*d*x + b*c)
/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*real_part(
cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*d^2*x^2*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 2*b^2*c*d*x*ima
g_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c*d*x
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 4*b^2*
c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*d^2
*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2
*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b
*c/d) + 8*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan
(1/2*a)*tan(1/2*b*c/d) - 8*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*
tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 16*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*r
eal_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*c^
2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*
b*c/d) + 2*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan
(1/2*a)^2*tan(1/2*b*c/d) - 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))
*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(-b*x
- b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(
cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*c^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 -
2*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*t
an(1/2*b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*ta
n(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2
 - b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2 + 2*b^2
*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 4*b^2*c*d*x*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 4*b^2*c*d*x*real_
part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2...
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x)^3,x)
[Out] int(sin(a + b*x)/(c + d*x)^3, x)
```

## 3.8 $\int (c+dx)^4 \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=161

$$\frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5}+\frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b^3}-\frac{(c+dx)^4}{}$$

```
[Out] 3/4*d^4*x/b^4-1/2*d*(d*x+c)^3/b^2+1/10*(d*x+c)^5/d-3/4*d^4*cos(b*x+a)*sin(b
*x+a)/b^5+3/2*d^2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3-1/2*(d*x+c)^4*cos(b*x
+a)*sin(b*x+a)/b-3/2*d^3*(d*x+c)*sin(b*x+a)^2/b^4+d*(d*x+c)^3*sin(b*x+a)^2/
b^2
```

**Rubi** [A]
time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$-\frac{3d^4\sin(a+bx)\cos(a+bx)}{4b^5}-\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b^3}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sin[a + b*x]^2,x]
```

```
[Out] (3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^4*C
os[a + b*x]*Sin[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a +
 b*x])/(2*b^3) - ((c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/(2*b) - (3*d^3*(c
+ d*x)*Sin[a + b*x]^2)/(2*b^4) + (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^4\sin^2(a+bx)\,dx = -\frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}+\frac{1}{2}\int (c+dx)$$

$$=\frac{(c+dx)^5}{10d}+\frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)\sin}{2b}$$

$$=-\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5}+\frac{3d^2(c+dx)^2\cos(a}{2b}$$

$$=\frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5}+\frac{3d^2(c+dx)}{}$$

**Mathematica [A]**
time = 0.36, size = 132, normalized size = 0.82

$$\frac{8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)-20bd(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))-10(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\sin(2(a+bx))}{80b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sin[a + b*x]^2,x]
```

```
[Out] (8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) - 20
*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 10*(3*d^4 -
6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(80*b^5)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. 2(147) = 294.
time = 0.13, size = 1030, normalized size = 6.40

| method | result |
|---|---|
| risch | $\frac{d^4x^5}{10}+\frac{c\,d^3x^4}{2}+d^2c^2x^3+d\,c^3x^2+\frac{c^4x}{2}+\frac{c^5}{10d}-\frac{d(2b^2d^3x^3+6b^2c\,d^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3c\,d^2)\cos}{4b^4}$ |
| norman | $\frac{\frac{2\left(2b^2c^3d-3c\,d^3\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^4}+\frac{d^4x^5\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{10}+\frac{d^4x^5\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5}+\frac{\left(2b^4c^4-6b^2c^2d^2+3d^4\right)\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^5}+\frac{cd(}{}}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^4*a^4*d^4*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-4/b^3*a^3*c*d
^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-4/b^4*a^3*d^4*((b*x+a)*(-1/2*
cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+6/b^2*
a^2*c^2*d^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+12/b^3*a^2*c*d^3*((b
*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+
a)^2)+6/b^4*a^2*d^4*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)
^3)-4/b*a*c^3*d*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-12/b^2*a*c^2*d^2
*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(
b*x+a)^2)-12/b^3*a*c*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2
*a)-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b
*x+a)^3)-4/b^4*a*d^4*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-
3/4*(b*x+a)^2*cos(b*x+a)^2+3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1
/2*a)-3/8*(b*x+a)^2-3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+c^4*(-1/2*cos(b*x+a)*si
n(b*x+a)+1/2*b*x+1/2*a)+4/b*c^3*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+6/b^2*c^2*d^2*((b*x+a)^2*(-1/2*c
os(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)
*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3)+4/b^3*c*d^3*((b*x+a)^3*(-1/2*cos(b
*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/4*(b*x+a)^2*cos(b*x+a)^2+3/2*(b*x+a)*(1/2
*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/8*(b*x+a)^2-3/8*sin(b*x+a)^2-3/8*(b
*x+a)^4)+1/b^4*d^4*((b*x+a)^4*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-(b
*x+a)^3*cos(b*x+a)^2+3*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+
3/2*(b*x+a)*cos(b*x+a)^2-3/4*cos(b*x+a)*sin(b*x+a)-3/4*b*x-3/4*a-(b*x+a)^3-
2/5*(b*x+a)^5))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(147) = 294$.
time = 0.39, size = 735, normalized size = 4.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/40*(10*(2*b*x + 2*a - sin(2*b*x + 2*a))*c^4 - 40*(2*b*x + 2*a - sin(2*b*x
 + 2*a))*a*c^3*d/b + 60*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2*c^2*d^2/b^2 -
40*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^3*c*d^3/b^3 + 10*(2*b*x + 2*a - sin(2
*b*x + 2*a))*a^4*d^4/b^4 + 20*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a)
 - cos(2*b*x + 2*a))*c^3*d/b - 60*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x +
2*a) - cos(2*b*x + 2*a))*a*c^2*d^2/b^2 + 60*(2*(b*x + a)^2 - 2*(b*x + a)*si
n(2*b*x + 2*a) - cos(2*b*x + 2*a))*a^2*c*d^3/b^3 - 20*(2*(b*x + a)^2 - 2*(b
*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a^3*d^4/b^4 + 10*(4*(b*x + a)^
3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*
c^2*d^2/b^2 - 20*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x
```

```
+ a)^2 - 1)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 10*(4*(b*x + a)^3 - 6*(b*x + a)
*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a^2*d^4/b^4 + 1
0*(2*(b*x + a)^4 - 3*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^
3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*c*d^3/b^3 - 10*(2*(b*x + a)^4 - 3*(2*(b*
x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x
+ 2*a))*a*d^4/b^4 + (4*(b*x + a)^5 - 10*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2
*b*x + 2*a) - 5*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*sin(2*b*x + 2*a))*d^4/b
^4)/b
```

**Fricas** [A]
time = 0.39, size = 286, normalized size = 1.78

$$\frac{2 b^5 d^4 x^5 + 10 b^5 c d^3 x^4 + 10 (2 b^5 c^2 d^2 + b^3 d^4) x^3 + 10 (2 b^5 c^3 d + 3 b^3 c d^3) x^2 - 10 (2 b^3 d^4 x^3 + 6 b^3 c d^3 x^2 + 2 b^3 c^3 d - 3 b c d^3 + 3 (2 b^3 c^2 d^2 - b d^4) x) \cos (b x + a)^2 - 5 (2 b^4 d^4 x^4 + 8 b^4 c d^3 x^3 + 2 b^4 c^4 - 6 b^2 c^2 d^2 + 3 d^4 + 6 (2 b^4 c^2 d^2 - b^2 d^4) x^2 + 4 (2 b^4 c^3 d - 3 b^2 c d^3) x) \cos (b x + a) \sin (b x + a) + 5 (2 b^5 c^4 + 6 b^3 c^2 d^2 - 3 b d^4) x}{20 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 10*(2*b^5*c^2*d^2 + b^3*d^4)*x^3 +
 10*(2*b^5*c^3*d + 3*b^3*c*d^3)*x^2 - 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 +
 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 - 5*
(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2
*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)
*sin(b*x + a) + 5*(2*b^5*c^4 + 6*b^3*c^2*d^2 - 3*b*d^4)*x)/b^5
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.
time = 0.55, size = 660, normalized size = 4.10

$$\begin{cases} \frac{c^4 x \sin^2(a+bx)}{2} + \frac{c^4 x \cos^2(a+bx)}{2} + c^3 d x^2 \sin^2(a+bx) + c^3 d x^2 \cos^2(a+bx) + c^2 d^2 x^3 \sin^2(a+bx) + c^2 d^2 x^3 \cos^2(a+bx) + \frac{c d^3 x^4 \sin^2(a+bx)}{2} + \frac{c d^3 x^4 \cos^2(a+bx)}{2} + \frac{d^4 x^5 \sin^2(a+bx)}{10} + \frac{d^4 x^5 \cos^2(a+bx)}{10} - \frac{c^4 \sin(a+bx) \cos(a+bx)}{2b} - \frac{2 c^3 d x \sin(a+bx) \cos(a+bx)}{b} - \frac{3 c^2 d^2 x^2 \sin(a+bx) \cos(a+bx)}{b} - \frac{2 c d^3 x^3 \sin(a+bx) \cos(a+bx)}{b} - \frac{d^4 x^4 \sin(a+bx) \cos(a+bx)}{2b} - \frac{c^3 d \cos^2(a+bx)}{b^2} + \frac{3 c^2 d^2 x \sin^2(a+bx)}{2b^2} - \frac{3 c^2 d^2 x \cos^2(a+bx)}{2b^2} + \frac{3 c d^3 x^2 \sin^2(a+bx)}{2b^2} - \frac{3 c d^3 x^2 \cos^2(a+bx)}{2b^2} + \frac{d^4 x^3 \sin^2(a+bx)}{2b^2} - \frac{d^4 x^3 \cos^2(a+bx)}{2b^2} + \frac{3 c^2 d^2 \sin(a+bx) \cos(a+bx)}{2b^3} + \frac{3 c d^3 x \sin(a+bx) \cos(a+bx)}{b^3} + \frac{3 d^4 x^2 \sin(a+bx) \cos(a+bx)}{2b^3} + \frac{3 c d^3 \cos^2(a+bx)}{2b^4} - \frac{3 d^4 x \sin^2(a+bx)}{4b^4} + \frac{3 d^4 x \cos^2(a+bx)}{4b^4} - \frac{3 d^4 \sin(a+bx) \cos(a+bx)}{4b^5} & \text{for}\: b \neq 0 \\ \left(c^4 x + 2 c^3 d x^2 + 2 c^2 d^2 x^3 + c d^3 x^4 + \frac{d^4 x^5}{5}\right) \sin^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sin(a + b*x)**2/2 + c**4*x*cos(a + b*x)**2/2 + c**3*d*x**
2*sin(a + b*x)**2 + c**3*d*x**2*cos(a + b*x)**2 + c**2*d**2*x**3*sin(a + b*
x)**2 + c**2*d**2*x**3*cos(a + b*x)**2 + c*d**3*x**4*sin(a + b*x)**2/2 + c*
d**3*x**4*cos(a + b*x)**2/2 + d**4*x**5*sin(a + b*x)**2/10 + d**4*x**5*cos(
a + b*x)**2/10 - c**4*sin(a + b*x)*cos(a + b*x)/(2*b) - 2*c**3*d*x*sin(a +
b*x)*cos(a + b*x)/b - 3*c**2*d**2*x**2*sin(a + b*x)*cos(a + b*x)/b - 2*c*d*
*3*x**3*sin(a + b*x)*cos(a + b*x)/b - d**4*x**4*sin(a + b*x)*cos(a + b*x)/(
2*b) - c**3*d*cos(a + b*x)**2/b**2 + 3*c**2*d**2*x*sin(a + b*x)**2/(2*b**2)
 - 3*c**2*d**2*x*cos(a + b*x)**2/(2*b**2) + 3*c*d**3*x**2*sin(a + b*x)**2/(
2*b**2) - 3*c*d**3*x**2*cos(a + b*x)**2/(2*b**2) + d**4*x**3*sin(a + b*x)**
2/(2*b**2) - d**4*x**3*cos(a + b*x)**2/(2*b**2) + 3*c**2*d**2*sin(a + b*x)*
cos(a + b*x)/(2*b**3) + 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)/b**3 + 3*d**4*
x**2*sin(a + b*x)*cos(a + b*x)/(2*b**3) + 3*c*d**3*cos(a + b*x)**2/(2*b**4)
```

```
 - 3*d**4*x*sin(a + b*x)**2/(4*b**4) + 3*d**4*x*cos(a + b*x)**2/(4*b**4) -
3*d**4*sin(a + b*x)*cos(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x
**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2, True))
```

**Giac** [A]
time = 3.69, size = 222, normalized size = 1.38

$$\frac{1}{10}d^4x^5 + \frac{1}{2}cd^3x^4 + c^2d^2x^3 + c^3dx^2 + \frac{1}{2}c^4x - \frac{(2b^3d^4x^3 + 6b^3cd^3x^2 + 6b^3c^2d^2x + 2b^3c^3d - 3bd^4x - 3bcd^3)\cos(2bx + 2a)}{4b^5} - \frac{(2b^4d^4x^4 + 8b^4cd^3x^3 + 12b^4c^2d^2x^2 + 8b^4c^3dx + 2b^4c^4 - 6b^2d^4x^2 - 12b^2cd^3x - 6b^2c^2d^2 + 3d^4)\sin(2bx + 2a)}{8b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x - 1/4*(2
*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*b^3*c^3*d - 3*b*d^4*x
- 3*b*c*d^3)*cos(2*b*x + 2*a)/b^5 - 1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 +
12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d
^3*x - 6*b^2*c^2*d^2 + 3*d^4)*sin(2*b*x + 2*a)/b^5
```

**Mupad** [B]
time = 1.09, size = 349, normalized size = 2.17

$$-\frac{\frac{15d^4\sin(2a+2bx)}{2} - 10b^5c^4x + 5b^4c^4\sin(2a+2bx) - 2b^5d^4x^5 + 10b^3c^3d\cos(2a+2bx) - 20b^5c^3dx^2 - 10b^5cd^3x^4 - 15b^2c^2d^2\sin(2a+2bx) + 10b^3d^4x^3\cos(2a+2bx) - 20b^5c^2d^2x^3 - 15b^2d^4x^2\sin(2a+2bx) + 5b^4d^4x^4\sin(2a+2bx) - 15bcd^3\cos(2a+2bx) - 15bd^4x\cos(2a+2bx) + 30b^4c^2d^2x^2\sin(2a+2bx) - 30b^2cd^3x\sin(2a+2bx) + 20b^4c^3dx\sin(2a+2bx) + 30b^3c^2d^2x\cos(2a+2bx) + 30b^3cd^3x^2\cos(2a+2bx) + 20b^4cd^3x^3\sin(2a+2bx)}{20b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] -((15*d^4*sin(2*a + 2*b*x))/2 - 10*b^5*c^4*x + 5*b^4*c^4*sin(2*a + 2*b*x) -
 2*b^5*d^4*x^5 + 10*b^3*c^3*d*cos(2*a + 2*b*x) - 20*b^5*c^3*d*x^2 - 10*b^5*
c*d^3*x^4 - 15*b^2*c^2*d^2*sin(2*a + 2*b*x) + 10*b^3*d^4*x^3*cos(2*a + 2*b*
x) - 20*b^5*c^2*d^2*x^3 - 15*b^2*d^4*x^2*sin(2*a + 2*b*x) + 5*b^4*d^4*x^4*s
in(2*a + 2*b*x) - 15*b*c*d^3*cos(2*a + 2*b*x) - 15*b*d^4*x*cos(2*a + 2*b*x)
 + 30*b^4*c^2*d^2*x^2*sin(2*a + 2*b*x) - 30*b^2*c*d^3*x*sin(2*a + 2*b*x) +
20*b^4*c^3*d*x*sin(2*a + 2*b*x) + 30*b^3*c^2*d^2*x*cos(2*a + 2*b*x) + 30*b^
3*c*d^3*x^2*cos(2*a + 2*b*x) + 20*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/(20*b^5)
```

# 3.9 $\int (c+dx)^3 \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=134

$$-\frac{3cd^2x}{4b^2}-\frac{3d^3x^2}{8b^2}+\frac{(c+dx)^4}{8d}+\frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b}-\frac{3d^3\,\mathrm{si}}{}$$

```
[Out] -3/4*c*d^2*x/b^2-3/8*d^3*x^2/b^2+1/8*(d*x+c)^4/d+3/4*d^2*(d*x+c)*cos(b*x+a)
*sin(b*x+a)/b^3-1/2*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b-3/8*d^3*sin(b*x+a)^2/
b^4+3/4*d*(d*x+c)^2*sin(b*x+a)^2/b^2
```

**Rubi** [A]
time = 0.05, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3392, 32, 3391}

$$-\frac{3d^3\sin^2(a+bx)}{8b^4}+\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}-\frac{3cd^2x}{4b^2}-\frac{3d^3x^2}{8b^2}+\frac{(c+dx)^4}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]^2,x]
```

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) + (3*d^2*(c
+ d*x)*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x]*Sin[a
 + b*x])/(2*b) - (3*d^3*Sin[a + b*x]^2)/(8*b^4) + (3*d*(c + d*x)^2*Sin[a +
b*x]^2)/(4*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^3 \sin^2(a+bx)\,dx = -\frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2} + \frac{1}{2}\int (c+dx)$$
$$= \frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3} - \frac{(c+dx)^3\cos(a+bx)\sin(a$$
$$= -\frac{3cd^2x}{4b^2} - \frac{3d^3x^2}{8b^2} + \frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3} - \frac{(c+dx}{}$$

**Mathematica** [A]
time = 0.27, size = 106, normalized size = 0.79

$$\frac{2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-3d(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))-2b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\sin(2(a+bx))}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]^2,x]
```

```
[Out] (2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 2*b^2*(c
 + d*x)^2)*Cos[2*(a + b*x)] - 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Si
n[2*(a + b*x)])/(16*b^4)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(120) = 240$.
time = 0.09, size = 587, normalized size = 4.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^3*a^3*d^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/b^2*a^2*c*
d^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/b^3*a^2*d^3*((b*x+a)*(-1/2
*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)-3/b*a
*c^2*d*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-6/b^2*a*c*d^2*((b*x+a)*(-
1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)-3/
b^3*a*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(b*x+a)
*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3)+c^3*(-
1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/b*c^2*d*((b*x+a)*(-1/2*cos(b*x+a
)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+3/b^2*c*d^2*((b
*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(b*x+a)*cos(b*x+a)^2
+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3)+1/b^3*d^3*((b*x+a)^
3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/4*(b*x+a)^2*cos(b*x+a)^2+3/2
```

```
*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/8*(b*x+a)^2-3/8*sin(b*
x+a)^2-3/8*(b*x+a)^4))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(120) = 240$.
time = 0.33, size = 442, normalized size = 3.30

$$\frac{4\left(2\,bx+2\,a-\sin\left(2\,bx+2\,a\right)\right)c^{3}-\frac{12\left(2\,bx+2\,a-\sin\left(2\,bx+2\,a\right)\right)ac^{2}d}{b}+\frac{12\left(2\,bx+2\,a-\sin\left(2\,bx+2\,a\right)\right)a^{2}cd^{2}}{b^{2}}-\frac{4\left(2\,bx+2\,a-\sin\left(2\,bx+2\,a\right)\right)a^{3}d^{3}}{b^{3}}+\frac{6\left(2\left(bx+a\right)^{2}-2\left(bx+a\right)\sin\left(2\,bx+2\,a\right)-\cos\left(2\,bx+2\,a\right)\right)c^{2}d}{b}-\frac{12\left(2\left(bx+a\right)^{2}-2\left(bx+a\right)\sin\left(2\,bx+2\,a\right)-\cos\left(2\,bx+2\,a\right)\right)acd^{2}}{b^{2}}+\frac{6\left(2\left(bx+a\right)^{2}-2\left(bx+a\right)\sin\left(2\,bx+2\,a\right)-\cos\left(2\,bx+2\,a\right)\right)a^{2}d^{3}}{b^{3}}+\frac{2\left(4\left(bx+a\right)^{3}-6\left(bx+a\right)\cos\left(2\,bx+2\,a\right)-3\left(2\left(bx+a\right)^{2}-1\right)\sin\left(2\,bx+2\,a\right)\right)cd^{2}}{b^{2}}-\frac{2\left(4\left(bx+a\right)^{3}-6\left(bx+a\right)\cos\left(2\,bx+2\,a\right)-3\left(2\left(bx+a\right)^{2}-1\right)\sin\left(2\,bx+2\,a\right)\right)ad^{3}}{b^{3}}+\frac{\left(2\left(bx+a\right)^{4}-3\left(2\left(bx+a\right)^{2}-1\right)\cos\left(2\,bx+2\,a\right)-2\left(2\left(bx+a\right)^{3}-3\,bx-3\,a\right)\sin\left(2\,bx+2\,a\right)\right)d^{3}}{b^{3}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*b*x + 2*a - sin(2*b*x + 2*a))*c^3 - 12*(2*b*x + 2*a - sin(2*b*x
+ 2*a))*a*c^2*d/b + 12*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2*c*d^2/b^2 - 4*(
2*b*x + 2*a - sin(2*b*x + 2*a))*a^3*d^3/b^3 + 6*(2*(b*x + a)^2 - 2*(b*x + a
)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)^2 - 2*(b*x
 + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)^2 -
 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a^2*d^3/b^3 + 2*(4*(b*x +
 a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*
a))*c*d^2/b^2 - 2*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x
 + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(b*x + a)^4 - 3*(2*(b*x + a)^
2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))
*d^3/b^3)/b
```

**Fricas** [A]
time = 0.48, size = 189, normalized size = 1.41

$$\frac{b^{4}d^{3}x^{4}+4\,b^{4}cd^{2}x^{3}+3\left(2\,b^{4}c^{2}d+b^{2}d^{3}\right)x^{2}-3\left(2\,b^{2}d^{3}x^{2}+4\,b^{2}cd^{2}x+2\,b^{2}c^{2}d-d^{3}\right)\cos\left(bx+a\right)^{2}-2\left(2\,b^{3}d^{3}x^{3}+6\,b^{3}cd^{2}x^{2}+2\,b^{3}c^{3}-3\,bcd^{2}+3\left(2\,b^{3}c^{2}d-bd^{3}\right)x\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+2\left(2\,b^{4}c^{3}+3\,b^{2}cd^{2}\right)x}{8\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 3*(2*b^4*c^2*d + b^2*d^3)*x^2 - 3*(2*b
^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 2*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*
x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3 + 3*b^2*c*d^2)*x)/b^4
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(131) = 262$.
time = 0.46, size = 456, normalized size = 3.40

$$\begin{cases} \frac{c^{3}x\sin^{2}\left(a+bx\right)}{2}+\frac{c^{3}x\cos^{2}\left(a+bx\right)}{2}+\frac{3c^{2}dx^{2}\sin^{2}\left(a+bx\right)}{4}+\frac{3c^{2}dx^{2}\cos^{2}\left(a+bx\right)}{4}+\frac{cd^{2}x^{3}\sin^{2}\left(a+bx\right)}{2}+\frac{cd^{2}x^{3}\cos^{2}\left(a+bx\right)}{2}+\frac{d^{3}x^{4}\sin^{2}\left(a+bx\right)}{8}+\frac{d^{3}x^{4}\cos^{2}\left(a+bx\right)}{8}-\frac{c^{3}\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b}-\frac{3c^{2}dx\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b}-\frac{3cd^{2}x^{2}\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b}-\frac{d^{3}x^{3}\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b}-\frac{3c^{2}d\cos^{2}\left(a+bx\right)}{4b^{2}}+\frac{3cd^{2}x\sin^{2}\left(a+bx\right)}{4b^{2}}-\frac{3cd^{2}x\cos^{2}\left(a+bx\right)}{4b^{2}}+\frac{3d^{3}x^{2}\sin^{2}\left(a+bx\right)}{8b^{2}}-\frac{3d^{3}x^{2}\cos^{2}\left(a+bx\right)}{8b^{2}}+\frac{3cd^{2}\sin\left(a+bx\right)\cos\left(a+bx\right)}{4b^{3}}+\frac{3d^{3}x\sin\left(a+bx\right)\cos\left(a+bx\right)}{4b^{3}}+\frac{3d^{3}\cos^{2}\left(a+bx\right)}{8b^{4}} & \text{for}\: b \neq 0 \\ \left(c^{3}x+\frac{3c^{2}dx^{2}}{2}+cd^{2}x^{3}+\frac{d^{3}x^{4}}{4}\right)\sin^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sin(a + b*x)**2/2 + c**3*x*cos(a + b*x)**2/2 + 3*c**2*d*x
**2*sin(a + b*x)**2/4 + 3*c**2*d*x**2*cos(a + b*x)**2/4 + c*d**2*x**3*sin(a
 + b*x)**2/2 + c*d**2*x**3*cos(a + b*x)**2/2 + d**3*x**4*sin(a + b*x)**2/8
+ d**3*x**4*cos(a + b*x)**2/8 - c**3*sin(a + b*x)*cos(a + b*x)/(2*b) - 3*c*
*2*d*x*sin(a + b*x)*cos(a + b*x)/(2*b) - 3*c*d**2*x**2*sin(a + b*x)*cos(a +
 b*x)/(2*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)/(2*b) - 3*c**2*d*cos(a +
b*x)**2/(4*b**2) + 3*c*d**2*x*sin(a + b*x)**2/(4*b**2) - 3*c*d**2*x*cos(a +
 b*x)**2/(4*b**2) + 3*d**3*x**2*sin(a + b*x)**2/(8*b**2) - 3*d**3*x**2*cos(
a + b*x)**2/(8*b**2) + 3*c*d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3) + 3*d**3
*x*sin(a + b*x)*cos(a + b*x)/(4*b**3) + 3*d**3*cos(a + b*x)**2/(8*b**4), Ne
(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2,
 True))
```

**Giac** [A]
time = 3.75, size = 153, normalized size = 1.14

$$\frac{1}{8}d^3x^4+\frac{1}{2}cd^2x^3+\frac{3}{4}c^2dx^2+\frac{1}{2}c^3x-\frac{3\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\cos\left(2\,bx+2\,a\right)}{16\,b^4}-\frac{(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-3\,bd^3x-3\,bcd^2)\sin\left(2\,bx+2\,a\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x - 3/16*(2*b^2*d^3*x
^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(2*b*x + 2*a)/b^4 - 1/8*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^
2)*sin(2*b*x + 2*a)/b^4
```

**Mupad** [B]
time = 0.85, size = 229, normalized size = 1.71

$$\frac{\frac{3\,d^3\cos\left(2\,a+2\,b\,x\right)}{2}+4\,b^4\,c^3\,x-2\,b^3\,c^3\,\sin\left(2\,a+2\,b\,x\right)+b^4\,d^3\,x^4-3\,b^2\,c^2\,d\,\cos\left(2\,a+2\,b\,x\right)+6\,b^4\,c^2\,d\,x^2+4\,b^4\,c\,d^2\,x^3-3\,b^2\,d^3\,x^2\,\cos\left(2\,a+2\,b\,x\right)-2\,b^3\,d^3\,x^3\,\sin\left(2\,a+2\,b\,x\right)+3\,b\,c\,d^2\,\sin\left(2\,a+2\,b\,x\right)+3\,b\,d^3\,x\,\sin\left(2\,a+2\,b\,x\right)-6\,b^2\,c\,d^2\,x\,\cos\left(2\,a+2\,b\,x\right)-6\,b^3\,c^2\,d\,x\,\sin\left(2\,a+2\,b\,x\right)-6\,b^3\,c\,d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] ((3*d^3*cos(2*a + 2*b*x))/2 + 4*b^4*c^3*x - 2*b^3*c^3*sin(2*a + 2*b*x) + b^
4*d^3*x^4 - 3*b^2*c^2*d*cos(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*d^2*x^
3 - 3*b^2*d^3*x^2*cos(2*a + 2*b*x) - 2*b^3*d^3*x^3*sin(2*a + 2*b*x) + 3*b*c
*d^2*sin(2*a + 2*b*x) + 3*b*d^3*x*sin(2*a + 2*b*x) - 6*b^2*c*d^2*x*cos(2*a
+ 2*b*x) - 6*b^3*c^2*d*x*sin(2*a + 2*b*x) - 6*b^3*c*d^2*x^2*sin(2*a + 2*b*x
))/(8*b^4)
```

# 3.10 $\int (c+dx)^2 \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=95

$$-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}+\frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}$$

```
[Out] -1/4*d^2*x/b^2+1/6*(d*x+c)^3/d+1/4*d^2*cos(b*x+a)*sin(b*x+a)/b^3-1/2*(d*x+c
)^2*cos(b*x+a)*sin(b*x+a)/b+1/2*d*(d*x+c)*sin(b*x+a)^2/b^2
```

**Rubi** [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$\frac{d^2\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]^2,x]
```

```
[Out] -1/4*(d^2*x)/b^2 + (c + d*x)^3/(6*d) + (d^2*Cos[a + b*x]*Sin[a + b*x])/(4*b
^3) - ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a +
b*x]^2)/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^2\sin^2(a+bx)\,dx = -\frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{1}{2}\int (c+dx)^2\,d$$
$$=\frac{(c+dx)^3}{6d}+\frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}+$$
$$=-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}+\frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{(c+dx)^2\cos(a+bx)\sin(a}{2b}$$

**Mathematica** [A]
time = 0.19, size = 77, normalized size = 0.81

$$\frac{4b^3x(3c^2+3cdx+d^2x^2)-6bd(c+dx)\cos(2(a+bx))-3(-d^2+2b^2(c+dx)^2)\sin(2(a+bx))}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]^2,x]
[Out] (4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cos[2*(a + b*x)] - 3
*(-d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(24*b^3)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(85) = 170$.
time = 0.06, size = 289, normalized size = 3.04

| method | result |
|---|---|
| risch | $\frac{d^2x^3}{6}+\frac{cd\,x^2}{2}+\frac{c^2x}{2}+\frac{c^3}{6d}-\frac{d(dx+c)\cos(2bx+2a)}{4b^2}-\frac{(2d^2x^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2bx+2a)}{8b^3}$ |
| derivativedivides | $\frac{\frac{a^2d^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\right.}{b^2}}{}$ |
| default | $\frac{\frac{a^2d^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\right.}{b^2}}{}$ |
| norman | $\frac{cd\,x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{d^2x^2\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{d^2x^3}{6}+\frac{cd\,x^2}{2}+\frac{d^2x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{d^2x^3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6}-\frac{(2b^2c^2-d^2)\tan\left(\frac{bx}{2}\right.}{2b^3}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^2*a^2*d^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2/b*a*c*d*(-1
/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2/b^2*a*d^2*((b*x+a)*(-1/2*cos(b*x+
a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+c^2*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+2/b*c*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+
a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+1/b^2*d^2*((b*x+a)^2*(-1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x
+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3))
```

**Maxima** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(85) = 170$.
time = 0.31, size = 232, normalized size = 2.44

$$\frac{6\,(2\,bx+2\,a-\sin\left(2\,bx+2\,a\right))c^2-\frac{12\,(2\,bx+2\,a-\sin(2\,bx+2\,a))acd}{b}+\frac{6\,(2\,bx+2\,a-\sin(2\,bx+2\,a))a^2d^2}{b^2}+\frac{6\left(2\,(bx+a)^2-2\,(bx+a)\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)\right)cd}{b}-\frac{6\left(2\,(bx+a)^2-2\,(bx+a)\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)\right)ad^2}{b^2}+\frac{\left(4\,(bx+a)^3-6\,(bx+a)\cos(2\,bx+2\,a)-3\left(2\,(bx+a)^2-1\right)\sin(2\,bx+2\,a)\right)d^2}{b^2}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*b*x + 2*a - sin(2*b*x + 2*a))*c^2 - 12*(2*b*x + 2*a - sin(2*b*x
+ 2*a))*a*c*d/b + 6*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2*d^2/b^2 + 6*(2*(b*
x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*c*d/b - 6*(2*(b
*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a*d^2/b^2 + (4
*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b
*x + 2*a))*d^2/b^2)/b
```

**Fricas** [**A**]
time = 0.46, size = 112, normalized size = 1.18

$$\frac{2\,b^3d^2x^3+6\,b^3cdx^2-6\,(bd^2x+bcd)\cos\left(bx+a\right)^2-3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos\left(bx+a\right)\sin\left(bx+a\right)+3\,(2\,b^3c^2+bd^2)x}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 - 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 -
3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a)
 + 3*(2*b^3*c^2 + b*d^2)*x)/b^3
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(85) = 170$.
time = 0.25, size = 264, normalized size = 2.78

$$\begin{cases} \frac{c^2x\sin^2\left(a+bx\right)}{2}+\frac{c^2x\cos^2\left(a+bx\right)}{2}+\frac{cdx^2\sin^2\left(a+bx\right)}{2}+\frac{cdx^2\cos^2\left(a+bx\right)}{2}+\frac{d^2x^3\sin^2\left(a+bx\right)}{6}+\frac{d^2x^3\cos^2\left(a+bx\right)}{6}-\frac{c^2\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b}-\frac{cdx\sin\left(a+bx\right)\cos\left(a+bx\right)}{b}-\frac{d^2x^2\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b}-\frac{cd\cos^2\left(a+bx\right)}{2b^2}+\frac{d^2x\sin^2\left(a+bx\right)}{4b^2}-\frac{d^2x\cos^2\left(a+bx\right)}{4b^2}+\frac{d^2\sin\left(a+bx\right)\cos\left(a+bx\right)}{4b^3} & \text{for}\; b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sin(a + b*x)**2/2 + c**2*x*cos(a + b*x)**2/2 + c*d*x**2*s
in(a + b*x)**2/2 + c*d*x**2*cos(a + b*x)**2/2 + d**2*x**3*sin(a + b*x)**2/6
 + d**2*x**3*cos(a + b*x)**2/6 - c**2*sin(a + b*x)*cos(a + b*x)/(2*b) - c*d
*x*sin(a + b*x)*cos(a + b*x)/b - d**2*x**2*sin(a + b*x)*cos(a + b*x)/(2*b)
- c*d*cos(a + b*x)**2/(2*b**2) + d**2*x*sin(a + b*x)**2/(4*b**2) - d**2*x*c
os(a + b*x)**2/(4*b**2) + d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)
), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2, True))
```

**Giac [A]**
time = 4.00, size = 94, normalized size = 0.99

$$\frac{1}{6}\,d^2x^3+\frac{1}{2}\,cdx^2+\frac{1}{2}\,c^2x-\frac{(bd^2x+bcd)\cos\left(2\,bx+2\,a\right)}{4\,b^3}-\frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\sin\left(2\,bx+2\,a\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x - 1/4*(b*d^2*x + b*c*d)*cos(2*b*x + 2
*a)/b^3 - 1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*sin(2*b*x + 2
*a)/b^3
```

**Mupad [B]**
time = 0.20, size = 179, normalized size = 1.88

$$x\left(\frac{c^2}{4}-\frac{d^2}{8\,b^2}\right)+x\left(\frac{c^2}{4}+\frac{d^2}{8\,b^2}\right)+\frac{d^2\,x^3}{6}+\frac{\sin\left(2\,a+2\,b\,x\right)\,\left(d^2-2\,b^2\,c^2\right)}{8\,b^3}+\frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{c^2}{2}-\frac{d^2}{4\,b^2}\right)}{2}-\frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{c^2}{2}+\frac{d^2}{4\,b^2}\right)}{2}+\frac{c\,d\,x^2}{2}-\frac{d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{4\,b}-\frac{c\,d\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{c\,d\,x\,\sin\left(2\,a+2\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x*(c^2/4 - d^2/(8*b^2)) + x*(c^2/4 + d^2/(8*b^2)) + (d^2*x^3)/6 + (sin(2*a
+ 2*b*x)*(d^2 - 2*b^2*c^2))/(8*b^3) + (x*cos(2*a + 2*b*x)*(c^2/2 - d^2/(4*b
^2)))/2 - (x*cos(2*a + 2*b*x)*(c^2/2 + d^2/(4*b^2)))/2 + (c*d*x^2)/2 - (d^2
*x^2*sin(2*a + 2*b*x))/(4*b) - (c*d*cos(2*a + 2*b*x))/(4*b^2) - (c*d*x*sin(
2*a + 2*b*x))/(2*b)
```

# 3.11 $\int (c+dx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=55

$$\frac{cx}{2}+\frac{dx^2}{4}-\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}+\frac{d\sin^2(a+bx)}{4b^2}$$

```
[Out] 1/2*c*x+1/4*d*x^2-1/2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b+1/4*d*sin(b*x+a)^2/b^
2
```

**Rubi** [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3391}

$$\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]^2,x]
```

```
[Out] (c*x)/2 + (d*x^2)/4 - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*Sin[
a + b*x]^2)/(4*b^2)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sin^2(a+bx)\,dx &= -\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}+\frac{d\sin^2(a+bx)}{4b^2}+\frac{1}{2}\int (c+dx)\,dx\\ &= \frac{cx}{2}+\frac{dx^2}{4}-\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}+\frac{d\sin^2(a+bx)}{4b^2}\end{aligned}$$

**Mathematica** [A]
time = 0.12, size = 52, normalized size = 0.95

$$\frac{-d\cos(2(a+bx))+2b(2ac+bx(2c+dx)-(c+dx)\sin(2(a+bx)))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]^2,x]
```

```
[Out] (-(d*Cos[2*(a + b*x)]) + 2*b*(2*a*c + b*x*(2*c + d*x) - (c + d*x)*Sin[2*(a
+ b*x)]))/(8*b^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.
time = 0.04, size = 112, normalized size = 2.04

| method | result |
|---|---|
| risch | $\frac{d\,x^2}{4}+\frac{cx}{2}-\frac{d\cos(2bx+2a)}{8b^2}-\frac{(dx+c)\sin(2bx+2a)}{4b}$ |
| derivativedivides | $\frac{-\frac{da\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{d\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)}{4}\right.}{b}}{b}$ |
| default | $\frac{-\frac{da\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{d\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)}{4}\right.}{b}}{b}$ |
| norman | $\frac{\frac{c\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+cx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{dx\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{cx}{2}+\frac{d\,x^2}{4}-\frac{c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{cx\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+c*(-1/2*cos(b*x+a)
*sin(b*x+a)+1/2*b*x+1/2*a)+1/b*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b
*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(47) = 94$.
time = 0.29, size = 96, normalized size = 1.75

$$\frac{2\,(2\,bx+2\,a-\sin\left(2\,bx+2\,a\right))c-\frac{2\,(2\,bx+2\,a-\sin(2\,bx+2\,a))ad}{b}+\frac{\left(2\,(bx+a)^2-2\,(bx+a)\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)\right)d}{b}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*b*x + 2*a - sin(2*b*x + 2*a))*c - 2*(2*b*x + 2*a - sin(2*b*x + 2*
a))*a*d/b + (2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a
))*d/b)/b
```

**Fricas** [A]
time = 0.41, size = 54, normalized size = 0.98

$$\frac{b^2dx^2+2\,b^2cx-d\cos\left(bx+a\right)^2-2\,(bdx+bc)\cos\left(bx+a\right)\sin\left(bx+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^2 + 2*b^2*c*x - d*cos(b*x + a)^2 - 2*(b*d*x + b*c)*cos(b*x + a
)*sin(b*x + a))/b^2
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(49) = 98$.
time = 0.13, size = 126, normalized size = 2.29

$$\begin{cases} \frac{cx\sin^2(a+bx)}{2} + \frac{cx\cos^2(a+bx)}{2} + \frac{dx^2\sin^2(a+bx)}{4} + \frac{dx^2\cos^2(a+bx)}{4} - \frac{c\sin(a+bx)\cos(a+bx)}{2b} - \frac{dx\sin(a+bx)\cos(a+bx)}{2b} - \frac{d\cos^2(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sin(a + b*x)**2/2 + c*x*cos(a + b*x)**2/2 + d*x**2*sin(a + b
*x)**2/4 + d*x**2*cos(a + b*x)**2/4 - c*sin(a + b*x)*cos(a + b*x)/(2*b) - d
*x*sin(a + b*x)*cos(a + b*x)/(2*b) - d*cos(a + b*x)**2/(4*b**2), Ne(b, 0)),
 ((c*x + d*x**2/2)*sin(a)**2, True))
```

**Giac** [A]
time = 3.39, size = 48, normalized size = 0.87

$$\frac{1}{4}dx^2 + \frac{1}{2}cx - \frac{d\cos(2bx + 2a)}{8b^2} - \frac{(bdx + bc)\sin(2bx + 2a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*x^2 + 1/2*c*x - 1/8*d*cos(2*b*x + 2*a)/b^2 - 1/4*(b*d*x + b*c)*sin(2*
b*x + 2*a)/b^2
```

**Mupad** [B]
time = 0.09, size = 57, normalized size = 1.04

$$\frac{c\,x}{2} + \frac{d\,x^2}{4} - \frac{d\,\cos(2\,a + 2\,b\,x)}{8\,b^2} - \frac{c\,\sin(2\,a + 2\,b\,x)}{4\,b} - \frac{d\,x\,\sin(2\,a + 2\,b\,x)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(c + d*x),x)
```

```
[Out] (c*x)/2 + (d*x^2)/4 - (d*cos(2*a + 2*b*x))/(8*b^2) - (c*sin(2*a + 2*b*x))/(
4*b) - (d*x*sin(2*a + 2*b*x))/(4*b)
```

# 3.12 $\int \frac{\sin^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=78

$$-\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] -1/2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d+1/2*ln(d*x+c)/d+1/2*Si(2*b*c/d+2*
b*x)*sin(2*a-2*b*c/d)/d
```

**Rubi** [A]
time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3384, 3380, 3383}

$$-\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x),x]
```

```
[Out] -1/2*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]
/(2*d) + (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{1}{2(c+dx)}-\frac{\cos(2a+2bx)}{2(c+dx)}\right)dx\\
&= \frac{\log(c+dx)}{2d}-\frac{1}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx\\
&= \frac{\log(c+dx)}{2d}-\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}\right.}{c+}\\
&= -\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}
\end{aligned}$$

**Mathematica [A]**
time = 0.07, size = 65, normalized size = 0.83

$$\frac{-\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)+\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x),x]
```

```
[Out] (-(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d]) + Log[c + d*x] + Si
n[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d)
```

**Maple [A]**
time = 0.05, size = 114, normalized size = 1.46

| method | result |
|---|---|
| risch | $\frac{\mathrm{e}^{-\frac{2i(da-cb)}{d}}\,\text{expIntegral}\left(1,2ibx+2ia-\frac{2i(da-cb)}{d}\right)}{4d}+\frac{\mathrm{e}^{\frac{2i(da-cb)}{d}}\,\text{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{4d}+\frac{\ln(dx+}{2d}$ |
| derivativedivides | $\frac{\frac{b\ln(-da+cb+d(bx+a))}{2d}-\frac{b\left(-\frac{2\,\text{sinIntegral}\left(-2bx-2a-\frac{2(-da+cb)}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{2\,\text{cosineIntegral}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-}{}\right.}{d}\right.}{4}}{b}$ |
| default | $\frac{\frac{b\ln(-da+cb+d(bx+a))}{2d}-\frac{b\left(-\frac{2\,\text{sinIntegral}\left(-2bx-2a-\frac{2(-da+cb)}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{2\,\text{cosineIntegral}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-}{}\right.}{d}\right.}{4}}{b}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*b*ln(-d*a+c*b+d*(b*x+a))/d-1/4*b*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)
*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d
))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.35, size = 162, normalized size = 2.08

$$\frac{b\left(E_1\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_1\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b\left(iE_1\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)-iE_1\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+2b\log\left(bc+(bx+a)d-ad\right)}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integ
ral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) + b*
(I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral
_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) + 2*b*l
og(b*c + (b*x + a)*d - a*d))/(b*d)
```

**Fricas** [A]
time = 0.39, size = 88, normalized size = 1.13

$$-\frac{\left(\operatorname{Ci}\left(\frac{2(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-2\sin\left(-\frac{2(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2(bdx+bc)}{d}\right)-2\log\left(dx+c\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/4*((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*
cos(-2*(b*c - a*d)/d) - 2*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c
)/d) - 2*log(d*x + c))/d
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x), x)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.24, size = 612, normalized size = 7.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(d*x + c))*tan(a)^2*tan(b*c/d)^2 - real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)^2*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*ta
n(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) - 2*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)*tan(b*c/d)^2 + 2*log(abs(d*x + c))*tan(a)^2 + real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)^2 + real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 2*log(ab
s(d*x + c))*tan(b*c/d)^2 + real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c
/d)^2 + real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 2*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(a) - 2*imag_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(a) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a) - 2*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 2*
log(abs(d*x + c)) - real_part(cos_integral(2*b*x + 2*b*c/d)) - real_part(co
s_integral(-2*b*x - 2*b*c/d)))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*ta
n(b*c/d)^2 + d)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin{(a+b\,x)}^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(c + d*x),x)
```

```
[Out] int(sin(a + b*x)^2/(c + d*x), x)
```

# 3.13 $\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=81

$$\frac{b\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2}-\frac{\sin^2(a+bx)}{d(c+dx)}+\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

```
[Out] b*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2+b*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/
d)/d^2-sin(b*x+a)^2/d/(d*x+c)
```

**Rubi** [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3394, 12, 3384, 3380, 3383}

$$\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] (b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d^2 - Sin[a + b*x]^
2/(d*(c + d*x)) + (b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx &= -\frac{\sin^2(a+bx)}{d(c+dx)}+\frac{(2b)\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d}\\ &= -\frac{\sin^2(a+bx)}{d(c+dx)}+\frac{b\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d}\\ &= -\frac{\sin^2(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}}{d}\\ &= \frac{b\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2}-\frac{\sin^2(a+bx)}{d(c+dx)}+\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}\end{aligned}$$

**Mathematica [A]**
time = 0.28, size = 75, normalized size = 0.93

$$\frac{b\text{Ci}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)-\frac{d\sin^2(a+bx)}{c+dx}+b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] (b*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - (d*Sin[a + b*x]^2)
/(c + d*x) + b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^2
```

**Maple [A]**
time = 0.08, size = 156, normalized size = 1.93

| method | result |
|---|---|
| risch | $-\frac{ib\,\mathrm{e}^{-\frac{2i(da-cb)}{d}}\,\text{expIntegral}\left(1,2ibx+2ia-\frac{2i(da-cb)}{d}\right)}{2d^2}+\frac{ib\,\mathrm{e}^{\frac{2i(da-cb)}{d}}\,\text{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^2}-$ |

| | |
|---|---|
| derivativedivides | $\frac{-\frac{b^2}{2(-da+cb+d(bx+a))d}-\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-da+cb+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-da+cb)}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}}{}\right)}{d}\right)}{4}}{b}$ |
| default | $\frac{-\frac{b^2}{2(-da+cb+d(bx+a))d}-\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-da+cb+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-da+cb)}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}}{}\right)}{d}\right)}{4}}{b}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*b^2/(-d*a+c*b+d*(b*x+a))/d-1/4*b^2*(-2*cos(2*b*x+2*a)/(-d*a+c*b+d
*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(
2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.36, size = 171, normalized size = 2.11

$$\frac{b^2\left(E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,b^2}{4\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) +
b^2*(I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_inte
gral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) - 2
*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

**Fricas** [A]
time = 0.44, size = 130, normalized size = 1.60

$$\frac{2\,d\cos\left(bx+a\right)^2+2\,(bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,d}{2\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integra
l(2*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*
d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - 2*d)/(
d^3*x + c*d^2)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**2, x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(81) = 162.
time = 2.87, size = 535, normalized size = 6.60

$$\frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\operatorname{Ci}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\,b^3c\operatorname{Ci}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,ab^2d\operatorname{Ci}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)-2\,b^3c\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)+2\,ab^2d\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)+b^2d\cos\left(-\frac{2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)-b^2d\right)d^2}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(2*((d
*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c - a
*d)/d) + 2*b^3*c*cos_integral(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
c)) + b*c - a*d)/d)*sin(-2*(b*c - a*d)/d) - 2*a*b^2*d*cos_integral(2*((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c - a*d)
/d) - 2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-2*(b*c - a*d
)/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c -
 a*d)/d) - 2*b^3*c*cos(-2*(b*c - a*d)/d)*sin_integral(-2*((d*x + c)*(b - b*
c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 2*a*b^2*d*cos(-2*(b*c - a*d)
/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c -
a*d)/d) + b^2*d*cos(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) - b
^2*d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a
*d^5)*b)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] int(sin(a + b*x)^2/(c + d*x)^2, x)
```

# 3.14 $\int \frac{\sin^2(a+bx)}{(c+dx)^3} dx$

**Optimal**. Leaf size=113

$$\frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{Ci}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3}$$

```
[Out] b^2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^3-b^2*Si(2*b*c/d+2*b*x)*sin(2*a-2*
b*c/d)/d^3-b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)-1/2*sin(b*x+a)^2/d/(d*x+c)^2
```

**Rubi** [A]
time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3395, 31, 3393, 3384, 3380, 3383}

$$\frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^3 - (b*Cos[a +
b*x]*Sin[a + b*x])/(d^2*(c + d*x)) - Sin[a + b*x]^2/(2*d*(c + d*x)^2) - (b^
2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx = -\frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\int \frac{1}{c+dx}\,dx}{d^2} - \frac{(2b^2)\int \frac{\sin^2(a+bx)}{c+dx}\,dx}{d^2}$$

$$= \frac{b^2\log(c+dx)}{d^3} - \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{(2b^2)\int \left(\frac{1}{2(c+dx)} - \frac{\cos(2a+}{2(c+d}\right.}{d^2}$$

$$= -\frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} - \frac{(b}{}$$

$$= \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\sin(2}{}$$

**Mathematica** [A]
time = 0.78, size = 101, normalized size = 0.89

$$-\frac{-2b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2b(c+dx)}{d}\right)+\frac{d\left(d\sin^2(a+bx)+b(c+dx)\sin(2(a+bx))\right)}{(c+dx)^2}+2b^2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] -1/2*(-2*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d*Si
n[a + b*x]^2 + b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sin[2*a -
 (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

**Maple [A]**
time = 0.12, size = 193, normalized size = 1.71

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{b^3}{4(-da+cb+d(bx+a))^2 d}-\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-da+cb+d(bx+a))^2 d}-\frac{-\frac{2\sin(2bx+2a)}{(-da+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-da+cb)}{d}\right)\sin\left(\frac{-2d}{}\right.}{d}}{d}}{}\right.}{4}}{b}$ |
| default | $\frac{-\frac{b^3}{4(-da+cb+d(bx+a))^2 d}-\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-da+cb+d(bx+a))^2 d}-\frac{-\frac{2\sin(2bx+2a)}{(-da+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-da+cb)}{d}\right)\sin\left(\frac{-2d}{}\right.}{d}}{d}}{}\right.}{4}}{b}$ |
| risch | $-\frac{b^2 e^{-\frac{2i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(da-cb)}{d}\right)}{2d^3}-\frac{b^2 e^{\frac{2i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^3}-\frac{}{4d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4*b^3/(-d*a+c*b+d*(b*x+a))^2/d-1/4*b^3*(-cos(2*b*x+2*a)/(-d*a+c*b+d
*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/(-d*a+c*b+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a
-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2
*(-a*d+b*c)/d)/d)/d)/d))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.41, size = 206, normalized size = 1.82

$$\frac{b^3\left(E_3\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_3\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(i\,E_3\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)-i\,E_3\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-b^3}{4\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\,(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) +
b^3*(I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_inte
gral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) - b
^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*b)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(111) = 222$.
time = 0.50, size = 223, normalized size = 1.97

$$\frac{d^2\cos\left(bx+a\right)^2 - 2\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right) - 2\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right) - d^2 + \left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right) + \left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - 2
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c - a*d)/d)*sin_integral(2
*(b*d*x + b*c)/d) - d^2 + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(
-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**3, x)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.29, size = 5141, normalized size = 45.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)
^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*si
n_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^
2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 -
 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*
tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan
(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-2*b*x -
```

```
 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b^2*d^2*x^2*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*real_part(cos_
integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b^2*d^2*x^2*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^2
*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c
/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*ta
n(a)^2*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(b*x)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^2*c*d*x*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*d^2*x^2*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^2*c^2
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2
 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)
 + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a
) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^2
*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^2
*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 2*b^
2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/
d) + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) +
8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(
b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*t
an(a)*tan(b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a
)^2*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x
)^2*tan(a)^2*tan(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/
d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*c*d*x*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b^2*c^2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*sin
_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*
```

```
x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b^2*d^2*x^2*real_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b^2*c*d*x*imag_part(cos
_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^2*c*d*x*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^2*c*d*x*sin_integral(2*(
b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b^2*d^2*x^2*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(a)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)^2 - b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*
tan(a)^2 - b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(a)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*
tan(b*c/d) + 8*b^2*c*d*x*sin_integral(2*(b*d*x ...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(c + d*x)^3,x)
[Out] int(sin(a + b*x)^2/(c + d*x)^3, x)
```

# 3.15 $\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=162

$$-\frac{b^2}{3d^3(c+dx)}-\frac{2b^3\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^4}-\frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}+\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}-$$

```
[Out] -1/3*b^2/d^3/(d*x+c)-2/3*b^3*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4-2/3*b^3
*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/3*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*
x+c)^2-1/3*sin(b*x+a)^2/d/(d*x+c)^3+2/3*b^2*sin(b*x+a)^2/d^3/(d*x+c)
```

**Rubi** [A]
time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3395, 32, 3394, 12, 3384, 3380, 3383}

$$-\frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] -1/3*b^2/(d^3*(c + d*x)) - (2*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a -
(2*b*c)/d])/(3*d^4) - (b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^2) - S
in[a + b*x]^2/(3*d*(c + d*x)^3) + (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c + d*x))
- (2*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{b^2\int \frac{1}{(c+dx)^2}\,dx}{3d^2} - \frac{(2b^2)\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx}{3d^2}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)} - \frac{(4b^3)}{}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)} - \frac{(2b^3)}{}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)} - \frac{(2b^3 \mathrm{c}}{}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{2b^3\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^4} - \frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a}{3d(c+}$$

**Mathematica [A]**
time = 0.80, size = 122, normalized size = 0.75

$$-\frac{4b^3\text{Ci}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{d\left(\left(-d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+d(d+b(c+dx)\sin(2(a+bx)))\right)}{(c+dx)^3}+4b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] -1/6*(4*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-d^2
 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(d + b*(c + d*x)*Sin[2*(a + b*x)
])))/(c + d*x)^3 + 4*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d
])/d^4
```

**Maple [A]**
time = 0.19, size = 229, normalized size = 1.41

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{b^4}{6(-da+cb+d(bx+a))^3 d}-\frac{b^4\left(-\frac{2\cos(2bx+2a)}{3(-da+cb+d(bx+a))^3 d}-\frac{2\left(-\frac{\sin(2bx+2a)}{(-da+cb+d(bx+a))^2 d}+\frac{-\frac{2\cos(2bx+2a)}{(-da+cb+d(bx+a))d}-\frac{2\left(-\frac{2\,\text{sinIntegral}\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}\right)}{\cdots}\right)}{4\cdots}}{b}$ |
| default | $\frac{-\frac{b^4}{6(-da+cb+d(bx+a))^3 d}-\frac{b^4\left(-\frac{2\cos(2bx+2a)}{3(-da+cb+d(bx+a))^3 d}-\frac{2\left(-\frac{\sin(2bx+2a)}{(-da+cb+d(bx+a))^2 d}+\frac{-\frac{2\cos(2bx+2a)}{(-da+cb+d(bx+a))d}-\frac{2\left(-\frac{2\,\text{sinIntegral}\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}\right)}{\cdots}\right)}{4\cdots}}{b}$ |
| risch | $\frac{ib^3 e^{-\frac{2i(da-cb)}{d}}\,\text{expIntegral}\left(1,2ibx+2ia-\frac{2i(da-cb)}{d}\right)}{3d^4}-\frac{ib^3 e^{\frac{2i(da-cb)}{d}}\,\text{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{3d^4}-\frac{\cdots}{6d(\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/6*b^4/(-d*a+c*b+d*(b*x+a))^3/d-1/4*b^4*(-2/3*cos(2*b*x+2*a)/(-d*a+c
*b+d*(b*x+a))^3/d-2/3*(-sin(2*b*x+2*a)/(-d*a+c*b+d*(b*x+a))^2/d+(-2*cos(2*b
*x+2*a)/(-d*a+c*b+d*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-
a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)/d
))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.53, size = 258, normalized size = 1.59

$$\frac{3\,b^4\left(E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+3\,b^4\left(i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,b^4}{12\,\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_
integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d)
 + 3*b^4*(I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp
_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d
) - 2*b^4)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4
- a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ a^2*d^4)*(b*x + a))*b)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(150) = 300.
time = 0.49, size = 341, normalized size = 2.10

$$\frac{b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-d^3-(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\cos\left(bx+a\right)^2-(bd^3x+bcd^2)\cos\left(bx+a\right)\sin\left(bx+a\right)-2\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)-\left((b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)}{3\,(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - d^3 - (2*b^2*d^3*x^2 + 4*b^2
*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - (b*d^3*x + b*c*d^2)*cos(b*x
+ a)*sin(b*x + a) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - ((b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d
) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(
-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*
d^5*x + c^3*d^4)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)**2/(d*x+c)**4,x)

[Out] Integral(sin(a + b*x)**2/(c + d*x)**4, x)

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.49, size = 7832, normalized size = 48.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")

[Out] -1/3*(b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 3*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*

```
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^3*d^3*x^3*real_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(
2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c*d^2*x^2*sin_integral(2
*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 - 2*b^3*d^3*x^3*real_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 12*b^3*c*d^2
*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)
 + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b
*c/d) + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan
(b*c/d) + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*
tan(b*c/d) + 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x
)^2*tan(a)^2*tan(b*c/d) + 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 3*b^3*c*d^2*x^2*imag_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*si
n_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^3*d^3*x^3*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 6*b^3*c^2*d*x*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 6
*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*ta
n(b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a
)^2*tan(b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*t
an(a)^2*tan(b*c/d)^2 + b^2*d^3*x^2*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^3*c
^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^
2 - b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*t
an(b*c/d)^2 + 2*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2
*tan(b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2 - b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 2
*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 6*b^3*c*d^2*x^2*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 6*b^3*c*d^2*x^2
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2}{{\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(c + d*x)^4,x)
[Out] int(sin(a + b*x)^2/(c + d*x)^4, x)
```

# 3.16 $\int (c+dx)^4 \sin^3(a+bx)\,dx$

**Optimal**. Leaf size=225

$$-\frac{488d^4\cos(a+bx)}{27b^5}+\frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3}-\frac{2(c+dx)^4\cos(a+bx)}{3b}+\frac{8d^4\cos^3(a+bx)}{81b^5}-\frac{160d^3(c+dx)\,\mathrm{s}}{9b^4}$$

```
[Out] -488/27*d^4*cos(b*x+a)/b^5+80/9*d^2*(d*x+c)^2*cos(b*x+a)/b^3-2/3*(d*x+c)^4*
cos(b*x+a)/b+8/81*d^4*cos(b*x+a)^3/b^5-160/9*d^3*(d*x+c)*sin(b*x+a)/b^4+8/3
*d*(d*x+c)^3*sin(b*x+a)/b^2+4/9*d^2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2/b^3-1
/3*(d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2/b-8/27*d^3*(d*x+c)*sin(b*x+a)^3/b^4+4/
9*d*(d*x+c)^3*sin(b*x+a)^3/b^2
```

**Rubi** [A]
time = 0.17, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2718, 2713}

$$\frac{8d^4\cos^3(a+bx)}{81b^5}-\frac{488d^4\cos(a+bx)}{27b^5}-\frac{8d^3(c+dx)\sin^3(a+bx)}{27b^4}-\frac{160d^3(c+dx)\sin(a+bx)}{9b^4}+\frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3}+\frac{4d^2(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{9b^3}+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{8d(c+dx)^3\sin(a+bx)}{3b^2}-\frac{2(c+dx)^4\cos(a+bx)}{3b}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sin[a + b*x]^3,x]
```

```
[Out] (-488*d^4*Cos[a + b*x])/(27*b^5) + (80*d^2*(c + d*x)^2*Cos[a + b*x])/(9*b^3
) - (2*(c + d*x)^4*Cos[a + b*x])/(3*b) + (8*d^4*Cos[a + b*x]^3)/(81*b^5) -
(160*d^3*(c + d*x)*Sin[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Sin[a + b*x])/(
3*b^2) + (4*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^3) - ((c + d*
x)^4*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) - (8*d^3*(c + d*x)*Sin[a + b*x]^3)/
(27*b^4) + (4*d*(c + d*x)^3*Sin[a + b*x]^3)/(9*b^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4 \sin^3(a+bx)\,dx &= -\frac{(c+dx)^4\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{2}{3}\int (c+d \\
&= -\frac{2(c+dx)^4\cos(a+bx)}{3b}+\frac{4d^2(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{9b^3}-\frac{(c+dx)}{} \\
&= \frac{8d^2(c+dx)^2\cos(a+bx)}{9b^3}-\frac{2(c+dx)^4\cos(a+bx)}{3b}+\frac{8d(c+dx)^3\sin(a+bx)}{3b^2} \\
&= -\frac{8d^4\cos(a+bx)}{27b^5}+\frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3}-\frac{2(c+dx)^4\cos(a+bx)}{3b}+\frac{8}{} \\
&= -\frac{56d^4\cos(a+bx)}{27b^5}+\frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3}-\frac{2(c+dx)^4\cos(a+bx)}{3b}+ \\
&= -\frac{488d^4\cos(a+bx)}{27b^5}+\frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3}-\frac{2(c+dx)^4\cos(a+bx)}{3b}+
\end{aligned}$$

**Mathematica [A]**
time = 0.61, size = 150, normalized size = 0.67

$$\frac{-243(24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cos(a+bx)+(8d^4-36b^2d^2(c+dx)^2+27b^4(c+dx)^4)\cos(3(a+bx))-24bd(c+dx)\left(242d^2-39b^2(c+dx)^2+(-2d^2+3b^2(c+dx)^2)\cos(2(a+bx))\right)\sin(a+bx)}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sin[a + b*x]^3,x]
```

```
[Out] (-243*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] + (8
*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)] - 24*b
*d*(c + d*x)*(242*d^2 - 39*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d*x)^2)*C
os[2*(a + b*x)])*Sin[a + b*x])/(324*b^5)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(205) = 410$.
time = 0.11, size = 1023, normalized size = 4.55

| method | result |
|---|---|

| | |
|---|---|
| risch | $-\frac{3(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2c\,d^3x-12b^2c^2d^2+24d^4)\cos(bx+a)}{4b^5}+\frac{3d(b^2d^3x^3+3\cdots}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*sin(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/b*(-1/3/b^4*a^4*d^4*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3/b^3*a^3*c*d^3*(2+sin(b*x+a)^2)*cos(b*x+a)-4/b^4*a^3*d^4*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))-2/b^2*a^2*c^2*d^2*(2+sin(b*x+a)^2)*cos(b*x+a)+12/b^3*a^2*c*d^3*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))+6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(b*x+a)+4/3*(b*x+a)*sin(b*x+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+4/3/b*a*c^3*d*(2+sin(b*x+a)^2)*cos(b*x+a)-12/b^2*a*c^2*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(b*x+a)+4/3*(b*x+a)*sin(b*x+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2)*cos(b*x+a))-4/b^4*a*d^4*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2*(b*x+a)^2*sin(b*x+a)-40/9*sin(b*x+a)+4*(b*x+a)*cos(b*x+a)+1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3)-1/3*c^4*(2+sin(b*x+a)^2)*cos(b*x+a)+4/b*c^3*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(b*x+a)+4/3*(b*x+a)*sin(b*x+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+4/b^3*c*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2*(b*x+a)^2*sin(b*x+a)-40/9*sin(b*x+a)+4*(b*x+a)*cos(b*x+a)+1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3)+1/b^4*d^4*(-1/3*(b*x+a)^4*(2+sin(b*x+a)^2)*cos(b*x+a)+8/3*(b*x+a)^3*sin(b*x+a)+8*(b*x+a)^2*cos(b*x+a)-160/9*cos(b*x+a)-160/9*(b*x+a)*sin(b*x+a)+4/9*(b*x+a)^3*sin(b*x+a)^3+4/9*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-8/27*(b*x+a)*sin(b*x+a)^3-8/81*(2+sin(b*x+a)^2)*cos(b*x+a)))

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(205) = 410$.
time = 0.34, size = 934, normalized size = 4.15

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/324*(108*(cos(b*x + a)^3 - 3*cos(b*x + a))*c^4 - 432*(cos(b*x + a)^3 - 3*cos(b*x + a))*a*c^3*d/b + 648*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^2*c^2*d^2/b^2 - 432*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^3*c*d^3/b^3 + 108*(cos(b*x +

```
a)^3 - 3*cos(b*x + a))*a^4*d^4/b^4 + 36*(3*(b*x + a)*cos(3*b*x + 3*a) - 27
*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*c^3*d/b - 108
*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*
a) + 27*sin(b*x + a))*a*c^2*d^2/b^2 + 108*(3*(b*x + a)*cos(3*b*x + 3*a) - 2
7*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*a^2*c*d^3/b^
3 - 36*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*
x + 3*a) + 27*sin(b*x + a))*a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x
 + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a)
+ 162*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 36*((9*(b*x + a)^2 - 2)*cos(3*b
*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a
) + 162*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 18*((9*(b*x + a)^2 - 2)*cos(3
*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3
*a) + 162*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 12*(3*(3*(b*x + a)^3 - 2*b*
x - 2*a)*cos(3*b*x + 3*a) - 81*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (
9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 243*((b*x + a)^2 - 2)*sin(b*x + a))*c
*d^3/b^3 - 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 81*((b*x
+ a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) +
 243*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + ((27*(b*x + a)^4 - 36*(b*x
 + a)^2 + 8)*cos(3*b*x + 3*a) - 243*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*cos
(b*x + a) - 12*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 972*((b*x +
 a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b
```

**Fricas** [A]
time = 0.45, size = 351, normalized size = 1.56

$$\frac{\left(27 b^{4} d^{4} x^{4} + 108 b^{4} c d^{3} x^{3} + 27 b^{4} c^{4} - 36 b^{2} c^{2} d^{2} + 8 d^{4} + 18 \left(9 b^{4} c^{2} d^{2} - 2 b^{2} d^{4}\right) x^{2} + 36 \left(3 b^{4} c^{3} d - 2 b^{2} c d^{3}\right) x\right) \cos \left(b x + a\right)^{3} - 3 \left(27 b^{4} d^{4} x^{4} + 108 b^{4} c d^{3} x^{3} + 27 b^{4} c^{4} - 252 b^{2} c^{2} d^{2} + 488 d^{4} + 18 \left(9 b^{4} c^{2} d^{2} - 14 b^{2} d^{4}\right) x^{2} + 36 \left(3 b^{4} c^{3} d - 14 b^{2} c d^{3}\right) x\right) \cos \left(b x + a\right) + 12 \left(21 b^{3} d^{4} x^{3} + 63 b^{3} c d^{3} x^{2} + 21 b^{3} c^{3} d - 122 b c d^{3} - \left(3 b^{3} d^{4} x^{3} + 9 b^{3} c d^{3} x^{2} + 3 b^{3} c^{3} d - 2 b c d^{3} + \left(9 b^{3} c^{2} d^{2} - 2 b d^{4}\right) x\right) \cos \left(b x + a\right)^{2} + \left(63 b^{3} c^{2} d^{2} - 122 b d^{4}\right) x\right) \sin \left(b x + a\right)}{81 b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8
*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*
x)*cos(b*x + a)^3 - 3*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 25
2*b^2*c^2*d^2 + 488*d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 + 36*(3*b^4*c
^3*d - 14*b^2*c*d^3)*x)*cos(b*x + a) + 12*(21*b^3*d^4*x^3 + 63*b^3*c*d^3*x^
2 + 21*b^3*c^3*d - 122*b*c*d^3 - (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c
^3*d - 2*b*c*d^3 + (9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + (63*b^3*c^
2*d^2 - 122*b*d^4)*x)*sin(b*x + a))/b^5
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(226) = 452$.
time = 0.83, size = 772, normalized size = 3.43

$$\begin{cases} - \frac{c^{4} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{b} - \frac{2 c^{4} \cos^{3}{\left(a + b x \right)}}{3 b} - \frac{4 c^{3} d x \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{b} - \frac{8 c^{3} d x \cos^{3}{\left(a + b x \right)}}{3 b} - \frac{6 c^{2} d^{2} x^{2} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{b} - \frac{4 c^{2} d^{2} x^{2} \cos^{3}{\left(a + b x \right)}}{b} - \frac{4 c d^{3} x^{3} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{b} - \frac{8 c d^{3} x^{3} \cos^{3}{\left(a + b x \right)}}{3 b} - \frac{d^{4} x^{4} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{b} - \frac{2 d^{4} x^{4} \cos^{3}{\left(a + b x \right)}}{3 b} + \frac{28 c^{3} d \sin^{3}{\left(a + b x \right)}}{9 b^{2}} + \frac{8 c^{3} d \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{3 b^{2}} + \frac{28 c^{2} d^{2} x \sin^{3}{\left(a + b x \right)}}{3 b^{2}} + \frac{8 c^{2} d^{2} x \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{b^{2}} + \frac{28 c d^{3} x^{2} \sin^{3}{\left(a + b x \right)}}{3 b^{2}} + \frac{8 c d^{3} x^{2} \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{b^{2}} + \frac{28 d^{4} x^{3} \sin^{3}{\left(a + b x \right)}}{9 b^{2}} + \frac{8 d^{4} x^{3} \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{3 b^{2}} + \frac{28 c^{2} d^{2} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{3 b^{3}} + \frac{80 c^{2} d^{2} \cos^{3}{\left(a + b x \right)}}{9 b^{3}} + \frac{56 c d^{3} x \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{3 b^{3}} + \frac{160 c d^{3} x \cos^{3}{\left(a + b x \right)}}{9 b^{3}} + \frac{28 d^{4} x^{2} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{3 b^{3}} + \frac{80 d^{4} x^{2} \cos^{3}{\left(a + b x \right)}}{9 b^{3}} - \frac{488 c d^{3} \sin^{3}{\left(a + b x \right)}}{27 b^{4}} - \frac{160 c d^{3} \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{9 b^{4}} - \frac{488 d^{4} x \sin^{3}{\left(a + b x \right)}}{27 b^{4}} - \frac{160 d^{4} x \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{9 b^{4}} - \frac{488 d^{4} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{27 b^{5}} - \frac{1456 d^{4} \cos^{3}{\left(a + b x \right)}}{81 b^{5}} & \text{for}\: b \neq 0 \\\left(c^{4} x + 2 c^{3} d x^{2} + 2 c^{2} d^{2} x^{3} + c d^{3} x^{4} + \frac{d^{4} x^{5}}{5}\right) \sin^{3}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**4*cos(a + b*x)**3/(3
*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)/b - 8*c**3*d*x*cos(a + b*x)**
3/(3*b) - 6*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)/b - 4*c**2*d**2*x**
2*cos(a + b*x)**3/b - 4*c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)/b - 8*c*d*
*3*x**3*cos(a + b*x)**3/(3*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*x)/b -
2*d**4*x**4*cos(a + b*x)**3/(3*b) + 28*c**3*d*sin(a + b*x)**3/(9*b**2) + 8*
c**3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 28*c**2*d**2*x*sin(a + b*x)*
*3/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 28*c*d**3*x
**2*sin(a + b*x)**3/(3*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b
**2 + 28*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 8*d**4*x**3*sin(a + b*x)*cos(
a + b*x)**2/(3*b**2) + 28*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) +
 80*c**2*d**2*cos(a + b*x)**3/(9*b**3) + 56*c*d**3*x*sin(a + b*x)**2*cos(a
+ b*x)/(3*b**3) + 160*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 28*d**4*x**2*sin(
a + b*x)**2*cos(a + b*x)/(3*b**3) + 80*d**4*x**2*cos(a + b*x)**3/(9*b**3) -
 488*c*d**3*sin(a + b*x)**3/(27*b**4) - 160*c*d**3*sin(a + b*x)*cos(a + b*x
)**2/(9*b**4) - 488*d**4*x*sin(a + b*x)**3/(27*b**4) - 160*d**4*x*sin(a + b
*x)*cos(a + b*x)**2/(9*b**4) - 488*d**4*sin(a + b*x)**2*cos(a + b*x)/(27*b*
*5) - 1456*d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x
**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**3, True))
```

**Giac [A]**
time = 3.24, size = 351, normalized size = 1.56

$$\frac{\left(27 b^4 d^4 x^4+108 b^4 c d^3 x^3+162 b^4 c^2 d^2 x^2+108 b^4 c^3 d x+27 b^4 c^4-36 b^2 d^4 x^2-72 b^2 c d^3 x-36 b^2 c^2 d^2+8 d^4\right) \cos (3 b x+3 a)}{324 b^5}-\frac{3\left(b^4 d^4 x^4+4 b^4 c d^3 x^3+6 b^4 c^2 d^2 x^2+4 b^4 c^3 d x+b^4 c^4-12 b^2 d^4 x^2-24 b^2 c d^3 x-12 b^2 c^2 d^2+24 d^4\right) \cos (b x+a)}{4 b^5}-\frac{\left(3 b^3 d^4 x^3+9 b^3 c d^3 x^2+9 b^3 c^2 d^2 x+3 b^3 c^3 d-2 b d^4 x-2 b c d^3\right) \sin (3 b x+3 a)}{27 b^5}+\frac{3\left(b^3 d^4 x^3+3 b^3 c d^3 x^2+3 b^3 c^2 d^2 x+b^3 c^3 d-6 b d^4 x-6 b c d^3\right) \sin (b x+a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c
^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*
d^4)*cos(3*b*x + 3*a)/b^5 - 3/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*
d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^
2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 - 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^
2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x + 3*a)
/b^5 + 3*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b
*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

**Mupad [B]**
time = 1.55, size = 533, normalized size = 2.37

$$\frac{8 x \cos (a+b x)^3 \left(20 c d^3-3 b^2 c^3 d\right)}{9 b^3}-\frac{2 \cos (a+b x)^3 \left(27 b^4 c^4-360 b^2 c^2 d^2+728 d^4\right)}{81 b^5}-\frac{\cos (a+b x) \sin (a+b x)^2 \left(27 b^4 c^4-252 b^2 c^2 d^2+488 d^4\right)}{27 b^5}-\frac{8 \cos (a+b x)^2 \sin (a+b x) \left(20 c d^3-3 b^2 c^3 d\right)}{9 b^4}-\frac{2 d^4 x^4 \cos (a+b x)^3}{3 b}-\frac{4 \sin (a+b x)^3 \left(122 c d^3-21 b^2 c^3 d\right)}{27 b^4}+\frac{28 d^4 x^3 \sin (a+b x)^3}{9 b^2}-\frac{4 x \sin (a+b x)^3 \left(122 d^4-63 b^2 c^2 d^2\right)}{27 b^4}+\frac{4 x^2 \cos (a+b x)^3 \left(20 d^4-9 b^2 c^2 d^2\right)}{9 b^3}+\frac{2 x^2 \cos (a+b x) \sin (a+b x)^2 \left(14 d^4-9 b^2 c^2 d^2\right)}{3 b^3}-\frac{8 c d^3 x^3 \cos (a+b x)^3}{3 b}-\frac{d^4 x^4 \cos (a+b x) \sin (a+b x)^2}{b}+\frac{8 d^4 x^3 \cos (a+b x)^2 \sin (a+b x)}{3 b^2}+\frac{28 c d^3 x^2 \sin (a+b x)^3}{3 b^2}-\frac{8 x \cos (a+b x)^2 \sin (a+b x) \left(20 d^4-9 b^2 c^2 d^2\right)}{9 b^4}+\frac{4 x \cos (a+b x) \sin (a+b x)^2 \left(14 c d^3-3 b^2 c^3 d\right)}{3 b^3}-\frac{4 c d^3 x^3 \cos (a+b x) \sin (a+b x)^2}{b}+\frac{8 c d^3 x^2 \cos (a+b x)^2 \sin (a+b x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] (8*x*cos(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (2*cos(a + b*x)^3*(
728*d^4 + 27*b^4*c^4 - 360*b^2*c^2*d^2))/(81*b^5) - (cos(a + b*x)*sin(a + b
*x)^2*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(27*b^5) - (8*cos(a + b*x)^
2*sin(a + b*x)*(20*c*d^3 - 3*b^2*c^3*d))/(9*b^4) - (2*d^4*x^4*cos(a + b*x)^
3)/(3*b) - (4*sin(a + b*x)^3*(122*c*d^3 - 21*b^2*c^3*d))/(27*b^4) + (28*d^4
*x^3*sin(a + b*x)^3)/(9*b^2) - (4*x*sin(a + b*x)^3*(122*d^4 - 63*b^2*c^2*d^
2))/(27*b^4) + (4*x^2*cos(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(9*b^3) + (2
*x^2*cos(a + b*x)*sin(a + b*x)^2*(14*d^4 - 9*b^2*c^2*d^2))/(3*b^3) - (8*c*d
^3*x^3*cos(a + b*x)^3)/(3*b) - (d^4*x^4*cos(a + b*x)*sin(a + b*x)^2)/b + (8
*d^4*x^3*cos(a + b*x)^2*sin(a + b*x))/(3*b^2) + (28*c*d^3*x^2*sin(a + b*x)^
3)/(3*b^2) - (8*x*cos(a + b*x)^2*sin(a + b*x)*(20*d^4 - 9*b^2*c^2*d^2))/(9*
b^4) + (4*x*cos(a + b*x)*sin(a + b*x)^2*(14*c*d^3 - 3*b^2*c^3*d))/(3*b^3) -
 (4*c*d^3*x^3*cos(a + b*x)*sin(a + b*x)^2)/b + (8*c*d^3*x^2*cos(a + b*x)^2*
sin(a + b*x))/b^2
```

# 3.17 $\int (c+dx)^3 \sin^3(a+bx)\,dx$

**Optimal**. Leaf size=175

$$\frac{40d^2(c+dx)\cos(a+bx)}{9b^3}-\frac{2(c+dx)^3\cos(a+bx)}{3b}-\frac{40d^3\sin(a+bx)}{9b^4}+\frac{2d(c+dx)^2\sin(a+bx)}{b^2}+\frac{2d^2(c+dx)\cos}{}$$

```
[Out] 40/9*d^2*(d*x+c)*cos(b*x+a)/b^3-2/3*(d*x+c)^3*cos(b*x+a)/b-40/9*d^3*sin(b*x
+a)/b^4+2*d*(d*x+c)^2*sin(b*x+a)/b^2+2/9*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)^
2/b^3-1/3*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2/b-2/27*d^3*sin(b*x+a)^3/b^4+1/3
*d*(d*x+c)^2*sin(b*x+a)^3/b^2
```

**Rubi** [A]
time = 0.11, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2717, 3391}

$$-\frac{2d^3\sin^3(a+bx)}{27b^4}-\frac{40d^3\sin(a+bx)}{9b^4}+\frac{40d^2(c+dx)\cos(a+bx)}{9b^3}+\frac{2d^2(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^3}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2d(c+dx)^2\sin(a+bx)}{b^2}-\frac{2(c+dx)^3\cos(a+bx)}{3b}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]^3,x]
```

```
[Out] (40*d^2*(c + d*x)*Cos[a + b*x])/(9*b^3) - (2*(c + d*x)^3*Cos[a + b*x])/(3*b
) - (40*d^3*Sin[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sin[a + b*x])/b^2 + (2
*d^2*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^3) - ((c + d*x)^3*Cos[a +
b*x]*Sin[a + b*x]^2)/(3*b) - (2*d^3*Sin[a + b*x]^3)/(27*b^4) + (d*(c + d*x)
^2*Sin[a + b*x]^3)/(3*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3 \sin^3(a+bx)\,dx &= -\frac{(c+dx)^3\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\int (c+dx\\ &= -\frac{2(c+dx)^3\cos(a+bx)}{3b}+\frac{2d^2(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^3}-\frac{(c+dx)^3\\ &= \frac{4d^2(c+dx)\cos(a+bx)}{9b^3}-\frac{2(c+dx)^3\cos(a+bx)}{3b}+\frac{2d(c+dx)^2\sin(a+bx)}{b^2}\\ &= \frac{40d^2(c+dx)\cos(a+bx)}{9b^3}-\frac{2(c+dx)^3\cos(a+bx)}{3b}-\frac{4d^3\sin(a+bx)}{9b^4}+\frac{2d(c\\ &= \frac{40d^2(c+dx)\cos(a+bx)}{9b^3}-\frac{2(c+dx)^3\cos(a+bx)}{3b}-\frac{40d^3\sin(a+bx)}{9b^4}+\frac{2d(\end{aligned}$$

**Mathematica [A]**
time = 0.60, size = 127, normalized size = 0.73

$$\frac{-162b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)+6b(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))-4d\left(242d^2-117b^2(c+dx)^2+\left(-2d^2+9b^2(c+dx)^2\right)\cos(2(a+bx))\right)\sin(a+bx)}{216b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]^3,x]
```

```
[Out] (-162*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 6*b*(c + d*x)*(
-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] - 4*d*(242*d^2 - 117*b^2*(c +
d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(216*
b^4)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(161) = 322$.
time = 0.09, size = 560, normalized size = 3.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/b^3*a^3*d^3*(2+sin(b*x+a)^2)*cos(b*x+a)-1/b^2*a^2*c*d^2*(2+sin(b*x
+a)^2)*cos(b*x+a)+3/b^3*a^2*d^3*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1
```

```
/9*sin(b*x+a)^3+2/3*sin(b*x+a))+1/b*a*c^2*d*(2+sin(b*x+a)^2)*cos(b*x+a)-6/b
^2*a*c*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*s
in(b*x+a))-3/b^3*a*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(
b*x+a)+4/3*(b*x+a)*sin(b*x+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2
)*cos(b*x+a))-1/3*c^3*(2+sin(b*x+a)^2)*cos(b*x+a)+3/b*c^2*d*(-1/3*(b*x+a)*(
2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))+3/b^2*c*d^2*(-1
/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(b*x+a)+4/3*(b*x+a)*sin(b*x
+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+1/b^3*d^3*(-
1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2*(b*x+a)^2*sin(b*x+a)-40/9*sin(b
*x+a)+4*(b*x+a)*cos(b*x+a)+1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*
x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(161) = 322$.
time = 0.32, size = 541, normalized size = 3.09

$$\frac{36\,\left(\cos\left(bx+a\right)^3-3\,\cos\left(bx+a\right)\right)c^3-\frac{108\,\left(\cos(bx+a)^3-3\,\cos(bx+a)\right)ac^2d}{b}+\frac{108\,\left(\cos(bx+a)^3-3\,\cos(bx+a)\right)a^2cd^2}{b^2}-\frac{36\,\left(\cos(bx+a)^3-3\,\cos(bx+a)\right)a^3d^3}{b^3}+\frac{9\,(3\,(bx+a)\cos(3\,bx+3\,a)-27\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)+27\,\sin(bx+a))c^2d}{b}-\frac{18\,(3\,(bx+a)\cos(3\,bx+3\,a)-27\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)+27\,\sin(bx+a))acd^2}{b^2}+\frac{9\,(3\,(bx+a)\cos(3\,bx+3\,a)-27\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)+27\,\sin(bx+a))a^2d^3}{b^3}+\frac{3\,\left(\left(9\,(bx+a)^2-2\right)\cos(3\,bx+3\,a)-81\,\left((bx+a)^2-2\right)\cos(bx+a)-6\,(bx+a)\sin(3\,bx+3\,a)+162\,(bx+a)\sin(bx+a)\right)cd^2}{b^2}-\frac{3\,\left(\left(9\,(bx+a)^2-2\right)\cos(3\,bx+3\,a)-81\,\left((bx+a)^2-2\right)\cos(bx+a)-6\,(bx+a)\sin(3\,bx+3\,a)+162\,(bx+a)\sin(bx+a)\right)ad^3}{b^3}+\frac{\left(3\,\left(3\,(bx+a)^3-2\,bx-2\,a\right)\cos(3\,bx+3\,a)-81\,\left((bx+a)^3-6\,bx-6\,a\right)\cos(bx+a)-\left(9\,(bx+a)^2-2\right)\sin(3\,bx+3\,a)+243\,\left((bx+a)^2-2\right)\sin(bx+a)\right)d^3}{b^3}}{108\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/108*(36*(cos(b*x + a)^3 - 3*cos(b*x + a))*c^3 - 108*(cos(b*x + a)^3 - 3*c
os(b*x + a))*a*c^2*d/b + 108*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^2*c*d^2/b^
2 - 36*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^3*d^3/b^3 + 9*(3*(b*x + a)*cos(3
*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a
))*c^2*d/b - 18*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) -
 sin(3*b*x + 3*a) + 27*sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*cos(3*b*x
 + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*a
^2*d^3/b^3 + 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)
*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*
c*d^2/b^2 - 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*
cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*a
*d^3/b^3 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 81*((b*x + a
)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 24
3*((b*x + a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

**Fricas** [A]
time = 0.37, size = 227, normalized size = 1.30

$$\frac{3\,(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+(9\,b^3c^2d-2\,bd^3)x)\cos\left(bx+a\right)^3-9\,(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-14\,bcd^2+(9\,b^3c^2d-14\,bd^3)x)\cos\left(bx+a\right)+\left(63\,b^2d^3x^2+126\,b^2cd^2x+63\,b^2c^2d-122\,d^3-(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)}{27\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c
^2*d - 2*b*d^3)*x)*cos(b*x + a)^3 - 9*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*
```

```
b^3*c^3 - 14*b*c*d^2 + (9*b^3*c^2*d - 14*b*d^3)*x)*cos(b*x + a) + (63*b^2*d
^3*x^2 + 126*b^2*c*d^2*x + 63*b^2*c^2*d - 122*d^3 - (9*b^2*d^3*x^2 + 18*b^2
*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.
time = 0.54, size = 495, normalized size = 2.83

$$\begin{cases} -\frac{c^3\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2c^3\cos^3(a+bx)}{3b} - \frac{3c^2dx\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2c^2dx\cos^3(a+bx)}{b} - \frac{3cd^2x^2\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2cd^2x^2\cos^3(a+bx)}{b} - \frac{d^3x^3\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2d^3x^3\cos^3(a+bx)}{3b} + \frac{7c^2d\sin^3(a+bx)}{3b^2} + \frac{2c^2d\sin(a+bx)\cos^2(a+bx)}{b^2} + \frac{14cd^2x\sin^3(a+bx)}{3b^2} + \frac{4cd^2x\sin(a+bx)\cos^2(a+bx)}{b^2} + \frac{7d^3x^2\sin^3(a+bx)}{3b^2} + \frac{2d^3x^2\sin(a+bx)\cos^2(a+bx)}{b^2} + \frac{14cd^2\sin^2(a+bx)\cos(a+bx)}{3b^3} + \frac{40cd^2\cos^3(a+bx)}{9b^3} + \frac{14d^3x\sin^2(a+bx)\cos(a+bx)}{3b^3} + \frac{40d^3x\cos^3(a+bx)}{9b^3} - \frac{122d^3\sin^3(a+bx)}{27b^4} - \frac{40d^3\sin(a+bx)\cos^2(a+bx)}{9b^4} & \text{for}\ b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**3*cos(a + b*x)**3/(3
*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**2*d*x*cos(a + b*x)**
3/b - 3*c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)/b - 2*c*d**2*x**2*cos(a +
b*x)**3/b - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)/b - 2*d**3*x**3*cos(a +
b*x)**3/(3*b) + 7*c**2*d*sin(a + b*x)**3/(3*b**2) + 2*c**2*d*sin(a + b*x)*c
os(a + b*x)**2/b**2 + 14*c*d**2*x*sin(a + b*x)**3/(3*b**2) + 4*c*d**2*x*sin
(a + b*x)*cos(a + b*x)**2/b**2 + 7*d**3*x**2*sin(a + b*x)**3/(3*b**2) + 2*d
**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b**2 + 14*c*d**2*sin(a + b*x)**2*cos(
a + b*x)/(3*b**3) + 40*c*d**2*cos(a + b*x)**3/(9*b**3) + 14*d**3*x*sin(a +
b*x)**2*cos(a + b*x)/(3*b**3) + 40*d**3*x*cos(a + b*x)**3/(9*b**3) - 122*d*
*3*sin(a + b*x)**3/(27*b**4) - 40*d**3*sin(a + b*x)*cos(a + b*x)**2/(9*b**4
), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a
)**3, True))
```

**Giac** [A]
time = 3.12, size = 231, normalized size = 1.32

$$\frac{(3b^3d^3x^3 + 9b^3cd^2x^2 + 9b^3c^2dx + 3b^3c^3 - 2bd^3x - 2bcd^2)\cos(3bx+3a)}{36b^4} - \frac{3(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3 - 6bd^3x - 6bcd^2)\cos(bx+a)}{4b^4} - \frac{(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 2d^3)\sin(3bx+3a)}{108b^4} + \frac{9(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\sin(bx+a)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3
*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 3/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 1/108*
(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^4
 + 9/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

**Mupad** [B]
time = 1.11, size = 365, normalized size = 2.09

$$\frac{2\cos(a+bx)^3(20cd^2-3b^2c^3)}{9b^3} - \frac{\sin(a+bx)^3(122d^3-63b^2c^2d)}{27b^4} + \frac{\cos(a+bx)\sin(a+bx)^2(14cd^2-3b^2c^3)}{3b^3} - \frac{2\cos(a+bx)^2\sin(a+bx)(20d^3-9b^2c^2d)}{9b^4} + \frac{2x\cos(a+bx)^3(20d^3-9b^2c^2d)}{9b^3} - \frac{2d^3x^3\cos(a+bx)^3}{3b} + \frac{7d^3x^2\sin(a+bx)^3}{3b^2} + \frac{14cd^2x\sin(a+bx)^3}{3b^2} + \frac{x\cos(a+bx)\sin(a+bx)^2(14d^3-9b^2c^2d)}{3b^3} - \frac{2cd^2x^2\cos(a+bx)^3}{b} - \frac{d^3x^3\cos(a+bx)\sin(a+bx)^2}{b} + \frac{2d^3x^2\cos(a+bx)^2\sin(a+bx)}{b^2} - \frac{3cd^2x^2\cos(a+bx)\sin(a+bx)^2}{b} + \frac{4cd^2x\cos(a+bx)^2\sin(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] (2*cos(a + b*x)^3*(20*c*d^2 - 3*b^2*c^3))/(9*b^3) - (sin(a + b*x)^3*(122*d^
3 - 63*b^2*c^2*d))/(27*b^4) + (cos(a + b*x)*sin(a + b*x)^2*(14*c*d^2 - 3*b^
2*c^3))/(3*b^3) - (2*cos(a + b*x)^2*sin(a + b*x)*(20*d^3 - 9*b^2*c^2*d))/(9
*b^4) + (2*x*cos(a + b*x)^3*(20*d^3 - 9*b^2*c^2*d))/(9*b^3) - (2*d^3*x^3*co
s(a + b*x)^3)/(3*b) + (7*d^3*x^2*sin(a + b*x)^3)/(3*b^2) + (14*c*d^2*x*sin(
a + b*x)^3)/(3*b^2) + (x*cos(a + b*x)*sin(a + b*x)^2*(14*d^3 - 9*b^2*c^2*d)
)/(3*b^3) - (2*c*d^2*x^2*cos(a + b*x)^3)/b - (d^3*x^3*cos(a + b*x)*sin(a +
b*x)^2)/b + (2*d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 - (3*c*d^2*x^2*cos(
a + b*x)*sin(a + b*x)^2)/b + (4*c*d^2*x*cos(a + b*x)^2*sin(a + b*x))/b^2
```

# 3.18 $\int (c+dx)^2 \sin^3(a+bx)\,dx$

**Optimal**. Leaf size=123

$$\frac{14d^2\cos(a+bx)}{9b^3}-\frac{2(c+dx)^2\cos(a+bx)}{3b}-\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d(c+dx)\sin(a+bx)}{3b^2}-\frac{(c+dx)^2\cos(a+bx)}{3b}$$

```
[Out] 14/9*d^2*cos(b*x+a)/b^3-2/3*(d*x+c)^2*cos(b*x+a)/b-2/27*d^2*cos(b*x+a)^3/b^
3+4/3*d*(d*x+c)*sin(b*x+a)/b^2-1/3*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2/b+2/9*
d*(d*x+c)*sin(b*x+a)^3/b^2
```

**Rubi** [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2718, 2713}

$$-\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{14d^2\cos(a+bx)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}+\frac{4d(c+dx)\sin(a+bx)}{3b^2}-\frac{2(c+dx)^2\cos(a+bx)}{3b}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]^3,x]
```

```
[Out] (14*d^2*Cos[a + b*x])/(9*b^3) - (2*(c + d*x)^2*Cos[a + b*x])/(3*b) - (2*d^2
*Cos[a + b*x]^3)/(27*b^3) + (4*d*(c + d*x)*Sin[a + b*x])/(3*b^2) - ((c + d*
x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (2*d*(c + d*x)*Sin[a + b*x]^3)/(9
*b^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^2\sin^3(a+bx)\,dx = -\frac{(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}+\frac{2}{3}\int (c+dx)$$
$$= -\frac{2(c+dx)^2\cos(a+bx)}{3b}-\frac{(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{2d(c+dx)\sin}{9b^2}$$
$$= \frac{2d^2\cos(a+bx)}{9b^3}-\frac{2(c+dx)^2\cos(a+bx)}{3b}-\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d(c+dx)\sin}{3b^2}$$
$$= \frac{14d^2\cos(a+bx)}{9b^3}-\frac{2(c+dx)^2\cos(a+bx)}{3b}-\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d(c+dx)\sin}{3b^2}$$

**Mathematica [A]**
time = 0.27, size = 86, normalized size = 0.70

$$\frac{-81(-2d^2+b^2(c+dx)^2)\cos(a+bx)+(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))-6bd(c+dx)(-27\sin(a+bx)+\sin(3(a+bx)))}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]^3,x]
[Out] (-81*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c + d*x)^2)
*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[a + b*x] + Sin[3*(a + b*x)]))/
(108*b^3)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(111) = 222.
time = 0.07, size = 265, normalized size = 2.15

| method | result |
|---|---|
| risch | $-\frac{3(d^2x^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{4b^3}+\frac{3d(dx+c)\sin(bx+a)}{2b^2}+\frac{(9d^2x^2b^2+18b^2cdx+9b^2c^2-2d^2)\cos(3bx+3a)}{108b^3}$ |
| derivativedivides | $\frac{-\frac{a^2d^2(2+\sin^2(bx+a))\cos(bx+a)}{3b^2}+\frac{2acd(2+\sin^2(bx+a))\cos(bx+a)}{3b}-\frac{2a\,d^2\left(-\frac{(bx+a)(2+\sin^2(bx+a))\cos(bx+a)}{3}+\frac{(\sin^3(bx+a))}{9}+2\right.}{b^2}}{}$ |
| default | $\frac{-\frac{a^2d^2(2+\sin^2(bx+a))\cos(bx+a)}{3b^2}+\frac{2acd(2+\sin^2(bx+a))\cos(bx+a)}{3b}-\frac{2a\,d^2\left(-\frac{(bx+a)(2+\sin^2(bx+a))\cos(bx+a)}{3}+\frac{(\sin^3(bx+a))}{9}+2\right.}{b^2}}{}$ |

| | |
|---|---|
| norman | $\frac{\frac{-36b^2c^2+80d^2}{27b^3} - \frac{2d^2x^2}{3b} + \frac{8d^2\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^3} + \frac{\left(-36b^2c^2+56d^2\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{9b^3} - \frac{4cdx}{3b} + \frac{8cd\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{3b^2} + \frac{64cd\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{9b^2}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3/b^2*a^2*d^2*(2+sin(b*x+a)^2)*cos(b*x+a)+2/3/b*a*c*d*(2+sin(b*x+a)
^2)*cos(b*x+a)-2/b^2*a*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*si
n(b*x+a)^3+2/3*sin(b*x+a))-1/3*c^2*(2+sin(b*x+a)^2)*cos(b*x+a)+2/b*c*d*(-1/
3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))+1/b^
2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(b*x+a)+4/3*(b*x+a
)*sin(b*x+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2)*cos(b*x+a)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(111) = 222$.
time = 0.30, size = 270, normalized size = 2.20

$$\frac{36\left(\cos\left(bx+a\right)^3-3\cos\left(bx+a\right)\right)c^2-\frac{72\left(\cos(bx+a)^3-3\cos(bx+a)\right)acd}{b}+\frac{36\left(\cos(bx+a)^3-3\cos(bx+a)\right)a^2d^2}{b^2}+\frac{6\left(3\left(bx+a\right)\cos(3bx+3a)-27\left(bx+a\right)\cos(bx+a)-\sin(3bx+3a)+27\sin(bx+a)\right)cd}{b}-\frac{6\left(3\left(bx+a\right)\cos(3bx+3a)-27\left(bx+a\right)\cos(bx+a)-\sin(3bx+3a)+27\sin(bx+a)\right)ad^2}{b^2}+\frac{\left(\left(9\left(bx+a\right)^2-2\right)\cos(3bx+3a)-81\left(\left(bx+a\right)^2-2\right)\cos(bx+a)-6\left(bx+a\right)\sin(3bx+3a)+162\left(bx+a\right)\sin(bx+a)\right)d^2}{b^2}}{108\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/108*(36*(cos(b*x + a)^3 - 3*cos(b*x + a))*c^2 - 72*(cos(b*x + a)^3 - 3*co
s(b*x + a))*a*c*d/b + 36*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^2*d^2/b^2 + 6*
(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a
) + 27*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)
*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*a*d^2/b^2 + ((9*(b*x +
a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a
)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

**Fricas** [A]
time = 0.43, size = 131, normalized size = 1.07

$$\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^3-3\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-14\,d^2\right)\cos\left(bx+a\right)+6\left(7\,bd^2x+7\,bcd-\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 - 3
*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 14*d^2)*cos(b*x + a) + 6*(7*b*
d^2*x + 7*b*c*d - (b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/b^3
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.
time = 0.34, size = 284, normalized size = 2.31

$$\begin{cases} -\frac{c^2\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{b}-\frac{2c^2\cos^3\left(a+bx\right)}{3b}-\frac{2cdx\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{b}-\frac{4cdx\cos^3\left(a+bx\right)}{3b}-\frac{d^2x^2\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{b}-\frac{2d^2x^2\cos^3\left(a+bx\right)}{3b}+\frac{14cd\sin^3\left(a+bx\right)}{9b^2}+\frac{4cd\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{3b^2}+\frac{14d^2x\sin^3\left(a+bx\right)}{9b^2}+\frac{4d^2x\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{3b^2}+\frac{14d^2\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{9b^3}+\frac{40d^2\cos^3\left(a+bx\right)}{27b^3} & \text{for}\ b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^3\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**2*cos(a + b*x)**3/(3
*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)/b - 4*c*d*x*cos(a + b*x)**3/(3*b
) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)/b - 2*d**2*x**2*cos(a + b*x)**3/
(3*b) + 14*c*d*sin(a + b*x)**3/(9*b**2) + 4*c*d*sin(a + b*x)*cos(a + b*x)**
2/(3*b**2) + 14*d**2*x*sin(a + b*x)**3/(9*b**2) + 4*d**2*x*sin(a + b*x)*cos
(a + b*x)**2/(3*b**2) + 14*d**2*sin(a + b*x)**2*cos(a + b*x)/(9*b**3) + 40*
d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/
3)*sin(a)**3, True))
```

**Giac [A]**
time = 3.82, size = 137, normalized size = 1.11

$$\frac{\left(9 b^{2} d^{2} x^{2}+18 b^{2} c d x+9 b^{2} c^{2}-2 d^{2}\right) \cos \left(3 b x+3 a\right)}{108 b^{3}}-\frac{3 \left(b^{2} d^{2} x^{2}+2 b^{2} c d x+b^{2} c^{2}-2 d^{2}\right) \cos \left(b x+a\right)}{4 b^{3}}-\frac{\left(b d^{2} x+b c d\right) \sin \left(3 b x+3 a\right)}{18 b^{3}}+\frac{3 \left(b d^{2} x+b c d\right) \sin \left(b x+a\right)}{2 b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)/b
^3 - 3/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 - 1
/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 3/2*(b*d^2*x + b*c*d)*sin(b*x
+ a)/b^3
```

**Mupad [B]**
time = 0.98, size = 174, normalized size = 1.41

$$\frac{\frac{3 d^{2} x \sin(a+b x)}{2}-\frac{d^{2} x \sin(3 a+3 b x)}{18}+\frac{3 c d \sin(a+b x)}{2}-\frac{c d \sin(3 a+3 b x)}{18}}{b^{2}}-\frac{\frac{3 c^{2} \cos(a+b x)}{4}-\frac{c^{2} \cos(3 a+3 b x)}{12}+\frac{3 d^{2} x^{2} \cos(a+b x)}{4}-\frac{d^{2} x^{2} \cos(3 a+3 b x)}{12}-\frac{c d x \cos(3 a+3 b x)}{6}+\frac{3 c d x \cos(a+b x)}{2}}{b}+\frac{3 d^{2} \cos\left(a+b x\right)}{2 b^{3}}-\frac{d^{2} \cos\left(3 a+3 b x\right)}{54 b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] ((3*d^2*x*sin(a + b*x))/2 - (d^2*x*sin(3*a + 3*b*x))/18 + (3*c*d*sin(a + b*
x))/2 - (c*d*sin(3*a + 3*b*x))/18)/b^2 - ((3*c^2*cos(a + b*x))/4 - (c^2*cos
(3*a + 3*b*x))/12 + (3*d^2*x^2*cos(a + b*x))/4 - (d^2*x^2*cos(3*a + 3*b*x))
/12 - (c*d*x*cos(3*a + 3*b*x))/6 + (3*c*d*x*cos(a + b*x))/2)/b + (3*d^2*cos
(a + b*x))/(2*b^3) - (d^2*cos(3*a + 3*b*x))/(54*b^3)
```

# 3.19 $\int (c+dx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=75

$$-\frac{2(c+dx)\cos(a+bx)}{3b}+\frac{2d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d\sin^3(a+bx)}{9b^2}$$

```
[Out] -2/3*(d*x+c)*cos(b*x+a)/b+2/3*d*sin(b*x+a)/b^2-1/3*(d*x+c)*cos(b*x+a)*sin(b
*x+a)^2/b+1/9*d*sin(b*x+a)^3/b^2
```

**Rubi** [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3391, 3377, 2717}

$$\frac{d\sin^3(a+bx)}{9b^2}+\frac{2d\sin(a+bx)}{3b^2}-\frac{2(c+dx)\cos(a+bx)}{3b}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]^3,x]
```

```
[Out] (-2*(c + d*x)*Cos[a + b*x])/(3*b) + (2*d*Sin[a + b*x])/(3*b^2) - ((c + d*x)
*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (d*Sin[a + b*x]^3)/(9*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)\sin^3(a+bx)\,dx = -\frac{(c+dx)\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{d\sin^3(a+bx)}{9b^2} + \frac{2}{3}\int (c+dx)\sin(a+bx)$$
$$= -\frac{2(c+dx)\cos(a+bx)}{3b} - \frac{(c+dx)\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{d\sin^3(a+bx)}{9b^2} +$$
$$= -\frac{2(c+dx)\cos(a+bx)}{3b} + \frac{2d\sin(a+bx)}{3b^2} - \frac{(c+dx)\cos(a+bx)\sin^2(a+bx)}{3b} +$$

**Mathematica** [A]
time = 0.13, size = 59, normalized size = 0.79

$$\frac{-27b(c+dx)\cos(a+bx) + 3b(c+dx)\cos(3(a+bx)) + d(27\sin(a+bx) - \sin(3(a+bx)))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]^3,x]
```

```
[Out] (-27*b*(c + d*x)*Cos[a + b*x] + 3*b*(c + d*x)*Cos[3*(a + b*x)] + d*(27*Sin[
a + b*x] - Sin[3*(a + b*x)]))/(36*b^2)
```

**Maple** [A]
time = 0.05, size = 95, normalized size = 1.27

| method | result |
|---|---|
| risch | $-\frac{3(dx+c)\cos(bx+a)}{4b}+\frac{3d\sin(bx+a)}{4b^2}+\frac{(dx+c)\cos(3bx+3a)}{12b}-\frac{d\sin(3bx+3a)}{36b^2}$ |
| derivativedivides | $\frac{\frac{da\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3b}-\frac{c\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3}+\frac{d\left(-\frac{(bx+a)\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3}+\frac{\left(\sin^3(bx+a)\right)}{9}+\frac{2\sin(bx+a)}{3}\right)}{b}}{b}$ |
| default | $\frac{\frac{da\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3b}-\frac{c\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3}+\frac{d\left(-\frac{(bx+a)\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3}+\frac{\left(\sin^3(bx+a)\right)}{9}+\frac{2\sin(bx+a)}{3}\right)}{b}}{b}$ |
| norman | $\frac{-\frac{4c\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4c}{3b}+\frac{4d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{3b^2}+\frac{32d\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{9b^2}+\frac{4d\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^2}-\frac{2dx}{3b}-\frac{2dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{2dx\left(\tan^4\left(\frac{bx}{2}\right.\right.}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/b*d*a*(2+sin(b*x+a)^2)*cos(b*x+a)-1/3*c*(2+sin(b*x+a)^2)*cos(b*x+a
)+1/b*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(
b*x+a)))
```

**Maxima** [A]
time = 0.33, size = 104, normalized size = 1.39

$$\frac{12\left(\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)\right)c-\frac{12\left(\cos(bx+a)^{3}-3\,\cos(bx+a)\right)ad}{b}+\frac{(3\,(bx+a)\cos(3\,bx+3\,a)-27\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)+27\,\sin(bx+a))d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(cos(b*x + a)^3 - 3*cos(b*x + a))*c - 12*(cos(b*x + a)^3 - 3*cos(b
*x + a))*a*d/b + (3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a)
- sin(3*b*x + 3*a) + 27*sin(b*x + a))*d/b)/b
```

**Fricas** [A]
time = 0.44, size = 62, normalized size = 0.83

$$\frac{3\left(bdx+bc\right)\cos\left(bx+a\right)^{3}-9\left(bdx+bc\right)\cos\left(bx+a\right)-\left(d\cos\left(bx+a\right)^{2}-7\,d\right)\sin\left(bx+a\right)}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*d*x + b*c)*cos(b*x + a)^3 - 9*(b*d*x + b*c)*cos(b*x + a) - (d*cos
(b*x + a)^2 - 7*d)*sin(b*x + a))/b^2
```

**Sympy** [A]
time = 0.19, size = 126, normalized size = 1.68

$$\begin{cases} -\frac{c\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)}{b}-\frac{2c\cos^{3}\left(a+bx\right)}{3b}-\frac{dx\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)}{b}-\frac{2dx\cos^{3}\left(a+bx\right)}{3b}+\frac{7d\sin^{3}\left(a+bx\right)}{9b^{2}}+\frac{2d\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b^{2}} & \text{for}\ b\neq 0\\ \left(cx+\frac{dx^{2}}{2}\right)\sin^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)/b - 2*c*cos(a + b*x)**3/(3*b) -
d*x*sin(a + b*x)**2*cos(a + b*x)/b - 2*d*x*cos(a + b*x)**3/(3*b) + 7*d*sin(
a + b*x)**3/(9*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b, 0))
, ((c*x + d*x**2/2)*sin(a)**3, True))
```

**Giac** [A]
time = 4.31, size = 69, normalized size = 0.92

$$\frac{\left(bdx+bc\right)\cos\left(3\,bx+3\,a\right)}{12\,b^{2}}-\frac{3\left(bdx+bc\right)\cos\left(bx+a\right)}{4\,b^{2}}-\frac{d\sin\left(3\,bx+3\,a\right)}{36\,b^{2}}+\frac{3\,d\sin\left(bx+a\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 3/4*(b*d*x + b*c)*cos(b*x + a)/b^
2 - 1/36*d*sin(3*b*x + 3*a)/b^2 + 3/4*d*sin(b*x + a)/b^2
```

**Mupad** [B]
time = 0.63, size = 79, normalized size = 1.05

$$\frac{7\,d\,\sin\left(a+b\,x\right)}{9\,b^2}-\frac{c\,\cos\left(a+b\,x\right)-\frac{c\,{\cos\left(a+b\,x\right)}^3}{3}+d\,x\,\cos\left(a+b\,x\right)-\frac{d\,x\,{\cos\left(a+b\,x\right)}^3}{3}}{b}-\frac{d\,{\cos\left(a+b\,x\right)}^2\,\sin\left(a+b\,x\right)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] (7*d*sin(a + b*x))/(9*b^2) - (c*cos(a + b*x) - (c*cos(a + b*x)^3)/3 + d*x*c
os(a + b*x) - (d*x*cos(a + b*x)^3)/3)/b - (d*cos(a + b*x)^2*sin(a + b*x))/(
9*b^2)
```

# 3.20 $\int \frac{\sin^3(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=121

$$-\frac{\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d}+\frac{3\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}\right.}{4d}$$

```
[Out] 3/4*cos(a-b*c/d)*Si(b*c/d+b*x)/d-1/4*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d-1
/4*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+3/4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

**Rubi** [A]
time = 0.17, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3393, 3384, 3380, 3383}

$$-\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x),x]
```

```
[Out] -1/4*(CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/d + (3*CosIntegr
al[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (3*Cos[a - (b*c)/d]*SinIntegral
[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*
x])/(4*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{3\sin(a+bx)}{4(c+dx)}-\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx \\
&= -\left(\frac{1}{4}\int \frac{\sin(3a+3bx)}{c+dx}\,dx\right)+\frac{3}{4}\int \frac{\sin(a+bx)}{c+dx}\,dx \\
&= -\left(\frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx\right)+\frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,d\cdots \\
&= -\frac{\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d}+\frac{3\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{4d}\cdots
\end{aligned}
$$

**Mathematica [A]**
time = 0.16, size = 102, normalized size = 0.84

$$
-\frac{\mathrm{Ci}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)-3\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)-3\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x),x]
[Out] -1/4*(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - 3*CosIntegral[b
*(c/d + x)]*Sin[a - (b*c)/d] - 3*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]
+ Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d
```

**Maple [A]**
time = 0.06, size = 172, normalized size = 1.42

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{b\left(-\frac{3\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+cb)}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\frac{3\,\mathrm{cosineIntegral}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}\right)}{12}+3b\left(-\frac{\mathrm{sinI}\cdots}{\cdots}\right.\cdots}{b}$ |
| default | $\frac{-\frac{b\left(-\frac{3\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+cb)}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\frac{3\,\mathrm{cosineIntegral}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}\right)}{12}+3b\left(-\frac{\mathrm{sinI}\cdots}{\cdots}\right.\cdots}{b}$ |
| risch | $-\frac{ie^{\frac{3i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,-3ibx-3ia-\frac{3(-iad+ibc)}{d}\right)}{8d}+\frac{ie^{-\frac{3i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,3ibx+3ia-\frac{3i(da-cb)}{d}\right)}{8d}-3ie^{-\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/12*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3
*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+3/4*b*(-Si(-b*x-a-(-a*d+b*c
)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.36, size = 279, normalized size = 2.31

$$-\frac{3\,b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(-i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+3\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(
-I*exp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral
_e(1, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b*(
exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -(
I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b*(exp_integral_e(
1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*(-I*b*c - I
*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

**Fricas** [A]
time = 0.40, size = 154, normalized size = 1.27

$$\frac{3\left(\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-\left(\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-2\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right)+6\,\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*sin
(-(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*
x + b*c)/d))*sin(-3*(b*c - a*d)/d) - 2*cos(-3*(b*c - a*d)/d)*sin_integral(3
*(b*d*x + b*c)/d) + 6*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x), x)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.76, size = 6296, normalized size = 52.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integral(b*x + b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*imag_part(cos
_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*sin_integra
l((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 6*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*real_part(cos_integral(-b*x - b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*real_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*real_part(co
s_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^
2 + 3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2 - 3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a
)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 6*sin_integral((b*d*x + b*c)/d)*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 12*imag_part(cos_integral(b*x + b*
c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*imag_par
t(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d) - 24*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integral(b*x + b*c/
d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*imag_part(cos_integral(-
b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*si
n_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 -
6*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2
+ 4*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/
```

```
2*b*c/d)*tan(1/2*b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan
(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + ima
g_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + 3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*sin_integr
al((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 - 3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 + 3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x
 + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*sin_integral(
(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
+ 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d) - 6*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a
)*tan(3/2*b*c/d)^2 - 6*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*t
an(1/2*a)*tan(3/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan
(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*real_part(cos_integra
l(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*real_p
art(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
 + 6*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d) - 6*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d) - 6*real_part(cos_integ...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (a+b x)^3}{c+d x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(c + d*x),x)
```

```
[Out] int(sin(a + b*x)^3/(c + d*x), x)
```

# 3.21 $\int \frac{\sin^3(a+bx)}{(c+dx)^2} dx$

**Optimal**. Leaf size=145

$$\frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin^3(a+bx)}{d(c+dx)}-\frac{3b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\text{s}}{}$$

```
[Out] -3/4*b*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^2+3/4*b*Ci(b*c/d+b*x)*cos(a-b*c
/d)/d^2+3/4*b*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-3/4*b*Si(b*c/d+b*x)*si
n(a-b*c/d)/d^2-sin(b*x+a)^3/d/(d*x+c)
```

**Rubi** [A]
time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3394, 3384, 3380, 3383}

$$\frac{3b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{3b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin^3(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] (3*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) - (3*b*Cos[3*a -
(3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]^3/(d*(c +
 d*x)) - (3*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*S
in[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx &= -\frac{\sin^3(a+bx)}{d(c+dx)}+\frac{(3b)\int \left(\frac{\cos(a+bx)}{4(c+dx)}-\frac{\cos(3a+3bx)}{4(c+dx)}\right)dx}{d}\\ &= -\frac{\sin^3(a+bx)}{d(c+dx)}+\frac{(3b)\int \frac{\cos(a+bx)}{c+dx}\,dx}{4d}-\frac{(3b)\int \frac{\cos(3a+3bx)}{c+dx}\,dx}{4d}\\ &= -\frac{\sin^3(a+bx)}{d(c+dx)}-\frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d}+\frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}}{4d}\\ &= \frac{3b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin^3(a+bx)}{d(c+dx)}-\frac{3b\sin}{}\end{aligned}$$

## Mathematica [A]
time = 0.69, size = 175, normalized size = 1.21

$$\frac{3b(c+dx)\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)-3b(c+dx)\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3b(c+dx)}{d}\right)-3d\cos(bx)\sin(a)+d\cos(3bx)\sin(3a)-3d\cos(a)\sin(bx)+d\cos(3a)\sin(3bx)-3b(c+dx)\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)+3b(c+dx)\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] (3*b*(c + d*x)*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - 3*b*(c + d*x)*Co
s[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - 3*d*Cos[b*x]*Sin[a] + d
*Cos[3*b*x]*Sin[3*a] - 3*d*Cos[a]*Sin[b*x] + d*Cos[3*a]*Sin[3*b*x] - 3*b*(c
 + d*x)*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*(c + d*x)*Sin[3*a -
 (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d^2*(c + d*x))
```

## Maple [A]
time = 0.10, size = 245, normalized size = 1.69

| method | result |
|---|---|
| derivativedivides | $-\frac{b^2\left(-\frac{3\sin(3bx+3a)}{(-da+cb+d(bx+a))d}+\frac{-\frac{9\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+cb)}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}}{d}+\frac{9\,\mathrm{cosineIntegral}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos}{d}\right.}{12}$ |

| | |
|---|---|
| default | $-\frac{b^2\left(-\frac{3\sin(3bx+3a)}{(-da+cb+d(bx+a))d}+\frac{-\frac{9\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+cb)}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}+\frac{9\,\mathrm{cosineIntegral}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos(}{d}}{d}\right.}{12}$ |
| risch | $\frac{3b\,e^{\frac{3i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,-3ibx-3ia-\frac{3(-iad+ibc)}{d}\right)}{8d^2}+\frac{3b\,e^{-\frac{3i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,3ibx+3ia-\frac{3i(da-cb)}{d}\right)}{8d^2}-\frac{3b\,e}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/12*b^2*(-3*sin(3*b*x+3*a)/(-d*a+c*b+d*(b*x+a))/d+3*(-3*Si(-3*b*x-3*
a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(
3*(-a*d+b*c)/d)/d)/d)+3/4*b^2*(-sin(b*x+a)/(-d*a+c*b+d*(b*x+a))/d+(-Si(-b*x
-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/
d)/d)/d))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.41, size = 306, normalized size = 2.11

$$-\frac{3\,b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(-i\,E_2\left(\frac{3(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{3(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+3\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b^2\left(E_2\left(\frac{3(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{3(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)}{8\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp
_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b
^2*(-I*exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_inte
gral_e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3
*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e
(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^2*(exp_int
egral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x + a
)*d^2 - a*d^2)*b)
```

**Fricas** [A]
time = 0.40, size = 238, normalized size = 1.64

$$\frac{6\,(bdx+bc)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-6\,(bdx+bc)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+8\left(d\cos(bx+a)^2-d\right)\sin(bx+a)}{8\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d)
- 6*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 3*((b
*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*
```

```
d*x + b*c)/d))*cos(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*cos_integral(3*(b*d*x
 + b*c)/d) + (b*d*x + b*c)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c -
a*d)/d) + 8*(d*cos(b*x + a)^2 - d)*sin(b*x + a))/(d^3*x + c*d^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x)**2, x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(137) = 274$.
time = 4.16, size = 1000, normalized size = 6.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-3*(b*c - a*d
)/d)*cos_integral(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c -
a*d)/d) + 3*b^3*c*cos(-3*(b*c - a*d)/d)*cos_integral(3*((d*x + c)*(b - b*c/
(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*a*b^2*d*cos(-3*(b*c - a*d)/d
)*cos_integral(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d
)/d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c - a*d)
/d)*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d
)/d) - 3*b^3*c*cos(-(b*c - a*d)/d)*cos_integral(((d*x + c)*(b - b*c/(d*x +
c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*cos(-(b*c - a*d)/d)*cos_int
egral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*(d
*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*sin(-(b*c - a*d)/d)*sin_int
egral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*b
^3*c*sin(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*sin(-(b*c - a*d)/d)*sin_integral(-((
d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*(d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*sin(-3*(b*c - a*d)/d)*sin_integral(
-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*b^3*c
*sin(-3*(b*c - a*d)/d)*sin_integral(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d) - 3*a*b^2*d*sin(-3*(b*c - a*d)/d)*sin_integral(-
3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*b^2*d*
sin(-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + b^2*d*sin(-3*(d*x +
```

```
c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x +
c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+bx)^3}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(c + d*x)^2,x)
[Out] int(sin(a + b*x)^3/(c + d*x)^2, x)
```

# 3.22 $\int \frac{\sin^3(a+bx)}{(c+dx)^3} \, dx$

**Optimal**. Leaf size=184

$$\frac{9b^2\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3}-\frac{3b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d^3}-\frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}-3$$

```
[Out] -3/8*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3+9/8*b^2*cos(3*a-3*b*c/d)*Si(3*b*c/d
+3*b*x)/d^3+9/8*b^2*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-3/8*b^2*Ci(b*c/d
+b*x)*sin(a-b*c/d)/d^3-3/2*b*cos(b*x+a)*sin(b*x+a)^2/d^2/(d*x+c)-1/2*sin(b*
x+a)^3/d/(d*x+c)^2
```

**Rubi** [A]
time = 0.24, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3395, 3384, 3380, 3383, 3393}

$$\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b\sin^2(a+bx)\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^3) - (3*b^
2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - (3*b*Cos[a + b*x]*
Sin[a + b*x]^2)/(2*d^2*(c + d*x)) - Sin[a + b*x]^3/(2*d*(c + d*x)^2) - (3*b
^2*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx = -\frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin^3(a+bx)}{2d(c+dx)^2} + \frac{(3b^2)\int \frac{\sin(a+bx)}{c+dx}\,dx}{d^2} - \frac{(9b^2)\int \frac{\sin^3(a+bx)}{c+dx}\,d}{2d^2}$$

$$= -\frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin^3(a+bx)}{2d(c+dx)^2} - \frac{(9b^2)\int \left(\frac{3\sin(a+bx)}{4(c+dx)} - \frac{\sin(3a+3bx)}{4(c+dx)}\right)dx}{2d^2} + $$

$$= \frac{3b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} - \frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cos\left(a-\right.}{}$$

$$= \frac{3b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} - \frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cos\left(a-\right.}{}$$

$$= \frac{9b^2\operatorname{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3} - \frac{3b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d^3} - \frac{3b\cos(a+bx)\sin^2(a+}{2d^2(c+dx)}$$

**Mathematica [A]**
time = 0.52, size = 221, normalized size = 1.20

$$\frac{-6d\cos(bx)(b(c+dx)\cos(a)+d\sin(a))+2d\cos(3bx)(3b(c+dx)\cos(3a)+d\sin(3a))+6d(-d\cos(a)+b(c+dx)\sin(a))\sin(bx)+2d(d\cos(3a)-3b(c+dx)\sin(3a))\sin(3bx)+6b^2(c+dx)^2\left(3\operatorname{Ci}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)-\operatorname{Ci}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)-\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\frac{c}{d}+x\right)\right)+3\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3b(c+dx)}{d}\right)\right)}{16d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (-6*d*Cos[b*x]*(b*(c + d*x)*Cos[a] + d*Sin[a]) + 2*d*Cos[3*b*x]*(3*b*(c + d
*x)*Cos[3*a] + d*Sin[3*a]) + 6*d*(-(d*Cos[a]) + b*(c + d*x)*Sin[a])*Sin[b*x
```

```
] + 2*d*(d*Cos[3*a] - 3*b*(c + d*x)*Sin[3*a])*Sin[3*b*x] + 6*b^2*(c + d*x)^
2*(3*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - CosIntegral[b*(c
/d + x)]*Sin[a - (b*c)/d] - Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*C
os[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]))/(16*d^3*(c + d*x)^2)
```

**Maple** [A]
time = 0.17, size = 318, normalized size = 1.73

| method | result |
|---|---|
| derivativedivides | $-\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2(-da+cb+d(bx+a))^2 d}+\frac{-\frac{9\cos(3bx+3a)}{2(-da+cb+d(bx+a))d}-\frac{9\left(-\frac{3\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+cb)}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\frac{3\,\mathrm{cosineI}\cdots}{\cdots}\right)}{2d}}{d}\cdots\right)}{12}\cdots$ |
| default | $-\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2(-da+cb+d(bx+a))^2 d}+\frac{-\frac{9\cos(3bx+3a)}{2(-da+cb+d(bx+a))d}-\frac{9\left(-\frac{3\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-da+cb)}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\frac{3\,\mathrm{cosineI}\cdots}{\cdots}\right)}{2d}}{d}\cdots\right)}{12}\cdots$ |
| risch | $\frac{9ib^2 e^{\frac{3i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,-3ibx-3ia-\frac{3(-iad+ibc)}{d}\right)}{16d^3}-\frac{9ib^2 e^{-\frac{3i(da-cb)}{d}}\,\mathrm{expIntegral}\left(1,3ibx+3ia-\frac{3i(da-cb)}{d}\right)}{16d^3}+\cdots$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/12*b^3*(-3/2*sin(3*b*x+3*a)/(-d*a+c*b+d*(b*x+a))^2/d+3/2*(-3*cos(3*
b*x+3*a)/(-d*a+c*b+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(
-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)+
3/4*b^3*(-1/2*sin(b*x+a)/(-d*a+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-d*a+c*
b+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d
+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)/d))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.52, size = 341, normalized size = 1.85

$$-\frac{3b^3\left(iE_3\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_3\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(-iE_3\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)+iE_3\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+3b^3\left(E_3\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+E_3\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b^3\left(E_3\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)+E_3\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)}{8\left(b^2c^2d-2abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp
_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b
^3*(-I*exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_inte
```

```
gral_e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3
*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e
(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^3*(exp_int
egral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*
b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(172) = 344$.
time = 0.38, size = 401, normalized size = 2.18

$$\frac{24\,(bd^2x+bcd)\cos(bx+a)^3+18\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-6\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-24\,(bd^2x+bcd)\cos(bx+a)+8\,(d^2\cos(bx+a)^2-d^2)\sin(bx+a)-3\,\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+9\,\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{16\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(24*(b*d^2*x + b*c*d)*cos(b*x + a)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 6*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*
c)/d) - 24*(b*d^2*x + b*c*d)*cos(b*x + a) + 8*(d^2*cos(b*x + a)^2 - d^2)*si
n(b*x + a) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x +
 b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c
)/d))*sin(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_in
tegral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integ
ral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*
d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x)**3, x)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.72, size = 116534, normalized size = 633.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 3*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^
2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*
x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x
^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*sin_integ
ral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*real_part(cos_integral(
b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d) + 18*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*ta
n(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*t
an(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1
/2*b*c/d)^2 + 6*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + 6*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*
b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*
d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d
*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*im
ag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*imag_part(c
os_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*imag_part(cos_inte
gral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*sin_integral(3*(b*d*x
 + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 + 9*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 3*b^2*d
^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2
```

```
+ 18*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2 - 12*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d) + 12*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2
4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*real_
part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*real_part(cos_i
ntegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 9*b^2*d^2*x^2*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*
d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*imag_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*t
an(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 6
*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^3}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(c + d*x)^3,x)
```

```
[Out] int(sin(a + b*x)^3/(c + d*x)^3, x)
```

# 3.23 $\int (c+dx)^3 \csc(a+bx)\,dx$

**Optimal**. Leaf size=185

$$-\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^2(c+dx)\text{Li}_3\left(-e^{i\ldots}\right.}{b^3}$$

```
[Out] -2*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*
x+a)))/b^2-3*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*poly
log(3,-exp(I*(b*x+a)))/b^3+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*
d^3*polylog(4,-exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

**Rubi** [A]
time = 0.09, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4268, 2611, 6744, 2320, 6724}

$$-\frac{6id^3\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[
2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))
])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d
*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x
))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3 \csc(a+bx)\,dx &= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)\,dx}{b} + \frac{(3d)\int(\ldots}{\ldots} \\
&= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\mathrm{Li}_2(\ldots}{b^2} \\
&= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\mathrm{Li}_2(\ldots}{b^2} \\
&= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\mathrm{Li}_2(\ldots}{b^2} \\
&= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\mathrm{Li}_2(\ldots}{b^2}
\end{aligned}
$$

**Mathematica [A]**
time = 0.34, size = 364, normalized size = 1.97

$$
\frac{-2b^3c^3\tanh^{-1}\left(e^{i(a+bx)}\right)+3b^3c^2dx\log\left(1-e^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1-e^{i(a+bx)}\right)+b^3d^3x^3\log\left(1-e^{i(a+bx)}\right)-3b^3c^2dx\log\left(1+e^{i(a+bx)}\right)-3b^3cd^2x^2\log\left(1+e^{i(a+bx)}\right)-b^3d^3x^3\log\left(1+e^{i(a+bx)}\right)+3ib^2d(c+dx)^2\mathrm{Li}_2\left(-e^{i(a+bx)}\right)-3ib^2d(c+dx)^2\mathrm{Li}_2\left(e^{i(a+bx)}\right)-6bcd^2\mathrm{Li}_3\left(-e^{i(a+bx)}\right)-6bd^3x\mathrm{Li}_3\left(-e^{i(a+bx)}\right)+6bcd^2\mathrm{Li}_3\left(e^{i(a+bx)}\right)+6bd^3x\mathrm{Li}_3\left(e^{i(a+bx)}\right)-6id^3\mathrm{Li}_4\left(-e^{i(a+bx)}\right)+6id^3\mathrm{Li}_4\left(e^{i(a+bx)}\right)}{b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x],x]
```

[Out] (-2*b^3*c^3*ArcTanh[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - E^(I*(a + b*x)
)] + 3*b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a
 + b*x))] - 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1
+ E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] + (3*I)*b^2*d*(c
+ d*x)^2*PolyLog[2, -E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2,
E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*PolyL
og[3, -E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^3*x
*PolyLog[3, E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] + (6*
I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(167) = 334$.
time = 0.07, size = 633, normalized size = 3.42

| method | result |
|---|---|
| risch | $-\frac{2c^3 \operatorname{arctanh}(e^{i(bx+a)})}{b} + \frac{6c\, d^2 \operatorname{polylog}(3, e^{i(bx+a)})}{b^3} - \frac{6c\, d^2 \operatorname{polylog}(3, -e^{i(bx+a)})}{b^3} + \frac{2d^3 a^3 \operatorname{arctanh}(e^{i(bx+a)})}{b^4} + \frac{6d^3 \operatorname{polylog}(}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*csc(b*x+a),x,method=_RETURNVERBOSE)

[Out] 6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a))
)+2/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))
*x-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x-2/b*c^3*arctanh(exp(I*(b*x+a)))+3
/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/b*
c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2-3/b*c^2
*d*ln(exp(I*(b*x+a))+1)*x-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+3/b*c^2*d*ln(1
-exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I/b^2*c*d^2*
polylog(2,-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+1/b*d^3*ln(
1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/b*d^3*ln(exp(I*(
b*x+a))+1)*x^3-1/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-6/b^3*c*d^2*a^2*arctanh(e
xp(I*(b*x+a)))+6/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,
exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*c^2*
d*polylog(2,exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-6*I*d^
3*polylog(4,-exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(161) = 322$.
time = 0.39, size = 716, normalized size = 3.87

$$-\frac{2c^3\log(\cot(bx+a)+\csc(bx+a)) - \frac{6ac^2d\log(\cot(bx+a)+\csc(bx+a))}{b} + \frac{6a^2cd^2\log(\cot(bx+a)+\csc(bx+a))}{b^2} - \frac{2a^3d^3\log(\cot(bx+a)+\csc(bx+a))}{b^3} + \frac{\text{[illegible]}}{2b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a),x, algorithm="maxima")

[Out] -1/2*(2*c^3*log(cot(b*x + a) + csc(b*x + a)) - 6*a*c^2*d*log(cot(b*x + a) +
 csc(b*x + a))/b + 6*a^2*c*d^2*log(cot(b*x + a) + csc(b*x + a))/b^2 - 2*a^3

```
*d^3*log(cot(b*x + a) + csc(b*x + a))/b^3 + (12*I*d^3*polylog(4, -e^(I*b*x
+ I*a)) - 12*I*d^3*polylog(4, e^(I*b*x + I*a)) - 2*(-I*(b*x + a)^3*d^3 + 3*
(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^
2*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)
^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c
*d^2 - I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 6*(
I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2
- I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 6*(-I*b^2*c^2*d + 2*I*a*b*c
*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*
dilog(e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + si
n(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^
3 - a*d^3)*polylog(3, -e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d
^3)*polylog(3, e^(I*b*x + I*a)))/b^3)/b
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(161) = 322$.
time = 0.40, size = 820, normalized size = 4.43

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d
^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)
) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) -
 I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1
/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^
2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3
 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^
3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x + b*c*d^2)*polylog(
3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*
x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) +
```

```
 I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b
*x + a)))/b^4
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \csc (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a),x)
[Out] Integral((c + d*x)**3*csc(a + b*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*csc(b*x + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\sin (a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/sin(a + b*x),x)
[Out] int((c + d*x)^3/sin(a + b*x), x)
```

# 3.24 $\int (c+dx)^2 \csc(a+bx)\,dx$

**Optimal**. Leaf size=123

$$-\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{2id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2}{}$$

```
[Out] -2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+
a)))/b^2-2*I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I
*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

**Rubi** [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4268, 2611, 2320, 6724}

$$-\frac{2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog[2,
 -E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b
^2 - (2*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a +
 b*x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2 \csc(a+bx)\,dx &= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-e^{i(a+bx)}\right)\,dx}{b} + \frac{(2d)\int\\ &= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2(}{b^2}\\ &= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2(}{b^2}\\ &= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2(}{b^2}\end{aligned}$$

**Mathematica** [A]
time = 0.19, size = 210, normalized size = 1.71

$$\frac{-2b^2c^2\tanh^{-1}\left(e^{i(a+bx)}\right)+2b^2cdx\log\left(1-e^{i(a+bx)}\right)+b^2d^2x^2\log\left(1-e^{i(a+bx)}\right)-2b^2cdx\log\left(1+e^{i(a+bx)}\right)-b^2d^2x^2\log\left(1+e^{i(a+bx)}\right)+2ibd(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)-2ibd(c+dx)\mathrm{Li}_2\left(e^{i(a+bx)}\right)-2d^2\mathrm{Li}_3\left(-e^{i(a+bx)}\right)+2d^2\mathrm{Li}_3\left(e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x],x]
```

```
[Out] (-2*b^2*c^2*ArcTanh[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))]
 + b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x
))] - b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2,
 -E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 2*d^
2*PolyLog[3, -E^(I*(a + b*x))] + 2*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(111) = 222.
time = 0.05, size = 361, normalized size = 2.93

| method | result |
|---|---|
| risch | $-\frac{2d^2a^2\,\mathrm{arctanh}\left(e^{i(bx+a)}\right)}{b^3}+\frac{4cda\,\mathrm{arctanh}\left(e^{i(bx+a)}\right)}{b^2}-\frac{2id^2\,\mathrm{polylog}\left(2,e^{i(bx+a)}\right)x}{b^2}+\frac{2id^2\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)x}{b^2}-\frac{2c^2\,\mathrm{arcta}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))+4/b^2*c*d*a*arctanh(exp(I*(b*x+a)))+
2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a
)))-2/b*c^2*arctanh(exp(I*(b*x+a)))-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b^3*
d^2*ln(exp(I*(b*x+a))+1)*a^2+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))+2*d^2*p
olylog(3,exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+1/b*d^2*l
n(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-2*I/b^2*d^2*poly
log(2,exp(I*(b*x+a)))*x-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*ln(exp(I*(
b*x+a))+1)*a+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*
a
```

**Maxima** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(107) = 214$.
time = 0.36, size = 398, normalized size = 3.24

$$-\frac{2\,c^2\log\left(\cot\left(bx+a\right)+\csc\left(bx+a\right)\right)-\frac{4\,acd\log(\cot(bx+a)+\csc(bx+a))}{b}+\frac{2\,a^2d^2\log(\cot(bx+a)+\csc(bx+a))}{b^2}+\frac{4\,d^2\mathrm{Li}_3(-e^{(i\,bx+i\,a)})-4\,d^2\mathrm{Li}_3(e^{(i\,bx+i\,a)})-2\left(-i\,(bx+a)^2d^2+2\left(-i\,bcd+i\,ad^2\right)(bx+a)\right)\arctan(\sin(bx+a),\cos(bx+a)+1)-2\left(-i\,(bx+a)^2d^2+2\left(-i\,bcd+i\,ad^2\right)(bx+a)\right)\arctan(\sin(bx+a),-\cos(bx+a)+1)-4\left(i\,bcd+i\,(bx+a)d^2-i\,ad^2\right)\mathrm{Li}_2\left(-e^{(i\,bx+i\,a)}\right)-4\left(-i\,bcd-i\,(bx+a)d^2+i\,ad^2\right)\mathrm{Li}_2\left(e^{(i\,bx+i\,a)}\right)+\left((bx+a)^2d^2+2\left(bcd-ad^2\right)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)-\left((bx+a)^2d^2+2\left(bcd-ad^2\right)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\cos(bx+a)+1\right)}{b^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^2*log(cot(b*x + a) + csc(b*x + a)) - 4*a*c*d*log(cot(b*x + a) + c
sc(b*x + a))/b + 2*a^2*d^2*log(cot(b*x + a) + csc(b*x + a))/b^2 + (4*d^2*po
lylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*a)) - 2*(-I*(b*x
 + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), cos(b
*x + a) + 1) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*ar
ctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(I*b*c*d + I*(b*x + a)*d^2 - I*a
*d^2)*dilog(-e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*di
log(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^2*d^2 +
2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*
x + a) + 1))/b^2)/b
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(107) = 214$.
time = 0.37, size = 504, normalized size = 4.10

$$\frac{2\,d^2\mathrm{polylog}(3,\cos(bx+a)+i\,\sin(bx+a))+2\,d^2\mathrm{polylog}(3,\cos(bx+a)-i\,\sin(bx+a))-2\,d^2\mathrm{polylog}(3,-\cos(bx+a)+i\,\sin(bx+a))-2\,d^2\mathrm{polylog}(3,-\cos(bx+a)-i\,\sin(bx+a))-2\,(i\,bd^2x+i\,bcd){\rm Li}_2(\cos(bx+a)+i\,\sin(bx+a))-2\,(-i\,bd^2x-i\,bcd){\rm Li}_2(\cos(bx+a)-i\,\sin(bx+a))-2\,(i\,bd^2x+i\,bcd){\rm Li}_2(-\cos(bx+a)+i\,\sin(bx+a))-2\,(-i\,bd^2x-i\,bcd){\rm Li}_2(-\cos(bx+a)-i\,\sin(bx+a))-(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(\cos(bx+a)+i\,\sin(bx+a)+1)-(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(\cos(bx+a)-i\,\sin(bx+a)+1)+(b^2c^2-2\,abcd+a^2d^2)\log(-\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}i\,\sin(bx+a)+\frac{1}{2})+(b^2c^2-2\,abcd+a^2d^2)\log(-\frac{1}{2}\,\cos(bx+a)-\frac{1}{2}i\,\sin(bx+a)+\frac{1}{2})+(b^2d^2x^2+2\,b^2cdx+2\,abcd-a^2d^2)\log(-\cos(bx+a)+i\,\sin(bx+a)+1)+(b^2d^2x^2+2\,b^2cdx+2\,abcd-a^2d^2)\log(-\cos(bx+a)-i\,\sin(bx+a)+1)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b
*c*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog
(cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x +
a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x + a) - I*sin
(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin
(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I
*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a)
+ 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c
*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^
2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/b^3
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \csc (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^2}{\sin (a + b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/sin(a + b*x),x)
```

```
[Out] int((c + d*x)^2/sin(a + b*x), x)
```

# 3.25 $\int (c+dx)\csc(a+bx)\,dx$

**Optimal**. Leaf size=67

$$-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}$$

```
[Out] -2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+I*d*polylog(2,-exp(I*(b*x+a)))/b^2-I*d
*polylog(2,exp(I*(b*x+a)))/b^2
```

**Rubi** [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4268, 2317, 2438}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x)
)])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\,dx = -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{d\int \log\left(1+e^{i(a+bx)}\right)}{b}$$

$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(id)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^2} - (id)\text{Subs}$$

$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}$$

**Mathematica** [A]
time = 0.06, size = 134, normalized size = 2.00

$$-\frac{c\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} + \frac{c\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} + \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)+i\left(\text{Li}_2\left(-e^{i(a+bx)}\right)-\text{Li}_2\left(e^{i(a+bx)}\right)\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x],x]
```

```
[Out] -((c*Log[Cos[a/2 + (b*x)/2]])/b) + (c*Log[Sin[a/2 + (b*x)/2]])/b + (d*((a +
 b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) - a*Log[Tan[(a
+ b*x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])
))/b^2
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(59) = 118.
time = 0.01, size = 124, normalized size = 1.85

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{da\ln(\csc(bx+a)-\cot(bx+a))}{b}+c\ln(\csc(bx+a)-\cot(bx+a))+\frac{d\left((bx+a)\ln\left(1-e^{i(bx+a)}\right)-(bx+a)\ln\left(e^{i(bx+a)}+1\right)+i\,\text{dilog}\left(e^{i(bx}\right.\right.}{b}}{b}$ |
| default | $\frac{-\frac{da\ln(\csc(bx+a)-\cot(bx+a))}{b}+c\ln(\csc(bx+a)-\cot(bx+a))+\frac{d\left((bx+a)\ln\left(1-e^{i(bx+a)}\right)-(bx+a)\ln\left(e^{i(bx+a)}+1\right)+i\,\text{dilog}\left(e^{i(bx}\right.\right.}{b}}{b}$ |
| risch | $-\frac{2c\,\text{arctanh}\left(e^{i(bx+a)}\right)}{b}+\frac{d\ln\left(1-e^{i(bx+a)}\right)x}{b}+\frac{d\ln\left(1-e^{i(bx+a)}\right)a}{b^2}-\frac{id\,\text{polylog}\left(2,e^{i(bx+a)}\right)}{b^2}-\frac{d\ln\left(e^{i(bx+a)}+1\right)a}{b}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*ln(csc(b*x+a)-cot(b*x+a))+c*ln(csc(b*x+a)-cot(b*x+a))+1/b*d*(
(b*x+a)*ln(1-exp(I*(b*x+a)))-(b*x+a)*ln(exp(I*(b*x+a))+1)+I*dilog(exp(I*(b*
x+a))+1)-I*dilog(1-exp(I*(b*x+a)))))
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(55) = 110.
time = 0.34, size = 175, normalized size = 2.61

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)-2\left(-i\,bdx-i\,bc\right)\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)+1\right)-2i\,d\text{Li}_2\left(-e^{(i\,bx+i\,a)}\right)+2i\,d\text{Li}_2\left(e^{(i\,bx+i\,a)}\right)+\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2+2\cos\left(bx+a\right)+1\right)-\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2-2\cos\left(bx+a\right)+1\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x - I*b*c)*arctan2(sin(b*x + a)
, cos(b*x + a) + 1) - 2*I*d*dilog(-e^(I*b*x + I*a)) + 2*I*d*dilog(e^(I*b*x
+ I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a
) + 1) - (b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
 + 1))/b^2
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(55) = 110$.
time = 0.38, size = 252, normalized size = 3.76

$$\frac{-i\,d\mathrm{Li}_2(\cos(bx+a)+i\,\sin(bx+a))+i\,d\mathrm{Li}_2(\cos(bx+a)-i\,\sin(bx+a))-i\,d\mathrm{Li}_2(-\cos(bx+a)+i\,\sin(bx+a))+i\,d\mathrm{Li}_2(-\cos(bx+a)-i\,\sin(bx+a))-(bdx+bc)\log(\cos(bx+a)+i\,\sin(bx+a)+1)-(bdx+bc)\log(\cos(bx+a)-i\,\sin(bx+a)+1)+(bc-ad)\log\left(-\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}i\,\sin(bx+a)+\frac{1}{2}\right)+(bc-ad)\log\left(-\frac{1}{2}\,\cos(bx+a)-\frac{1}{2}i\,\sin(bx+a)+\frac{1}{2}\right)+(bdx+ad)\log(-\cos(bx+a)+i\,\sin(bx+a)+1)+(bdx+ad)\log(-\cos(bx+a)-i\,\sin(bx+a)+1)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(-cos
(b*x + a) - I*sin(b*x + a)) - (b*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x +
a) + 1) - (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b*c - a*d
)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*
cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a)
+ I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) +
1))/b^2
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/sin(a + b*x),x)
[Out] int((c + d*x)/sin(a + b*x), x)
```

## 3.26 $\int \frac{\csc(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=17

$$\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(csc(b*x+a)/(d*x+c),x)
```

**Rubi** [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)}{c+dx}\,dx$$

**Mathematica** [A]
time = 4.02, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Csc[a + b*x]/(c + d*x), x]
```

**Maple** [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)/(d*x+c),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/(d*x + c), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)/(d*x + c), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (a+bx)}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)/(c + d*x), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/(d*x + c), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sin \left(a+b\,x\right)\,\left(c+d\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(c + d*x)),x)
[Out] int(1/(sin(a + b*x)*(c + d*x)), x)
```

# 3.27 $\int \frac{\csc(a+bx)}{(c+dx)^2} dx$

**Optimal**. Leaf size=17

$$\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(csc(b*x+a)/(d*x+c)^2,x)
```

**Rubi** [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{(c+dx)^2} dx = \int \frac{\csc(a+bx)}{(c+dx)^2} dx$$

**Mathematica** [A]
time = 4.79, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Csc[a + b*x]/(c + d*x)^2, x]
```

**Maple** [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*x+c)^2,x)
[Out] int(csc(b*x+a)/(d*x+c)^2,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(csc(b*x + a)/(d*x + c)^2, x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc{\left(a + bx \right)}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)/(c + d*x)**2, x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(csc(b*x + a)/(d*x + c)^2, x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(1/(sin(a + b*x)*(c + d*x)^2), x)
```

# 3.28 $\int (c+dx)^3 \csc^2(a+bx)\, dx$

**Optimal**. Leaf size=113

$$-\frac{i(c+dx)^3}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{3id^2(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{Li}_3\left(e^{2i(\right.}}{2b^4}$$

```
[Out] -I*(d*x+c)^3/b-(d*x+c)^3*cot(b*x+a)/b+3*d*(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/
b^2-3*I*d^2*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,exp(2
*I*(b*x+a)))/b^4
```

**Rubi** [A]
time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4269, 3798, 2221, 2611, 2320, 6724}

$$\frac{3d^3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{i(c+dx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - ((c + d*x)^3*Cot[a + b*x])/b + (3*d*(c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*PolyLog[2, E^((2*I)*(a
 + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\csc^2(a+bx)\,dx &= -\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\cot(a+bx)\,dx}{b}\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx}{b}\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(6d\cdots}{}\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{3id\cdots}{}\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{3id\cdots}{}\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{3id\cdots}{}
\end{aligned}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 384 vs. 2(113) = 226.
time = 6.45, size = 384, normalized size = 3.40

$$-\frac{d^3e^{-ia}\csc(a)\left(2b^2x^2\left(2be^{2ia}x+3i\left(-1+e^{2ia}\right)\log\left(1-e^{2i(a+bx)}\right)\right)+6b\left(-1+e^{2ia}\right)x\,\mathrm{Li}_2\left(e^{2i(a+bx)}\right)+3i\left(-1+e^{2ia}\right)\mathrm{Li}_3\left(e^{2i(a+bx)}\right)\right)}{4b^4}+\frac{3c^2d\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)}+\frac{\csc(a)\csc(a+bx)\left(c^3\sin(bx)+3c^2dx\sin(bx)+3cd^2x^2\sin(bx)+d^3x^3\sin(bx)\right)}{b}-\frac{3cd^2\csc(a)\sec(a)\left(b^2e^{i\tan^{-1}(\tan(a))}x^2+\frac{\left(ibx\left(-\pi+2\tan^{-1}(\tan(a))\right)-\pi\log\left(1+e^{-2ibx}\right)-2\left(bx+\tan^{-1}(\tan(a))\right)\log\left(1-e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+\pi\log(\cos(bx))+2\tan^{-1}(\tan(a))\log\left(\sin\left(bx+\tan^{-1}(\tan(a))\right)\right)+i\mathrm{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)\tan(a)}{\sqrt{1+\tan^2(a)}}\right)}{b^3\sqrt{\sec^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2,x]

[Out] -1/4*(d^3*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a))*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((2*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*PolyLog[3, E^((2*I)*(a + b*x))]))/(b^4*E^(I*a)) + (3*c^2*d*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c^3*Sin[b*x] + 3*c^2*d*x*Sin[b*x] + 3*c*d^2*x^2*Sin[b*x] + d^3*x^3*Sin[b*x]))/b - (3*c*d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(103) = 206$.
time = 0.07, size = 541, normalized size = 4.79

| method | result |
|---|---|
| risch | $\frac{6d^3 \operatorname{polylog}(3,-e^{i(bx+a)})}{b^4}+\frac{6d^3 \operatorname{polylog}(3,e^{i(bx+a)})}{b^4}+\frac{3d c^2 \ln(e^{i(bx+a)}+1)}{b^2}-\frac{6d^3 a^2 \ln(e^{i(bx+a)})}{b^4}+\frac{3d^3 a^2 \ln(e^{i(bx+a)}-1)}{b^4}+$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*csc(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 6*d^3/b^4*polylog(3,-exp(I*(b*x+a)))+6*d^3/b^4*polylog(3,exp(I*(b*x+a)))+3*d/b^2*c^2*ln(exp(I*(b*x+a))+1)-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+3*d^3/b^4*a^2*ln(exp(I*(b*x+a))-1)+3*d/b^2*c^2*ln(exp(I*(b*x+a))-1)-6*d/b^2*c^2*ln(exp(I*(b*x+a)))+3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2-3*d^3/b^4*a^2*ln(1-exp(I*(b*x+a)))+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2-2*I*d^3/b*x^3+4*I*d^3/b^4*a^3-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))-1)-12*I*d^2/b^2*c*a*x+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a+6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x+12*d^2/b^3*c*a*ln(exp(I*(b*x+a)))-6*d^2/b^3*c*a*ln(exp(I*(b*x+a))-1)-6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))-6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))+6*I*d^3/b^3*a^2*x-6*I*d^2/b*c*x^2-6*I*d^2/b^3*c*a^2-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(100) = 200$.
time = 0.43, size = 1654, normalized size = 14.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + si
n(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*((
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a
)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^2/((cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) + 3*((cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x +
2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*c^3/tan(b*x
+ a) + 6*a*c^2*d/(b*tan(b*x + a)) - 6*a^2*c*d^2/(b^2*tan(b*x + a)) + 2*a^3*
d^3/(b^3*tan(b*x + a)) - 2*(6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
 a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) +
(-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2
- a*d^3)*(b*x + a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(
2*b*x + 2*a) - (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*((b*x + a)^3*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x +
 a)*d^3 - a*d^3 - (b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (I*b
*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a
)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 - (b*c*d^2 + (b*x + a)*d^3 - a*d^3
)*cos(2*b*x + 2*a) - (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin(2*b*x + 2*
a))*dilog(e^(I*b*x + I*a)) + 3*(-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^
3)*(b*x + a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*cos(
2*b*x + 2*a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x
+ 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 3*(-I*(
b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a) + (I*(b*x + a)^2*d^3 +
2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^3 + 2*
(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
 + a)^2 - 2*cos(b*x + a) + 1) + 12*(I*d^3*cos(2*b*x + 2*a) - d^3*sin(2*b*x
+ 2*a) - I*d^3)*polylog(3, -e^(I*b*x + I*a)) + 12*(I*d^3*cos(2*b*x + 2*a) -
 d^3*sin(2*b*x + 2*a) - I*d^3)*polylog(3, e^(I*b*x + I*a)) + 4*(I*(b*x + a)
^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-2*I*b^3*c
os(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) + 2*I*b^3))/b
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than

twice the leaf count of optimal. 676 vs. $2(100) = 200$.
time = 0.40, size = 676, normalized size = 5.98

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^3*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*p
olylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -c
os(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a)
 - I*sin(b*x + a))*sin(b*x + a) - 6*(I*b*d^3*x + I*b*c*d^2)*dilog(cos(b*x +
 a) + I*sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(cos(b
*x + a) - I*sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(-
cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*(I*b*d^3*x + I*b*c*d^2)*dil
og(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*
d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(b*x + a)
 + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a)
+ 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a)
 + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*
b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 2
*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a))/(b
^4*sin(b*x + a))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \csc^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*csc(a + b*x)**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/sin(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^3/sin(a + b*x)^2, x)
```

# 3.29 $\int (c+dx)^2 \csc^2(a+bx)\,dx$

**Optimal**. Leaf size=83

$$-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}$$

```
[Out] -I*(d*x+c)^2/b-(d*x+c)^2*cot(b*x+a)/b+2*d*(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b^
2-I*d^2*polylog(2,exp(2*I*(b*x+a)))/b^3
```

**Rubi** [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4269, 3798, 2221, 2317, 2438}

$$-\frac{id^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b - ((c + d*x)^2*Cot[a + b*x])/b + (2*d*(c + d*x)*Log[1
- E^((2*I)*(a + b*x))])/b^2 - (I*d^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
```

```
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx)\,dx = -\frac{(c+dx)^2\cot(a+bx)}{b} + \frac{(2d)\int (c+dx)\cot(a+bx)\,dx}{b}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^2\cot(a+bx)}{b} - \frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^2\cot(a+bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{(2d^2)}{}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^2\cot(a+bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2} + \frac{(id^2)}{}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^2\cot(a+bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\mathrm{L}}{}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 181 vs. $2(83) = 166$.
time = 2.54, size = 181, normalized size = 2.18

$$\frac{\csc(a)\left(-2bcd(bx\cos(a)-\log(\sin(a+bx))\sin(a))+d^2\left(-b^2e^{i\tan^{-1}(\tan(a))}x^2\cos(a)\sqrt{\sec^2(a)}-\left(-ibx(\pi-2\tan^{-1}(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\tan^{-1}(\tan(a)))\log\left(1-e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+\pi\log(\cos(bx))+2\tan^{-1}(\tan(a))\log\left(\sin\left(bx+\tan^{-1}(\tan(a))\right)\right)+i\mathrm{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)\sin(a)\right)+b^2(c+dx)^2\csc(a+bx)\sin(bx)\right)}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2,x]
```

```
[Out] (Csc[a]*(-2*b*c*d*(b*x*Cos[a] - Log[Sin[a + b*x]]*Sin[a]) + d^2*(-(b^2*E^(I
*ArcTan[Tan[a]])*x^2*Cos[a]*Sqrt[Sec[a]^2]) - ((-I)*b*x*(Pi - 2*ArcTan[Tan[
a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*
I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b
*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Sin
[a]) + b^2*(c + d*x)^2*Csc[a + b*x]*Sin[b*x]))/b^3
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(77) = 154$.
time = 0.06, size = 276, normalized size = 3.33

| method | result |
|---|---|
| risch | $-\frac{2i(d^2x^2+2cdx+c^2)}{b(\mathrm{e}^{2i(bx+a)}-1)}+\frac{2dc\ln(\mathrm{e}^{i(bx+a)}-1)}{b^2}-\frac{4dc\ln(\mathrm{e}^{i(bx+a)})}{b^2}+\frac{2dc\ln(\mathrm{e}^{i(bx+a)}+1)}{b^2}-\frac{2id^2x^2}{b}-\frac{4id^2ax}{b^2}-\frac{2id^2a^2}{b^3}+\frac{2d^2}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))-1)+2*d/b^2*c*ln(exp(I*(b*x+a
))-1)-4*d/b^2*c*ln(exp(I*(b*x+a)))+2*d/b^2*c*ln(exp(I*(b*x+a))+1)-2*I*d^2/b
*x^2-4*I*d^2/b^2*a*x-2*I*d^2/b^3*a^2+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2
/b^3*ln(1-exp(I*(b*x+a)))*a-2*I*d^2/b^3*polylog(2,exp(I*(b*x+a)))+2*d^2/b^2
*ln(exp(I*(b*x+a))+1)*x-2*I*d^2/b^3*polylog(2,-exp(I*(b*x+a)))-2*d^2/b^3*a*
ln(exp(I*(b*x+a))-1)+4*d^2/b^3*a*ln(exp(I*(b*x+a)))
```

**Maxima** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(74) = 148$.
time = 0.38, size = 552, normalized size = 6.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b^2*c^2 + 2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (-I
*b*d^2*x - I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) +
1) - 2*(b*c*d*cos(2*b*x + 2*a) + I*b*c*d*sin(2*b*x + 2*a) - b*c*d)*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d^2*x*cos(2*b*x + 2*a) + I*b*d^2*x*s
in(2*b*x + 2*a) - b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*(b^
2*d^2*x^2 + 2*b^2*c*d*x)*cos(2*b*x + 2*a) + 2*(d^2*cos(2*b*x + 2*a) + I*d^2
*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) + 2*(d^2*cos(2*b*x + 2*a)
+ I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x + I*b*c
*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)*sin(2*b*x
+ 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*b*d^
2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)
*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1
) + 2*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x)*sin(2*b*x + 2*a))/(-I*b^3*cos(2*b*x +
 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(74) = 148$.
time = 0.38, size = 379, normalized size = 4.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (-I*d^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + I*d^2*dilog(cos
(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + I*d^2*dilog(-cos(b*x + a) + I*si
n(b*x + a))*sin(b*x + a) - I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(
b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x
 + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x +
a) + (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(
b*x + a) + (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2
)*sin(b*x + a) + (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*
sin(b*x + a) + (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*si
n(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a))/(b^3*sin(b
*x + a))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \csc^2 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^2}{\sin (a + b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/sin(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^2/sin(a + b*x)^2, x)
```

# 3.30 $\int (c+dx)\csc^2(a+bx)\,dx$

**Optimal**. Leaf size=29

$$-\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}$$

```
[Out] -(d*x+c)*cot(b*x+a)/b+d*ln(sin(b*x+a))/b^2
```

**Rubi** [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4269, 3556}

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2,x]
```

```
[Out] -(((c + d*x)*Cot[a + b*x])/b) + (d*Log[Sin[a + b*x]])/b^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\csc^2(a+bx)\,dx &= -\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\int\cot(a+bx)\,dx}{b}\\ &= -\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}\end{aligned}$$

**Mathematica** [A]
time = 0.08, size = 52, normalized size = 1.79

$$-\frac{dx\cot(a)}{b}-\frac{c\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}+\frac{dx\csc(a)\csc(a+bx)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*x*Cot[a])/b) - (c*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2 + (d*x*C
sc[a]*Csc[a + b*x]*Sin[b*x])/b
```

**Maple** [A]
time = 0.02, size = 53, normalized size = 1.83

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{da\cot(bx+a)}{b}-c\cot(bx+a)+\frac{d(-(bx+a)\cot(bx+a)+\ln(\sin(bx+a)))}{b}}{b}$ | 53 |
| default | $\frac{\frac{da\cot(bx+a)}{b}-c\cot(bx+a)+\frac{d(-(bx+a)\cot(bx+a)+\ln(\sin(bx+a)))}{b}}{b}$ | 53 |
| risch | $-\frac{2idx}{b}-\frac{2ida}{b^2}-\frac{2i(dx+c)}{b(e^{2i(bx+a)}-1)}+\frac{d\ln(e^{2i(bx+a)}-1)}{b^2}$ | 59 |
| norman | $\frac{-\frac{c}{2b}+\frac{c\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}-\frac{dx}{2b}+\frac{dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}-\frac{d\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}$ | 98 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b*d*a*cot(b*x+a)-c*cot(b*x+a)+1/b*d*(-(b*x+a)*cot(b*x+a)+ln(sin(b*x+
a))))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(29) = 58.
time = 0.29, size = 217, normalized size = 7.48

$$\frac{\frac{\left(\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)+\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\cos(bx+a)+1\right)-4(bx+a)\sin(2bx+2a)\right)d}{\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)b}-\frac{2c}{\tan(bx+a)}+\frac{2ad}{b\tan(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 2*c/tan(b*
x + a) + 2*a*d/(b*tan(b*x + a)))/b
```

**Fricas** [A]
time = 0.37, size = 46, normalized size = 1.59

$$\frac{d\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-\left(bdx+bc\right)\cos\left(bx+a\right)}{b^2\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (d*log(1/2*sin(b*x + a))*sin(b*x + a) - (b*d*x + b*c)*cos(b*x + a))/(b^2*si
n(b*x + a))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**2, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. 2(29) = 58.
time = 4.42, size = 1251, normalized size = 43.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 -
b*d*x*tan(1/2*b*x)^2 - 4*b*d*x*tan(1/2*b*x)*tan(1/2*a) + d*log(16*(tan(1/2*
b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/
2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6
- 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b
*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan
(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan
(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan
(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) - b*d*x*tan(1/2*
a)^2 + d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^
3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b
*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1
/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*t
an(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 +
 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/
2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(
1/2*a)^2 - b*c*tan(1/2*b*x)^2 - 4*b*c*tan(1/2*b*x)*tan(1/2*a) - b*c*tan(1/2
*a)^2 + b*d*x - d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*ta
```

```
n(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*
tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1
/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2
*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*
x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*
b*x) - d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^
3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b
*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1
/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*t
an(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 +
 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/
2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*a) + b*c)
/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2*a)^2 - b^2*tan(1
/2*b*x) - b^2*tan(1/2*a))
```

**Mupad [B]**
time = 1.18, size = 55, normalized size = 1.90

$$\frac{d\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^2}-\frac{\left(c+d\,x\right)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}-\frac{d\,x\,2\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/sin(a + b*x)^2,x)
```

```
[Out] (d*log(exp(a*2i)*exp(b*x*2i) - 1))/b^2 - ((c + d*x)*2i)/(b*(exp(a*2i + b*x*
2i) - 1)) - (d*x*2i)/b
```

# 3.31 $\int \frac{\csc^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\csc^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(csc(b*x+a)^2/(d*x+c),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Csc[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)}{c+dx}\,dx$$

**Mathematica** [A]
time = 3.98, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Csc[a + b*x]^2/(c + d*x), x]
```

**Maple** [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2/(d*x+c),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d
)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(
b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*c
os(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*x
^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^2*x + b*c*d + (b*d^2*x + b
*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*
x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x
+ b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b
*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x
+ a)), x) - 2*sin(2*b*x + 2*a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 +
 (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2/(d*x + c), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**2/(c + d*x), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*x + c), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(sin(a + b*x)^2*(c + d*x)), x)
```

# 3.32 $\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\csc^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(csc(b*x+a)^2/(d*x+c)^2,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Csc[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica** [A]
time = 4.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Csc[a + b*x]^2/(c + d*x)^2, x]
```

**Maple** [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2/(d*x+c)^2,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*((b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)
*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^
2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(sin(b
*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b
*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x
^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3
*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) - (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d
+ (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b
*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2
*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3
*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(
b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)
^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x)
- sin(2*b*x + 2*a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x
 + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x +
2*a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2{(a + bx)}}{(c + dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2/(c + d*x)**2, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*x + c)^2, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(1/(sin(a + b*x)^2*(c + d*x)^2), x)
```

# 3.33 $\int (c+dx)^3 \csc^3(a+bx)\,dx$

**Optimal**. Leaf size=309

$$-\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)}{2b}$$

```
[Out] -6*d^2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^3-(d*x+c)^3*arctanh(exp(I*(b*x+a))
)/b-3/2*d*(d*x+c)^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)*csc(b*x+a)/b+3*
I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*
x+a)))/b^2-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(
2,exp(I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3+3*d^2*(d
*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+
3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

**Rubi** [A]

time = 0.16, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4271, 4268, 2317, 2438, 2611, 6744, 2320, 6724}

$$\frac{3id^3\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{3id^3\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3,x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 - ((c + d*x)^3*ArcTanh[E^(I
*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot
[a + b*x]*Csc[a + b*x])/(2*b) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^
4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3
*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(
I*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (
3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((3*I)*d^3*PolyLog[4, -E
^(I*(a + b*x))])/b^4 + ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3 \csc^3(a+bx)\,dx &= -\frac{3d(c+dx)^2\csc(a+bx)}{2b^2} - \frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^3 \\
&= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\operatorname{cs}}{2b^2} \\
&= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\operatorname{cs}}{2b^2} \\
&= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\operatorname{cs}}{2b^2} \\
&= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\operatorname{cs}}{2b^2} \\
&= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\operatorname{cs}}{2b^2}
\end{aligned}
$$

**Mathematica** [A]
time = 2.52, size = 478, normalized size = 1.55

$$
-\frac{2b^3c^3\tanh^{-1}\left(e^{i(a+bx)}\right)+12bcd^2\tanh^{-1}\left(e^{i(a+bx)}\right)+b^2(c+dx)^2(3d+b(c+dx)\cot(a+bx))\csc(a+bx)-3b^3c^2dx\log\left(1-e^{i(a+bx)}\right)-6bd^3x\log\left(1-e^{i(a+bx)}\right)-3b^3cd^2x^2\log\left(1-e^{i(a+bx)}\right)-b^3d^3x^3\log\left(1-e^{i(a+bx)}\right)+3b^3c^2dx\log\left(1+e^{i(a+bx)}\right)+6bd^3x\log\left(1+e^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1+e^{i(a+bx)}\right)+b^3d^3x^3\log\left(1+e^{i(a+bx)}\right)-3id(2d^2+b^2(c+dx)^2)\operatorname{Li}_2\left(-e^{i(a+bx)}\right)+3id(2d^2+b^2(c+dx)^2)\operatorname{Li}_2\left(e^{i(a+bx)}\right)+6bcd^2\operatorname{Li}_3\left(-e^{i(a+bx)}\right)+6bd^3x\operatorname{Li}_3\left(-e^{i(a+bx)}\right)-6bcd^2\operatorname{Li}_3\left(e^{i(a+bx)}\right)-6bd^3x\operatorname{Li}_3\left(e^{i(a+bx)}\right)+6id^3\operatorname{Li}_4\left(-e^{i(a+bx)}\right)-6id^3\operatorname{Li}_4\left(e^{i(a+bx)}\right)}{2b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3,x]
```

```
[Out] -1/2*(2*b^3*c^3*ArcTanh[E^(I*(a + b*x))] + 12*b*c*d^2*ArcTanh[E^(I*(a + b*x
))] + b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] - 3*b^3
*c^2*d*x*Log[1 - E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 - E^(I*(a + b*x))] - 3*
b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - E^(I*(a + b*x)
)] + 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 + E^(I*(a + b
*x))] + 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 + E^(I
*(a + b*x))] - (3*I)*d*(2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x)
)] + (3*I)*d*(2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] + 6*b*c*
d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*x))] -
 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, E^(I*(a + b*x
))] + (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, E^(I*(a
 + b*x))])/b^4
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1055 vs. 2(275) = 550.
time = 0.13, size = 1056, normalized size = 3.42

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1056 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3/2*I/b^2*d^3*polylog(2,exp(I*
(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+3/2*I/b^2*c^2*d*pol
ylog(2,-exp(I*(b*x+a)))+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b*exp(3*I*(b*
x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))+d^3*x^3*b
*exp(I*(b*x+a))+b*c^3*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3
*x^2*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a
))+b*c^3*exp(I*(b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a
))+6*I*c*d^2*x*exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))+3/b^3*c*d^2*polylog
(3,exp(I*(b*x+a)))-3/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+1/b^4*d^3*a^3*arc
tanh(exp(I*(b*x+a)))+3/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-3/b^3*d^3*polylo
g(3,-exp(I*(b*x+a)))*x-1/b*c^3*arctanh(exp(I*(b*x+a)))-3*I/b^2*polylog(2,ex
p(I*(b*x+a)))*c*d^2*x+3*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2*x+3/2/b*c*d^
2*ln(1-exp(I*(b*x+a)))*x^2-3/2/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/2/b*c*d
^2*ln(exp(I*(b*x+a))+1)*x^2+3/2/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2-3/2/b*c^
2*d*ln(exp(I*(b*x+a))+1)*x-3/2/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+3/2/b*c^2*d
*ln(1-exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+1/2/b*d^3*ln(1
-exp(I*(b*x+a)))*x^3+1/2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/2/b*d^3*ln(exp(
I*(b*x+a))+1)*x^3-1/2/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-3/b^3*c*d^2*a^2*arct
anh(exp(I*(b*x+a)))+3/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+3/b^4*d^3*ln(1-ex
p(I*(b*x+a)))*a-3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x-3/b^4*d^3*ln(exp(I*(b*x+a)
)+1)*a+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x-6/b^3*c*d^2*arctanh(exp(I*(b*x+a)))
+6/b^4*d^3*a*arctanh(exp(I*(b*x+a)))-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-
3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b
^4+3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3886 vs. $2(265) = 530$.
time = 1.61, size = 3886, normalized size = 12.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log
(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(
cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*cos(b*x + a)/
(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b^2 -
```

a^3*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log
(cos(b*x + a) - 1))/b^3 - 4*(2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*
(b*x + a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(4*b*
x + 4*a) - 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(
b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(2*b
*x + 2*a) - (-I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(-I*b*c*d^2 +
I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 - 2*I)*d^
3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a
*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2
+ (I*a^2 + 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), co
s(b*x + a) + 1) - 12*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*cos(4*b*x + 4*a)
- 2*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) + (I*b*c*d^2 - I*a*d^3)*sin(4*b*x +
4*a) + 2*(-I*b*c*d^2 + I*a*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos
(b*x + a) - 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(
b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*
(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x
+ 2*a) - (-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I
*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 - 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a
) - 2*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c
^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arct
an2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(-I*(b*x + a)^3*d^3 - 3*b^2*c^2*d
+ 6*a*b*c*d^2 - 3*a^2*d^3 + 3*(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a)^2 + 3*
(-I*b^2*c^2*d + 2*(I*a - 1)*b*c*d^2 + (-I*a^2 + 2*a)*d^3)*(b*x + a))*cos(3*
b*x + 3*a) - 4*(-I*(b*x + a)^3*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2 + 3*a^2*d^3
+ 3*(-I*b*c*d^2 + (I*a + 1)*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*(I*a + 1
)*b*c*d^2 + (-I*a^2 - 2*a)*d^3)*(b*x + a))*cos(b*x + a) - 6*(b^2*c^2*d - 2*
a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
+ (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2
- a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x +
a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a)
+ (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (I*a^2 + 2*I)*d^3 + 2*
(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-I*b^2*c^2*d + 2*I*a
*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I*a^2 - 2*I)*d^3 + 2*(-I*b*c*d^2 + I*a*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + 6*(b^2*c^2*d - 2*a
*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
+ (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 -
a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a
)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I*a^2 - 2*I)*d^3 + 2
*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*b^2*c^2*d - 2*I*
a*b*c*d^2 + I*(b*x + a)^2*d^3 + (I*a^2 + 2*I)*d^3 + 2*(I*b*c*d^2 - I*a*d^3)

```
*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^3*d^3
- 6*I*b*c*d^2 + 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 - 3*(I*b^2*
c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3
- 6*I*b*c*d^2 + 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 - 3*(I*b^2*
c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*
(-I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b
*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 - 2*I)*d^3)*(b*x + a)
)*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))
*sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 -
 a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) +
1) + (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 - 3*(-I*b*c*d^2 + I*a*d^3
)*(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 - 2*I)*d^3)*(b*x
+ a) + (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 - 3*(-I*b*c*d^2 + I*a*d
^3)*(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 - 2*I)*d^3)*(b*
x + a))*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 +
 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2...
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1744 vs. $2(265) = 530$.
time = 0.42, size = 1744, normalized size = 5.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x +
a) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3 + (I*b^2*d
^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(cos
(b*x + a) + I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^
2*d + 2*I*d^3 + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3)*
cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 -
2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x
+ I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x +
a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + 2*I*d^3 + (-I*b^2*
d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2)*dilog(-c
os(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 +
6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c
^2*d + 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + 2*b*d^3)*x)*log(cos(b*x
+ a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b
*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*
d + 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + 2*b*d^3)*x)*log(cos(b*x + a
) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 -
```

```
(a^3 + 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6
*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)
- (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3 - (b^3*c
^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2)
*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 - (b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3
*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + 2*b*d^3)*x)*log(-cos(b
*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*
cos(b*x + a)^2 + 3*(b^3*c^2*d + 2*b*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1) - 6*(-I*d^3*cos(b*x + a)^2 + I*d^3)*polylog(4, cos(b*x + a) + I*si
n(b*x + a)) - 6*(I*d^3*cos(b*x + a)^2 - I*d^3)*polylog(4, cos(b*x + a) - I*
sin(b*x + a)) - 6*(-I*d^3*cos(b*x + a)^2 + I*d^3)*polylog(4, -cos(b*x + a)
+ I*sin(b*x + a)) - 6*(I*d^3*cos(b*x + a)^2 - I*d^3)*polylog(4, -cos(b*x +
a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x +
 a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (
b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)
) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -
cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)
*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 6*(b^2*d^3*x^
2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(b*x + a))/(b^4*cos(b*x + a)^2 - b^4)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \csc^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3,x)
[Out] Integral((c + d*x)**3*csc(a + b*x)**3, x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^3*csc(b*x + a)^3, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/sin(a + b*x)^3,x)
[Out] \text{Hanged}
```

# 3.34 $\int (c+dx)^2 \csc^3(a+bx)\,dx$

**Optimal**. Leaf size=180

$$-\frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+$$

```
[Out] -(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-d^2*arctanh(cos(b*x+a))/b^3-d*(d*x+c)*
csc(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)*csc(b*x+a)/b+I*d*(d*x+c)*polylog(2,
-exp(I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2-d^2*polylog(
3,-exp(I*(b*x+a)))/b^3+d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

**Rubi** [A]
time = 0.10, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4271, 3855, 4268, 2611, 2320, 6724}

$$-\frac{d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3,x]
```

```
[Out] -(((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b) - (d^2*ArcTanh[Cos[a + b*x]])/b
^3 - (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x
])/(2*b) + (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*(c + d*x
)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3
 + (d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2 \csc^3(a+bx)\,dx &= -\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}+\frac{1}{2}\int (c+dx)^2\\
&= -\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+b}{b^2}\\
&= -\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+b}{b^2}\\
&= -\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+b}{b^2}\\
&= -\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+b}{b^2}\end{aligned}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 423 vs. $2(180) = 360$.

time = 6.59, size = 423, normalized size = 2.35

$$-\frac{d(c+dx)\csc(a)}{b^2}+\frac{(-c^2-2cdx-d^2x^2)\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}-\frac{2b^2c^2\tanh^{-1}\left(e^{i(a+bx)}\right)+4d^2\tanh^{-1}\left(e^{i(a+bx)}\right)-2b^2cdx\log\left(1-e^{i(a+bx)}\right)-b^2d^2x^2\log\left(1-e^{i(a+bx)}\right)+2b^2cdx\log\left(1+e^{i(a+bx)}\right)+b^2d^2x^2\log\left(1+e^{i(a+bx)}\right)-2ibd(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)+2ibd(c+dx)\mathrm{Li}_2\left(e^{i(a+bx)}\right)+2d^2\mathrm{Li}_3\left(-e^{i(a+bx)}\right)-2d^2\mathrm{Li}_3\left(e^{i(a+bx)}\right)}{2b^3}+\frac{(c^2+2cdx+d^2x^2)\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}+\frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(-cd\sin\left(\frac{bx}{2}\right)-d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}+\frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3,x]
```

```
[Out] -((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2
]^2)/(8*b) - (2*b^2*c^2*ArcTanh[E^(I*(a + b*x))] + 4*d^2*ArcTanh[E^(I*(a +
b*x))] - 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 - E^(I*(a
 + b*x))] + 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 + E^(I
*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (2*I)*b*d
*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + 2*d^2*PolyLog[3, -E^(I*(a + b*x))]
 - 2*d^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*
Sec[a/2 + (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)
/2]) - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin
[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(166) = 332.
time = 0.08, size = 548, normalized size = 3.04

| method | result |
|---|---|
| risch | $\frac{d^2x^2b\,\mathrm{e}^{3i(bx+a)}+2cdxb\,\mathrm{e}^{3i(bx+a)}+b\,c^2\mathrm{e}^{3i(bx+a)}+d^2x^2b\,\mathrm{e}^{i(bx+a)}+2cdxb\,\mathrm{e}^{i(bx+a)}-2id^2x\,\mathrm{e}^{3i(bx+a)}+b\,c^2\mathrm{e}^{i(bx+a)}-2idc\,\mathrm{e}^{3i(bx+a)}+2id^2x}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*
(b*x+a))+b*c^2*exp(3*I*(b*x+a))+d^2*x^2*b*exp(I*(b*x+a))+2*c*d*x*b*exp(I*(b
*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))+b*c^2*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x
+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))+d^2*polylog(3,exp(I*(
b*x+a)))/b^3-d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2/b^3*d^2*arctanh(exp(I*(b*
x+a)))+1/2/b^3*ln(exp(I*(b*x+a))+1)*a^2*d^2-1/2/b^3*ln(1-exp(I*(b*x+a)))*a^
2*d^2-1/b*c^2*arctanh(exp(I*(b*x+a)))-1/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))
-1/b*ln(exp(I*(b*x+a))+1)*c*d*x-1/b^2*ln(exp(I*(b*x+a))+1)*a*c*d+1/b*ln(1-e
xp(I*(b*x+a)))*c*d*x+1/b^2*ln(1-exp(I*(b*x+a)))*a*c*d+2/b^2*c*d*a*arctanh(e
xp(I*(b*x+a)))-I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+I/b^2*polylog(2,-exp(I*(
b*x+a)))*d^2*x-I/b^2*polylog(2,exp(I*(b*x+a)))*d^2*x+1/2/b*ln(1-exp(I*(b*x+
a)))*d^2*x^2+I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-1/2/b*ln(exp(I*(b*x+a))+1
)*d^2*x^2
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. 2(162) = 324.

time = 0.62, size = 1938, normalized size = 10.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log
(cos(b*x + a) - 1)) - 2*a*c*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(co
s(b*x + a) + 1) + log(cos(b*x + a) - 1))/b + a^2*d^2*(2*cos(b*x + a)/(cos(b
*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b^2 - 4*(2*
((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + ((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*
d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) - 2*(I*(
b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(2*b*x + 2*a
))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(d^2*cos(4*b*x + 4*a) - 2*d^
2*cos(2*b*x + 2*a) + I*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*b*x + 2*a) + d^
2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d
- a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*
b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x +
2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(4*b*x +
4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x +
2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(-I*(b*x + a)^2*d^2 - 2*
b*c*d + 2*a*d^2 + 2*(-I*b*c*d + (I*a - 1)*d^2)*(b*x + a))*cos(3*b*x + 3*a)
- 4*(-I*(b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(-I*b*c*d + (I*a + 1)*d^2)*
(b*x + a))*cos(b*x + a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x
+ a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(
2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x + 4*a) + 2*(
-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I
*a)) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*c
os(4*b*x + 4*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-I*
b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) - 2*(I*b*c*d + I*(b*x +
 a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)
^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2 + (-I*(b*x + a)^2*d^2 -
2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*a) - 2*(-I*(b*x +
a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) + (
(b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) - 2
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*
d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2 + (I*(b*x + a)^2*d^2 - 2*(
-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^
2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) + 2*((b
*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(
```

```
cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(I*d^2*cos(4*b*x
+ 4*a) - 2*I*d^2*cos(2*b*x + 2*a) - d^2*sin(4*b*x + 4*a) + 2*d^2*sin(2*b*x
+ 2*a) + I*d^2)*polylog(3, -e^(I*b*x + I*a)) - 4*(-I*d^2*cos(4*b*x + 4*a) +
 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) - 2*d^2*sin(2*b*x + 2*a) -
 I*d^2)*polylog(3, e^(I*b*x + I*a)) - 4*((b*x + a)^2*d^2 - 2*I*b*c*d + 2*I*
a*d^2 + 2*(b*c*d - (a + I)*d^2)*(b*x + a))*sin(3*b*x + 3*a) - 4*((b*x + a)^
2*d^2 + 2*I*b*c*d - 2*I*a*d^2 + 2*(b*c*d - (a - I)*d^2)*(b*x + a))*sin(b*x
+ a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b
*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(162) = 324$.
time = 0.40, size = 972, normalized size = 5.40

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) - 2*(-I*b*d^2*x -
 I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin
(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2
)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2
*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 2*(I*
b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x +
 a) - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cos(b*x + a)^2 + 2*d^2)*log(cos(b*x + a)
+ I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cos(b*x + a)^2 + 2*d^2)*log(cos(b*x + a)
- I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2 - (b^2*c^2 - 2
*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin
(b*x + a) + 1/2) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2 - (b^2*c^2 - 2*a*b*
c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x
^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) +
 I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b
*x + a) - I*sin(b*x + a) + 1) + 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, cos
(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, cos(b
*x + a) - I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, -cos(b*
x + a) + I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, -cos(b*x
 + a) - I*sin(b*x + a)) + 4*(b*d^2*x + b*c*d)*sin(b*x + a))/(b^3*cos(b*x +
a)^2 - b^3)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3,x)
[Out] Integral((c + d*x)**2*csc(a + b*x)**3, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^2*csc(b*x + a)^3, x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/sin(a + b*x)^3,x)
[Out] \text{Hanged}
```

# 3.35 $\int (c+dx)\csc^3(a+bx)\,dx$

**Optimal**. Leaf size=109

$$-\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}+\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(e^{i(a+}\right.}{2b^2}$$

```
[Out] -(d*x+c)*arctanh(exp(I*(b*x+a)))/b-1/2*d*csc(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x
+a)*csc(b*x+a)/b+1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/2*I*d*polylog(2,e
xp(I*(b*x+a)))/b^2
```

**Rubi** [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4270, 4268, 2317, 2438}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3,x]
```

```
[Out] -(((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b) - (d*Csc[a + b*x])/(2*b^2) - ((c
+ d*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) + ((I/2)*d*PolyLog[2, -E^(I*(a + b*
x))])/b^2 - ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\csc^3(a+bx)\,dx &= -\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}+\frac{1}{2}\int (c+dx)\csc(a+bx\\ &= -\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx}{2b}\\ &= -\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx}{2b}\\ &= -\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx}{2b}\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 292 vs. 2(109) = 218.
time = 1.23, size = 292, normalized size = 2.68

$$-\frac{dx\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}-\frac{c\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)+i\left(\text{Li}_2\left(-e^{i(a+bx)}\right)-\text{Li}_2\left(e^{i(a+bx)}\right)\right)\right)}{2b^2}+\frac{dx\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}+\frac{c\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{d\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{4b^2}-\frac{d\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3,x]
```

```
[Out] -1/8*(d*x*Csc[a/2 + (b*x)/2]^2)/b - (c*Csc[(a + b*x)/2]^2)/(8*b) - (c*Log[C
os[(a + b*x)/2]])/(2*b) + (c*Log[Sin[(a + b*x)/2]])/(2*b) + (d*((a + b*x)*(
Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) - a*Log[Tan[(a + b*x)/
2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/(2*b
^2) + (d*x*Sec[a/2 + (b*x)/2]^2)/(8*b) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*
Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) - (d*Sec[a/2]*Sec[a/2 + (
b*x)/2]*Sin[(b*x)/2])/(4*b^2)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(93) = 186.
time = 0.05, size = 246, normalized size = 2.26

| method | result |
|---|---|
| risch | $\frac{dxb\,e^{3i(bx+a)}+cb\,e^{3i(bx+a)}+dxb\,e^{i(bx+a)}+cb\,e^{i(bx+a)}-id\,e^{3i(bx+a)}+id\,e^{i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2}-\frac{c\,\text{arctanh}\left(e^{i(bx+a)}\right)}{b}+\frac{d\ln\left(1-e^{i(bx+a)}\right)x}{2b}+$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d*x*b*exp(3*I*(b*x+a))+c*b*exp(3*I*(b*x+a))+d
*x*b*exp(I*(b*x+a))+c*b*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*x+
a)))-1/b*c*arctanh(exp(I*(b*x+a)))+1/2/b*d*ln(1-exp(I*(b*x+a)))*x+1/2/b^2*d
*ln(1-exp(I*(b*x+a)))*a-1/2*I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/2/b*d*ln(ex
p(I*(b*x+a))+1)*x-1/2/b^2*d*ln(exp(I*(b*x+a))+1)*a+1/2*I*d*polylog(2,-exp(I
*(b*x+a)))/b^2+1/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(89) = 178$.
time = 0.44, size = 763, normalized size = 7.00

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*b
*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*sin(2
*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x + 4
*a) - 2*b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) - 2*I*b*c*sin(2*b*x +
 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b*x +
 4*a) - 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) - 2*I*b*d*x*sin
(2*b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(-I*b*d
*x - I*b*c - d)*cos(3*b*x + 3*a) - 4*(-I*b*d*x - I*b*c + d)*cos(b*x + a) -
2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d
*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x + 4*a) - 2*
d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dil
og(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*
a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a)
 - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) +
2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*c
os(b*x + a) + 1) - 4*(b*d*x + b*c - I*d)*sin(3*b*x + 3*a) - 4*(b*d*x + b*c
+ I*d)*sin(b*x + a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a)
+ 4*b^2*sin(4*b*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(89) = 178$.
time = 0.38, size = 452, normalized size = 4.15

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d*x + b*c)*cos(b*x + a) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) + I*sin(
b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)
) + (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) + (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a
) - I*sin(b*x + a) + 1) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + ((b*c - a*d)*cos(b*x + a)^2 - b
*c + a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b*d*x - (b*d
*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (
b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1) + 2*d*sin(b*x + a))/(b^2*cos(b*x + a)^2 - b^2)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**3, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3, x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/sin(a + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

# 3.36 $\int \frac{\csc^3(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=19

$$\operatorname{Int}\left(\frac{\csc^3(a+bx)}{c+dx},x\right)$$

[Out] Unintegrable(csc(b*x+a)^3/(d*x+c),x)

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

[In] Int[Csc[a + b*x]^3/(c + d*x),x]

[Out] Defer[Int][Csc[a + b*x]^3/(c + d*x), x]

Rubi steps

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

**Mathematica** [A]
time = 21.02, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Csc[a + b*x]^3/(c + d*x),x]

[Out] Integrate[Csc[a + b*x]^3/(c + d*x), x]

**Maple** [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3/(d*x+c),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*x
 + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*c)
*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x + b*
c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(b*x
+ a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 -
4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b
*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*si
n(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*
a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*sin(b*x +
 a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) + (d*cos(
3*b*x + 3*a) - d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*x + b
*c)*sin(b*x + a))*sin(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b*d*x + b*c
)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (b*d*x + b*c
)*sin(b*x + a))*sin(2*b*x + 2*a) + d*sin(b*x + a))/(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 +
```

```
2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3/(d*x + c), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*x+c),x)
[Out] Integral(csc(a + b*x)**3/(c + d*x), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3/(d*x + c), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*(c + d*x)),x)
[Out] int(1/(sin(a + b*x)^3*(c + d*x)), x)
```

## 3.37 $\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\csc^3(a+bx)}{(c+dx)^2},x\right)$$

`[Out] Unintegrable(csc(b*x+a)^3/(d*x+c)^2,x)`

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

`[In] Int[Csc[a + b*x]^3/(c + d*x)^2,x]`

`[Out] Defer[Int][Csc[a + b*x]^3/(c + d*x)^2, x]`

Rubi steps

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica** [A]
time = 23.18, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[Csc[a + b*x]^3/(c + d*x)^2,x]`

`[Out] Integrate[Csc[a + b*x]^3/(c + d*x)^2, x]`

**Maple** [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^3/(d*x+c)^2,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*b
*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x +
b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x
+ b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*co
s(b*x + a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 +
 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*
a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x
+ 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 + 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x
^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*s
in(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^
2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)
)*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 6*d^2)*sin(b*x + a)/
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^
2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^
```

```
3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (2*d*c
os(3*b*x + 3*a) - 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*
x + b*c)*sin(b*x + a))*sin(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a) - (b*d*x
+ b*c)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(2*d*cos(b*x + a) - (b*d*
x + b*c)*sin(b*x + a))*sin(2*b*x + 2*a) + 2*d*sin(b*x + a))/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x +
 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*c
os(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(2*b*x + 2*a))
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**Sympy** [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)**3/(c + d*x)**2, x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(1/(sin(a + b*x)^3*(c + d*x)^2), x)
```

# 3.38 $\int (c+dx)^{5/2}\sin(a+bx)\,dx$

**Optimal**. Leaf size=195

$$\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{4b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\!-\ 15$$

```
[Out] -(d*x+c)^(5/2)*cos(b*x+a)/b+5/2*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2-15/8*d^(5/2)
*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1
/2)*Pi^(1/2)/b^(7/2)+15/8*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)
^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/4*d^2*cos(b*x+a)*(
d*x+c)^(1/2)/b^3
```

**Rubi** [A]
time = 0.31, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{4b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{2b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(4*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x]
)/b - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(4*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqr
t[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(7/2)) + (5*
d*(c + d*x)^(3/2)*Sin[a + b*x])/(2*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx = -\frac{(c+dx)^{5/2}\cos(a+bx)}{b}+\frac{(5d)\int (c+dx)^{3/2}\cos(a+bx)\,dx}{2b}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{b}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{2b^2}-\frac{(15d^2)\int \sqrt{c+dx}\,\sin\ldots}{4b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{4b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}+\frac{5d(c+dx)^{3/2}\sin(a+\ldots}{2b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{4b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}+\frac{5d(c+dx)^{3/2}\sin(a+\ldots}{2b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{4b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}+\frac{5d(c+dx)^{3/2}\sin(a+\ldots}{2b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{4b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{b\ldots}{\ldots}\right.}{\ldots}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.08, size = 124, normalized size = 0.64

$$\frac{d^2 e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}+\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{7}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sin[a + b*x],x]
```

```
[Out] (d^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[7/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d] + (E^(((2*I)*b*c)/d)*Gamma[7/2, (I*b*(c + d*x))/d])/Sqrt
[(I*b*(c + d*x))/d]))/(2*b^3*E^((I*(b*c + a*d))/d))
```

## Maple [A]

time = 0.04, size = 233, normalized size = 1.19

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{\cdots}}{\cdots}\right)}{\cdots}\right)}{\cdots}}{d}$ |

| | |
|---|---|
| default | $\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/2/b*d*(d*x+c)^(5/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+5/2/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.31, size = 263, normalized size = 1.35

$$\frac{\sqrt{2}\left(40\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)-4\left(4\sqrt{2}(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}\,bd^2\right)\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-15\left(-(i-1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-15\left((i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)\right)}{32b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(2)*(40*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(((d*x + c)*b - b*c + a*d
)/d) - 4*(4*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*d^2)*c
os(((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)
*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(pi)*d^3*(b^2/d^2)^(
1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/b^4
```

**Fricas** [A]
time = 0.37, size = 190, normalized size = 0.97

$$-\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+2\sqrt{dx+c}\left(\left(4b^3d^2x^2+8b^3cdx+4b^3c^2-15bd^2\right)\cos(bx+a)-10\left(b^2d^2x+b^2cd\right)\sin(bx+a)\right)}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresne
l_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 2*sqrt(d*
x + c)*((4*b^3*d^2*x^2 + 8*b^3*c*d*x + 4*b^3*c^2 - 15*b*d^2)*cos(b*x + a) -
 10*(b^2*d^2*x + b^2*c*d)*sin(b*x + a)))/b^4
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

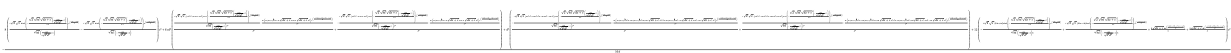
$$\int (c+dx)^{\frac{5}{2}}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(5/2)*sin(a + b*x), x)
```

**Giac** [C] Result contains complex when optimal does not.
time = 3.14, size = 1246, normalized size = 6.39



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(8*(I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)))*c^3 + 6*c*d^2*((I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d
 - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*
I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)
*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(4*
b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d
- 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + d^
3*((-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3
)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(
d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^
2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d
*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (I*sqrt(2)*s
```

```
qrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2
)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*
(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*
e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 12*(-I*sqrt(2)*sqrt(pi)*(
2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I
*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d
)/b + 2*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(sin(a + b*x)*(c + d*x)^(5/2), x)
```

# 3.39 $\int (c+dx)^{3/2}\sin(a+bx)\,dx$

**Optimal**. Leaf size=170

$$-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+}}{\sqrt{d}}\right.}{2b^{5/2}}$$

```
[Out] -(d*x+c)^(3/2)*cos(b*x+a)/b-3/4*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/4*d^(3/2)*Fre
snelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*
Pi^(1/2)/b^(5/2)+3/2*d*sin(b*x+a)*(d*x+c)^(1/2)/b^2
```

**Rubi** [A]
time = 0.18, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{2b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sin[a + b*x],x]
```

```
[Out] -(((c + d*x)^(3/2)*Cos[a + b*x])/b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*b^(5/2)) - (3*d^
(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
 - (b*c)/d])/(2*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x])/(2*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}+\frac{(3d)\int \sqrt{c+dx}\,\cos(a+bx)\,dx}{2b}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}+\frac{3d\sqrt{c+dx}\,\sin(a+bx)}{2b^2}-\frac{(3d^2)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{4b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}+\frac{3d\sqrt{c+dx}\,\sin(a+bx)}{2b^2}-\frac{\left(3d^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin}{\sqrt{}}}{4b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}+\frac{3d\sqrt{c+dx}\,\sin(a+bx)}{2b^2}-\frac{\left(3d\cos\left(a-\frac{bc}{d}\right)\right)\text{Subs}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}-$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.07, size = 125, normalized size = 0.74

$$-\frac{ide^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{5}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sin[a + b*x],x]
```

```
[Out] ((-1/2*I)*d*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[5/2, ((-I)*b*(c + d*x))/d])/S
qrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[5/2, (I*b*(c + d*x))/d
])/Sqrt[(I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d))
```

**Maple [A]**
time = 0.04, size = 188, normalized size = 1.11

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{b}}{d}$ |
| default | $\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{b}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/2/b*d*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d))))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.32, size = 242, normalized size = 1.42

$$-\frac{\sqrt{2}\left(8\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-12\sqrt{2}\,\sqrt{dx+c}\,bd\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+3\left((i+1)\sqrt{\pi}\,d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i-1)\sqrt{\pi}\,d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{ib}{d}}\right)+3\left(-(i-1)\sqrt{\pi}\,d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{\pi}\,d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{ib}{d}}\right)\right)}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*(8*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(((d*x + c)*b - b*c + a*d)/
d) - 12*sqrt(2)*sqrt(d*x + c)*b*d*sin(((d*x + c)*b - b*c + a*d)/d) + 3*((I
+ 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*d^
2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + 3*(
-(I - 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi
)*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))
/b^3
```

**Fricas** [A]
time = 0.36, size = 156, normalized size = 0.92

$$-\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\left(3bd\sin(bx+a)-2\left(b^2dx+b^2c\right)\cos(bx+a)\right)\sqrt{dx+c}}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_
cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*(3*b*d*si
n(b*x + a) - 2*(b^2*d*x + b^2*c)*cos(b*x + a))*sqrt(d*x + c))/b^3
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x), x)
```

**Giac** [C] Result contains complex when optimal does not.
time = 4.27, size = 779, normalized size = 4.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(4*(I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
```

```
^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)))*c^2 + d^2*((I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*
d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*
I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((
-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(4*b^2*c
^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*s
qrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 4*(-I*s
qrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b)*c)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)\,\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(sin(a + b*x)*(c + d*x)^(3/2), x)
```

# 3.40 $\int \sqrt{c+dx}\,\sin(a+bx)\,dx$

**Optimal**. Leaf size=142

$$-\frac{\sqrt{c+dx}\,\cos(a+bx)}{b}+\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)\ldots}{b^{3/2}}$$

```
[Out] 1/2*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d
^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/2*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-cos(b*x+a)*
(d*x+c)^(1/2)/b
```

**Rubi** [A]
time = 0.13, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{d}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{d}\,\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{c+dx}\,\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sin[a + b*x],x]
```

```
[Out] -((Sqrt[c + d*x]*Cos[a + b*x])/b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fr
esnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2) - (Sqrt[d]*Sqrt
[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d
])/b^(3/2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx}\,\sin(a+bx)\,dx &= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{b}+\frac{d\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{2b}\\
&= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{b}+\frac{\left(d\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b}-\frac{\left(d\sin\left(a-\frac{bc}{d}\right)\right)}{2b}\\
&= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{b}+\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Subst}\left(\int\cos\left(\frac{bx^2}{d}\right)\,dx,x,\sqrt{c+dx}\right)}{b}-\\
&= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{b}+\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.06, size = 123, normalized size = 0.87

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sin[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(2*b*E^((I*(b*c + a*d))/d))
```

**Maple** [A]
time = 0.04, size = 145, normalized size = 1.02

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b \ldots}{\sqrt{\ldots}}\right.\right.}{2b\sqrt{\frac{b}{d}}}}{d}$ |
| default | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b \ldots}{\sqrt{\ldots}}\right.\right.}{2b\sqrt{\frac{b}{d}}}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/2/b*d*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d)))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.33, size = 196, normalized size = 1.38

$$-\frac{\sqrt{2}\left(4\sqrt{2}\,\sqrt{dx+c}\,b\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left((i-1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+\left(-(i+1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i-1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(4*sqrt(2)*sqrt(d*x + c)*b*cos(((d*x + c)*b - b*c + a*d)/d) +
((I - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*
d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (-(
I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*d*
(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

**Fricas** [**A**]
time = 0.35, size = 127, normalized size = 0.89

$$\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\sqrt{dx+c}\,b\cos\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sq
rt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*sqrt(d*x + c)*b*co
s(b*x + a))/b^2
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c+dx}\,\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x), x)
```

**Giac** [**C**] Result contains complex when optimal does not.
time = 5.07, size = 426, normalized size = 3.00

$$-\frac{-\frac{i\sqrt{2}\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{i\sqrt{2}\sqrt{\pi}\,(2\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+2\left(\frac{i\sqrt{2}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c+\frac{2\sqrt{dx+c}\,de^{\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}\,de^{\left(\frac{-i\,(dx+c)b+i\,bc-i\,ad}{d}\right)}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d
```

```
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*(I*sqrt(2)*sqrt(pi)*d*erf
(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*
erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 2*sqrt(d*x +
 c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-I*(d
*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(c + d*x)^(1/2),x)
[Out] int(sin(a + b*x)*(c + d*x)^(1/2), x)
```

# 3.41 $\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=117

$$\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/
2)*Pi^(1/2)/b^(1/2)/d^(1/2)+FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx = \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx + \sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx$$

$$= \frac{\left(2\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d} + \frac{\left(2\sin\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right.\right.}{d}$$

$$= \frac{\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\,\sqrt{d}} + \frac{\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right.}{\sqrt{b}\,\sqrt{d}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.04, size = 121, normalized size = 1.03

$$-\frac{e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] -1/2*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d*x))/
d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d
])/(b*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

**Maple [A]**
time = 0.03, size = 99, normalized size = 0.85

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$ | 99 |
| default | $\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$ | 99 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)
/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.30, size = 159, normalized size = 1.36

$$-\frac{\sqrt{2}\left(\left(-(i+1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+\left((i-1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*((-(I + 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I -
 1)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*
b/d)) + ((I - 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqr
t(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))
/b
```

**Fricas [A]**
time = 0.37, size = 107, normalized size = 0.91

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\,\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + sqrt(2)*pi*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d))/b
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)/sqrt(c + d*x), x)
```

**Giac** [C] Result contains complex when optimal does not.
time = 2.44, size = 168, normalized size = 1.44

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)))/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)/(c + d*x)^(1/2), x)
```

# 3.42 $\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=139

$$\frac{2\sqrt{b}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{b}\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}}-\frac{2\sin(a}{d\sqrt{c}}$$

```
[Out] 2*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(
1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*sin(a-b*c/d)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*sin(b*x+a)/d/
(d*x+c)^(1/2)
```

**Rubi** [A]
time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3387, 3386, 3432, 3385, 3433}

$$\frac{2\sqrt{2\pi}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{2\pi}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/d^(3/2) - (2*Sqrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(3/2) - (2*Sin[a + b*x])/(d
*Sqrt[c + d*x])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\sin(a+bx)}{d\sqrt{c+dx}} + \frac{(2b)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\sin(a+bx)}{d\sqrt{c+dx}} + \frac{\left(2b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d} - \frac{\left(2b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\sin(a+bx)}{d\sqrt{c+dx}} + \frac{\left(4b\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{(4b\sin\left(a-\right.}{}$$

$$= \frac{2\sqrt{b}\,\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\,\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.21, size = 148, normalized size = 1.06

$$\frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+2ie^{\frac{i(bc+ad)}{d}}\sin(a+bx)\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (I*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d*x))/
d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/
d] + (2*I)*E^((I*(b*c + a*d))/d)*Sin[a + b*x]))/(d*E^((I*(b*c + a*d))/d)*Sq
rt[c + d*x])
```

**Maple [A]**
time = 0.02, size = 140, normalized size = 1.01

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right.\right.}{d\sqrt{\frac{b}{d}}}}{d}$ |
| default | $\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right.\right.}{d\sqrt{\frac{b}{d}}}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(
b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.60, size = 129, normalized size = 0.93

$$-\frac{\left(\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{i\,(dx+c)b}{d}\right)-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{i\,(dx+c)b}{d}\right)-(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{i\,(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\sqrt{\frac{(dx+c)b}{d}}}{4\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(((I - 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma
(-1/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-1/2
, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(-(b
*c - a*d)/d))*sqrt((d*x + c)*b/d)/(sqrt(d*x + c)*d)
```

**Fricas** [**A**]
time = 0.35, size = 146, normalized size = 1.05

$$\frac{2\left(\sqrt{2}\,(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{dx+c}\,\sin\left(bx+a\right)\right)}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(s
qrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d
))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) -
sqrt(d*x + c)*sin(b*x + a))/(d^2*x + c*d)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)}{\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x)**(3/2), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*x + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x)^(3/2),x)
```

```
[Out] int(sin(a + b*x)/(c + d*x)^(3/2), x)
```

# 3.43 $\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=168

$$-\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{4b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b^{3/2}\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin}{3d^{5/2}}$$

```
[Out] -2/3*sin(b*x+a)/d/(d*x+c)^(3/2)-4/3*b^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)-4/3*b^(3/2)
*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1
/2)*Pi^(1/2)/d^(5/2)-4/3*b*cos(b*x+a)/d^2/(d*x+c)^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3387, 3386, 3432, 3385, 3433}

$$-\frac{4\sqrt{2\pi}\,b^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4\sqrt{2\pi}\,b^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (-4*b*Cos[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (4*b^(3/2)*Sqrt[2*Pi]*Cos[a - (
b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) -
 (4*b^(3/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]
*Sin[a - (b*c)/d])/(3*d^(5/2)) - (2*Sin[a + b*x])/(3*d*(c + d*x)^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}} + \frac{(2b)\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin(a+bx)}{3d(c+dx)^{3/2}} - \frac{(4b^2)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin(a+bx)}{3d(c+dx)^{3/2}} - \frac{\left(4b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} - \frac{\left(4b^2\sin\left(a-\frac{bc}{d}\right) \ldots\right.}{3 \ldots}$$

$$= -\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin(a+bx)}{3d(c+dx)^{3/2}} - \frac{\left(8b^2\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx} \ldots\right.}{3d^3}$$

$$= -\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{4b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b^{3/2}\sqrt{2\pi}\,C\left(\frac{\sqrt{b} \ldots}{\ldots}\right.}{\ldots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.41, size = 162, normalized size = 0.96

$$\frac{2\left(-b(c+dx)\left(-e^{i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{-i(a+bx)}\left(1+e^{2i(a+bx)}-e^{\frac{ib(c+dx)}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)\right)-d\sin(a+bx)\right)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-(b*(c + d*x)*(-(E^(I*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1
/2, ((-I)*b*(c + d*x))/d]) + (1 + E^((2*I)*(a + b*x)) - E^((I*b*(c + d*x))/
d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*(a + b*x))))
 - d*Sin[a + b*x]))/(3*d^2*(c + d*x)^(3/2))
```

**Maple** [A]
time = 0.04, size = 180, normalized size = 1.07

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$ |
| default | $\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/3/(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^
(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.63, size = 129, normalized size = 0.77

$$-\frac{\left(\left(-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{i(dx+c)b}{d}\right)+(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{i(dx+c)b}{d}\right)-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}}{4\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*((-(I + 1)*sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamm
a(-3/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-3/
2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin(-(
b*c - a*d)/d))*((d*x + c)*b/d)^(3/2)/((d*x + c)^(3/2)*d)
```

**Fricas [A]**
time = 0.38, size = 208, normalized size = 1.24

$$-\frac{2\left(2\sqrt{2}\left(\pi b d^2 x^2+2\pi b c d x+\pi b c^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+2\sqrt{2}\left(\pi b d^2 x^2+2\pi b c d x+\pi b c^2\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{dx+c}\left(2\left(bdx+bc\right)\cos\left(bx+a\right)+d\sin\left(bx+a\right)\right)\right)}{3\left(d^4x^2+2\,cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 2*sqrt
(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*(2*(
b*d*x + b*c)*cos(b*x + a) + d*sin(b*x + a)))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2
)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x)**(5/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*x + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x)^(5/2),x)
[Out] int(sin(a + b*x)/(c + d*x)^(5/2), x)
```

# 3.44 $\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx$

**Optimal**. Leaf size=193

$$-\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{8b^{5/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^{5/2}\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin$$

```
[Out] -4/15*b*cos(b*x+a)/d^2/(d*x+c)^(3/2)-2/5*sin(b*x+a)/d/(d*x+c)^(5/2)-8/15*b^
(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^2*sin(b*x
+a)/d^3/(d*x+c)^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3387, 3386, 3432, 3385, 3433}

$$-\frac{8\sqrt{2\pi}\,b^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}\,b^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (-4*b*Cos[a + b*x])/(15*d^2*(c + d*x)^(3/2)) - (8*b^(5/2)*Sqrt[2*Pi]*Cos[a
- (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2
)) + (8*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt
[d]]*Sin[a - (b*c)/d])/(15*d^(7/2)) - (2*Sin[a + b*x])/(5*d*(c + d*x)^(5/2)
) + (8*b^2*Sin[a + b*x])/(15*d^3*Sqrt[c + d*x])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx &= -\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} + \frac{(2b)\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx}{5d} \\
&= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} - \frac{(4b^2)\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2} \\
&= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}} - \frac{(8b^3)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{15d^3} \\
&= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}} - \frac{\left(8b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{15d^3} \\
&= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}} - \frac{\left(16b^3\cos\left(a-\frac{bc}{d}\right)\right)\mathrm{Subst}\left(\int \cos\right.}{15d^4} \\
&= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{8b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}\sqrt{2\pi}\,S\left(\right.}{}
\end{aligned}
$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.33, size = 208, normalized size = 1.08

$$
-\frac{i\left(b(c+dx)\left(2e^{i\left(a-\frac{bc}{d}\right)}\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)-ie^{-i(a+bx)}\left(2d-4ib(c+dx)+4de^{\frac{ib(c+dx)}{d}}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)\right)-6id^2\sin(a+bx)\right)}{15d^3(c+dx)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] ((-1/15*I)*(b*(c + d*x)*(2*E^(I*(a - (b*c)/d))*(E^((I*b*(c + d*x))/d)*((-I)
*d + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-I)
*b*(c + d*x))/d]) - (I*(2*d - (4*I)*b*(c + d*x) + 4*d*E^((I*b*(c + d*x))/d)
*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, (I*b*(c + d*x))/d]))/E^(I*(a + b*x)))
 - (6*I)*d^2*Sin[a + b*x]))/(d^3*(c + d*x)^(5/2))
```

**Maple [A]**
time = 0.02, size = 220, normalized size = 1.14

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\right.}{}\right.}{}\right.}{5d}}{d}$ |
| default | $\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\right.}{}\right.}{}\right.}{5d}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/5/(d*x+c)^(5/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+2/5*b/d*(-1/3/(d*x+c
)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^(1/2)*sin(1/d*b*
(d*x+c)+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*Fre
snelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*Fres
nelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.60, size = 129, normalized size = 0.67

$$\frac{\left(\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{i(dx+c)b}{d}\right)-(i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}}{4\,(dx+c)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/4*(((I - 1)*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(
-5/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-5/2,
 I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-(b*
c - a*d)/d))*((d*x + c)*b/d)^(5/2)/((d*x + c)^(5/2)*d)
```

**Fricas** [A]
time = 0.39, size = 297, normalized size = 1.54

$$-\frac{2\left(4\sqrt{2}\left(\pi b^{2}d^{3}x^{3}+3\pi b^{2}cd^{2}x^{2}+3\pi b^{2}c^{2}dx+\pi b^{2}c^{3}\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4\sqrt{2}\left(\pi b^{2}d^{3}x^{3}+3\pi b^{2}cd^{2}x^{2}+3\pi b^{2}c^{2}dx+\pi b^{2}c^{3}\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{dx+c}\left(2\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)-\left(4b^{2}d^{2}x^{2}+8b^{2}cdx+4b^{2}c^{2}-3d^{2}\right)\sin\left(bx+a\right)\right)\right)}{15\left(d^{6}x^{3}+3cd^{5}x^{2}+3c^{2}d^{4}x+c^{3}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x +
pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) - 4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*
pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*(2*(b*d^2*x + b*c*d)
*cos(b*x + a) - (4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 3*d^2)*sin(b*x +
 a)))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)}{\left(c+dx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**(7/2),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x)**(7/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*x + c)^(7/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x)^(7/2),x)
[Out] int(sin(a + b*x)/(c + d*x)^(7/2), x)
```

# 3.45 $\int (c+dx)^{5/2} \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=231

$$-\frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}-\frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}-\frac{15d^{5/2}\sqrt{\pi}\,C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}$$

```
[Out] -5/16*d*(d*x+c)^(3/2)/b^2+1/7*(d*x+c)^(7/2)/d-1/2*(d*x+c)^(5/2)*cos(b*x+a)*
sin(b*x+a)/b+5/8*d*(d*x+c)^(3/2)*sin(b*x+a)^2/b^2-15/128*d^(5/2)*cos(2*a-2*
b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)-
15/128*d^(5/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2
*b*c/d)*Pi^(1/2)/b^(7/2)+15/64*d^2*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^3
```

**Rubi** [A]
time = 0.30, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3392, 32, 3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}+\frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}-\frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sin[a + b*x]^2,x]
```

```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) - (15*d^(5/2)*Sqrt[
Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])])/(128*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(128*b^(7/2)) - ((c + d*x)^(5/
2)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x]^2)/
(8*b^2) + (15*d^2*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(64*b^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx = -\frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2} + \frac{1}{2}\int (c$$

$$= \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{5d(c+dx)^{3/2}\sin^2(a+b}{8b^2}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + 5d(c$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + 5d(c$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + 5d(c$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + 5d(c$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right.}{128b^{7/2}}$$

**Mathematica** [A]
time = 1.41, size = 194, normalized size = 0.84

$$\frac{\sqrt{\frac{b}{d}}\left(-105d^3\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-105d^3\sqrt{\pi}\,C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(64b^3(c+dx)^3-140bd^2(c+dx)\cos(2(a+bx))-7d(-15d^2+16b^2(c+dx)^2)\sin(2(a+bx))\right)\right)}{896b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sq
rt[c + d*x])/Sqrt[Pi]] - 105*d^3*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*
x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(64*b^3*(c +
 d*x)^3 - 140*b*d^2*(c + d*x)*Cos[2*(a + b*x)] - 7*d*(-15*d^2 + 16*b^2*(c +
 d*x)^2)*Sin[2*(a + b*x)])))/(896*b^4)
```

**Maple** [A]
time = 0.04, size = 242, normalized size = 1.05

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{\frac{(dx+c)^{\frac{7}{2}}}{7}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c}{d}\cdots\right.}{4b}\cdots\right.}{\cdots}\right.}{\cdots}\cdots}{d}$ |
| default | $\frac{\frac{(dx+c)^{\frac{7}{2}}}{7}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c}{d}\cdots\right.}{4b}\cdots\right.}{\cdots}\right.}{\cdots}\cdots}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/14*(d*x+c)^(7/2)-1/8/b*d*(d*x+c)^(5/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)
/d)+5/8/b*d*(-1/4/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/4/b*
d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/
(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)))))
```

**<u>Maxima</u>** **[C]** Result contains complex when optimal does not.
time = 0.55, size = 295, normalized size = 1.28

$$\frac{\sqrt{2}\left(\frac{512\sqrt{2}\,(dx+c)^{\frac{7}{2}}b^4}{d}-1120\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+105\left(-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{2ib}{d}}\right)+105\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{2ib}{d}}\right)-56\left(16\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\,\sqrt{dx+c}\,bd^2\right)\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)\right)}{7168\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7168*sqrt(2)*(512*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 1120*sqrt(2)*(d*x + c)^
(3/2)*b^2*d*cos(2*((d*x + c)*b - b*c + a*d)/d) + 105*(-(I + 1)*4^(1/4)*sqrt
(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(pi)*d
^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d))
+ 105*((I - 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
 (I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)) - 56*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(
2)*sqrt(d*x + c)*b*d^2)*sin(2*((d*x + c)*b - b*c + a*d)/d))/b^4
```

**Fricas** [**A**]
time = 0.38, size = 258, normalized size = 1.12

$$-\frac{105\,\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+105\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(32\,b^4d^3x^3+96\,b^4cd^2x^2+32\,b^4c^3+70\,b^2cd^2-140\,(b^2d^3x+b^2cd^2)\cos\left(bx+a\right)^2-7\,(16\,b^3d^3x^2+32\,b^3cd^2x+16\,b^3c^2d-15\,bd^3)\cos\left(bx+a\right)\sin\left(bx+a\right)+2\,(48\,b^4c^2d+35\,b^2d^3)x\right)\sqrt{dx+c}}{896\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/896*(105*pi*d^4*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) + 105*pi*d^4*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(32*b^4*d^3*x^3 + 96*b^4*c*
d^2*x^2 + 32*b^4*c^3 + 70*b^2*c*d^2 - 140*(b^2*d^3*x + b^2*c*d^2)*cos(b*x +
 a)^2 - 7*(16*b^3*d^3*x^2 + 32*b^3*c*d^2*x + 16*b^3*c^2*d - 15*b*d^3)*cos(b
*x + a)*sin(b*x + a) + 2*(48*b^4*c^2*d + 35*b^2*d^3)*x)*sqrt(d*x + c))/(b^4
*d)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{5}{2}}\sin^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(5/2)*sin(a + b*x)**2, x)
```

**Giac** [**C**] Result contains complex when optimal does not.
time = 3.87, size = 1324, normalized size = 5.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8960*(2240*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
 + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) +
sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(d*x + c)
)*c^3 + 28*c*d^2*(64*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*
x + c)*c^2)/d^2 + 15*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*(-4*I*(d*x + c)^(3/2)*b*d +
8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b^2)/d^2 + 15*(sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*(4*I*(d*x + c)^(3/2)*b
*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b -
I*b*c + I*a*d)/d)/b^2)/d^2) + d^3*(256*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5
/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3 - 35*(sqrt(pi)*(
64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(-16*I*(d*x + c)^(5/2)*b^2*d + 48*I*(d*
x + c)^(3/2)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*
d^2 - 36*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-2*(-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 35*(sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*
d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b^3) + 2*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*
sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d
^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/
d^3) - 560*(3*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b) + 3*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2) + 48*sqrt(d*x + c)*c + 6*I*sqrt(d*x +
 c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^(-
2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

# 3.46 $\int (c+dx)^{3/2} \sin^2(a+bx)\, dx$

**Optimal**. Leaf size=203

$$-\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}+\frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}-\frac{3d^{3/2}\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin}{32b^{5/2}}$$

```
[Out] 1/5*(d*x+c)^(5/2)/d-1/2*(d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)/b+3/32*d^(3/2)*
cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2
)/b^(5/2)-3/32*d^(3/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*s
in(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)-3/16*d*(d*x+c)^(1/2)/b^2+3/8*d*sin(b*x+a)^
2*(d*x+c)^(1/2)/b^2
```

**Rubi** [A]
time = 0.25, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 32, 3393, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}-\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) + (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(32*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(S
qrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) - ((c + d*x)^(3/2)*Cos
[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]^2)/(8*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+dx}\,\sin^2(a+bx)}{8b^2} + \frac{1}{2}\int (c+$$

$$= \frac{(c+dx)^{5/2}}{5d} - \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+dx}\,\sin^2(a+bx)}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+}}{}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+}}{}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+}}{}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right) C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}$$

**Mathematica [A]**
time = 1.07, size = 175, normalized size = 0.86

$$\frac{\sqrt{\frac{b}{d}}\left(15d^2\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right) C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 15d^2\sqrt{\pi}\, S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right) + 2\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(-15d^2\cos(2(a+bx)) + 4b(c+dx)(4b(c+dx) - 5d\sin(2(a+bx))))\right)}{160b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]] - 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Cos[2*
(a + b*x)] + 4*b*(c + d*x)*(4*b*(c + d*x) - 5*d*Sin[2*(a + b*x)]))))/(160*b
^3)
```

**Maple [A]**
time = 0.03, size = 197, normalized size = 0.97

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{\frac{(dx+c)^{\frac{5}{2}}}{5}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{Fr}\cdots\right.}{\cdots}\right.}{4b\cdots}}{d}$ |
| default | $\frac{\frac{(dx+c)^{\frac{5}{2}}}{5}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{Fre}\cdots\right.}{\cdots}\right.}{4b\cdots}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/10*(d*x+c)^(5/2)-1/8/b*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)
/d)+3/8/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*
d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d))))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.53, size = 274, normalized size = 1.35

$$\frac{\sqrt{2}\left(\frac{128\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d}-160\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-120\sqrt{2}\sqrt{dx+c}\,bd\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+15\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)+15\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)}{1280b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1280*sqrt(2)*(128*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 160*sqrt(2)*(d*x + c)^(
3/2)*b^2*sin(2*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*b*d
*cos(2*((d*x + c)*b - b*c + a*d)/d) + 15*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 15*((I + 1
)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1
/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(-2*I*b/d)))/b^3
```

**Fricas** [A]
time = 0.37, size = 195, normalized size = 0.96

$$\frac{15\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+2\left(16b^3d^2x^2+32b^3cdx+16b^3c^2-30bd^2\cos(bx+a)^2+15bd^2-40(b^2d^2x+b^2cd)\cos(bx+a)\sin(bx+a)\right)\sqrt{dx+c}}{160b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/160*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x
 + 16*b^3*c^2 - 30*b*d^2*cos(b*x + a)^2 + 15*b*d^2 - 40*(b^2*d^2*x + b^2*c*
d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(b^3*d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}} \sin^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2, x)
```

**Giac** [C] Result contains complex when optimal does not.
time = 4.94, size = 816, normalized size = 4.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/960*(240*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sq
rt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(d*x + c))*
c^2 + d^2*(64*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*
c^2)/d^2 + 15*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqr
t(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a
*d)/d)/b^2)/d^2 + 15*(sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b^2)/d^2) - 40*(3*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sq
```

```
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2) + 48*sqrt(d*x + c)*c +
 6*I*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(
d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin (a+b x)^2 (c+d x)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

# 3.47 $\int \sqrt{c+dx}\,\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=158

$$\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}\,\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{d}\,\sqrt{\pi}\,C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}}-\frac{\sqrt{c+}}{}$$

```
[Out] 1/3*(d*x+c)^(3/2)/d+1/8*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)+1/8*FresnelC(2*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/4*sin(2*b
*x+2*a)*(d*x+c)^(1/2)/b
```

**Rubi** [A]
time = 0.19, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\pi}\,\sqrt{d}\,\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}\,\sqrt{d}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\,\sin(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(3*d) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]
*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d
])/(8*b^(3/2)) - (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+dx}\,\sin^2(a+bx)\,dx &= \int \left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\,\cos(2a+2bx)\right)dx\\
&= \frac{(c+dx)^{3/2}}{3d}-\frac{1}{2}\int \sqrt{c+dx}\,\cos(2a+2bx)\,dx\\
&= \frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{c+dx}\,\sin(2a+2bx)}{4b}+\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b}\\
&= \frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{c+dx}\,\sin(2a+2bx)}{4b}+\frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b}\\
&= \frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{c+dx}\,\sin(2a+2bx)}{4b}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\right.}{4b}\\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}\,\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{d}\,\sqrt{\pi}\,C\left(\frac{2\sqrt{}}{}\right.}{}\end{aligned}$$

**Mathematica** [A]

time = 0.31, size = 149, normalized size = 0.94

$$\frac{3d\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+3d\sqrt{\pi}\,C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(4b(c+dx)-3d\sin(2(a+bx)))}{24\left(\frac{b}{d}\right)^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqr
t[Pi]] + 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x) - 3*d*Sin[2*(a +
b*x)]))/(24*(b/d)^(3/2)*d^2)
```

**Maple [A]**
time = 0.04, size = 150, normalized size = 0.95

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b}{\cdots}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$ |
| default | $\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b}{\cdots}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/6*(d*x+c)^(3/2)-1/8/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/
d)+1/16/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.52, size = 229, normalized size = 1.45

$$\frac{\sqrt{2}\left(\frac{32\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2}{d}-24\sqrt{2}\sqrt{dx+c}\,b\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)+3\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)}{192\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/192*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c)*
b*sin(2*((d*x + c)*b - b*c + a*d)/d) + 3*((I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d
^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4
)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 3*(-(I - 1)*4^(
1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqr
t(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*
b/d)))/b^2
```

**Fricas [A]**
time = 0.37, size = 148, normalized size = 0.94

$$\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+4\,(2\,b^2dx-3\,bd\cos\left(bx+a\right)\sin\left(bx+a\right)+2\,b^2c)\sqrt{dx+c}}{24\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(2*b^2*d*x - 3*b*d*cos(b*x + a)*s
in(b*x + a) + 2*b^2*c)*sqrt(d*x + c))/(b^2*d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c+dx}\,\sin^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2, x)
```

**Giac [C]** Result contains complex when optimal does not.
time = 4.29, size = 434, normalized size = 2.75

$$\frac{12\left(\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+4\,\sqrt{dx+c}\right)c-\frac{3\,\sqrt{\pi}\,(4\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{3\,\sqrt{\pi}\,(4\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+16\,(dx+c)^{\frac{3}{2}}-48\,\sqrt{dx+c}\,c-\frac{6i\,\sqrt{dx+c}\,de^{\left(-\frac{2\,(i\,(dx+c)b-i\,bc+i\,ad)}{d}\right)}}{b}+\frac{6i\,\sqrt{dx+c}\,de^{\left(-\frac{2\,(-i\,(dx+c)b+i\,bc-i\,ad)}{d}\right)}}{b}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/48*(12*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt
(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*
```

```
b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(d*x + c))*c
- 3*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
*b) - 3*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
 1)*b) + 16*(d*x + c)^(3/2) - 48*sqrt(d*x + c)*c - 6*I*sqrt(d*x + c)*d*e^(-
2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-2*(-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^2\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

# 3.48 $\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=130

$$\frac{\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

```
[Out] -1/2*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi
^(1/2)/b^(1/2)/d^(1/2)+1/2*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2
))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(1/2)/d^(1/2)+(d*x+c)^(1/2)/d
```

**Rubi** [A]
time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3393, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] Sqrt[c + d*x]/d - (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelS[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*
Sqrt[d])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx &= \int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)\,dx\\
&= \frac{\sqrt{c+dx}}{d}-\frac{1}{2}\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx\\
&= \frac{\sqrt{c+dx}}{d}-\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx+\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+\right.}{\sqrt{c+d}}\\
&= \frac{\sqrt{c+dx}}{d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int\cos\left(\frac{2bx^2}{d}\right)\,dx,x,\sqrt{c+dx}\right)}{d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Subs}}{}\\
&= \frac{\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a\right.}{2\sqrt{b}\sqrt{d}}
\end{aligned}$$

**Mathematica [A]**
time = 0.15, size = 126, normalized size = 0.97

$$\frac{\sqrt{\frac{b}{d}}\left(2\sqrt{\frac{b}{d}}\sqrt{c+dx}-\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+\sqrt{\pi}S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[b/d]*(2*Sqrt[b/d]*Sqrt[c + d*x] - Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresn
elC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*
Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d]))/(2*b)
```

**Maple** [A]
time = 0.04, size = 108, normalized size = 0.83

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\sqrt{dx+c}-\frac{\sqrt{\pi}\,\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\operatorname{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$ | 108 |
| default | $\frac{\sqrt{dx+c}-\frac{\sqrt{\pi}\,\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\operatorname{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$ | 108 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2*(d*x+c)^(1/2)-1/4*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*Fresnel
C(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/P
i^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.50, size = 187, normalized size = 1.44

$$\frac{\sqrt{2}\left(\left(\left(i-1\right)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+\left(i+1\right)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{2i\,b}{d}}\right)+\left(-\left(i+1\right)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(i-1\right)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{2i\,b}{d}}\right)+\frac{8\sqrt{2}\,\sqrt{dx+c}\,b}{d}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*(((I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/
d) + (I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*cos
(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) + 8*sqrt(2)*sqrt(d*x + c)*b/d)/
b
```

**Fricas [A]**
time = 0.36, size = 114, normalized size = 0.88

$$-\frac{\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2(bc-ad)}{d}\right) \mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \pi d \sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right) - 2\sqrt{dx+c}\,b}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-2*(b*c - a*d)/d) - 2*sqrt(d*x + c)*b)/(b*d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/sqrt(c + d*x), x)
```

**Giac [C]** Result contains complex when optimal does not.
time = 4.47, size = 165, normalized size = 1.27

$$\frac{\frac{\sqrt{\pi}\, d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)} + \frac{\sqrt{\pi}\, d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)} + 4\sqrt{dx+c}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(d*x + c))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(c + d*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2/(c + d*x)^(1/2), x)
```

# 3.49 $\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=135

$$\frac{2\sqrt{b}\,\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{d^{3/2}}+\frac{2\sqrt{b}\,\sqrt{\pi}\,C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^{3/2}}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] 2*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*b^(1/
2)*Pi^(1/2)/d^(3/2)+2*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*si
n(2*a-2*b*c/d)*b^(1/2)*Pi^(1/2)/d^(3/2)-2*sin(b*x+a)^2/d/(d*x+c)^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3394, 12, 3387, 3386, 3432, 3385, 3433}

$$\frac{2\sqrt{\pi}\,\sqrt{b}\,\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{d^{3/2}}+\frac{2\sqrt{\pi}\,\sqrt{b}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{d^{3/2}}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])])/d^(3/2) + (2*Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/d^(3/2) - (2*Sin[a +
b*x]^2)/(d*Sqrt[c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}+\frac{(4b)\int \frac{\sin(2a+2bx)}{2\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}+\frac{(2b)\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}+\frac{\left(2b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{d}+\frac{\left(2b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2\right.}{\sqrt{c+d}}
\\
&= -\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}+\frac{\left(4b\cos\left(2a-\frac{2bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{d^2}+\frac{(4b\sin(}{}
\\
&= \frac{2\sqrt{b}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}}+\frac{2\sqrt{b}\sqrt{\pi}\,C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin(2a-}{d^{3/2}}
\end{aligned}$$

**Mathematica [A]**
time = 0.24, size = 149, normalized size = 1.10

$$\frac{-1+\cos(2(a+bx))+2\sqrt{\frac{b}{d}}\sqrt{\pi}\sqrt{c+dx}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+2\sqrt{\frac{b}{d}}\sqrt{\pi}\sqrt{c+dx}\,C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (-1 + Cos[2*(a + b*x)] + 2*Sqrt[b/d]*Sqrt[Pi]*Sqrt[c + d*x]*Cos[2*a - (2*b*
c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 2*Sqrt[b/d]*Sqrt[Pi]
*Sqrt[c + d*x]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*
b*c)/d])/(d*Sqrt[c + d*x])
```

**Maple [A]**
time = 0.05, size = 145, normalized size = 1.07

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{1}{\sqrt{dx+c}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{d}}{\sqrt{\pi}}\right.\right.}{d\sqrt{\frac{b}{d}}}}{d}$ |
| default | $\frac{-\frac{1}{\sqrt{dx+c}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{d}}{\sqrt{\pi}}\right.\right.}{d\sqrt{\frac{b}{d}}}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/2/(d*x+c)^(1/2)+1/2/(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+
b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d)))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.60, size = 135, normalized size = 1.00

$$-\frac{\sqrt{2}\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{2i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{2i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{2i(dx+c)b}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\sqrt{\frac{(dx+c)b}{d}}+8}{8\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) + (I - 1)*s
qrt(2)*gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + ((I - 1)*sq
rt(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*(d*
x + c)*b/d))*sin(-2*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) + 8)/(sqrt(d*x + c)
*d)
```

**Fricas [A]**
time = 0.36, size = 138, normalized size = 1.02

$$\frac{2\left((\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+\sqrt{dx+c}\left(\cos(bx+a)^2-1\right)\right)}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) + (pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(2*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + sqrt(d*x + c)*(cos(b*x +
 a)^2 - 1))/(d^2*x + c*d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*x + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(c + d*x)^(3/2),x)
```

```
[Out] int(sin(a + b*x)^2/(c + d*x)^(3/2), x)
```

# 3.50 $\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=170

$$\frac{8b^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}}-\frac{8b^{3/2}\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^{5/2}}-\frac{8b\cos(a+bx)\sin}{3d^2\sqrt{c+d}}$$

```
[Out] -2/3*sin(b*x+a)^2/d/(d*x+c)^(3/2)+8/3*b^(3/2)*cos(2*a-2*b*c/d)*FresnelC(2*b
^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/d^(5/2)-8/3*b^(3/2)*Fresnel
S(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(5/
2)-8/3*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

**Rubi** [A]
time = 0.22, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 32, 3393, 3387, 3386, 3432, 3385, 3433}

$$\frac{8\sqrt{\pi}\,b^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{3d^{5/2}}-\frac{8\sqrt{\pi}\,b^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (8*b^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])])/(3*d^(5/2)) - (8*b^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(3*d^(5/2)) - (8*
b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sin[a + b*x]^2)/(3*
d*(c + d*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\int\frac{1}{\sqrt{c+dx}}\,dx}{3d^2} - \frac{(16b^2)\int\frac{\sin^2(a+}{\sqrt{c+}}}{3d^2}$$

$$= \frac{16b^2\sqrt{c+dx}}{3d^3} - \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{(16b^2)\int\left(\frac{1}{2\sqrt{c+dx}} - \right.}{3d^2}$$

$$= -\frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\int\frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(8b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} -$$

$$= -\frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(16b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int\cos\left(\frac{2bx^2}{d}\right.\right.}{3d^3}$$

$$= \frac{8b^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}} - \frac{8b^{3/2}\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\right.}{3d^{5/2}}$$

**Mathematica [A]**
time = 0.92, size = 158, normalized size = 0.93

$$\frac{2\left(4b\sqrt{\frac{b}{d}}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 4b\sqrt{\frac{b}{d}}\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right) - \frac{\sin(a+bx)(4b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^{3/2}}\right)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (2*(4*b*Sqrt[b/d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[
c + d*x])/Sqrt[Pi]] - 4*b*Sqrt[b/d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c +
 d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] - (Sin[a + b*x]*(4*b*(c + d*x)*Cos[a
+ b*x] + d*Sin[a + b*x]))/(c + d*x)^(3/2)))/(3*d^2)
```

**Maple [A]**
time = 0.05, size = 189, normalized size = 1.11

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{1}{3(dx+c)^{\frac{3}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}\cdots}{\sqrt{\pi}\sqrt{\frac{b}{d}}\cdots}\right)\cdots\right)}{d\sqrt{\frac{b}{d}}\cdots}\right)}{3d}\cdots}{d}$ |
| default | $\frac{-\frac{1}{3(dx+c)^{\frac{3}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}\cdots}{\sqrt{\pi}\sqrt{\frac{b}{d}}\cdots}\right)\cdots\right)}{d\sqrt{\frac{b}{d}}\cdots}\right)}{3d}\cdots}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/6/(d*x+c)^(3/2)+1/6/(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+
2/3*b/d*(-1/(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(
b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
))))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.61, size = 136, normalized size = 0.80

$$-\frac{3\sqrt{2}\left(\left(-(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{2i(dx+c)b}{d}\right)+(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{2i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{2i(dx+c)b}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}+4}{12(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I + 1
)*sqrt(2)*gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I + 1
)*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-3/2, -2*I
*(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) + 4)/((d*x +
c)^(3/2)*d)
```

**Fricas** [A]
time = 0.37, size = 209, normalized size = 1.23

$$\frac{2\left(4\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+\left(d\cos(bx+a)^2-4(bdx+bc)\cos(bx+a)\sin(bx+a)-d\right)\sqrt{dx+c}\right)}{3\left(d^4x^2+2cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(4*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-2*(b*c
- a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 4*(pi*b*d^2*x^2 + 2
*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-2*(b*c - a*d)/d) + (d*cos(b*x + a)^2 - 4*(b*d*x + b*c)*cos(b*x
 + a)*sin(b*x + a) - d)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**(5/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*x + c)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(c + d*x)^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2/(c + d*x)^(5/2), x)
```

# 3.51 $\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx$

**Optimal**. Leaf size=216

$$-\frac{16b^2}{15d^3\sqrt{c+dx}}-\frac{32b^{5/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{15d^{7/2}}-\frac{32b^{5/2}\sqrt{\pi}\,C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)\sin\left(2a-\right.}{15d^{7/2}}$$

```
[Out] -8/15*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(3/2)-2/5*sin(b*x+a)^2/d/(d*x+c)^
(5/2)-32/15*b^(5/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))*Pi^(1/2)/d^(7/2)-32/15*b^(5/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(7/2)-16/15*b^2/d^3/(d*x+c)
^(1/2)+32/15*b^2*sin(b*x+a)^2/d^3/(d*x+c)^(1/2)
```

**Rubi** [A]
time = 0.23, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3395, 32, 3394, 12, 3387, 3386, 3432, 3385, 3433}

$$-\frac{32\sqrt{\pi}\,b^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{15d^{7/2}}-\frac{32\sqrt{\pi}\,b^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{15d^{7/2}}+\frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (32*b^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(15*d^(7/2)) - (3
2*b^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(15*d^(7/2)) - (8*b*Cos[a + b*x]*Sin[a + b*x])/(15*d^2
*(c + d*x)^(3/2)) - (2*Sin[a + b*x]^2)/(5*d*(c + d*x)^(5/2)) + (32*b^2*Sin[
a + b*x]^2)/(15*d^3*Sqrt[c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{(8b^2)\int \frac{1}{(c+dx)^{3/2}}\,dx}{15d^2} - \frac{(16b^2)\int \frac{\sin^2(a+b}{(c+dx)^{3/}}}{15d^2}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}} -$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}} -$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}} -$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}} -$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{32b^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}} - \frac{32b^{5/2}\sqrt{\pi}\,C\left(\frac{2\sqrt{b}}{}\right.}{}$$

**Mathematica** [A]
time = 1.31, size = 244, normalized size = 1.13

$$-\frac{3d^2+16b^2c^2\cos(2(a+bx))-3d^2\cos(2(a+bx))+32b^2cdx\cos(2(a+bx))+16b^2d^2x^2\cos(2(a+bx))+32b\left(\frac{b}{d}\right)^{3/2}d\sqrt{\pi}(c+dx)^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+32b\left(\frac{b}{d}\right)^{3/2}d\sqrt{\pi}(c+dx)^{5/2}C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+4bcd\sin(2(a+bx))+4bd^2x\sin(2(a+bx))}{15d^3(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] -1/15*(3*d^2 + 16*b^2*c^2*Cos[2*(a + b*x)] - 3*d^2*Cos[2*(a + b*x)] + 32*b^
2*c*d*x*Cos[2*(a + b*x)] + 16*b^2*d^2*x^2*Cos[2*(a + b*x)] + 32*b*(b/d)^(3/
2)*d*Sqrt[Pi]*(c + d*x)^(5/2)*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sq
rt[c + d*x])/Sqrt[Pi]] + 32*b*(b/d)^(3/2)*d*Sqrt[Pi]*(c + d*x)^(5/2)*Fresne
lC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 4*b*c*d*Sin
[2*(a + b*x)] + 4*b*d^2*x*Sin[2*(a + b*x)])/(d^3*(c + d*x)^(5/2))
```

**Maple** [A]
time = 0.05, size = 230, normalized size = 1.06

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{1}{5(dx+c)^{\frac{5}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2d}{}\right.\right.}{}\right.}{}\right.}{5d}}{d}$ |
| default | $\frac{-\frac{1}{5(dx+c)^{\frac{5}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2d}{}\right.\right.}{}\right.}{}\right.}{5d}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/10/(d*x+c)^(5/2)+1/10/(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d
)+2/5*b/d*(-1/3/(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+4/3*b/d*(-1/
(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-2*b/d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.62, size = 136, normalized size = 0.63

$$-\frac{5\sqrt{2}\left(\left((i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{2i(dx+c)b}{d}\right)-(i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left(-(i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{2i(dx+c)b}{d}\right)+(i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{2i(dx+c)b}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}+2}{10\,(dx+c)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/10*(5*sqrt(2)*(((I + 1)*sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) - (I - 1)
*sqrt(2)*gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I - 1)
*sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) + (I + 1)*sqrt(2)*gamma(-5/2, -2*I*
(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) + 2)/((d*x + c
)^(5/2)*d)
```

**Fricas [A]**
time = 0.42, size = 328, normalized size = 1.52

$$-\frac{2\left(16\left(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+16\left(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-\left(16\,b^2d^2x^2+32\,b^2cdx+16\,b^2c^2-3\,d^2\right)\cos\left(bx+a\right)^2-4\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-3\,d^2\right)\sqrt{dx+c}\right)}{15\left(d^6x^3+3\,cd^5x^2+3\,c^2d^4x+c^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*
c^3)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) + 16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + p
i*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-
2*(b*c - a*d)/d) - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - (16*b^2*d^2*
x^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 3*d^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*
d)*cos(b*x + a)*sin(b*x + a) - 3*d^2)*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2
 + 3*c^2*d^4*x + c^3*d^3)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left(a + b x \right)}}{\left(c + d x\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(7/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**(7/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*x + c)^(7/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a+b\,x)^2}{(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(c + d*x)^(7/2),x)
```

```
[Out] int(sin(a + b*x)^2/(c + d*x)^(7/2), x)
```

# 3.52 $\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx$

**Optimal**. Leaf size=247

$$-\frac{16b^2}{105d^3(c+dx)^{3/2}}-\frac{128b^{7/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}}+\frac{128b^{7/2}\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin(2\ldots}{105d^{9/2}}$$

```
[Out] -16/105*b^2/d^3/(d*x+c)^(3/2)-8/35*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(5/2
)-2/7*sin(b*x+a)^2/d/(d*x+c)^(7/2)+32/105*b^2*sin(b*x+a)^2/d^3/(d*x+c)^(3/2
)-128/105*b^(7/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/Pi^(1/2))*Pi^(1/2)/d^(9/2)+128/105*b^(7/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(9/2)+128/105*b^3*cos(b*x+a
)*sin(b*x+a)/d^4/(d*x+c)^(1/2)
```

**Rubi** [A]

time = 0.28, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 32, 3393, 3387, 3386, 3432, 3385, 3433}

$$-\frac{128\sqrt{\pi}b^{7/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{105d^{9/2}}+\frac{128\sqrt{\pi}b^{7/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}}+\frac{128b^3\sin(a+bx)\cos(a+bx)}{105d^4\sqrt{c+dx}}+\frac{32b^2\sin^2(a+bx)}{105d^3(c+dx)^{3/2}}-\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (128*b^(7/2)*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(105*d^(9/2))
 + (128*b^(7/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])]*Sin[2*a - (2*b*c)/d])/(105*d^(9/2)) - (8*b*Cos[a + b*x]*Sin[a + b*x])/
(35*d^2*(c + d*x)^(5/2)) + (128*b^3*Cos[a + b*x]*Sin[a + b*x])/(105*d^4*Sqr
t[c + d*x]) - (2*Sin[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) + (32*b^2*Sin[a + b*
x]^2)/(105*d^3*(c + d*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{(8b^2)\int \frac{1}{(c+dx)^{5/2}}\,dx}{35d^2} - \frac{(16b^2)\int \frac{\sin^2(a+b}{(c+dx)^{5/}}}{35d^2}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\,\mathrm{s}}{7c}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{256b^4\sqrt{c+dx}}{105d^5} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx}{105d^4\sqrt{c}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\,\mathrm{s}}{7c}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\,\mathrm{s}}{7c}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\,\mathrm{s}}{7c}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{128b^{7/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}} + \frac{128b^{7/2}\sqrt{\pi}\,S}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 661 vs. 2(247) = 494.
time = 2.93, size = 661, normalized size = 2.68

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (-30*d^3 + Cos[2*a]*(4*Cos[(b*c)/d]*Sin[(b*c)/d]*(15*d^3*Sin[(2*b*(c + d*x)
)/d] + 4*b*(c + d*x)*(3*d^2*Cos[(2*b*(c + d*x))/d] - 4*b*(c + d*x)*(4*b*(c
+ d*x)*Cos[(2*b*(c + d*x))/d] + 8*b*Sqrt[b/d]*Sqrt[Pi]*(c + d*x)^(3/2)*Fres
nelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + d*Sin[(2*b*(c + d*x))/d]))) +
2*Cos[(2*b*c)/d]*(15*d^3*Cos[(2*b*(c + d*x))/d] - 4*b*(c + d*x)*(3*d^2*Sin[
(2*b*(c + d*x))/d] + 4*b*(c + d*x)*(d*Cos[(2*b*(c + d*x))/d] + 8*b*Sqrt[b/d
]*Sqrt[Pi]*(c + d*x)^(3/2)*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] -
 4*b*(c + d*x)*Sin[(2*b*(c + d*x))/d])))) - 2*Cos[a]*Sin[a]*(2*(Cos[(b*c)/d
] - Sin[(b*c)/d])*(Cos[(b*c)/d] + Sin[(b*c)/d])*(15*d^3*Sin[(2*b*(c + d*x))
/d] + 4*b*(c + d*x)*(3*d^2*Cos[(2*b*(c + d*x))/d] - 4*b*(c + d*x)*(4*b*(c +
```

```
 d*x)*Cos[(2*b*(c + d*x))/d] + 8*b*Sqrt[b/d]*Sqrt[Pi]*(c + d*x)^(3/2)*Fresn
elS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + d*Sin[(2*b*(c + d*x))/d]))) - 2
*Sin[(2*b*c)/d]*(15*d^3*Cos[(2*b*(c + d*x))/d] - 4*b*(c + d*x)*(3*d^2*Sin[(
2*b*(c + d*x))/d] + 4*b*(c + d*x)*(d*Cos[(2*b*(c + d*x))/d] + 8*b*Sqrt[b/d]
*Sqrt[Pi]*(c + d*x)^(3/2)*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] -
4*b*(c + d*x)*Sin[(2*b*(c + d*x))/d])))))/(210*d^4*(c + d*x)^(7/2))
```

**Maple** [A]
time = 0.07, size = 273, normalized size = 1.11

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{7(dx+c)^{\frac{7}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+}{d}\right.}{\sqrt{dx}}\right.}{}\right.}{}\right.}{}}{d}$ |

| | |
|---|---|
| default | $\frac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{7(dx+c)^{\frac{7}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2da-2cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx}{d}\right.}{\sqrt{}}\right.}{}\right.}{}\right.}{}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/14/(d*x+c)^(7/2)+1/14/(d*x+c)^(7/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d
)+2/7*b/d*(-1/5/(d*x+c)^(5/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+4/5*b/d*(-1/
3/(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-4/3*b/d*(-1/(d*x+c)^(1/2)*
sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c
)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*
FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.67, size = 136, normalized size = 0.55

$$\frac{7\sqrt{2}\left(\left(-(i-1)\sqrt{2}\,\Gamma\left(-\frac{7}{2},\frac{2i(dx+c)b}{d}\right)+(i+1)\sqrt{2}\,\Gamma\left(-\frac{7}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{7}{2},\frac{2i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{7}{2},-\frac{2i(dx+c)b}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}-1}{7(dx+c)^{\frac{7}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*(7*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I + 1)*
sqrt(2)*gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I + 1)*
sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-7/2, -2*I*(
d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(7/2) - 1)/((d*x + c)
^(7/2)*d)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(195) = 390$.
time = 0.42, size = 422, normalized size = 1.71

$$-\frac{2\left(64\left(\pi b^3d^4x^4+4\pi b^3cd^3x^3+6\pi b^3c^2d^2x^2+4\pi b^3c^3dx+\pi b^3c^4\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)C\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-64\left(\pi b^3d^4x^4+4\pi b^3cd^3x^3+6\pi b^3c^2d^2x^2+4\pi b^3c^3dx+\pi b^3c^4\right)\sqrt{\frac{b}{\pi d}}S\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-\left(8b^2d^3x^2+16b^2cd^2x+8b^2c^2d-15d^3-\left(16b^2d^3x^2+32b^2cd^2x+16b^2c^2d-15d^3\right)\cos\left(bx+a\right)^2+4\left(16b^3d^3x^3+48b^3cd^2x^2+16b^3c^3-3bcd^2+3\left(16b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}\right)}{105\left(d^8x^4+4cd^7x^3+6c^2d^6x^2+4c^3d^5x+c^4d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3 + 6*pi*b^3*c^2*d^2*x^2 + 4*
pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_c
os(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^
3 + 6*pi*b^3*c^2*d^2*x^2 + 4*pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*fr
esnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - (8*b^2*d^
3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 15*d^3 - (16*b^2*d^3*x^2 + 32*b^2*c*
d^2*x + 16*b^2*c^2*d - 15*d^3)*cos(b*x + a)^2 + 4*(16*b^3*d^3*x^3 + 48*b^3*
c*d^2*x^2 + 16*b^3*c^3 - 3*b*c*d^2 + 3*(16*b^3*c^2*d - b*d^3)*x)*cos(b*x +
a)*sin(b*x + a))*sqrt(d*x + c))/(d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*
c^3*d^5*x + c^4*d^4)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*x + c)^(9/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,x\right)^2}{{\left(c+d\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(c + d*x)^(9/2),x)
[Out] int(sin(a + b*x)^2/(c + d*x)^(9/2), x)
```

# 3.53 $\int (c+dx)^{5/2} \sin^3(a+bx)\,dx$

**Optimal**. Leaf size=410

$$\frac{45d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{2(c+dx)^{5/2}\cos(a+bx)}{3b}-\frac{5d^2\sqrt{c+dx}\,\cos(3a+3bx)}{144b^3}-\frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)\cdots}{16\cdots}$$

```
[Out] -2/3*(d*x+c)^(5/2)*cos(b*x+a)/b+5/3*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2-1/3*(d*x
+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2/b+5/18*d*(d*x+c)^(3/2)*sin(b*x+a)^3/b^2+5
/864*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/864*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-
45/32*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+45/32*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+45/16*d
^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3-5/144*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

**Rubi** [A]
time = 0.80, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 3377, 3387, 3386, 3432, 3385, 3433, 3393}

$$-\frac{45\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{45\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{45d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{5d^2\sqrt{c+dx}\,\cos(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{3b^2}-\frac{2(c+dx)^{5/2}\cos(a+bx)}{3b}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - (2*(c + d*x)^(5/2)*Cos[a + b
*x])/(3*b) - (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - (45*d^(5/2)
*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(16*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[
(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(144*b^(7/2)) - (5*d^(5/2)*Sqr
t[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b
*c)/d])/(144*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3
/2)*Sin[a + b*x])/(3*b^2) - ((c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2)/(
3*b) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x]^3)/(18*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx = -\frac{(c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2} + \frac{2}{3}\int (c$$

$$= -\frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)}{3b} + 5d(c+d$$

$$= -\frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{3b^2} - \frac{(c+dx)^{5/2}\cos(a+}{3b}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\,\cos(3a}{144b^3}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\,\cos(3a}{144b^3}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\,\cos(3a}{144b^3}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\,\cos(3a}{144b^3}$$

**Mathematica [A]**
time = 2.06, size = 542, normalized size = 1.32

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sin[a + b*x]^3,x]
```

```
[Out] (-648*b^3*c^2*Sqrt[c + d*x]*Cos[a + b*x] + 2430*b*d^2*Sqrt[c + d*x]*Cos[a +
 b*x] - 1296*b^3*c*d*x*Sqrt[c + d*x]*Cos[a + b*x] - 648*b^3*d^2*x^2*Sqrt[c
+ d*x]*Cos[a + b*x] + 72*b^3*c^2*Sqrt[c + d*x]*Cos[3*(a + b*x)] - 30*b*d^2*
Sqrt[c + d*x]*Cos[3*(a + b*x)] + 144*b^3*c*d*x*Sqrt[c + d*x]*Cos[3*(a + b*x
)] + 72*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[3*(a + b*x)] - 1215*Sqrt[b/d]*d^3*Sqr
t[2*Pi]*Cos[a - (b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 5*S
qrt[b/d]*d^3*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]] - 5*Sqrt[b/d]*d^3*Sqrt[6*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*S
qrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 1215*Sqrt[b/d]*d^3*Sqrt[2*Pi]*FresnelS
[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] + 1620*b^2*c*d*Sqrt[c
 + d*x]*Sin[a + b*x] + 1620*b^2*d^2*x*Sqrt[c + d*x]*Sin[a + b*x] - 60*b^2*c
```

```
*d*Sqrt[c + d*x]*Sin[3*(a + b*x)] - 60*b^2*d^2*x*Sqrt[c + d*x]*Sin[3*(a + b
*x)])/(864*b^4)
```

## Maple [A]

time = 0.03, size = 476, normalized size = 1.16

| method | result |
|---|---|
| derivativedivides | $-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}+\cdots\right.}{\cdots}\right.}{\cdots}$ |
| default | $-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}+\cdots\right.}{\cdots}\right.}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/8/b*d*(d*x+c)^(5/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+15/8/b*d*(1/2/b*
d*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1
/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin
((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1
/24/b*d*(d*x+c)^(5/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-5/24/b*d*(1/6/b*d*(d
*x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2
```

```
)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*b*(d*x+c)^(1/2)/d)))))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.55, size = 547, normalized size = 1.33

$$-\frac{240\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-6480\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)-24\left(\frac{12\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-5\,\sqrt{dx+c}\,b^{2}d\right)\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)+648\left(\frac{4\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-15\,\sqrt{dx+c}\,b^{2}d\right)\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-5\left(-(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{3\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{3i\,b}{d}}\right)-1215\left((i-1)\sqrt{2}\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{2}\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{i\,b}{d}}\right)-1215\left(-(i+1)\sqrt{2}\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i-1)\sqrt{2}\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{i\,b}{d}}\right)-5\left((i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)+(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{3\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3i\,b}{d}}\right)\right)d}{3456\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) - 6480*
(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5/
2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) + 648*
(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*c +
 a*d)/d) - 5*(-(I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-
3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*s
in(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 1215*((I - 1)*sqrt
(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*sq
rt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*
b/d)) - 1215*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 5*((I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)
*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b
/d)))*d/b^5
```

**Fricas** [A]
time = 0.39, size = 371, normalized size = 0.90

$$\frac{5\,\sqrt{6}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-1215\,\sqrt{2}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+1215\,\sqrt{2}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\,S\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-5\,\sqrt{6}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\,S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+24\left(\left(12\,b^{3}d^{2}x^{2}+24\,b^{3}cdx+12\,b^{3}c^{2}-5\,bd^{2}\right)\cos\left(bx+a\right)^{3}-3\left(12\,b^{3}d^{2}x^{2}+24\,b^{3}cdx+12\,b^{3}c^{2}-35\,bd^{2}\right)\cos\left(bx+a\right)+10\left(7\,b^{2}d^{2}x+7\,b^{2}cd-\left(b^{2}d^{2}x+b^{2}cd\right)\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{864\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 1215*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 1215*
sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*((12*b^3*d^2*x
^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^3 - 3*(12*b^3*d^2*x^
2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2)*cos(b*x + a) + 10*(7*b^2*d^2*x +
```

```
7*b^2*c*d - (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x +
c))/b^4
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

**Giac** [**C**] Result contains complex when optimal does not.
time = 3.95, size = 2479, normalized size = 6.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/1728*(72*(9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(27*(I*sqrt(2)*sqrt(
pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*
c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2
+ (I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-2*I*(d*x + c)^(3/2)*b*d
 + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d -
3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*
I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)
*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12*
b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b^2) - 6*I*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d
```

```
- sqrt(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + d
^3*(81*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I
*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4
*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c
)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sq
rt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (-I*sqrt
(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/
2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(-12*I*(d*x
+ c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^
2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x +
c)*d^3)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 81*(I*sqrt(2)*
sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/
2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10
*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)
*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + (I*sqrt(6)*sqrt(pi)*(72*b
^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(-12*I*(d*x + c)^(5/2)*b^2*d +
36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^
(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^(-3*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 36*(-27*I*sqrt(2)*sqrt(pi)*(2*b*c
 + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt
(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b) + 27*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)
*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 54*sqrt(d
*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-3
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c
- I*a*d)/d)/b)*c^2)/d
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(c + d*x)^(5/2),x)
[Out] int(sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

# 3.54 $\int (c+dx)^{3/2} \sin^3(a+bx)\,dx$

**Optimal**. Leaf size=354

$$-\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b}-\frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{\cdots}}{\cdots}\right)}{24b^{5/2}}$$

```
[Out] -2/3*(d*x+c)^(3/2)*cos(b*x+a)/b-1/3*(d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2/b
+1/144*d^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)+1/144*d^(3/2)*FresnelC(b^(1/2)*6^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2
)-9/16*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-9/16*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+d*sin(b
*x+a)*(d*x+c)^(1/2)/b^2+1/6*d*sin(b*x+a)^3*(d*x+c)^(1/2)/b^2
```

**Rubi** [A]
time = 0.68, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$,
Rules used = {3392, 3377, 3387, 3386, 3432, 3385, 3433, 3393}

$$\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}+\frac{d\sqrt{c+dx}\sin(a+bx)}{b^2}-\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sin[a + b*x]^3,x]
```

```
[Out] (-2*(c + d*x)^(3/2)*Cos[a + b*x])/(3*b) - (9*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*
c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (
d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(24*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) - (9*d^(
3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(5/2)) + (d*Sqrt[c + d*x]*Sin[a + b*x])/b^2 - ((c + d*x)^(
3/2)*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (d*Sqrt[c + d*x]*Sin[a + b*x]^3)/
(6*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{d\sqrt{c+dx}\,\sin^3(a+bx)}{6b^2} + \frac{2}{3}\int (c+$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b} - \frac{(c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{d\sqrt{c+d}}{}$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b} + \frac{d\sqrt{c+dx}\,\sin(a+bx)}{b^2} - \frac{(c+dx)^{3/2}\cos(a+bx}{3b}$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b} + \frac{d\sqrt{c+dx}\,\sin(a+bx)}{b^2} - \frac{(c+dx)^{3/2}\cos(a+bx}{3b}$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b} + \frac{d\sqrt{c+dx}\,\sin(a+bx)}{b^2} - \frac{(c+dx)^{3/2}\cos(a+bx}{3b}$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b} - \frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}$$

**Mathematica** [A]
time = 1.04, size = 389, normalized size = 1.10

$$\frac{-108bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(a+bx)-108b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(a+bx)+12bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(3(a+bx))+12b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(3(a+bx))-81d\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+d\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+d\sqrt{6\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)-81d\sqrt{2\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right)+162\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(a+bx)-6\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(3(a+bx))}{144b^2\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sin[a + b*x]^3,x]
```

```
[Out] (-108*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[a + b*x] - 108*b*Sqrt[b/d]*d*x*Sqrt[c
 + d*x]*Cos[a + b*x] + 12*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] + 12
*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[3*(a + b*x)] - 81*d*Sqrt[2*Pi]*Cos[a - (
b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + d*Sqrt[6*Pi]*Cos[3*a
 - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + d*Sqrt[6*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] - 81*d*Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] + 162
*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[a + b*x] - 6*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[3
*(a + b*x)])/(144*b^2*Sqrt[b/d])
```

**Maple** [A]
time = 0.03, size = 384, normalized size = 1.08

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right)\right.}{}\right)}{4b}}{}$ |
| default | $\frac{-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right)\right.}{}\right)}{4b}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/8/b*d*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+9/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))+1/24/b*d*(d*x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.57, size = 499, normalized size = 1.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d - 432*(d
*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*b^2
*sin(3*((d*x + c)*b - b*c + a*d)/d) + 648*sqrt(d*x + c)*b^2*sin(((d*x + c)*
b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 81*(-(I + 1)*sq
```

```
rt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sq
rt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/
d)) + 81*((I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)
- (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^
(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2
)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^4
```

**Fricas** [**A**]
time = 0.39, size = 300, normalized size = 0.85

$$\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) - 81\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) - 81\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right) + \sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right) + 24\left(2\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^3 - 6\left(b^2dx+b^2c\right)\cos\left(bx+a\right) - \left(bd\cos\left(bx+a\right)^2 - 7\,bd\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{144\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*c
os(b*x + a)^3 - 6*(b^2*d*x + b^2*c)*cos(b*x + a) - (b*d*cos(b*x + a)^2 - 7*
b*d)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**3, x)
```

**Giac** [**C**] Result contains complex when optimal does not.
time = 5.09, size = 1548, normalized size = 4.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/288*(12*(9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
```

```
b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(27*(I*sqrt(2)*sqrt(pi)*(4
*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d +
3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (I*s
qrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-2*I*(d*x + c)^(3/2)*b*d + 4*I
*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I
*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12*b^2*c^
2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) - 6*I*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - sqrt
(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 4*(-27*
I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(2)*sqrt(pi)*(2*b*c - I
*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(
6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + 54*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b
- 6*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*
x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-3
*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

# 3.55 $\int \sqrt{c+dx}\,\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=304

$$-\frac{3\sqrt{c+dx}\,\cos(a+bx)}{4b}+\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{3\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{d}\ \cdots}{}$$

```
[Out] -1/72*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/72*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+
3/8*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d
^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-3/8*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-3/4*cos(b*x
+a)*(d*x+c)^(1/2)/b+1/12*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

**Rubi** [A]
time = 0.33, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{3\sqrt{c+dx}\cos(a+bx)}{4b}+\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cos[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(1
2*b) + (3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c
)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin
[3*a - (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*S
qrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\,\sin^3(a+bx)\,dx = \int \left(\frac{3}{4}\sqrt{c+dx}\,\sin(a+bx) - \frac{1}{4}\sqrt{c+dx}\,\sin(3a+3bx)\right)\,dx$$

$$= -\left(\frac{1}{4}\int \sqrt{c+dx}\,\sin(3a+3bx)\,dx\right) + \frac{3}{4}\int \sqrt{c+dx}\,\sin(a+bx)\,dx$$

$$= -\frac{3\sqrt{c+dx}\,\cos(a+bx)}{4b} + \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b} - \frac{d\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx}{24b} + \ldots$$

$$= -\frac{3\sqrt{c+dx}\,\cos(a+bx)}{4b} + \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b} - \frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \ldots}{24b}$$

$$= -\frac{3\sqrt{c+dx}\,\cos(a+bx)}{4b} + \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{3\sqrt{c+dx}\,\cos(a+bx)}{4b} + \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b} + \frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\ldots}{\ldots}$$

**Mathematica** [A]
time = 0.47, size = 266, normalized size = 0.88

$$\frac{-54\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(a+bx)+6\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(3(a+bx))+27\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{6\pi}S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)-27\sqrt{2\pi}S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right)}{72b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[a + b*x] + 6*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3
*(a + b*x)] + 27*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]] - Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]] + Sqrt[6*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]
]*Sin[3*a - (3*b*c)/d] - 27*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]]*Sin[a - (b*c)/d])/(72*b*Sqrt[b/d])
```

**Maple** [A]
time = 0.03, size = 296, normalized size = 0.97

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{3d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\operatorname{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$ |
| default | $\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{3d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\operatorname{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/8/b*d*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))+1/24/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/
d)-1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.53, size = 424, normalized size = 1.39

$$\frac{\left(\frac{24\sqrt{dx+c}\,b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{216\sqrt{dx+c}\,b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac14}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i+1)\cdot 9^{\frac14}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{3ib}{d}}\right)-27\left((i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-27\left(-(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{bc-ad}{d}\right)-(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)+\left(-(i+1)\cdot 9^{\frac14}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i-1)\cdot 9^{\frac14}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3ib}{d}}\right)\right)d}{288\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d - 216*sqrt
(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) - 27*((I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) + (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(I*b/d)) - 27*(-(I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*
cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt
(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt
(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b
/d)))*d/b^3
```

**Fricas** [**A**]
time = 0.38, size = 246, normalized size = 0.81

$$-\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-24\,\left(b\cos\left(bx+a\right)^{3}-3\,b\cos\left(bx+a\right)\right)\sqrt{dx+c}}{72\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b
*c - a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c
)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^3 - 3*b*cos(b*
x + a))*sqrt(d*x + c))/b^2
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3, x)
```

**Giac** [**C**] Result contains complex when optimal does not.
time = 4.73, size = 848, normalized size = 2.79

$$-\frac{-\frac{27i\sqrt{2}\sqrt{\pi}(2bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\sqrt{6}\sqrt{\pi}(6bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{3(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{27i\sqrt{2}\sqrt{\pi}(2bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{i\sqrt{6}\sqrt{\pi}(6bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{3(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+6\left(\frac{9i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{i\sqrt{6}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{3(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{9i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{6}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{3(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c+\frac{54\sqrt{dx+c}\,de^{\left(\frac{i(dx+c)b-ibc+iad}{d}\right)}}{b}-\frac{6\sqrt{dx+c}\,de^{\left(-\frac{3(i(dx+c)b-ibc+iad)}{d}\right)}}{b}+\frac{54\sqrt{dx+c}\,de^{\left(\frac{-i(dx+c)b+ibc-iad}{d}\right)}}{b}-\frac{6\sqrt{dx+c}\,de^{\left(-\frac{3(-i(dx+c)b+ibc-iad)}{d}\right)}}{b}}{144\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/144*(-27*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(2)*sqrt(pi
)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b) + I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 6*(9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt
```

```
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*er
f(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 54*sqrt(d*x +
 c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-3*(I*
(d*x + c)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b +
 I*b*c - I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*
a*d)/d)/b)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(c + d*x)^(1/2),x)
[Out] int(sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

## 3.56 $\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=257

$$\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}C\left(\frac{\sqrt{b}}{}\right.}{}$$

```
[Out] -1/12*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-1/12*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)+
3/4*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2
^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)+3/4*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)
```

**Rubi** [A]
time = 0.27, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(
Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/
6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(2*Sqrt[b]*Sqrt[d]) + (3*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx &= \int \left(\frac{3\sin(a+bx)}{4\sqrt{c+dx}}-\frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)dx\\
&= -\left(\frac{1}{4}\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx\right)+\frac{3}{4}\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx\\
&= -\left(\frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx\right)+\frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx\\
&= -\frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)\,dx,x,\sqrt{c+dx}\right)}{2d}+\frac{\left(3\cos\left(a-\frac{bc}{d}\right)\right)\mathrm{Subst}\left(\int \sin\left(\right.\right.}{2d}\\
&= \frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+\right.}{\sqrt{d}}}{2\sqrt{b}\sqrt{d}}
\end{aligned}$$

**Mathematica** [A]
time = 0.36, size = 202, normalized size = 0.79

$$-\frac{\sqrt{\frac{b}{d}}\sqrt{\frac{\pi}{2}}\left(-9\cos\left(a-\frac{bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+\sqrt{3}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{3}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)-9C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] -1/6*(Sqrt[b/d]*Sqrt[Pi/2]*(-9*Cos[a - (b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] + Sqrt[3]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]] + Sqrt[3]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*S
in[3*a - (3*b*c)/d] - 9*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a
- (b*c)/d]))/b
```

**Maple** [A]
time = 0.03, size = 210, normalized size = 0.82

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4\sqrt{\frac{b}{d}}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\ldots}{\ldots}}{d}$ |
| default | $\frac{\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4\sqrt{\frac{b}{d}}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\ldots}{\ldots}}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/8*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^
(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.54, size = 377, normalized size = 1.47

$$\frac{\left(\left(-\frac{(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)}{d}+\frac{(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)}{d}\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{3ib}{d}}\right)-9\left(-\frac{(i+1)\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)}{d}+\frac{(i-1)\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)}{d}\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{ib}{d}}\right)-9\left(\frac{(i-1)\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)}{d}-\frac{(i+1)\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)}{d}\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{ib}{d}}\right)+\left(\frac{(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)}{d}-\frac{(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)}{d}\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{3ib}{d}}\right)\right)d}{48\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/48*((-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*
d)/d)/d + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c -
a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 9*(-(I + 1)*sqrt(2)*sqrt(pi)*
b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I - 1
)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (I + 1)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c
- a*d)/d)/d - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*
c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^2
```

**Fricas [A]**
time = 0.36, size = 212, normalized size = 0.82

$$-\frac{\sqrt{6}\,\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{6}\,\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(6)*pi*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*
sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d
)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sqrt(
b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d
)/d) + sqrt(6)*pi*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-3*(b*c - a*d)/d))/b
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**3/sqrt(c + d*x), x)
```

**Giac [C]** Result contains complex when optimal does not.
time = 4.61, size = 332, normalized size = 1.29

$$-\frac{\frac{9i\,\sqrt{2}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{i\,\sqrt{6}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{9i\,\sqrt{2}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\,\sqrt{6}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,x\right)^3}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(c + d*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)^3/(c + d*x)^(1/2), x)
```

# 3.57 $\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=270

$$\frac{3\sqrt{b}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{b}\,\sqrt{\frac{3\pi}{2}}\,\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}\,\sqrt{\ }\cdots}{}$$

```
[Out] 3/2*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b
^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-3/2*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*sin(a-b*c/d)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-1/2*cos(3*a
-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*
6^(1/2)*Pi^(1/2)/d^(3/2)+1/2*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*sin(3*a-3*b*c/d)*b^(1/2)*6^(1/2)*Pi^(1/2)/d^(3/2)-2*sin(b*x+a)^3
/d/(d*x+c)^(1/2)
```

**Rubi** [A]
time = 0.37, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3394, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[3*a - (3*b*c)/d]*Fr
esnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]*Sqrt
[(3*Pi)/2]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/d^(3/2) - (3*Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(3/2) - (2*Sin[a + b*x]^3)/(d*Sqrt
[c + d*x])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}} + \frac{(6b)\int \left(\frac{\cos(a+bx)}{4\sqrt{c+dx}} - \frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d} \\
&= -\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}} + \frac{(3b)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{2d} - \frac{(3b)\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx}{2d} \\
&= -\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{2d} + \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}}{2d} \\
&= -\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d^2} + \frac{(3b\cos(}{} \\
&= \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{}}{}\right.}{d^{3/2}}
\end{aligned}
$$

**Mathematica** [A]
time = 0.62, size = 300, normalized size = 1.11

$$
\frac{3\sqrt{\frac{b}{d}}\sqrt{2\pi}\sqrt{c+dx}\cos\left(a-\frac{bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - \sqrt{\frac{b}{d}}\sqrt{6\pi}\sqrt{c+dx}\cos\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) + \sqrt{\frac{b}{d}}\sqrt{6\pi}\sqrt{c+dx}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right) - 3\sqrt{\frac{b}{d}}\sqrt{2\pi}\sqrt{c+dx}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right) - 3\sin(a+bx) + \sin(3(a+bx))}{2d\sqrt{c+dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (3*Sqrt[b/d]*Sqrt[2*Pi]*Sqrt[c + d*x]*Cos[a - (b*c)/d]*FresnelC[Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - Sqrt[b/d]*Sqrt[6*Pi]*Sqrt[c + d*x]*Cos[3*a - (3*
b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[b/d]*Sqrt[6*Pi]
*Sqrt[c + d*x]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*
c)/d] - 3*Sqrt[b/d]*Sqrt[2*Pi]*Sqrt[c + d*x]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[a - (b*c)/d] - 3*Sin[a + b*x] + Sin[3*(a + b*x)])/(2*d*S
qrt[c + d*x])
```

**Maple** [A]
time = 0.03, size = 288, normalized size = 1.07

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{3\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2\sqrt{dx+c}}+\frac{3b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\text{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+\cdots}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\cdots\right.\right.}{2d\sqrt{\frac{b}{d}}}\cdots}{\cdots}$ |
| default | $\frac{-\frac{3\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2\sqrt{dx+c}}+\frac{3b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\text{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+\cdots}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\cdots\right.\right.}{2d\sqrt{\frac{b}{d}}}\cdots}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/4/(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/4*b/d*2^(1/2)*Pi^(
1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d))+1/4/(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/4*b/d
*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS
(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.67, size = 253, normalized size = 0.94

$$\frac{\sqrt{3}\left(\left(\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{3i(dx+c)b}{d}\right)-\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+\left(\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{3i(dx+c)b}{d}\right)-\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{3i(dx+c)b}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\sqrt{\frac{(dx+c)b}{d}}-3\left(\left(\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{i(dx+c)b}{d}\right)-\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{i(dx+c)b}{d}\right)-\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\sqrt{\frac{(dx+c)b}{d}}}{16\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(3)*(((I - 1)*sqrt(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) - (I + 1)*sq
rt(2)*gamma(-1/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I + 1)*sqr
t(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-1/2, -3*I*(d*x
 + c)*b/d))*sin(-3*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) - 3*(((I - 1)*sqrt(2
)*gamma(-1/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b
/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)*b/d) -
(I - 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*sqrt((d
*x + c)*b/d))/(sqrt(d*x + c)*d)
```

**Fricas** [A]
time = 0.38, size = 274, normalized size = 1.01

$$-\frac{\sqrt{6}\left(\pi dx+\pi c\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-3\sqrt{2}\left(\pi dx+\pi c\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\left(\pi dx+\pi c\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{6}\left(\pi dx+\pi c\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-4\sqrt{dx+c}\left(\cos\left(bx+a\right)^{2}-1\right)\sin\left(bx+a\right)}{2\left(d^{2}x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(6)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_
cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(
b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*
d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - sqrt(6)*(pi*d*x + pi*c)*sqrt(b
/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) - 4*sqrt(d*x + c)*(cos(b*x + a)^2 - 1)*sin(b*x + a))/(d^2*x + c*d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x)**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*x + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(c + d*x)^(3/2),x)
```

```
[Out] int(sin(a + b*x)^3/(c + d*x)^(3/2), x)
```

# 3.58 $\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=292

$$-\frac{b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{b^{3/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{b^{3/2}\sqrt{}}{}$$

```
[Out] -2/3*sin(b*x+a)^3/d/(d*x+c)^(3/2)-b^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)-b^(3/2)*Fresn
elC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi
^(1/2)/d^(5/2)+b^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/d^(5/2)+b^(3/2)*FresnelC(b^(1/2)*6^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/d^(5
/2)-4*b*cos(b*x+a)*sin(b*x+a)^2/d^2/(d*x+c)^(1/2)
```

**Rubi** [A]
time = 0.49, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3395, 3387, 3386, 3432, 3385, 3433, 3393}

$$\frac{\sqrt{6\pi}\,b^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{2\pi}\,b^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{2\pi}\,b^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{\sqrt{6\pi}\,b^{3/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] -((b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/d^(5/2)) + (b^(3/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*Fresn
elS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(5/2) + (b^(3/2)*Sqrt[6*
Pi]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/
d])/d^(5/2) - (b^(3/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x
])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(5/2) - (4*b*Cos[a + b*x]*Sin[a + b*x]^2)/(
d^2*Sqrt[c + d*x]) - (2*Sin[a + b*x]^3)/(3*d*(c + d*x)^(3/2))
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{4b\cos(a+bx)\sin^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(12b^2)\int \frac{\sin^3(}{\sqrt{c}}}{d^2}$$

$$= -\frac{4b\cos(a+bx)\sin^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{(12b^2)\int \left(\frac{3\sin(a+bx)}{4\sqrt{c+dx}} - \frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)}{d^2}$$

$$= -\frac{4b\cos(a+bx)\sin^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{(3b^2)\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(9b^2)\int \frac{\sin(a}{\sqrt{c+}}}{d^2}$$

$$= \frac{8b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{8b^{3/2}\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right.}{d^{5/2}}$$

$$= \frac{8b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{8b^{3/2}\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right.}{d^{5/2}}$$

$$= -\frac{b^{3/2}\sqrt{2\pi}\,\cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{b^{3/2}\sqrt{6\pi}\,\cos\left(3a-\frac{3bc}{d}\right) S\left(\frac{\sqrt{b}\,\sqrt{}\right.}{d^{5/2}}$$

**<u>Mathematica</u> [A]**
time = 1.53, size = 496, normalized size = 1.70

$$\frac{-6bc\cos(a+bx) - 6bdx\cos(a+bx) + 6bc\cos(3(a+bx)) + 6bdx\cos(3(a+bx)) - 6b\sqrt{\frac{b}{d}}\sqrt{2\pi}(c+dx)^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) + 6b\sqrt{\frac{b}{d}}\sqrt{6\pi}(c+dx)^{3/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) + 6bc\sqrt{\frac{b}{d}}\sqrt{6\pi}\sqrt{c+dx}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right) + 6b\sqrt{\frac{b}{d}}\,d\sqrt{6\pi}\,x\sqrt{c+dx}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right) - 6bc\sqrt{\frac{b}{d}}\sqrt{2\pi}\sqrt{c+dx}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right) - 6b\sqrt{\frac{b}{d}}\,d\sqrt{2\pi}\,x\sqrt{c+dx}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(a-\frac{bc}{d}\right) - 3d\sin(a+bx) + d\sin(3(a+bx))}{6d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-6*b*c*Cos[a + b*x] - 6*b*d*x*Cos[a + b*x] + 6*b*c*Cos[3*(a + b*x)] + 6*b*
d*x*Cos[3*(a + b*x)] - 6*b*Sqrt[b/d]*Sqrt[2*Pi]*(c + d*x)^(3/2)*Cos[a - (b*
c)/d]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 6*b*Sqrt[b/d]*Sqrt[6*P
i]*(c + d*x)^(3/2)*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[
c + d*x]] + 6*b*c*Sqrt[b/d]*Sqrt[6*Pi]*Sqrt[c + d*x]*FresnelC[Sqrt[b/d]*Sqr
t[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 6*b*Sqrt[b/d]*d*Sqrt[6*Pi]*x*
Sqrt[c + d*x]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c
```

```
)/d] - 6*b*c*Sqrt[b/d]*Sqrt[2*Pi]*Sqrt[c + d*x]*FresnelC[Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] - 6*b*Sqrt[b/d]*d*Sqrt[2*Pi]*x*Sqrt[c +
d*x]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] - 3*d*Si
n[a + b*x] + d*Sin[3*(a + b*x)])/(6*d^2*(c + d*x)^(3/2))
```

**Maple** [A]
time = 0.03, size = 368, normalized size = 1.26

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2(dx+c)^{\frac{3}{2}}}+\frac{b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)S\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{Fres}\cdots\right.}{d\sqrt{\frac{b}{d}}}\cdots\right.}{d}\cdots}{\cdots}$ |
| default | $\frac{-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{2(dx+c)^{\frac{3}{2}}}+\frac{b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)S\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{Fres}\cdots\right.}{d\sqrt{\frac{b}{d}}}\cdots\right.}{d}\cdots}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/4/(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/2*b/d*(-1/(d*x+c)^
(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/12
/(d*x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^(1/2)*c
os(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(c
os(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d))))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.68, size = 253, normalized size = 0.87

$$\frac{3\left(\sqrt{3}\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{3i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{3i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{3i(dx+c)b}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}-\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}\right)}{16\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/16*(sqrt(3)*((-(I + 1)*sqrt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) + (I - 1)*s
qrt(2)*gamma(-3/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I - 1)*sq
rt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-3/2, -3*I*(d*
x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) - ((-(I + 1)*sqrt
(2)*gamma(-3/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)
*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d)
- (I + 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*((d*x
 + c)*b/d)^(3/2))/((d*x + c)^(3/2)*d)
```

**Fricas** [A]
time = 0.42, size = 388, normalized size = 1.33

$$\frac{3\sqrt{6}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-3\sqrt{2}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-3\sqrt{2}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+3\sqrt{6}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+2\left(6(bdx+bc)\cos(bx+a)^3-6(bdx+bc)\cos(bx+a)+\left(d\cos(bx+a)^2-d\right)\sin(bx+a)\right)\sqrt{dx+c}}{3\left(d^4x^2+2cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(6)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(
-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqr
t(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-(b*c - a*
d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(pi*b*d
^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d
*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 3*sqrt(6)*(pi*b*d^2*x^2 + 2*p
i*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 2*(6*(b*d*x + b*c)*cos(b*x + a)^3 - 6*(
b*d*x + b*c)*cos(b*x + a) + (d*cos(b*x + a)^2 - d)*sin(b*x + a))*sqrt(d*x +
 c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x)**(5/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*x + c)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a+b\,x)^3}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(c + d*x)^(5/2),x)
```

```
[Out] int(sin(a + b*x)^3/(c + d*x)^(5/2), x)
```

# 3.59 $\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx$

**Optimal**. Leaf size=356

$$-\frac{2b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{6b^{5/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{6b}{}$$

```
[Out] -4/5*b*cos(b*x+a)*sin(b*x+a)^2/d^2/(d*x+c)^(3/2)-2/5*sin(b*x+a)^3/d/(d*x+c)
^(5/2)-2/5*b^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)+2/5*b^(5/2)*FresnelS(b^(1/2)*2^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/d^(7/2)+6/5*
b^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*6^(1/2)*Pi^(1/2)/d^(7/2)-6/5*b^(5/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/d^(7/2)-16/5*b^
2*sin(b*x+a)/d^3/(d*x+c)^(1/2)+24/5*b^2*sin(b*x+a)^3/d^3/(d*x+c)^(1/2)
```

**Rubi** [A]

time = 0.54, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 3378, 3387, 3386, 3432, 3385, 3433, 3394}

$$-\frac{2\sqrt{2\pi}\,b^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{6\sqrt{6\pi}\,b^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{6\sqrt{6\pi}\,b^{5/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2\sqrt{2\pi}\,b^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{24b^2\sin^3(a+bx)}{5d^3\sqrt{c+dx}}-\frac{16b^2\sin(a+bx)}{5d^3\sqrt{c+dx}}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (-2*b^(5/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) - (6*b^(
5/2)*Sqrt[6*Pi]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*
a - (3*b*c)/d])/(5*d^(7/2)) + (2*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(5*d^(7/2)) - (16*b^2*Sin[a
 + b*x])/(5*d^3*Sqrt[c + d*x]) - (4*b*Cos[a + b*x]*Sin[a + b*x]^2)/(5*d^2*(
c + d*x)^(3/2)) - (2*Sin[a + b*x]^3)/(5*d*(c + d*x)^(5/2)) + (24*b^2*Sin[a
+ b*x]^3)/(5*d^3*Sqrt[c + d*x])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{4b\cos(a+bx)\sin^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{(8b^2)\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2} - \frac{(12b^2)\int \frac{\sin^3(a+}{(c+dx)}}{5d^2} \cdots$$

$$= -\frac{16b^2\sin(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cos(a+bx)\sin^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\sin^3(a+bx)}{5d^3\sqrt{c+dx}} \cdots$$

$$= -\frac{16b^2\sin(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cos(a+bx)\sin^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\sin^3(a+bx)}{5d^3\sqrt{c+dx}} \cdots$$

$$= -\frac{16b^2\sin(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cos(a+bx)\sin^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\sin^3(a+bx)}{5d^3\sqrt{c+dx}} \cdots$$

$$= \frac{16b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{16b^{5/2}\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}} \cdots\right.}{5d^{7/2}} \cdots$$

$$= -\frac{2b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{6b^{5/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\cdots}{\cdots}\right.}{5d^{7/2}} \cdots$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 1429 vs. $2(356) = 712$.
time = 6.22, size = 1429, normalized size = 4.01

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (3*(Cos[a]*((2*(b/d)^(5/2)*Sin[(b*c)/d]*(Cos[(b*(c + d*x))/d]/((b/d)^(5/2)*
(c + d*x)^(5/2)) - (2*(2*(Cos[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt[c + d*x]) +
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]) + Sin[(b*(c + d*x)
)/d]/((b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5*d) - (2*(b/d)^(5/2)*Cos[(b*c)/d
```

```
]*(Sin[(b*(c + d*x))/d]/((b/d)^(5/2)*(c + d*x)^(5/2)) + (2*(Cos[(b*(c + d*x
))/d]/((b/d)^(3/2)*(c + d*x)^(3/2)) - 2*(-(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sq
rt[2/Pi]*Sqrt[c + d*x]]) + Sin[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt[c + d*x])))
)/3))/(5*d)) + Sin[a]*((-2*(b/d)^(5/2)*Cos[(b*c)/d]*(Cos[(b*(c + d*x))/d]/(
(b/d)^(5/2)*(c + d*x)^(5/2)) - (2*(2*(Cos[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt[
c + d*x]) + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]) + Sin[
(b*(c + d*x))/d]/((b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5*d) - (2*(b/d)^(5/2)
*Sin[(b*c)/d]*(Sin[(b*(c + d*x))/d]/((b/d)^(5/2)*(c + d*x)^(5/2)) + (2*(Cos
[(b*(c + d*x))/d]/((b/d)^(3/2)*(c + d*x)^(3/2)) - 2*(-(Sqrt[2*Pi]*FresnelC[
Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]) + Sin[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt
[c + d*x]))))/3))/(5*d))))/4 + (-(Cos[3*a]*((18*Sqrt[3]*(b/d)^(5/2)*Sin[(3*
b*c)/d]*(Cos[(3*b*(c + d*x))/d]/(9*Sqrt[3]*(b/d)^(5/2)*(c + d*x)^(5/2)) - (
2*(2*(Cos[(3*b*(c + d*x))/d]/(Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[2*Pi]
*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) + Sin[(3*b*(c + d*x))/d]/(3*
Sqrt[3]*(b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5*d) - (18*Sqrt[3]*(b/d)^(5/2)*
Cos[(3*b*c)/d]*(Sin[(3*b*(c + d*x))/d]/(9*Sqrt[3]*(b/d)^(5/2)*(c + d*x)^(5/
2)) + (2*(Cos[(3*b*(c + d*x))/d]/(3*Sqrt[3]*(b/d)^(3/2)*(c + d*x)^(3/2)) -
2*(-(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) + Sin[(3*b*(c
 + d*x))/d]/(Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]))))/3))/(5*d))) - Sin[3*a]*((-
18*Sqrt[3]*(b/d)^(5/2)*Cos[(3*b*c)/d]*(Cos[(3*b*(c + d*x))/d]/(9*Sqrt[3]*(b
/d)^(5/2)*(c + d*x)^(5/2)) - (2*(2*(Cos[(3*b*(c + d*x))/d]/(Sqrt[3]*Sqrt[b/
d]*Sqrt[c + d*x]) + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]
) + Sin[(3*b*(c + d*x))/d]/(3*Sqrt[3]*(b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5
*d) - (18*Sqrt[3]*(b/d)^(5/2)*Sin[(3*b*c)/d]*(Sin[(3*b*(c + d*x))/d]/(9*Sqr
t[3]*(b/d)^(5/2)*(c + d*x)^(5/2)) + (2*(Cos[(3*b*(c + d*x))/d]/(3*Sqrt[3]*(
b/d)^(3/2)*(c + d*x)^(3/2)) - 2*(-(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]) + Sin[(3*b*(c + d*x))/d]/(Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x])
)))/3))/(5*d)))/4
```

**Maple [A]**
time = 0.03, size = 450, normalized size = 1.26

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $$\frac{-\frac{3\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{10(dx+c)^{\frac{5}{2}}}+\frac{3b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\right.}{}\right.}{}\right.}{5d}}{}$$ |
| default | $$\frac{-\frac{3\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{10(dx+c)^{\frac{5}{2}}}+\frac{3b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{da-cb}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\right.}{}\right.}{}\right.}{5d}}{}$$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/20/(d*x+c)^(5/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/10*b/d*(-1/3/(d*x
+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^(1/2)*sin(1/d*
b*(d*x+c)+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*F
resnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*Fr
esnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/20/(d*x+c)^(5/2
)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x+c)^(3/2)*cos(3/d*b*(
d*x+c)+3*(a*d-b*c)/d)-2*b/d*(-1/(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)
/d)+b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2
^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*F
resnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

**Maxima** [C] Result contains complex when optimal does not.

time = 0.68, size = 254, normalized size = 0.71

$$-\frac{3\left(3\sqrt{3}\left(\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{3i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+\left((i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{3i(dx+c)b}{d}\right)-(i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{3i(dx+c)b}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}-\left(\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{i(dx+c)b}{d}\right)-(i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{i(dx+c)b}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}\right)}{16\,(dx+c)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -3/16*(3*sqrt(3)*(((I - 1)*sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) - (I + 1)
*sqrt(2)*gamma(-5/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I + 1)*
sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-5/2, -3*I*(
d*x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) - (((I - 1)*sqr
t(2)*gamma(-5/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c
)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/d)
 - (I - 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*((d*
x + c)*b/d)^(5/2))/((d*x + c)^(5/2)*d)
```

**Fricas [A]**
time = 0.42, size = 549, normalized size = 1.54

$$\frac{2\left(3\sqrt{6}\left(\pi b^2d^3x^3+3\pi b^2cd^2x^2+3\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\left(\pi b^2d^3x^3+3\pi b^2cd^2x^2+3\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\left(\pi b^2d^3x^3+3\pi b^2cd^2x^2+3\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-3\sqrt{6}\left(\pi b^2d^3x^3+3\pi b^2cd^2x^2+3\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+\left(2\left(bd^2x+bcd\right)\cos(bx+a)^3-2\left(bd^2x+bcd\right)\cos(bx+a)+\left(4b^2d^2x^2+8b^2cdx+4b^2c^2-\left(12b^2d^2x^2+24b^2cdx+12b^2c^2-d^2\right)\cos(bx+a)^2-d^2\right)\sin(bx+a)\right)\sqrt{dx+c}\right)}{5\left(d^6x^3+3cd^5x^2+3c^2d^4x+c^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(6)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi
*b^2*c^3)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) - sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi
*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(s
qrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c
*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 3*sqrt(6)*(pi*b^2*d^3
*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*f
resnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + (2
*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 2*(b*d^2*x + b*c*d)*cos(b*x + a) + (4*b
^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - (12*b^2*d^2*x^2 + 24*b^2*c*d*x + 12*
b^2*c^2 - d^2)*cos(b*x + a)^2 - d^2)*sin(b*x + a))*sqrt(d*x + c))/(d^6*x^3
+ 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**(7/2),x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x)**(7/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*x + c)^(7/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,x\right)^3}{{\left(c+d\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(c + d*x)^(7/2),x)
```

```
[Out] int(sin(a + b*x)^3/(c + d*x)^(7/2), x)
```

# 3.60 $\int (dx)^{3/2}\sin(fx)\,dx$

**Optimal**. Leaf size=87

$$-\frac{(dx)^{3/2}\cos(fx)}{f}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{2f^{5/2}}+\frac{3d\sqrt{dx}\,\sin(fx)}{2f^2}$$

```
[Out] -(d*x)^(3/2)*cos(f*x)/f-3/4*d^(3/2)*FresnelS(f^(1/2)*2^(1/2)/Pi^(1/2)*(d*x)
^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/f^(5/2)+3/2*d*sin(f*x)*(d*x)^(1/2)/f^2
```

**Rubi** [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {3377, 3386, 3432}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{2f^{5/2}}+\frac{3d\sqrt{dx}\,\sin(fx)}{2f^2}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sin[f*x],x]
```

```
[Out] -(((d*x)^(3/2)*Cos[f*x])/f) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[f]*Sqrt[
2/Pi]*Sqrt[d*x])/Sqrt[d]])/(2*f^(5/2)) + (3*d*Sqrt[d*x]*Sin[f*x])/(2*f^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int (dx)^{3/2}\sin(fx)\,dx &= -\frac{(dx)^{3/2}\cos(fx)}{f}+\frac{(3d)\int \sqrt{dx}\,\cos(fx)\,dx}{2f}\\
&= -\frac{(dx)^{3/2}\cos(fx)}{f}+\frac{3d\sqrt{dx}\,\sin(fx)}{2f^2}-\frac{(3d^2)\int \frac{\sin(fx)}{\sqrt{dx}}\,dx}{4f^2}\\
&= -\frac{(dx)^{3/2}\cos(fx)}{f}+\frac{3d\sqrt{dx}\,\sin(fx)}{2f^2}-\frac{(3d)\text{Subst}\left(\int \sin\left(\frac{fx^2}{d}\right)\,dx,x,\sqrt{dx}\right)}{2f^2}\\
&= -\frac{(dx)^{3/2}\cos(fx)}{f}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,S\left(\frac{\sqrt{f}\,\sqrt{\frac{2}{\pi}}\,\sqrt{dx}}{\sqrt{d}}\right)}{2f^{5/2}}+\frac{3d\sqrt{dx}\,\sin(fx)}{2f^2}
\end{aligned}
$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.01, size = 60, normalized size = 0.69

$$
\frac{d^2\left(\sqrt{-ifx}\,\Gamma\left(\frac{5}{2},-ifx\right)+\sqrt{ifx}\,\Gamma\left(\frac{5}{2},ifx\right)\right)}{2f^3\sqrt{dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Sin[f*x],x]
```

```
[Out] (d^2*(Sqrt[(-I)*f*x]*Gamma[5/2, (-I)*f*x] + Sqrt[I*f*x]*Gamma[5/2, I*f*x]))
/(2*f^3*Sqrt[d*x])
```

**Maple** [A]
time = 0.04, size = 87, normalized size = 1.00

| method | result | size |
|---|---|---|
| meijerg | $\frac{2(dx)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\left(-\frac{x^{\frac{3}{2}}\sqrt{2}\,f^{\frac{3}{2}}\cos(fx)}{4\sqrt{\pi}}+\frac{3\sqrt{x}\,\sqrt{2}\,\sqrt{f}\,\sin(fx)}{8\sqrt{\pi}}-\frac{3\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt{f}}{\sqrt{\pi}}\right)}{8}\right)}{x^{\frac{3}{2}}f^{\frac{5}{2}}}$ | 73 |

| | | |
|---|---|---|
| derivativedivides | $\frac{-\frac{d(dx)^{\frac{3}{2}}\cos(fx)}{f}+\frac{3d\left(\frac{d\sqrt{dx}\,\sin(fx)}{2f}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{4f\sqrt{\frac{f}{d}}}\right)}{f}}{d}$ | 87 |
| default | $\frac{-\frac{d(dx)^{\frac{3}{2}}\cos(fx)}{f}+\frac{3d\left(\frac{d\sqrt{dx}\,\sin(fx)}{2f}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{4f\sqrt{\frac{f}{d}}}\right)}{f}}{d}$ | 87 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*sin(f*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/2/f*d*(d*x)^(3/2)*cos(f*x)+3/2/f*d*(1/2/f*d*(d*x)^(1/2)*sin(f*x)-1/
4/f*d*2^(1/2)*Pi^(1/2)/(f/d)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(f/d)^(1/2)*f*
(d*x)^(1/2)/d)))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.29, size = 106, normalized size = 1.22

$$-\frac{\sqrt{2}\left(8\sqrt{2}\,(dx)^{\frac{3}{2}}f^2\cos(fx)-12\sqrt{2}\,\sqrt{dx}\,df\sin(fx)+(3i+3)\,\sqrt{\pi}\,d^2\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{if}{d}}\right)-(3i-3)\,\sqrt{\pi}\,d^2\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{if}{d}}\right)\right)}{16\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sin(f*x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*(8*sqrt(2)*(d*x)^(3/2)*f^2*cos(f*x) - 12*sqrt(2)*sqrt(d*x)*d*
f*sin(f*x) + (3*I + 3)*sqrt(pi)*d^2*(f^2/d^2)^(1/4)*erf(sqrt(d*x)*sqrt(I*f/
d)) - (3*I - 3)*sqrt(pi)*d^2*(f^2/d^2)^(1/4)*erf(sqrt(d*x)*sqrt(-I*f/d)))/f
^3
```

**Fricas [A]**
time = 0.35, size = 72, normalized size = 0.83

$$-\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{f}{\pi d}}\;\mathrm{S}\left(\sqrt{2}\,\sqrt{dx}\,\sqrt{\frac{f}{\pi d}}\right)+2\left(2\,df^2x\cos(fx)-3\,df\sin(fx)\right)\sqrt{dx}}{4\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sin(f*x),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(2)*pi*d^2*sqrt(f/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x)*sqrt(f/
(pi*d))) + 2*(2*d*f^2*x*cos(f*x) - 3*d*f*sin(f*x))*sqrt(d*x))/f^3
```

**Sympy** [A]
time = 16.08, size = 117, normalized size = 1.34

$$-\frac{7d^{\frac{3}{2}}x^{\frac{3}{2}}\cos(fx)\Gamma\left(\frac{7}{4}\right)}{4f\Gamma\left(\frac{11}{4}\right)}+\frac{21d^{\frac{3}{2}}\sqrt{x}\,\sin(fx)\Gamma\left(\frac{7}{4}\right)}{8f^2\Gamma\left(\frac{11}{4}\right)}-\frac{21\sqrt{2}\,\sqrt{\pi}\,d^{\frac{3}{2}}S\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{7}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*sin(f*x),x)
```

```
[Out] -7*d**(3/2)*x**(3/2)*cos(f*x)*gamma(7/4)/(4*f*gamma(11/4)) + 21*d**(3/2)*sq
rt(x)*sin(f*x)*gamma(7/4)/(8*f**2*gamma(11/4)) - 21*sqrt(2)*sqrt(pi)*d**(3/
2)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(7/4)/(16*f**(5/2)*gamma
(11/4))
```

**Giac** [C] Result contains complex when optimal does not.
time = 3.28, size = 220, normalized size = 2.53

$$-\frac{1}{8}d\left(\frac{-\frac{3i\sqrt{2}\,\sqrt{\pi}\,d^3\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{df}\,\sqrt{dx}\left(\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}{2\,d}\right)}{\sqrt{df}\left(\frac{i\,df}{\sqrt{d^2f^2}}+1\right)f^2}-\frac{2i\left(2i\sqrt{dx}\,d^2fx+3\sqrt{dx}\,d^2\right)e^{(-i\,fx)}}{f^2}}{d^2}+\frac{\frac{3i\sqrt{2}\,\sqrt{\pi}\,d^3\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{df}\,\sqrt{dx}\left(-\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}{2\,d}\right)}{\sqrt{df}\left(-\frac{i\,df}{\sqrt{d^2f^2}}+1\right)f^2}-\frac{2i\left(2i\sqrt{dx}\,d^2fx-3\sqrt{dx}\,d^2\right)e^{(i\,fx)}}{f^2}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sin(f*x),x, algorithm="giac")
```

```
[Out] -1/8*d*((-3*I*sqrt(2)*sqrt(pi)*d^3*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(I*
d*f/sqrt(d^2*f^2) + 1)/d)/(sqrt(d*f)*(I*d*f/sqrt(d^2*f^2) + 1)*f^2) - 2*I*(
2*I*sqrt(d*x)*d^2*f*x + 3*sqrt(d*x)*d^2)*e^(-I*f*x)/f^2)/d^2 + (3*I*sqrt(2)
*sqrt(pi)*d^3*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(-I*d*f/sqrt(d^2*f^2) +
1)/d)/(sqrt(d*f)*(-I*d*f/sqrt(d^2*f^2) + 1)*f^2) - 2*I*(2*I*sqrt(d*x)*d^2*f
*x - 3*sqrt(d*x)*d^2)*e^(I*f*x)/f^2)/d^2)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(f\,x)\,\left(d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)*(d*x)^(3/2),x)
```

```
[Out] int(sin(f*x)*(d*x)^(3/2), x)
```

# 3.61 $\int \sqrt{dx}\,\sin(fx)\,dx$

**Optimal**. Leaf size=65

$$-\frac{\sqrt{dx}\,\cos(fx)}{f}+\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,C\left(\frac{\sqrt{f}\,\sqrt{\frac{2}{\pi}}\,\sqrt{dx}}{\sqrt{d}}\right)}{f^{3/2}}$$

```
[Out] 1/2*FresnelC(f^(1/2)*2^(1/2)/Pi^(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*
Pi^(1/2)/f^(3/2)-cos(f*x)*(d*x)^(1/2)/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3377, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{d}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{f^{3/2}}-\frac{\sqrt{dx}\,\cos(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sin[f*x],x]
```

```
[Out] -((Sqrt[d*x]*Cos[f*x])/f) + (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[f]*Sqrt[2/Pi
]*Sqrt[d*x])/Sqrt[d]])/f^(3/2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{dx}\,\sin(fx)\,dx &= -\frac{\sqrt{dx}\,\cos(fx)}{f} + \frac{d\int \frac{\cos(fx)}{\sqrt{dx}}\,dx}{2f} \\
&= -\frac{\sqrt{dx}\,\cos(fx)}{f} + \frac{\mathrm{Subst}\left(\int \cos\left(\frac{fx^2}{d}\right)\,dx, x, \sqrt{dx}\right)}{f} \\
&= -\frac{\sqrt{dx}\,\cos(fx)}{f} + \frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,C\left(\frac{\sqrt{f}\,\sqrt{\frac{2}{\pi}}\,\sqrt{dx}}{\sqrt{d}}\right)}{f^{3/2}}
\end{aligned}
$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.01, size = 69, normalized size = 1.06

$$
-\frac{\sqrt{dx}\,\Gamma\left(\frac{3}{2}, -ifx\right)}{2f\sqrt{-ifx}} - \frac{\sqrt{dx}\,\Gamma\left(\frac{3}{2}, ifx\right)}{2f\sqrt{ifx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Sin[f*x],x]
```

```
[Out] -1/2*(Sqrt[d*x]*Gamma[3/2, (-I)*f*x])/(f*Sqrt[(-I)*f*x]) - (Sqrt[d*x]*Gamma
[3/2, I*f*x])/(2*f*Sqrt[I*f*x])
```

**Maple** [A]
time = 0.02, size = 65, normalized size = 1.00

| method | result | size |
|---|---|---|
| meijerg | $\frac{\sqrt{dx}\,\sqrt{2}\,\sqrt{\pi}\left(-\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{f}\,\cos(fx)}{2\sqrt{\pi}}+\frac{\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt{f}}{\sqrt{\pi}}\right)}{2}\right)}{\sqrt{x}\,f^{\frac{3}{2}}}$ | 54 |
| derivativedivides | $\frac{-\frac{d\sqrt{dx}\,\cos(fx)}{f}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{2f\sqrt{\frac{f}{d}}}}{d}$ | 65 |

| | | |
|---|---|---|
| default | $\frac{-\frac{d\sqrt{dx}\,\cos(fx)}{f}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{2f\sqrt{\frac{f}{d}}}}{d}$ | 65 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*sin(f*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/2/f*d*(d*x)^(1/2)*cos(f*x)+1/4/f*d*2^(1/2)*Pi^(1/2)/(f/d)^(1/2)*Fre
snelC(2^(1/2)/Pi^(1/2)/(f/d)^(1/2)*f*(d*x)^(1/2)/d))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.33, size = 84, normalized size = 1.29

$$-\frac{\sqrt{2}\left(4\sqrt{2}\sqrt{dx}\,f\cos(fx)+(i-1)\sqrt{\pi}\,d\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{if}{d}}\right)-(i+1)\sqrt{\pi}\,d\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{if}{d}}\right)\right)}{8f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*sin(f*x),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(4*sqrt(2)*sqrt(d*x)*f*cos(f*x) + (I - 1)*sqrt(pi)*d*(f^2/d^2)
^(1/4)*erf(sqrt(d*x)*sqrt(I*f/d)) - (I + 1)*sqrt(pi)*d*(f^2/d^2)^(1/4)*erf(
sqrt(d*x)*sqrt(-I*f/d)))/f^2
```

**Fricas** [A]
time = 0.35, size = 54, normalized size = 0.83

$$\frac{\sqrt{2}\,\pi d\sqrt{\frac{f}{\pi d}}\;\mathrm{C}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right)-2\sqrt{dx}\,f\cos(fx)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*sin(f*x),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*d*sqrt(f/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x)*sqrt(f/(pi*d
))) - 2*sqrt(d*x)*f*cos(f*x))/f^2
```

**Sympy** [A]
time = 1.08, size = 85, normalized size = 1.31

$$-\frac{5\sqrt{d}\sqrt{x}\cos(fx)\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt{2}\sqrt{\pi}\sqrt{d}\,C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*sin(f*x),x)
```

```
[Out] -5*sqrt(d)*sqrt(x)*cos(f*x)*gamma(5/4)/(4*f*gamma(9/4)) + 5*sqrt(2)*sqrt(pi
)*sqrt(d)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(5/4)/(8*f**(3/2)
*gamma(9/4))
```

**Giac** [C] Result contains complex when optimal does not.
time = 3.55, size = 176, normalized size = 2.71

$$-\frac{\frac{\sqrt{2}\,\sqrt{\pi}\,d^2\operatorname{erf}\left(-\frac{\sqrt{2}\,\sqrt{df}\,\sqrt{dx}\left(\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}{2\,d}\right)}{\sqrt{df}\left(\frac{i\,df}{\sqrt{d^2f^2}}+1\right)f}+\frac{\sqrt{2}\,\sqrt{\pi}\,d^2\operatorname{erf}\left(-\frac{\sqrt{2}\,\sqrt{df}\,\sqrt{dx}\left(-\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}{2\,d}\right)}{\sqrt{df}\left(-\frac{i\,df}{\sqrt{d^2f^2}}+1\right)f}+\frac{2\,\sqrt{dx}\,de^{(i\,fx)}}{f}+\frac{2\,\sqrt{dx}\,de^{(-i\,fx)}}{f}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*sin(f*x),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(I*d*f/sqrt
(d^2*f^2) + 1)/d)/(sqrt(d*f)*(I*d*f/sqrt(d^2*f^2) + 1)*f) + sqrt(2)*sqrt(pi
)*d^2*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(-I*d*f/sqrt(d^2*f^2) + 1)/d)/(s
qrt(d*f)*(-I*d*f/sqrt(d^2*f^2) + 1)*f) + 2*sqrt(d*x)*d*e^(I*f*x)/f + 2*sqrt
(d*x)*d*e^(-I*f*x)/f)/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(f\,x\right)\,\sqrt{d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)*(d*x)^(1/2),x)
```

```
[Out] int(sin(f*x)*(d*x)^(1/2), x)
```

## 3.62 $\int \frac{\sin(fx)}{\sqrt{dx}}\,dx$

**Optimal**. Leaf size=46

$$\frac{\sqrt{2\pi}\,S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{f}}$$

```
[Out] FresnelS(f^(1/2)*2^(1/2)/Pi^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(
1/2)/f^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3386, 3432}

$$\frac{\sqrt{2\pi}\,S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[f*x]/Sqrt[d*x],x]
```

```
[Out] (Sqrt[2*Pi]*FresnelS[(Sqrt[f]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt
[f])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \frac{2\text{Subst}\left(\int \sin\left(\frac{fx^2}{d}\right)\,dx, x, \sqrt{dx}\right)}{d}$$

$$= \frac{\sqrt{2\pi}\,S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{f}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.01, size = 59, normalized size = 1.28

$$\frac{-\sqrt{-ifx}\,\Gamma\left(\frac{1}{2},-ifx\right)-\sqrt{ifx}\,\Gamma\left(\frac{1}{2},ifx\right)}{2f\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[f*x]/Sqrt[d*x],x]
```

```
[Out] (-(Sqrt[(-I)*f*x]*Gamma[1/2, (-I)*f*x]) - Sqrt[I*f*x]*Gamma[1/2, I*f*x])/(2
*f*Sqrt[d*x])
```

**Maple** [A]
time = 0.01, size = 42, normalized size = 0.91

| method | result | size |
|---|---|---|
| meijerg | $\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt{f}}{\sqrt{\pi}}\right)}{\sqrt{dx}\,\sqrt{f}}$ | 33 |
| derivativedivides | $\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{d\sqrt{\frac{f}{d}}}$ | 42 |
| default | $\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{d\sqrt{\frac{f}{d}}}$ | 42 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)*Pi^(1/2)/(f/d)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(f/d)^(1/2)*f*(d
*x)^(1/2)/d)
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.29, size = 67, normalized size = 1.46

$$\frac{\sqrt{2}\left((i+1)\sqrt{\pi}\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{if}{d}}\right)-(i-1)\sqrt{\pi}\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{if}{d}}\right)\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*((I + 1)*sqrt(pi)*(f^2/d^2)^(1/4)*erf(sqrt(d*x)*sqrt(I*f/d)) -
(I - 1)*sqrt(pi)*(f^2/d^2)^(1/4)*erf(sqrt(d*x)*sqrt(-I*f/d)))/f
```

**Fricas** [A]
time = 0.35, size = 38, normalized size = 0.83

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{f}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx}\,\sqrt{\frac{f}{\pi d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*pi*sqrt(f/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x)*sqrt(f/(pi*d)))/f
```

**Sympy** [A]
time = 0.58, size = 54, normalized size = 1.17

$$\frac{3\sqrt{2}\,\sqrt{\pi}\,S\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{4\sqrt{d}\,\sqrt{f}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)**(1/2),x)
```

```
[Out] 3*sqrt(2)*sqrt(pi)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(3/4)/(4
*sqrt(d)*sqrt(f)*gamma(7/4))
```

**Giac** [C] Result contains complex when optimal does not.

time = 4.74, size = 136, normalized size = 2.96

$$-\frac{\frac{i\sqrt{2}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{df}\,\sqrt{dx}\,\left(\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}{2\,d}\right)}{\sqrt{df}\,\left(\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}-\frac{i\sqrt{2}\,\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{df}\,\sqrt{dx}\,\left(-\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}{2\,d}\right)}{\sqrt{df}\,\left(-\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(I*d*f/sqrt
(d^2*f^2) + 1)/d)/(sqrt(d*f)*(I*d*f/sqrt(d^2*f^2) + 1)) - I*sqrt(2)*sqrt(pi
)*d*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(-I*d*f/sqrt(d^2*f^2) + 1)/d)/(sqr
t(d*f)*(-I*d*f/sqrt(d^2*f^2) + 1)))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(f\,x)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)/(d*x)^(1/2),x)
```

```
[Out] int(sin(f*x)/(d*x)^(1/2), x)
```

# 3.63 $\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx$

**Optimal**. Leaf size=64

$$\frac{2\sqrt{f}\,\sqrt{2\pi}\,C\left(\frac{\sqrt{f}\,\sqrt{\frac{2}{\pi}}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin(fx)}{d\sqrt{dx}}$$

```
[Out] 2*FresnelC(f^(1/2)*2^(1/2)/Pi^(1/2)*(d*x)^(1/2)/d^(1/2))*f^(1/2)*2^(1/2)*Pi
^(1/2)/d^(3/2)-2*sin(f*x)/d/(d*x)^(1/2)
```

**Rubi** [A]

time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3378, 3385, 3433}

$$\frac{2\sqrt{2\pi}\,\sqrt{f}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin(fx)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[f*x]/(d*x)^(3/2),x]
```

```
[Out] (2*Sqrt[f]*Sqrt[2*Pi]*FresnelC[(Sqrt[f]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/d^(
3/2) - (2*Sin[f*x])/(d*Sqrt[d*x])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx &= -\frac{2\sin(fx)}{d\sqrt{dx}} + \frac{(2f)\int \frac{\cos(fx)}{\sqrt{dx}}\,dx}{d}\\ &= -\frac{2\sin(fx)}{d\sqrt{dx}} + \frac{(4f)\text{Subst}\left(\int \cos\left(\frac{fx^2}{d}\right)\,dx, x, \sqrt{dx}\right)}{d^2}\\ &= \frac{2\sqrt{f}\,\sqrt{2\pi}\,C\left(\frac{\sqrt{f}\,\sqrt{\frac{2}{\pi}}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sin(fx)}{d\sqrt{dx}}\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.02, size = 64, normalized size = 1.00

$$\frac{x\left(-i\sqrt{-ifx}\,\Gamma\left(\frac{1}{2},-ifx\right)+i\sqrt{ifx}\,\Gamma\left(\frac{1}{2},ifx\right)-2\sin(fx)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[f*x]/(d*x)^(3/2),x]
```

```
[Out] (x*((-I)*Sqrt[(-I)*f*x]*Gamma[1/2, (-I)*f*x] + I*Sqrt[I*f*x]*Gamma[1/2, I*f
*x] - 2*Sin[f*x]))/(d*x)^(3/2)
```

**Maple** [A]
time = 0.01, size = 60, normalized size = 0.94

| method | result | size |
|---|---|---|
| meijerg | $\frac{\sqrt{\pi}\,x^{\frac{3}{2}}\sqrt{2}\,\sqrt{f}\left(-\frac{4\sqrt{2}\,\sin(fx)}{\sqrt{\pi}\,\sqrt{x}\,\sqrt{f}}+8\,\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt{f}}{\sqrt{\pi}}\right)\right)}{4(dx)^{\frac{3}{2}}}$ | 55 |
| derivativedivides | $\frac{-\frac{2\sin(fx)}{\sqrt{dx}}+\frac{2f\sqrt{2}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{d\sqrt{\frac{f}{d}}}}{d}$ | 60 |

| | | |
|---|---|---|
| default | $\frac{-\frac{2\sin(fx)}{\sqrt{dx}}+\frac{2f\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{d\sqrt{\frac{f}{d}}}}{d}$ | 60 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-sin(f*x)/(d*x)^(1/2)+f/d*2^(1/2)*Pi^(1/2)/(f/d)^(1/2)*FresnelC(2^(1/2
)/Pi^(1/2)/(f/d)^(1/2)*f*(d*x)^(1/2)/d))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.56, size = 38, normalized size = 0.59

$$-\frac{\sqrt{fx}\,\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},i\,fx\right)-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-i\,fx\right)\right)}{4\,\sqrt{dx}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(f*x)*((I - 1)*sqrt(2)*gamma(-1/2, I*f*x) - (I + 1)*sqrt(2)*gamma(
-1/2, -I*f*x))/(sqrt(d*x)*d)
```

**Fricas [A]**
time = 0.34, size = 57, normalized size = 0.89

$$\frac{2\left(\sqrt{2}\,\pi dx\sqrt{\frac{f}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx}\,\sqrt{\frac{f}{\pi d}}\right)-\sqrt{dx}\,\sin\left(fx\right)\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*pi*d*x*sqrt(f/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x)*sqrt(f/(pi*d
))) - sqrt(d*x)*sin(f*x))/(d^2*x)
```

**Sympy [A]**
time = 2.30, size = 80, normalized size = 1.25

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{f}\,C\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}-\frac{\sin\left(fx\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)**(3/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*sqrt(f)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(1
/4)/(2*d**(3/2)*gamma(5/4)) - sin(f*x)*gamma(1/4)/(2*d**(3/2)*sqrt(x)*gamma
(5/4))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x)/(d*x)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(f\,x\right)}{{\left(d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)/(d*x)^(3/2),x)
```

```
[Out] int(sin(f*x)/(d*x)^(3/2), x)
```

# 3.64 $\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx$

**Optimal**. Leaf size=87

$$-\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{4f^{3/2}\sqrt{2\pi}\,S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

```
[Out] -2/3*sin(f*x)/d/(d*x)^(3/2)-4/3*f^(3/2)*FresnelS(f^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)-4/3*f*cos(f*x)/d^2/(d*x)^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3378, 3386, 3432}

$$-\frac{4\sqrt{2\pi}\,f^{3/2}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[f*x]/(d*x)^(5/2),x]
```

```
[Out] (-4*f*Cos[f*x])/(3*d^2*Sqrt[d*x]) - (4*f^(3/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[f]
*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (2*Sin[f*x])/(3*d*(d*x)^(3/2
))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx &= -\frac{2\sin(fx)}{3d(dx)^{3/2}}+\frac{(2f)\int \frac{\cos(fx)}{(dx)^{3/2}}\,dx}{3d}\\
&= -\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}-\frac{(4f^2)\int \frac{\sin(fx)}{\sqrt{dx}}\,dx}{3d^2}\\
&= -\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}-\frac{(8f^2)\,\text{Subst}\left(\int \sin\left(\frac{fx^2}{d}\right)\,dx,x,\sqrt{dx}\right)}{3d^3}\\
&= -\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{4f^{3/2}\sqrt{2\pi}\,S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.06, size = 111, normalized size = 1.28

$$\frac{2fx^{5/2}\left(-\frac{e^{ifx}-\sqrt{-ifx}\,\Gamma\left(\frac{1}{2},-ifx\right)}{\sqrt{x}}+\frac{-e^{-ifx}+\sqrt{ifx}\,\Gamma\left(\frac{1}{2},ifx\right)}{\sqrt{x}}\right)}{3(dx)^{5/2}}-\frac{2x\sin(fx)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[f*x]/(d*x)^(5/2),x]
```

```
[Out] (2*f*x^(5/2)*(-((E^(I*f*x) - Sqrt[(-I)*f*x]*Gamma[1/2, (-I)*f*x])/Sqrt[x])
+ (-E^((-I)*f*x) + Sqrt[I*f*x]*Gamma[1/2, I*f*x])/Sqrt[x]))/(3*(d*x)^(5/2))
 - (2*x*Sin[f*x])/(3*(d*x)^(5/2))
```

**Maple** [A]
time = 0.02, size = 79, normalized size = 0.91

| method | result | size |
|---|---|---|

| | | |
|---|---|---|
| meijerg | $\frac{\sqrt{\pi}\, x^{\frac{5}{2}} \sqrt{2}\, f^{\frac{3}{2}} \left(-\frac{16\sqrt{2}\, \cos(fx)}{3\sqrt{\pi}\, \sqrt{x}\, \sqrt{f}}-\frac{8\sqrt{2}\, \sin(fx)}{3\sqrt{\pi}\, x^{\frac{3}{2}} f^{\frac{3}{2}}}-\frac{32\, \mathrm{S}\left(\frac{\sqrt{2}\, \sqrt{x}\, \sqrt{f}}{\sqrt{\pi}}\right)}{3}\right)}{8(dx)^{\frac{5}{2}}}$ | 73 |
| derivativedivides | $\frac{-\frac{2\sin(fx)}{3(dx)^{\frac{3}{2}}}+\frac{4f\left(-\frac{\cos(fx)}{\sqrt{dx}}-\frac{f\sqrt{2}\, \sqrt{\pi}\, \mathrm{S}\left(\frac{\sqrt{2}\, f\sqrt{dx}}{\sqrt{\pi}\, \sqrt{\frac{f}{d}}\, d}\right)}{d\sqrt{\frac{f}{d}}}\right)}{3d}}{d}$ | 79 |
| default | $\frac{-\frac{2\sin(fx)}{3(dx)^{\frac{3}{2}}}+\frac{4f\left(-\frac{\cos(fx)}{\sqrt{dx}}-\frac{f\sqrt{2}\, \sqrt{\pi}\, \mathrm{S}\left(\frac{\sqrt{2}\, f\sqrt{dx}}{\sqrt{\pi}\, \sqrt{\frac{f}{d}}\, d}\right)}{d\sqrt{\frac{f}{d}}}\right)}{3d}}{d}$ | 79 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/3*sin(f*x)/(d*x)^(3/2)+2/3*f/d*(-1/(d*x)^(1/2)*cos(f*x)-f/d*2^(1/2)
*Pi^(1/2)/(f/d)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(f/d)^(1/2)*f*(d*x)^(1/2)/d
)))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.59, size = 38, normalized size = 0.44

$$-\frac{(fx)^{\frac{3}{2}}\left(-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},i\,fx\right)+(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},-i\,fx\right)\right)}{4\,(dx)^{\frac{3}{2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(f*x)^(3/2)*(-(I + 1)*sqrt(2)*gamma(-3/2, I*f*x) + (I - 1)*sqrt(2)*gam
ma(-3/2, -I*f*x))/((d*x)^(3/2)*d)
```

**Fricas** [A]

time = 0.35, size = 69, normalized size = 0.79

$$-\frac{2\left(2\sqrt{2}\,\pi d f x^{2}\sqrt{\frac{f}{\pi d}}\;\mathrm{S}\left(\sqrt{2}\,\sqrt{dx}\,\sqrt{\frac{f}{\pi d}}\right)+\left(2\,fx\cos\left(fx\right)+\sin\left(fx\right)\right)\sqrt{dx}\right)}{3\,d^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*pi*d*f*x^2*sqrt(f/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x)*sqr
t(f/(pi*d))) + (2*f*x*cos(f*x) + sin(f*x))*sqrt(d*x))/(d^3*x^2)
```

**Sympy** [A]
time = 17.75, size = 114, normalized size = 1.31

$$\frac{\sqrt{2}\,\sqrt{\pi}\,f^{\frac{3}{2}}S\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{f\cos\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{\sin\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{6d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)**(5/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*f**(3/2)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(
-1/4)/(3*d**(5/2)*gamma(3/4)) + f*cos(f*x)*gamma(-1/4)/(3*d**(5/2)*sqrt(x)*
gamma(3/4)) + sin(f*x)*gamma(-1/4)/(6*d**(5/2)*x**(3/2)*gamma(3/4))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x)/(d*x)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(f\,x\right)}{{\left(d\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)/(d*x)^(5/2),x)
```

```
[Out] int(sin(f*x)/(d*x)^(5/2), x)
```

# 3.65 $\int \sqrt{c+dx}\, \csc(a+bx)\, dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\sqrt{c+dx}\, \csc(a+bx), x\right)$$

```
[Out] Unintegrable(csc(b*x+a)*(d*x+c)^(1/2),x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{c+dx}\, \csc(a+bx)\, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + d*x]*Csc[a + b*x],x]
[Out] Defer[Int][Sqrt[c + d*x]*Csc[a + b*x], x]
```

Rubi steps

$$\int \sqrt{c+dx}\, \csc(a+bx)\, dx = \int \sqrt{c+dx}\, \csc(a+bx)\, dx$$

**Mathematica** [A]
time = 10.48, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\, \csc(a+bx)\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x]*Csc[a + b*x],x]
[Out] Integrate[Sqrt[c + d*x]*Csc[a + b*x], x]
```

**Maple** [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right) \sqrt{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(csc(b*x+a)*(d*x+c)^(1/2),x)
```

**Maxima** [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x + c)*csc(b*x + a), x)
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x + c)*csc(b*x + a), x)
```

**Sympy** [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx}\csc{(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*x+c)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*csc(a + b*x), x)
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x + c)*csc(b*x + a), x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c+d\,x}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/sin(a + b*x),x)
[Out] int((c + d*x)^(1/2)/sin(a + b*x), x)
```

# 3.66 $\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=19

$$\mathrm{Int}\left(\frac{\csc(a+bx)}{\sqrt{c+dx}},x\right)$$

```
[Out] Unintegrable(csc(b*x+a)/(d*x+c)^(1/2),x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[a + b*x]/Sqrt[c + d*x],x]
[Out] Defer[Int][Csc[a + b*x]/Sqrt[c + d*x], x]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$$

**Mathematica** [A]
time = 10.39, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[a + b*x]/Sqrt[c + d*x],x]
[Out] Integrate[Csc[a + b*x]/Sqrt[c + d*x], x]
```

**Maple** [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*x+c)^(1/2),x)
[Out] int(csc(b*x+a)/(d*x+c)^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)/sqrt(d*x + c), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(csc(b*x + a)/sqrt(d*x + c), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)**(1/2),x)
[Out] Integral(csc(a + b*x)/sqrt(c + d*x), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)/sqrt(d*x + c), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sin (a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(c + d*x)^(1/2)),x)
[Out] int(1/(sin(a + b*x)*(c + d*x)^(1/2)), x)
```

# 3.67 $\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)} + x\sqrt{\sin(e+fx)}\right) dx$

**Optimal**. Leaf size=38

$$-\frac{2x\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{4\sqrt{\sin(e+fx)}}{f^2}$$

```
[Out] -2*x*cos(f*x+e)/f/sin(f*x+e)^(1/2)+4*sin(f*x+e)^(1/2)/f^2
```

**Rubi** [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{3396\}$

$$\frac{4\sqrt{\sin(e+fx)}}{f^2}-\frac{2x\cos(e+fx)}{f\sqrt{\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sin[e + f*x]^(3/2) + x*Sqrt[Sin[e + f*x]],x]
```

```
[Out] (-2*x*Cos[e + f*x])/(f*Sqrt[Sin[e + f*x]]) + (4*Sqrt[Sin[e + f*x]])/f^2
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right) dx &= \int \frac{x}{\sin^{\frac{3}{2}}(e+fx)}\, dx+\int x\sqrt{\sin(e+fx)}\, dx\\ &= -\frac{2x\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{4\sqrt{\sin(e+fx)}}{f^2}\end{aligned}$$

**Mathematica** [A]
time = 0.24, size = 33, normalized size = 0.87

$$\frac{-2fx\cos(e+fx)+4\sin(e+fx)}{f^2\sqrt{\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sin[e + f*x]^(3/2) + x*Sqrt[Sin[e + f*x]],x]
```

```
[Out] (-2*f*x*Cos[e + f*x] + 4*Sin[e + f*x])/(f^2*Sqrt[Sin[e + f*x]])
```

**Maple** [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x}{\sin \left(f x+e\right)^{\frac{3}{2}}}+x\left(\sqrt{\sin}\left(f x+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x)
```

```
[Out] int(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(3/2), x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(\sin^{2}\left(e+f x\right)+1)}{\sin^{\frac{3}{2}}\left(e+f x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)**(3/2)+x*sin(f*x+e)**(1/2),x)
```

```
[Out] Integral(x*(sin(e + f*x)**2 + 1)/sin(e + f*x)**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x, algorithm="giac")
[Out] integrate(x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(3/2), x)
```

**Mupad [B]**
time = 1.05, size = 36, normalized size = 0.95

$$\frac{4\sin\left(e+f\,x\right)^2-f\,x\,\sin\left(2\,e+2\,f\,x\right)}{f^2\sin\left(e+f\,x\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(e + f*x)^(1/2) + x/sin(e + f*x)^(3/2),x)
[Out] (4*sin(e + f*x)^2 - f*x*sin(2*e + 2*f*x))/(f^2*sin(e + f*x)^(3/2))
```

# 3.68 $\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)\,dx$

**Optimal**. Leaf size=62

$$-\frac{16E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f^3}-\frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{8x\sqrt{\sin(e+fx)}}{f^2}$$

```
[Out] 16*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f^3-2*x^2*cos(f*x+e)/f/sin(f*x+e)^(1/2)+
8*x*sin(f*x+e)^(1/2)/f^2
```

**Rubi** [A]

time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.069$, Rules used = {3397, 2719}

$$-\frac{16E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f^3}+\frac{8x\sqrt{\sin(e+fx)}}{f^2}-\frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sin[e + f*x]^(3/2) + x^2*Sqrt[Sin[e + f*x]],x]
```

```
[Out] (-16*EllipticE[(e - Pi/2 + f*x)/2, 2])/f^3 - (2*x^2*Cos[e + f*x])/(f*Sqrt[S
in[e + f*x]]) + (8*x*Sqrt[Sin[e + f*x]])/f^2
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3397

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1)
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[d^2*m*((m - 1)/(b^2*f^2*(n + 1)*(n + 2))), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[d*m*(c + d*x)^(m - 1)*((b*
Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)dx &= \int \frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}\,dx+\int x^2\sqrt{\sin(e+fx)}\,dx \\
&= -\frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{8x\sqrt{\sin(e+fx)}}{f^2}-\frac{8\int\sqrt{\sin(e+fx)}}{f^2} \\
&= -\frac{16E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f^3}-\frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{8x\sqrt{\sin(e+}}{f^2}
\end{aligned}
$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.87, size = 185, normalized size = 2.98

$$
\frac{8e^{-ifx}\sqrt{2-2e^{2i(e+fx)}}\left(3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2i(e+fx)}\right)+e^{2ifx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(e+fx)}\right)\right)\sec(e)}{3\sqrt{-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)}\,f^3}-\frac{\sec(e)\left((8+f^2x^2)\cos(fx)+(-8+f^2x^2)\cos(2e+fx)-8fx\cos(e)\sin(e+fx)\right)}{f^3\sqrt{\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sin[e + f*x]^(3/2) + x^2*Sqrt[Sin[e + f*x]],x]
```

```
[Out] (8*Sqrt[2 - 2*E^((2*I)*(e + f*x))]*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^(
(2*I)*(e + f*x))] + E^((2*I)*f*x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)
*(e + f*x))])*Sec[e])/(3*E^(I*f*x)*Sqrt[((-I)*(-1 + E^((2*I)*(e + f*x))))/E
^(I*(e + f*x))]*f^3) - (Sec[e]*((8 + f^2*x^2)*Cos[f*x] + (-8 + f^2*x^2)*Cos
[2*e + f*x] - 8*f*x*Cos[e]*Sin[e + f*x]))/(f^3*Sqrt[Sin[e + f*x]])
```

**Maple** [F]
time = 0.12, size = 0, normalized size = 0.00

$$
\int \frac{x^2}{\sin\left(fx+e\right)^{\frac{3}{2}}}+x^2\left(\sqrt{\sin}\left(fx+e\right)\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x)
```

```
[Out] int(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(sin(f*x + e)) + x^2/sin(f*x + e)^(3/2), x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(\sin^2(e+fx)+1)}{\sin^{\frac{3}{2}}(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sin(f*x+e)**(3/2)+x**2*sin(f*x+e)**(1/2),x)
```

```
[Out] Integral(x**2*(sin(e + f*x)**2 + 1)/sin(e + f*x)**(3/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(sin(f*x + e)) + x^2/sin(f*x + e)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\sqrt{\sin(e+f\,x)}+\frac{x^2}{{\sin(e+f\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(e + f*x)^(1/2) + x^2/sin(e + f*x)^(3/2),x)
```

```
[Out] int(x^2*sin(e + f*x)^(1/2) + x^2/sin(e + f*x)^(3/2), x)
```

# 3.69 $\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right)\,dx$

**Optimal**. Leaf size=42

$$-\frac{2x\cos(e+fx)}{3f\sin^{\frac{3}{2}}(e+fx)}-\frac{4}{3f^2\sqrt{\sin(e+fx)}}$$

```
[Out] -2/3*x*cos(f*x+e)/f/sin(f*x+e)^(3/2)-4/3/f^2/sin(f*x+e)^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.036$, Rules used = {3396}

$$-\frac{4}{3f^2\sqrt{\sin(e+fx)}}-\frac{2x\cos(e+fx)}{3f\sin^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sin[e + f*x]^(5/2) - x/(3*Sqrt[Sin[e + f*x]]),x]
```

```
[Out] (-2*x*Cos[e + f*x])/(3*f*Sin[e + f*x]^(3/2)) - 4/(3*f^2*Sqrt[Sin[e + f*x]])
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right)dx &= -\left(\frac{1}{3}\int \frac{x}{\sqrt{\sin(e+fx)}}\,dx\right)+\int \frac{x}{\sin^{\frac{5}{2}}(e+fx)}\,dx\\ &= -\frac{2x\cos(e+fx)}{3f\sin^{\frac{3}{2}}(e+fx)}-\frac{4}{3f^2\sqrt{\sin(e+fx)}}\end{aligned}$$

**Mathematica** [A]
time = 0.19, size = 35, normalized size = 0.83

$$-\frac{2(fx\cos(e+fx)+2\sin(e+fx))}{3f^2\sin^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sin[e + f*x]^(5/2) - x/(3*Sqrt[Sin[e + f*x]]),x]
```

```
[Out] (-2*(f*x*Cos[e + f*x] + 2*Sin[e + f*x]))/(3*f^2*Sin[e + f*x]^(3/2))
```

**Maple** [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x}{\sin\left(fx+e\right)^{\frac{5}{2}}}-\frac{x}{3\sqrt{\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x)
```

```
[Out] int(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x/sqrt(sin(f*x + e)) + x/sin(f*x + e)^(5/2), x)
```

**Fricas** [A]
time = 0.36, size = 52, normalized size = 1.24

$$\frac{2\left(fx\cos\left(fx+e\right)+2\sin\left(fx+e\right)\right)\sqrt{\sin\left(fx+e\right)}}{3\left(f^{2}\cos\left(fx+e\right)^{2}-f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(f*x*cos(f*x + e) + 2*sin(f*x + e))*sqrt(sin(f*x + e))/(f^2*cos(f*x + e
)^2 - f^2)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3x}{\sin^{\frac{5}{2}}\left(e+fx\right)}\right)dx+\int\frac{x}{\sqrt{\sin\left(e+fx\right)}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)**(5/2)-1/3*x/sin(f*x+e)**(1/2),x)
```

```
[Out] -(Integral(-3*x/sin(e + f*x)**(5/2), x) + Integral(x/sqrt(sin(e + f*x)), x)
)/3
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x/sqrt(sin(f*x + e)) + x/sin(f*x + e)^(5/2), x)
```

**Mupad [B]**
time = 3.11, size = 140, normalized size = 3.33

$$-\frac{4\,\sqrt{\sin\left(e+f\,x\right)}\,\left(20\,\sin\left(e+f\,x\right)-10\,\sin\left(3\,e+3\,f\,x\right)+2\,\sin\left(5\,e+5\,f\,x\right)-2\,f\,x\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+3\,f\,x\,\left(2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2-1\right)-f\,x\,\left(2\sin\left(\frac{5\,e}{2}+\frac{5\,f\,x}{2}\right)^2-1\right)\right)}{3\,f^2\,\left(30\sin\left(e+f\,x\right)^2-12\sin\left(2\,e+2\,f\,x\right)^2+2\sin\left(3\,e+3\,f\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(e + f*x)^(5/2) - x/(3*sin(e + f*x)^(1/2)),x)
```

```
[Out] -(4*sin(e + f*x)^(1/2)*(20*sin(e + f*x) - 10*sin(3*e + 3*f*x) + 2*sin(5*e +
 5*f*x) - 2*f*x*(2*sin(e/2 + (f*x)/2)^2 - 1) + 3*f*x*(2*sin((3*e)/2 + (3*f*
x)/2)^2 - 1) - f*x*(2*sin((5*e)/2 + (5*f*x)/2)^2 - 1)))/(3*f^2*(2*sin(3*e +
 3*f*x)^2 - 12*sin(2*e + 2*f*x)^2 + 30*sin(e + f*x)^2))
```

# 3.70 $\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)\,dx$

**Optimal**. Leaf size=83

$$-\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)}-\frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)}-\frac{6x\cos(e+fx)}{5f\sqrt{\sin(e+fx)}}+\frac{12\sqrt{\sin(e+fx)}}{5f^2}$$

```
[Out] -2/5*x*cos(f*x+e)/f/sin(f*x+e)^(5/2)-4/15/f^2/sin(f*x+e)^(3/2)-6/5*x*cos(f*
x+e)/f/sin(f*x+e)^(1/2)+12/5*sin(f*x+e)^(1/2)/f^2
```

**Rubi** [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.036$, Rules used = {3396}

$$-\frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)}+\frac{12\sqrt{\sin(e+fx)}}{5f^2}-\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)}-\frac{6x\cos(e+fx)}{5f\sqrt{\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sin[e + f*x]^(7/2) + (3*x*Sqrt[Sin[e + f*x]])/5,x]
```

```
[Out] (-2*x*Cos[e + f*x])/(5*f*Sin[e + f*x]^(5/2)) - 4/(15*f^2*Sin[e + f*x]^(3/2)
) - (6*x*Cos[e + f*x])/(5*f*Sqrt[Sin[e + f*x]]) + (12*Sqrt[Sin[e + f*x]])/(
5*f^2)
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)dx &= \frac{3}{5}\int x\sqrt{\sin(e+fx)}\,dx+\int \frac{x}{\sin^{\frac{7}{2}}(e+fx)}\,dx\\ &= -\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)}-\frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)}+\frac{3}{5}\int \frac{x}{\sin^{\frac{3}{2}}(e+fx)}\,d\\ &= -\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)}-\frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)}-\frac{6x\cos(e+fx)}{5f\sqrt{\sin(e+fx)}}\end{aligned}$$

**Mathematica [A]**
time = 0.38, size = 58, normalized size = 0.70

$$\frac{-21fx\cos(e+fx)+9fx\cos(3(e+fx))+46\sin(e+fx)-18\sin(3(e+fx))}{30f^2\sin^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sin[e + f*x]^(7/2) + (3*x*Sqrt[Sin[e + f*x]])/5,x]
```

```
[Out] (-21*f*x*Cos[e + f*x] + 9*f*x*Cos[3*(e + f*x)] + 46*Sin[e + f*x] - 18*Sin[3
*(e + f*x)])/(30*f^2*Sin[e + f*x]^(5/2))
```

**Maple [F]**
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x}{\sin\left(fx+e\right)^{\frac{7}{2}}}+\frac{3x\left(\sqrt{\sin}\left(fx+e\right)\right)}{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x)
```

```
[Out] int(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(7/2), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{5x}{\sin^{\frac{7}{2}}(e+fx)}\,dx + \int 3x\sqrt{\sin(e+fx)}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)**(7/2)+3/5*x*sin(f*x+e)**(1/2),x)
```

```
[Out] (Integral(5*x/sin(e + f*x)**(7/2), x) + Integral(3*x*sqrt(sin(e + f*x)), x)
)/5
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(7/2), x)
```

**Mupad [B]**
time = 4.49, size = 253, normalized size = 3.05

$$\left(\frac{12}{5f^2}+\frac{x\,6\mathrm{i}}{5f}\right)\sqrt{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}}-\frac{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(\frac{x\,3\mathrm{i}}{5f}-\frac{32+f\,x\,66\mathrm{i}}{30f^2}\right)}{{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^2}-\frac{x\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}}\,12\mathrm{i}}{5f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}+\frac{x\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}}\,16\mathrm{i}}{5f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x*sin(e + f*x)^(1/2))/5 + x/sin(e + f*x)^(7/2),x)
```

```
[Out] ((x*6i)/(5*f) + 12/(5*f^2))*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2)^(1/2) - (exp(e*2i + f*x*2i)*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2)^(1/2)*((x*3i)/(5*f) - (f*x*66i + 32)/(30*f^2)))/(exp(
e*2i + f*x*2i) - 1)^2 - (x*exp(e*2i + f*x*2i)*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2)^(1/2)*12i)/(5*f*(exp(e*2i + f*x*2i) - 1)) + (x
*exp(e*2i + f*x*2i)*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/
2)^(1/2)*16i)/(5*f*(exp(e*2i + f*x*2i) - 1)^3)
```

# 3.71 $\int (c+dx)^m (b\sin(e+fx))^n \, dx$

**Optimal**. Leaf size=21

$$\text{Int}((c+dx)^m (b\sin(e+fx))^n, x)$$

```
[Out] Unintegrable((d*x+c)^m*(b*sin(f*x+e))^n,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m (b\sin(e+fx))^n \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(b*Sin[e + f*x])^n,x]
[Out] Defer[Int][(c + d*x)^m*(b*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m (b\sin(e+fx))^n \, dx = \int (c+dx)^m (b\sin(e+fx))^n \, dx$$

**Mathematica** [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int (c+dx)^m (b\sin(e+fx))^n \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(b*Sin[e + f*x])^n,x]
[Out] Integrate[(c + d*x)^m*(b*Sin[e + f*x])^n, x]
```

**Maple** [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(b\sin\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(b*sin(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(b*sin(f*x+e))^n,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(b*sin(f*x + e))^n, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(b*sin(f*x + e))^n, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(e+fx))^n (c+dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(b*sin(f*x+e))**n,x)
[Out] Integral((b*sin(e + f*x))**n*(c + d*x)**m, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(b*sin(f*x + e))^n, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (b\,\sin(e+f\,x))^n\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^n*(c + d*x)^m,x)
[Out] int((b*sin(e + f*x))^n*(c + d*x)^m, x)
```

## 3.72 $\int (c+dx)^m \sin^3(a+bx)\,dx$

**Optimal**. Leaf size=267

$$-\frac{3e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right.}{8b}$$

```
[Out] -3/8*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/
d)^m)-3/8*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+
c)/d)^m)+1/8*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x+c)
/d)/b/((-I*b*(d*x+c)/d)^m)+1/8*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+c)/d
)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi** [A]
time = 0.22, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3389, 2212}

$$-\frac{3e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}-\frac{3e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}+\frac{3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]^3,x]
```

```
[Out] (-3*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8*
b*(((-I)*b*(c + d*x))/d)^m) - (3*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d
])/(8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (3^(-1 - m)*E^((3*I)*(
a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-I)
*b*(c + d*x))/d)^m) + (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*
x))/d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \sin^3(a+bx)\,dx &= \int \left(\frac{3}{4}(c+dx)^m \sin(a+bx) - \frac{1}{4}(c+dx)^m \sin(3a+3bx)\right)\,dx\\ &= -\left(\frac{1}{4}\int (c+dx)^m \sin(3a+3bx)\,dx\right) + \frac{3}{4}\int (c+dx)^m \sin(a+bx)\,dx\\ &= -\left(\frac{1}{8}i\int e^{-i(3a+3bx)}(c+dx)^m\,dx\right) + \frac{1}{8}i\int e^{i(3a+3bx)}(c+dx)^m\,dx + \frac{3}{8}i\int e^{-i(a\cdots}\\ &= -\frac{3e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} - \frac{3e^{-i\left(a-\frac{bc}{d}\right)}(c+dx\cdots}{\cdots}\end{aligned}$$

**Mathematica [A]**
time = 9.42, size = 251, normalized size = 0.94

$$\frac{3^{-1-m}e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{2+m}e^{2i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-3^{2+m}e^{2ia+\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)+e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((2*I)*(2*a + (b*c)/d))*((I*b*(c + d
*x))/d)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]) - 3^(2 + m)*E^((2*I)*a + ((4*
I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, (I*b*(c + d*x))/d] + E^((6
*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(((6*
I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d]))/(
8*b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

**Maple [F]**
time = 0.13, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\sin^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*sin(b*x+a)^3,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)^3, x)
```

**Fricas [A]**
time = 0.15, size = 188, normalized size = 0.70

$$-\frac{9\,e^{\left(-\frac{dm\log\left(\frac{i\,b}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)-e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right)+9\,e^{\left(-\frac{dm\log\left(-\frac{i\,b}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-i\,bdx-i\,bc}{d}\right)-e^{\left(-\frac{dm\log\left(\frac{3i\,b}{d}\right)-3i\,bc+3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(-i\,bdx-i\,bc)}{d}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(9*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*
b*c)/d) - e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, -3*(I
*b*d*x + I*b*c)/d) + 9*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1
, (-I*b*d*x - I*b*c)/d) - e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gam
ma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin^3{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)**3, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(a+bx)^3 (c+dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(c + d*x)^m,x)
[Out] int(sin(a + b*x)^3*(c + d*x)^m, x)
```

# 3.73 $\int (c+dx)^m \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=162

$$\frac{(c+dx)^{1+m}}{2d(1+m)}+\frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{i2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m}{}$$

```
[Out] 1/2*(d*x+c)^(1+m)/d/(1+m)+I*2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m
,-2*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-I*2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*
I*b*(d*x+c)/d)/b/exp(2*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi** [A]

time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3388, 2212}

$$\frac{i2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{i2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) + (I*2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c
+ d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)
 - (I*2^(-3 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*
I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \sin^2(a+bx)\,dx &= \int \left(\frac{1}{2}(c+dx)^m - \frac{1}{2}(c+dx)^m \cos(2a+2bx)\right)\,dx\\ &= \frac{(c+dx)^{1+m}}{2d(1+m)} - \frac{1}{2}\int (c+dx)^m \cos(2a+2bx)\,dx\\ &= \frac{(c+dx)^{1+m}}{2d(1+m)} - \frac{1}{4}\int e^{-i(2a+2bx)}(c+dx)^m\,dx - \frac{1}{4}\int e^{i(2a+2bx)}(c+dx)^m\,dx\\ &= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}\end{aligned}$$

**Mathematica [A]**
time = 0.42, size = 211, normalized size = 1.30

$$\frac{2^{-3-m}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(2^{2+m}b(c+dx)\left(\frac{b^2(c+dx)^2}{d^2}\right)^m - id(1+m)\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)\left(\cos\left(2a-\frac{2bc}{d}\right)-i\sin\left(2a-\frac{2bc}{d}\right)\right)+id(1+m)\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)\left(\cos\left(2a-\frac{2bc}{d}\right)+i\sin\left(2a-\frac{2bc}{d}\right)\right)\right)}{bd(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]^2,x]
[Out] (2^(-3 - m)*(c + d*x)^m*(2^(2 + m)*b*(c + d*x)*((b^2*(c + d*x)^2)/d^2)^m -
I*d*(1 + m)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d]*(C
os[2*a - (2*b*c)/d] - I*Sin[2*a - (2*b*c)/d]) + I*d*(1 + m)*((I*b*(c + d*x)
)/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]*(Cos[2*a - (2*b*c)/d] + I*Sin[2
*a - (2*b*c)/d])))/(b*d*(1 + m)*((b^2*(c + d*x)^2)/d^2)^m)
```

**Maple [F]**
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\sin^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a)^2,x)
[Out] int((d*x+c)^m*sin(b*x+a)^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*b*x + 2*a), x) - e^(m*log(d*x +
 c) + log(d*x + c)))/(d*m + d)
```

**Fricas** [A]
time = 0.10, size = 136, normalized size = 0.84

$$\frac{(i\,dm + i\,d)e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1, -\frac{2\,(i\,bdx+i\,bc)}{d}\right) + (-i\,dm - i\,d)e^{\left(-\frac{dm\log\left(\frac{2i\,b}{d}\right)-2i\,bc+2i\,ad}{d}\right)}\Gamma\left(m+1, -\frac{2\,(-i\,bdx-i\,bc)}{d}\right) + 4\,(bdx+bc)(dx+c)^m}{8\,(bdm+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((I*d*m + I*d)*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m +
 1, -2*(I*b*d*x + I*b*c)/d) + (-I*d*m - I*d)*e^(-(d*m*log(2*I*b/d) - 2*I*b*
c + 2*I*a*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) + 4*(b*d*x + b*c)*(d*
x + c)^m)/(b*d*m + b*d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)^2, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(sin(a + b*x)^2*(c + d*x)^m, x)
```

# 3.74 $\int (c+dx)^m \sin(a+bx)\,dx$

**Optimal**. Leaf size=127

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b}$$

```
[Out] -1/2*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/
d)^m)-1/2*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+
c)/d)^m)
```

**Rubi** [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3389, 2212}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sin[a + b*x],x]
```

```
[Out] -1/2*(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(
b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])
/(2*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\sin(a+bx)\,dx &= \frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx-\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx\\ &= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\end{aligned}$$

**Mathematica [A]**
time = 0.03, size = 121, normalized size = 0.95

$$\frac{e^{-\frac{i(bc+ad)}{d}}(c+dx)^m\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x],x]
```

```
[Out] ((c + d*x)^m*(-((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((-I)*b*(
c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d])/((I*b
*(c + d*x))/d)^m))/(2*b*E^((I*(b*c + a*d))/d))
```

**Maple [F]**
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sin(b*x+a),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a), x)
```

**Fricas [A]**
time = 0.10, size = 94, normalized size = 0.74

$$-\frac{e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+ibc-iad}{d}\right)}\Gamma\left(m+1,\frac{-ibdx-ibc}{d}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*c
)/d) + e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*
b*c)/d))/b
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(c + d*x)^m,x)
```

```
[Out] int(sin(a + b*x)*(c + d*x)^m, x)
```

## 3.75 $\int (c+dx)^m \csc(a+bx)\,dx$

**Optimal**. Leaf size=17

$$\text{Int}((c+dx)^m\csc(a+bx),x)$$

```
[Out] Unintegrable((d*x+c)^m*csc(b*x+a),x)
```

**Rubi** [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m\csc(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m\csc(a+bx)\,dx = \int (c+dx)^m\csc(a+bx)\,dx$$

**Mathematica** [A]
time = 5.38, size = 0, normalized size = 0.00

$$\int (c+dx)^m\csc(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x], x]
```

**Maple** [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx+c)^m\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a),x)
[Out] Integral((c + d*x)**m*csc(a + b*x), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{(c+d\,x)^m}{\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/sin(a + b*x),x)
[Out] int((c + d*x)^m/sin(a + b*x), x)
```

# 3.76 $\int (c+dx)^m \csc^2(a+bx)\,dx$

**Optimal**. Leaf size=19

$$\mathrm{Int}\left((c+dx)^m \csc^2(a+bx),x\right)$$

```
[Out] Unintegrable((d*x+c)^m*csc(b*x+a)^2,x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\,dx$$

**Mathematica** [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2, x]
```

**Maple** [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^2,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^2, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*csc(a + b*x)**2, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(c+d\,x)^m}{\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/sin(a + b*x)^2,x)
[Out] int((c + d*x)^m/sin(a + b*x)^2, x)
```

# 3.77 $\int x^{3+m}\sin(a+bx)\,dx$

**Optimal**. Leaf size=79

$$\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}$$

```
[Out] 1/2*I*exp(I*a)*x^m*GAMMA(4+m,-I*b*x)/b^4/((-I*b*x)^m)-1/2*I*x^m*GAMMA(4+m,I
*b*x)/b^4/exp(I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3389, 2212}

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+4,-ibx)}{2b^4}-\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+4,ibx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Sin[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[4 + m, I*b*x])/(b^4*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{3+m}\sin(a+bx)\,dx&=\frac{1}{2}i\int e^{-i(a+bx)}x^{3+m}\,dx-\frac{1}{2}i\int e^{i(a+bx)}x^{3+m}\,dx\\&=\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}\end{aligned}$$

**Mathematica [A]**
time = 0.01, size = 79, normalized size = 1.00

$$\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Sin[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[4 + m, I*b*x])/(b^4*E^(I*a)*(I*b*x)^m)
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.05, size = 454, normalized size = 5.75

| method | result |
|---|---|
| meijerg | $2^{3+m}b^{-4-m}\sqrt{\pi}\,\left(\frac{2^{-3-m}x^{2+m}b^{2+m}(m^2+7m+10)\sin(bx)}{\sqrt{\pi}\,(5+m)}-\frac{2^{-3-m}x^{2+m}b^{2+m}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}}-\frac{2^{-3-m}x^{2+m}b^{2+m}m}{}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(3+m)*b^(-4-m)*Pi^(1/2)*(2^(-3-m)/Pi^(1/2)/(5+m)*x^(2+m)*b^(2+m)*(m^2+7*m
+10)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(cos(b*x)*x*b-sin(b*x))-2^(
-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*m*(3+m)*(2+m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,
3/2,b*x)*sin(b*x)+2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(3+m)*(2+m)*(b*x)^(-5/2
-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)+2^(3+m)/b^4*(b^
2)^(-1/2*m)*Pi^(1/2)*(3*2^(-4-m)/Pi^(1/2)/(m+4)*x^(3+m)*b^3*(b^2)^(1/2*m)*(
2/3*m+8/3)*sin(b*x)-2^(-3-m)/Pi^(1/2)/(m+4)*x^(1+m)*b*(b^2)^(1/2*m)*(-m^2-7
*m-12)*(cos(b*x)*x*b-sin(b*x))+2^(-3-m)/Pi^(1/2)/(m+4)*x^(2+m)*b^2*(b^2)^(1
/2*m)*(-m^3-8*m^2-19*m-12)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)-
2^(-3-m)/Pi^(1/2)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2+m)*(1+m)*(3+m)*(b*x)^(-5/2-m
)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin(a)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 3)*sin(b*x + a), x)
```

**Fricas** [A]
time = 0.09, size = 52, normalized size = 0.66

$$-\frac{e^{\left(-(m+3)\log(i\,b)-i\,a\right)}\Gamma(m+4,i\,bx)+e^{\left(-(m+3)\log(-i\,b)+i\,a\right)}\Gamma(m+4,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*(e^(-(m + 3)*log(I*b) - I*a)*gamma(m + 4, I*b*x) + e^(-(m + 3)*log(-I*
b) + I*a)*gamma(m + 4, -I*b*x))/b
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+3}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*sin(b*x+a),x)
[Out] Integral(x**(m + 3)*sin(a + b*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m + 3)*sin(b*x + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+3}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 3)*sin(a + b*x),x)
[Out] int(x^(m + 3)*sin(a + b*x), x)
```

# 3.78 $\int x^{2+m}\sin(a+bx)\,dx$

**Optimal**. Leaf size=75

$$\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}$$

```
[Out] 1/2*exp(I*a)*x^m*GAMMA(3+m,-I*b*x)/b^3/((-I*b*x)^m)+1/2*x^m*GAMMA(3+m,I*b*x
)/b^3/exp(I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3389, 2212}

$$\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+3,-ibx)}{2b^3}+\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+3,ibx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Sin[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(2*b^3*((-I)*b*x)^m) + (x^m*Gamma[3 +
m, I*b*x])/(2*b^3*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{2+m}\sin(a+bx)\,dx&=\frac{1}{2}i\int e^{-i(a+bx)}x^{2+m}\,dx-\frac{1}{2}i\int e^{i(a+bx)}x^{2+m}\,dx\\&=\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}\end{aligned}$$

**Mathematica [A]**
time = 0.01, size = 75, normalized size = 1.00

$$\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Sin[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(2*b^3*((-I)*b*x)^m) + (x^m*Gamma[3 + m, I*b*x])/(2*b^3*E^(I*a)*(I*b*x)^m)
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.04, size = 353, normalized size = 4.71

| method | result |
|---|---|
| meijerg | $2^{2+m}b^{-3-m}\sqrt{\pi}\,\left(-\frac{2^{-2-m}x^{1+m}b^{1+m}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}}+\frac{2^{-2-m}x^{2+m}b^{2+m}(m^2+5m+4)(bx)^{-\frac{3}{2}-m}\operatorname{LommelS1}(m+\frac{3}{2}}{\sqrt{\pi}\,(m+4)}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(2+m)*b^(-3-m)*Pi^(1/2)*(-2^(-2-m)/Pi^(1/2)*x^(1+m)*b^(1+m)*(cos(b*x)*x*b
-sin(b*x))+2^(-2-m)/Pi^(1/2)/(m+4)*x^(2+m)*b^(2+m)*(m^2+5*m+4)*(b*x)^(-3/2-
m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+2^(-2-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(2+m)
*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(
a)+2^(2+m)/b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-3-m)/Pi^(1/2)/(3+m)*x^(2+
m)*(b^2)^(3/2+1/2*m)*(2+2/3*m)/b*sin(b*x)-2^(-2-m)/Pi^(1/2)*x^(2+m)*(b^2)^(
3/2+1/2*m)/b*(2+m)*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(-2-
m)/Pi^(1/2)*x^(2+m)*(b^2)^(3/2+1/2*m)/b*(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-
sin(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2)*sin(b*x + a), x)
```

**Fricas [A]**
time = 0.09, size = 52, normalized size = 0.69

$$-\frac{e^{(-(m+2)\log(ib)-ia)}\Gamma(m+3,ibx)+e^{(-(m+2)\log(-ib)+ia)}\Gamma(m+3,-ibx)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m + 2)*log(I*b) - I*a)*gamma(m + 3, I*b*x) + e^(-(m + 2)*log(-I*
b) + I*a)*gamma(m + 3, -I*b*x))/b
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+2}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*sin(b*x+a),x)
```

```
[Out] Integral(x**(m + 2)*sin(a + b*x), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*sin(b*x + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+2}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)*sin(a + b*x),x)
```

```
[Out] int(x^(m + 2)*sin(a + b*x), x)
```

# 3.79 $\int x^{1+m}\sin(a+bx)\,dx$

**Optimal**. Leaf size=79

$$-\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2}+\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}$$

```
[Out] -1/2*I*exp(I*a)*x^m*GAMMA(2+m,-I*b*x)/b^2/((-I*b*x)^m)+1/2*I*x^m*GAMMA(2+m,
I*b*x)/b^2/exp(I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3389, 2212}

$$\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+2,ibx)}{2b^2}-\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+2,-ibx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Sin[a + b*x],x]
```

```
[Out] ((-1/2*I)*E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(b^2*((-I)*b*x)^m) + ((I/2)*x
^m*Gamma[2 + m, I*b*x])/(b^2*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{1+m}\sin(a+bx)\,dx&=\frac{1}{2}i\int e^{-i(a+bx)}x^{1+m}\,dx-\frac{1}{2}i\int e^{i(a+bx)}x^{1+m}\,dx\\&=-\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2}+\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}\end{aligned}$$

**Mathematica** [A]
time = 0.01, size = 79, normalized size = 1.00

$$-\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2}+\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Sin[a + b*x],x]
```

```
[Out] ((-1/2*I)*E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(b^2*((-I)*b*x)^m) + ((I/2)*x
^m*Gamma[2 + m, I*b*x])/(b^2*E^(I*a)*(I*b*x)^m)
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.04, size = 290, normalized size = 3.67

| method | result |
|---|---|
| meijerg | $2^{1+m}b^{-2-m}\sqrt{\pi}\,\left(\frac{2^{-1-m}x^{2+m}b^{2+m}m(bx)^{-\frac{3}{2}-m}\,\mathrm{LommelS1}\left(m+\frac{1}{2},\frac{3}{2},bx\right)\sin(bx)}{\sqrt{\pi}}-\frac{2^{-1-m}x^{2+m}b^{2+m}(bx)^{-\frac{5}{2}-m}(\cos(bx)xb-}{\sqrt{\pi}}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1+m)*b^(-2-m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*m*(b*x)^(-3/2-
m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-1-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(b*x)
^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)+2^(1+m)/b
^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)/(2+m)*x^(1+m)*b*(b^2)^(1/2*m)
*sin(b*x)+3*2^(-2-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2/3+2/3*m)*(
b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+2^(-1-m)/Pi^(1/2)*x^(2+m)*b^
2*(b^2)^(1/2*m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2
,1/2,b*x))*sin(a)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 1)*sin(b*x + a), x)
```

**Fricas** [A]
time = 0.11, size = 52, normalized size = 0.66

$$-\frac{e^{(-(m+1)\log(i\,b)-i\,a)}\Gamma(m+2,i\,bx)+e^{(-(m+1)\log(-i\,b)+i\,a)}\Gamma(m+2,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m + 1)*log(I*b) - I*a)*gamma(m + 2, I*b*x) + e^(-(m + 1)*log(-I*
b) + I*a)*gamma(m + 2, -I*b*x))/b
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+1}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*sin(b*x+a),x)
```

```
[Out] Integral(x**(m + 1)*sin(a + b*x), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*sin(b*x + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+1}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)*sin(a + b*x),x)
```

```
[Out] int(x^(m + 1)*sin(a + b*x), x)
```

# 3.80 $\int x^m \sin(a+bx)\,dx$

**Optimal**. Leaf size=75

$$-\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}$$

```
[Out] -1/2*exp(I*a)*x^m*GAMMA(1+m,-I*b*x)/b/((-I*b*x)^m)-1/2*x^m*GAMMA(1+m,I*b*x)
/b/exp(I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3389, 2212}

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+1,-ibx)}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+1,ibx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x],x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) - (x^m*Gamma[1 +
 m, I*b*x])/(2*b*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^m\sin(a+bx)\,dx &= \frac{1}{2}i\int e^{-i(a+bx)}x^m\,dx-\frac{1}{2}i\int e^{i(a+bx)}x^m\,dx\\ &= -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}\end{aligned}$$

**Mathematica [A]**
time = 0.01, size = 75, normalized size = 1.00

$$-\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x],x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) - (x^m*Gamma[1 +
 m, I*b*x])/(2*b*E^(I*a)*(I*b*x)^m)
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.04, size = 378, normalized size = 5.04

| method | result |
|---|---|
| meijerg | $2^m b^{-1-m}\sqrt{\pi}\left(\frac{x^{1+m}b^{1+m}2^{-m}\sin(bx)}{\sqrt{\pi}\,(2+m)}-\frac{2^{-m}x^{2+m}b^{2+m}(bx)^{-\frac{3}{2}-m}\,\mathrm{LommelS1}(m+\frac{3}{2},\frac{3}{2},bx)\sin(bx)}{\sqrt{\pi}\,(2+m)}-\frac{3\,2^{-1-m}x^{2+m}b^{2+}}{}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^m*b^(-1-m)*Pi^(1/2)*(1/Pi^(1/2)/(2+m)*x^(1+m)*b^(1+m)*2^(-m)*sin(b*x)-2^(
-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*s
in(b*x)-3*2^(-1-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)*(4/3+2/3*m)*(b*x)^(-5/2-m
)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(a)+2^m*(b^2)^(-1/2-1
/2*m)*Pi^(1/2)*(3*2^(-1-m)/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m)*x^m*(6+2*m)/(9+
3*m)/b*sin(b*x)+1/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m)*x^m*2^(-m)/b*(cos(b*x)*x
*b-sin(b*x))+2^(-m)/Pi^(1/2)/(1+m)*x^(2+m)*(b^2)^(1/2+1/2*m)*b*m*(b*x)^(-3/
2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-m)/Pi^(1/2)/(1+m)*x^(2+m)*(b^2)^(
1/2+1/2*m)*b*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x)
)*sin(a)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sin(b*x + a), x)
```

**Fricas [A]**
time = 0.08, size = 48, normalized size = 0.64

$$-\frac{e^{(-m\log(i\,b)-i\,a)}\Gamma(m+1,i\,bx)+e^{(-m\log(-i\,b)+i\,a)}\Gamma(m+1,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) + e^(-m*log(-I*b) + I*a)*ga
mma(m + 1, -I*b*x))/b
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(b*x+a),x)
```

```
[Out] Integral(x**m*sin(a + b*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + b*x),x)
```

```
[Out] int(x^m*sin(a + b*x), x)
```

# 3.81 $\int x^{-1+m}\sin(a+bx)\,dx$

**Optimal**. Leaf size=69

$$\frac{1}{2}ie^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx)-\frac{1}{2}ie^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)$$

```
[Out] 1/2*I*exp(I*a)*x^m*GAMMA(m,-I*b*x)/((-I*b*x)^m)-1/2*I*x^m*GAMMA(m,I*b*x)/ex
p(I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3389, 2212}

$$\frac{1}{2}ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m,-ibx)-\frac{1}{2}ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Sin[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*x^m*Gamma[m, I
*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\sin(a+bx)\,dx&=\frac{1}{2}i\int e^{-i(a+bx)}x^{-1+m}\,dx-\frac{1}{2}i\int e^{i(a+bx)}x^{-1+m}\,dx\\&=\frac{1}{2}ie^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx)-\frac{1}{2}ie^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)\end{aligned}$$

**Mathematica [A]**
time = 0.02, size = 63, normalized size = 0.91

$$\frac{1}{2}ie^{-ia}x^m\left(e^{2ia}(-ibx)^{-m}\Gamma(m,-ibx)-(ibx)^{-m}\Gamma(m,ibx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Sin[a + b*x],x]
[Out] ((I/2)*x^m*((E^((2*I)*a)*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - Gamma[m, I*b*x]
/(I*b*x)^m))/E^(I*a)
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.05, size = 426, normalized size = 6.17

| method | result |
|---|---|
| meijerg | $2^{-1+m}b^{-m}\sqrt{\pi}\,\left(\frac{2^{1-m}x^m b^m\sin(bx)}{\sqrt{\pi}\,(1+m)}-\frac{2^{1-m}x^m b^m(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,(1+m)m}-\frac{x^{2+m}b^{2+m}2^{1-m}(bx)^{-\frac{3}{2}-m}\operatorname{LommelS1}(m+\frac{1}{2},\frac{3}{2},b}{\sqrt{\pi}\,(1+m)}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 2^(-1+m)*b^(-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)/(1+m)*x^m*b^m*sin(b*x)-2^(1-m)/P
i^(1/2)/(1+m)*x^m*b^m/m*(cos(b*x)*x*b-sin(b*x))-1/Pi^(1/2)/(1+m)*x^(2+m)*b^
(2+m)*2^(1-m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+1/Pi^(1/2)/(1
+m)*x^(2+m)*b^(2+m)*2^(1-m)/m*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lommel
S1(m+3/2,1/2,b*x))*cos(a)+2^(-1+m)*(b^2)^(-1/2*m)*Pi^(1/2)*(3/Pi^(1/2)/m*x^
(-1+m)*2^(-m)*(b^2)^(1/2*m)*(2*b^2*x^2+2*m+4)/(6+3*m)/b*sin(b*x)+2^(1-m)/Pi
^(1/2)/m*x^(-1+m)*(b^2)^(1/2*m)/b*(cos(b*x)*x*b-sin(b*x))-3/Pi^(1/2)/m*x^(2
+m)*2^(1-m)*(b^2)^(1/2*m)*b^2/(6+3*m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x
)*sin(b*x)-1/Pi^(1/2)/m*x^(2+m)*2^(1-m)*(b^2)^(1/2*m)*b^2*(b*x)^(-5/2-m)*(c
os(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin(a)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x^(m - 1)*sin(b*x + a), x)
```

**Fricas [A]**
time = 0.09, size = 48, normalized size = 0.70

$$-\frac{e^{(-(m-1)\log(i\,b)-i\,a)}\Gamma(m,i\,bx)+e^{(-(m-1)\log(-i\,b)+i\,a)}\Gamma(m,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m - 1)*log(I*b) - I*a)*gamma(m, I*b*x) + e^(-(m - 1)*log(-I*b) +
 I*a)*gamma(m, -I*b*x))/b
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-1}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*sin(b*x+a),x)
```

```
[Out] Integral(x**(m - 1)*sin(a + b*x), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*sin(b*x + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-1}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*sin(a + b*x),x)
```

```
[Out] int(x^(m - 1)*sin(a + b*x), x)
```

# 3.82 $\int x^{-2+m} \sin(a+bx)\,dx$

**Optimal**. Leaf size=71

$$\frac{1}{2}be^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx)+\frac{1}{2}be^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)$$

```
[Out] 1/2*b*exp(I*a)*x^m*GAMMA(-1+m,-I*b*x)/((-I*b*x)^m)+1/2*b*x^m*GAMMA(-1+m,I*b
*x)/exp(I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3389, 2212}

$$\frac{1}{2}e^{ia}bx^m(-ibx)^{-m}\text{Gamma}(m-1,-ibx)+\frac{1}{2}e^{-ia}bx^m(ibx)^{-m}\text{Gamma}(m-1,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Sin[a + b*x],x]
```

```
[Out] (b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b*x^m*Gamma[-1
+ m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{-2+m}\sin(a+bx)\,dx &= \frac{1}{2}i\int e^{-i(a+bx)}x^{-2+m}\,dx-\frac{1}{2}i\int e^{i(a+bx)}x^{-2+m}\,dx\\ &= \frac{1}{2}be^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx)+\frac{1}{2}be^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)\end{aligned}$$

**Mathematica [A]**
time = 0.01, size = 65, normalized size = 0.92

$$\frac{1}{2}be^{-ia}x^m\left(e^{2ia}(-ibx)^{-m}\Gamma(-1+m,-ibx)+(ibx)^{-m}\Gamma(-1+m,ibx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Sin[a + b*x],x]
```

```
[Out] (b*x^m*((E^((2*I)*a)*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m + Gamma[-1 + m,
I*b*x]/(I*b*x)^m))/(2*E^(I*a))
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.05, size = 529, normalized size = 7.45

| method | result |
|---|---|
| meijerg | $2^{m-2}b^{1-m}\sqrt{\pi}\,\left(\frac{2^{1-m}x^{-1+m}b^{-1+m}(-2x^2b^2+2m^2+2m-4)\sin(bx)}{\sqrt{\pi}\,m(2+m)(-1+m)}-\frac{3\,2^{2-m}x^{-1+m}b^{-1+m}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,m(-3+3m)}+\frac{2^{2-m}}{}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(m-2)*b^(1-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)/m*x^(-1+m)*b^(-1+m)*(-2*b^2*x^2+
2*m^2+2*m-4)/(2+m)/(-1+m)*sin(b*x)-3*2^(2-m)/Pi^(1/2)/m*x^(-1+m)*b^(-1+m)/(
-3+3*m)*(cos(b*x)*x*b-sin(b*x))+2^(2-m)/Pi^(1/2)/m*x^(2+m)*b^(2+m)/(2+m)/(-
1+m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+3*2^(2-m)/Pi^(1/2)/m*x
^(2+m)*b^(2+m)/(-3+3*m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1
/2,1/2,b*x))*cos(a)+2^(m-2)*(b^2)^(-1/2-1/2*m)*b^2*Pi^(1/2)*(3*2^(1-m)/Pi^(
1/2)/(-1+m)*x^(m-2)*(b^2)^(-1/2+1/2*m)*(2*b^2*x^2+2*m+2)/(3+3*m)/b*sin(b*x)
-2^(2-m)/Pi^(1/2)/(-1+m)*x^(m-2)*(b^2)^(-1/2+1/2*m)/b*(b^2*x^2-m^2-m)/m/(1+
m)*(cos(b*x)*x*b-sin(b*x))-3*2^(2-m)/Pi^(1/2)/(-1+m)*x^(2+m)*(b^2)^(-1/2+1/
2*m)*b^3/(3+3*m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(2-m)/Pi
^(1/2)/(-1+m)*x^(2+m)*(b^2)^(-1/2+1/2*m)*b^3/m/(1+m)*(b*x)^(-5/2-m)*(cos(b*
x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2)*sin(b*x + a), x)
```

**Fricas [A]**
time = 0.10, size = 52, normalized size = 0.73

$$-\frac{e^{\left(-(m-2)\log(i\,b)-i\,a\right)}\Gamma(m-1,i\,bx)+e^{\left(-(m-2)\log(-i\,b)+i\,a\right)}\Gamma(m-1,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m - 2)*log(I*b) - I*a)*gamma(m - 1, I*b*x) + e^(-(m - 2)*log(-I*
b) + I*a)*gamma(m - 1, -I*b*x))/b
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-2}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*sin(b*x+a),x)
```

```
[Out] Integral(x**(m - 2)*sin(a + b*x), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*sin(b*x + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-2}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)*sin(a + b*x),x)
```

```
[Out] int(x^(m - 2)*sin(a + b*x), x)
```

# 3.83 $\int x^{-3+m} \sin(a+bx)\,dx$

**Optimal**. Leaf size=79

$$-\frac{1}{2}ib^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx)+\frac{1}{2}ib^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)$$

```
[Out] -1/2*I*b^2*exp(I*a)*x^m*GAMMA(-2+m,-I*b*x)/((-I*b*x)^m)+1/2*I*b^2*x^m*GAMMA
(-2+m,I*b*x)/exp(I*a)/((I*b*x)^m)
```

**Rubi** [A]

time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3389, 2212}

$$\frac{1}{2}ie^{-ia}b^2x^m(ibx)^{-m}\text{Gamma}(m-2,ibx)-\frac{1}{2}ie^{ia}b^2x^m(-ibx)^{-m}\text{Gamma}(m-2,-ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Sin[a + b*x],x]
```

```
[Out] ((-1/2*I)*b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/((-I)*b*x)^m + ((I/2)*b^
2*x^m*Gamma[-2 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{-3+m}\sin(a+bx)\,dx&=\frac{1}{2}i\int e^{-i(a+bx)}x^{-3+m}\,dx-\frac{1}{2}i\int e^{i(a+bx)}x^{-3+m}\,dx\\&=-\frac{1}{2}ib^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx)+\frac{1}{2}ib^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)\end{aligned}$$

**Mathematica** [A]
time = 0.01, size = 79, normalized size = 1.00

$$-\frac{1}{2}ib^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx)+\frac{1}{2}ib^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Sin[a + b*x],x]
```

```
[Out] ((-1/2*I)*b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/((-I)*b*x)^m + ((I/2)*b^
2*x^m*Gamma[-2 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.05, size = 599, normalized size = 7.58

| method | result |
|---|---|
| meijerg | $2^{m-3}b^{2-m}\sqrt{\pi}\left(\frac{2^{2-m}x^{m-2}b^{m-2}(-2x^2b^2+2m^2-2m-4)\sin(bx)}{\sqrt{\pi}\,(-1+m)(1+m)(m-2)}+\frac{2^{-m+3}x^{m-2}b^{m-2}(x^2b^2-m^2-m)(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,(-1+m)(1+m)m(m-2)}+\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-3)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(m-3)*b^(2-m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-1+m)*x^(m-2)*b^(m-2)*(-2*b^2*x
^2+2*m^2-2*m-4)/(1+m)/(m-2)*sin(b*x)+2^(-m+3)/Pi^(1/2)/(-1+m)*x^(m-2)*b^(m-
2)*(b^2*x^2-m^2-m)/(1+m)/m/(m-2)*(cos(b*x)*x*b-sin(b*x))+2^(-m+3)/Pi^(1/2)/
(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(m-2)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*s
in(b*x)-2^(-m+3)/Pi^(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/m/(m-2)*(b*x)^(-5/2-
m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)+2^(m-3)*(b^2)^(-
1/2*m)*b^2*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(m-2)*x^(m-3)*(b^2)^(1/2*m)/b^3*(-2*b
^4*x^4+2*b^2*m^2*x^2+2*b^2*m*x^2-4*b^2*x^2+2*m^3+2*m^2-4*m)/m/(2+m)/(-1+m)*
sin(b*x)-2^(-m+3)/Pi^(1/2)/(m-2)*x^(m-3)*(b^2)^(1/2*m)/b^3*(b^2*x^2-m^2+m)/
(-1+m)/m*(cos(b*x)*x*b-sin(b*x))+2^(-m+3)/Pi^(1/2)/(m-2)*x^(2+m)*(b^2)^(1/2
*m)*b^2/m/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+2^(-
m+3)/Pi^(1/2)/(m-2)*x^(2+m)*(b^2)^(1/2*m)*b^2/(-1+m)/m*(b*x)^(-5/2-m)*(cos(
b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin(a)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 3)*sin(b*x + a), x)
```

**Fricas** [A]
time = 0.11, size = 52, normalized size = 0.66

$$-\frac{e^{\left(-(m-3)\log(i\,b)-i\,a\right)}\Gamma(m-2,i\,bx)+e^{\left(-(m-3)\log(-i\,b)+i\,a\right)}\Gamma(m-2,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*(e^(-(m - 3)*log(I*b) - I*a)*gamma(m - 2, I*b*x) + e^(-(m - 3)*log(-I*
b) + I*a)*gamma(m - 2, -I*b*x))/b
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-3}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*sin(b*x+a),x)
[Out] Integral(x**(m - 3)*sin(a + b*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 3)*sin(b*x + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-3}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)*sin(a + b*x),x)
[Out] int(x^(m - 3)*sin(a + b*x), x)
```

# 3.84 $\int x^{3+m}\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=97

$$\frac{x^{4+m}}{2(4+m)}+\frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4}+\frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(4+m,2ibx)}{b^4}$$

```
[Out] 1/2*x^(4+m)/(4+m)+2^(-6-m)*exp(2*I*a)*x^m*GAMMA(4+m,-2*I*b*x)/b^4/((-I*b*x)
^m)+2^(-6-m)*x^m*GAMMA(4+m,2*I*b*x)/b^4/exp(2*I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {3393, 3388, 2212}

$$\frac{e^{2ia}2^{-m-6}x^m(-ibx)^{-m}\text{Gamma}(m+4,-2ibx)}{b^4}+\frac{e^{-2ia}2^{-m-6}x^m(ibx)^{-m}\text{Gamma}(m+4,2ibx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Sin[a + b*x]^2,x]
```

```
[Out] x^(4 + m)/(2*(4 + m)) + (2^(-6 - m)*E^((2*I)*a)*x^m*Gamma[4 + m, (-2*I)*b*x
])/(b^4*((-I)*b*x)^m) + (2^(-6 - m)*x^m*Gamma[4 + m, (2*I)*b*x])/(b^4*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^{3+m}\sin^2(a+bx)\,dx &= \int\left(\frac{x^{3+m}}{2}-\frac{1}{2}x^{3+m}\cos(2a+2bx)\right)dx\\ &= \frac{x^{4+m}}{2(4+m)}-\frac{1}{2}\int x^{3+m}\cos(2a+2bx)\,dx\\ &= \frac{x^{4+m}}{2(4+m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{3+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{3+m}\,dx\\ &= \frac{x^{4+m}}{2(4+m)}+\frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4}+\frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(4}{b^4}\end{aligned}$$

**Mathematica [A]**
time = 0.23, size = 118, normalized size = 1.22

$$\frac{2^{-6-m}x^m(b^2x^2)^{-m}\left(2^{5+m}b^4x^4(b^2x^2)^m+(4+m)(-ibx)^m\Gamma(4+m,2ibx)(\cos(a)-i\sin(a))^2+(4+m)(ibx)^m\Gamma(4+m,-2ibx)(\cos(a)+i\sin(a))^2\right)}{b^4(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-6 - m)*x^m*(2^(5 + m)*b^4*x^4*(b^2*x^2)^m + (4 + m)*((-I)*b*x)^m*Gamma
[4 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + (4 + m)*(I*b*x)^m*Gamma[4 + m, (
-2*I)*b*x]*(Cos[a] + I*Sin[a])^2))/(b^4*(4 + m)*(b^2*x^2)^m)
```

**Maple [F]**
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{3+m}\left(\sin^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(3+m)*sin(b*x+a)^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m + 4)*integrate(x^3*x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + 4*log(
x)))/(m + 4)
```

**Fricas [A]**
time = 0.13, size = 77, normalized size = 0.79

$$\frac{4\,bxx^{m+3} + (-i\,m - 4i)e^{\left(-(m+3)\log(2i\,b) - 2i\,a\right)}\Gamma(m+4, 2i\,bx) + (i\,m + 4i)e^{\left(-(m+3)\log(-2i\,b) + 2i\,a\right)}\Gamma(m+4, -2i\,bx)}{8\,(bm + 4\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 3) + (-I*m - 4*I)*e^(-(m + 3)*log(2*I*b) - 2*I*a)*gamma(m
 + 4, 2*I*b*x) + (I*m + 4*I)*e^(-(m + 3)*log(-2*I*b) + 2*I*a)*gamma(m + 4,
-2*I*b*x))/(b*m + 4*b)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+3}\sin^2{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*sin(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 3)*sin(a + b*x)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*sin(b*x + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+3}\,{\sin\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 3)*sin(a + b*x)^2,x)
```

```
[Out] int(x^(m + 3)*sin(a + b*x)^2, x)
```

# 3.85 $\int x^{2+m} \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=103

$$\frac{x^{3+m}}{2(3+m)}-\frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3}+\frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(3+m,2ibx)}{b^3}$$

```
[Out] 1/2*x^(3+m)/(3+m)-I*2^(-5-m)*exp(2*I*a)*x^m*GAMMA(3+m,-2*I*b*x)/b^3/((-I*b*
x)^m)+I*2^(-5-m)*x^m*GAMMA(3+m,2*I*b*x)/b^3/exp(2*I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$-\frac{ie^{2ia}2^{-m-5}x^m(-ibx)^{-m}\text{Gamma}(m+3,-2ibx)}{b^3}+\frac{ie^{-2ia}2^{-m-5}x^m(ibx)^{-m}\text{Gamma}(m+3,2ibx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Sin[a + b*x]^2,x]
```

```
[Out] x^(3 + m)/(2*(3 + m)) - (I*2^(-5 - m)*E^((2*I)*a)*x^m*Gamma[3 + m, (-2*I)*b
*x])/(b^3*((-I)*b*x)^m) + (I*2^(-5 - m)*x^m*Gamma[3 + m, (2*I)*b*x])/(b^3*E
^((2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^{2+m}\sin^2(a+bx)\,dx &= \int\left(\frac{x^{2+m}}{2}-\frac{1}{2}x^{2+m}\cos(2a+2bx)\right)dx\\ &= \frac{x^{3+m}}{2(3+m)}-\frac{1}{2}\int x^{2+m}\cos(2a+2bx)\,dx\\ &= \frac{x^{3+m}}{2(3+m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{2+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{2+m}\,dx\\ &= \frac{x^{3+m}}{2(3+m)}-\frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3}+\frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(3}{b^3}\end{aligned}$$

**Mathematica [A]**
time = 0.22, size = 120, normalized size = 1.17

$$\frac{2^{-5-m}x^m(b^2x^2)^{-m}\left(2^{4+m}bx(b^2x^2)^{1+m}+(3+m)(ibx)^m\Gamma(3+m,-2ibx)(-i\cos(2a)+\sin(2a))+(3+m)(-ibx)^m\Gamma(3+m,2ibx)(i\cos(2a)+\sin(2a))\right)}{b^3(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-5 - m)*x^m*(2^(4 + m)*b*x*(b^2*x^2)^(1 + m) + (3 + m)*(I*b*x)^m*Gamma[
3 + m, (-2*I)*b*x]*((-I)*Cos[2*a] + Sin[2*a]) + (3 + m)*((-I)*b*x)^m*Gamma[
3 + m, (2*I)*b*x]*(I*Cos[2*a] + Sin[2*a])))/(b^3*(3 + m)*(b^2*x^2)^m)
```

**Maple [F]**
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{2+m}\left(\sin^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(2+m)*sin(b*x+a)^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m + 3)*integrate(x^2*x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + 3*log(
x)))/(m + 3)
```

**Fricas [A]**
time = 0.10, size = 77, normalized size = 0.75

$$\frac{4\,bxx^{m+2} + (-i\,m - 3i)e^{(-(m+2)\log(2i\,b) - 2i\,a)}\Gamma(m+3, 2i\,bx) + (i\,m + 3i)e^{(-(m+2)\log(-2i\,b) + 2i\,a)}\Gamma(m+3, -2i\,bx)}{8\,(bm + 3\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 2) + (-I*m - 3*I)*e^(-(m + 2)*log(2*I*b) - 2*I*a)*gamma(m
 + 3, 2*I*b*x) + (I*m + 3*I)*e^(-(m + 2)*log(-2*I*b) + 2*I*a)*gamma(m + 3,
-2*I*b*x))/(b*m + 3*b)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+2}\sin^2{(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*sin(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 2)*sin(a + b*x)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*sin(b*x + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+2}\,{\sin\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)*sin(a + b*x)^2,x)
```

```
[Out] int(x^(m + 2)*sin(a + b*x)^2, x)
```

# 3.86 $\int x^{1+m}\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=99

$$\frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2}-\frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(2+m,2ibx)}{b^2}$$

```
[Out] 1/2*x^(2+m)/(2+m)-2^(-4-m)*exp(2*I*a)*x^m*GAMMA(2+m,-2*I*b*x)/b^2/((-I*b*x)
^m)-2^(-4-m)*x^m*GAMMA(2+m,2*I*b*x)/b^2/exp(2*I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {3393, 3388, 2212}

$$-\frac{e^{2ia}2^{-m-4}x^m(-ibx)^{-m}\text{Gamma}(m+2,-2ibx)}{b^2}-\frac{e^{-2ia}2^{-m-4}x^m(ibx)^{-m}\text{Gamma}(m+2,2ibx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Sin[a + b*x]^2,x]
```

```
[Out] x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^((2*I)*a)*x^m*Gamma[2 + m, (-2*I)*b*x
])/(b^2*((-I)*b*x)^m) - (2^(-4 - m)*x^m*Gamma[2 + m, (2*I)*b*x])/(b^2*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int x^{1+m}\sin^2(a+bx)\,dx &= \int \left(\frac{x^{1+m}}{2}-\frac{1}{2}x^{1+m}\cos(2a+2bx)\right)\,dx \\
&= \frac{x^{2+m}}{2(2+m)}-\frac{1}{2}\int x^{1+m}\cos(2a+2bx)\,dx \\
&= \frac{x^{2+m}}{2(2+m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{1+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{1+m}\,dx \\
&= \frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2}-\frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(2
\end{aligned}
$$

**Mathematica** [A]
time = 0.22, size = 116, normalized size = 1.17

$$
\frac{2^{-4-m}x^m(b^2x^2)^{-m}\left(2^{3+m}(b^2x^2)^{1+m}-(2+m)(-ibx)^m\Gamma(2+m,2ibx)(\cos(a)-i\sin(a))^2-(2+m)(ibx)^m\Gamma(2+m,-2ibx)(\cos(a)+i\sin(a))^2\right)}{b^2(2+m)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-4 - m)*x^m*(2^(3 + m)*(b^2*x^2)^(1 + m) - (2 + m)*((-I)*b*x)^m*Gamma[2
 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 - (2 + m)*(I*b*x)^m*Gamma[2 + m, (-2
*I)*b*x]*(Cos[a] + I*Sin[a])^2))/(b^2*(2 + m)*(b^2*x^2)^m)
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$
\int x^{1+m}\left(\sin^2(bx+a)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(1+m)*sin(b*x+a)^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m + 2)*integrate(x*x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + 2*log(x)
))/(m + 2)
```

**Fricas [A]**
time = 0.10, size = 77, normalized size = 0.78

$$\frac{4\,bxx^{m+1}+(-i\,m-2i)e^{(-(m+1)\log(2i\,b)-2i\,a)}\Gamma(m+2,2i\,bx)+(i\,m+2i)e^{(-(m+1)\log(-2i\,b)+2i\,a)}\Gamma(m+2,-2i\,bx)}{8\,(bm+2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 1) + (-I*m - 2*I)*e^(-(m + 1)*log(2*I*b) - 2*I*a)*gamma(m
 + 2, 2*I*b*x) + (I*m + 2*I)*e^(-(m + 1)*log(-2*I*b) + 2*I*a)*gamma(m + 2,
-2*I*b*x))/(b*m + 2*b)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+1}\sin^2{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*sin(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 1)*sin(a + b*x)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*sin(b*x + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+1}\,{\sin\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)*sin(a + b*x)^2,x)
```

```
[Out] int(x^(m + 1)*sin(a + b*x)^2, x)
```

# 3.87 $\int x^m \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=103

$$\frac{x^{1+m}}{2(1+m)}+\frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(1+m,-2ibx)}{b}-\frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(1+m,2ibx)}{b}$$

```
[Out] 1/2*x^(1+m)/(1+m)+I*2^(-3-m)*exp(2*I*a)*x^m*GAMMA(1+m,-2*I*b*x)/b/((-I*b*x)
^m)-I*2^(-3-m)*x^m*GAMMA(1+m,2*I*b*x)/b/exp(2*I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3393, 3388, 2212}

$$\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\text{Gamma}(m+1,-2ibx)}{b}-\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\text{Gamma}(m+1,2ibx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) + (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamma[1 + m, (-2*I)*b
*x])/(b*((-I)*b*x)^m) - (I*2^(-3 - m)*x^m*Gamma[1 + m, (2*I)*b*x])/(b*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^m \sin^2(a+bx)\,dx &= \int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2a+2bx)\right)dx\\ &= \frac{x^{1+m}}{2(1+m)}-\frac{1}{2}\int x^m\cos(2a+2bx)\,dx\\ &= \frac{x^{1+m}}{2(1+m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^m\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^m\,dx\\ &= \frac{x^{1+m}}{2(1+m)}+\frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(1+m,-2ibx)}{b}-\frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(1+}{b}\end{aligned}$$

**Mathematica** [A]
time = 0.19, size = 120, normalized size = 1.17

$$\frac{2^{-3-m}x^m(b^2x^2)^{-m}\left(2^{2+m}bx(b^2x^2)^m-i(1+m)(-ibx)^m\Gamma(1+m,2ibx)(\cos(a)-i\sin(a))^2+i(1+m)(ibx)^m\Gamma(1+m,-2ibx)(\cos(a)+i\sin(a))^2\right)}{b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x]^2,x]
[Out] (2^(-3 - m)*x^m*(2^(2 + m)*b*x*(b^2*x^2)^m - I*(1 + m)*((-I)*b*x)^m*Gamma[1
 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + I*(1 + m)*(I*b*x)^m*Gamma[1 + m, (
-2*I)*b*x]*(Cos[a] + I*Sin[a])^2))/(b*(1 + m)*(b^2*x^2)^m)
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^m\left(\sin^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(b*x+a)^2,x)
[Out] int(x^m*sin(b*x+a)^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a)^2,x, algorithm="maxima")
[Out] -1/2*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + log(x)))/(
m + 1)
```

**Fricas [A]**
time = 0.10, size = 69, normalized size = 0.67

$$\frac{4\,bxx^{m} + (-i\,m - i)e^{(-m\log(2i\,b) - 2i\,a)}\Gamma(m+1, 2i\,bx) + (i\,m + i)e^{(-m\log(-2i\,b) + 2i\,a)}\Gamma(m+1, -2i\,bx)}{8\,(bm + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^m + (-I*m - I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x)
 + (I*m + I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))/(b*m + b)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \sin^{2}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sin(a + b*x)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,{\sin\left(a+b\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + b*x)^2,x)
```

```
[Out] int(x^m*sin(a + b*x)^2, x)
```

# 3.88 $\int x^{-1+m} \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=83

$$\frac{x^m}{2m}+2^{-2-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(m,-2ibx)+2^{-2-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(m,2ibx)$$

```
[Out] 1/2*x^m/m+2^(-2-m)*exp(2*I*a)*x^m*GAMMA(m,-2*I*b*x)/((-I*b*x)^m)+2^(-2-m)*x
^m*GAMMA(m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$e^{2ia}2^{-m-2}x^m(-ibx)^{-m}\text{Gamma}(m,-2ibx)+e^{-2ia}2^{-m-2}x^m(ibx)^{-m}\text{Gamma}(m,2ibx)+\frac{x^m}{2m}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Sin[a + b*x]^2,x]
```

```
[Out] x^m/(2*m) + (2^(-2 - m)*E^((2*I)*a)*x^m*Gamma[m, (-2*I)*b*x])/((-I)*b*x)^m
+ (2^(-2 - m)*x^m*Gamma[m, (2*I)*b*x])/(E^((2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\sin^2(a+bx)\,dx &= \int\left(\frac{x^{-1+m}}{2}-\frac{1}{2}x^{-1+m}\cos(2a+2bx)\right)dx\\ &= \frac{x^m}{2m}-\frac{1}{2}\int x^{-1+m}\cos(2a+2bx)\,dx\\ &= \frac{x^m}{2m}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{-1+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{-1+m}\,dx\\ &= \frac{x^m}{2m}+2^{-2-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(m,-2ibx)+2^{-2-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(m,2ibx)\end{aligned}$$

**Mathematica** [**A**]
time = 0.16, size = 99, normalized size = 1.19

$$\frac{2^{-2-m}x^m(b^2x^2)^{-m}\left(2^{1+m}(b^2x^2)^m+m(-ibx)^m\Gamma(m,2ibx)(\cos(a)-i\sin(a))^2+m(ibx)^m\Gamma(m,-2ibx)(\cos(a)+i\sin(a))^2\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-2 - m)*x^m*(2^(1 + m)*(b^2*x^2)^m + m*((-I)*b*x)^m*Gamma[m, (2*I)*b*x]
*(Cos[a] - I*Sin[a])^2 + m*(I*b*x)^m*Gamma[m, (-2*I)*b*x]*(Cos[a] + I*Sin[a
])^2))/(m*(b^2*x^2)^m)
```

**Maple** [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{-1+m}\left(\sin^2\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(-1+m)*sin(b*x+a)^2,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(m*integrate(x^m*cos(2*b*x + 2*a)/x, x) - x^m)/m
```

**Fricas** [A]
time = 0.12, size = 64, normalized size = 0.77

$$\frac{4\,bxx^{m-1} - i\,me^{(-(m-1)\log(2i\,b)-2i\,a)}\Gamma(m, 2i\,bx) + i\,me^{(-(m-1)\log(-2i\,b)+2i\,a)}\Gamma(m, -2i\,bx)}{8\,bm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 1) - I*m*e^(-(m - 1)*log(2*I*b) - 2*I*a)*gamma(m, 2*I*b*x
) + I*m*e^(-(m - 1)*log(-2*I*b) + 2*I*a)*gamma(m, -2*I*b*x))/(b*m)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-1}\sin^2{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*sin(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 1)*sin(a + b*x)**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*sin(b*x + a)^2, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-1}\,{\sin\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*sin(a + b*x)^2,x)
```

```
[Out] int(x^(m - 1)*sin(a + b*x)^2, x)
```

# 3.89 $\int x^{-2+m}\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=101

$$-\frac{x^{-1+m}}{2(1-m)}-i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\Gamma(-1+m,-2ibx)+i2^{-1-m}be^{-2ia}x^m(ibx)^{-m}\Gamma(-1+m,2ibx)$$

```
[Out] -1/2*x^(-1+m)/(1-m)-I*2^(-1-m)*b*exp(2*I*a)*x^m*GAMMA(-1+m,-2*I*b*x)/((-I*b
*x)^m)+I*2^(-1-m)*b*x^m*GAMMA(-1+m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$-ie^{2ia}b2^{-m-1}x^m(-ibx)^{-m}\text{Gamma}(m-1,-2ibx)+ie^{-2ia}b2^{-m-1}x^m(ibx)^{-m}\text{Gamma}(m-1,2ibx)-\frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Sin[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-1 + m)/(1 - m) - (I*2^(-1 - m)*b*E^((2*I)*a)*x^m*Gamma[-1 + m, (-2
*I)*b*x])/((-I)*b*x)^m + (I*2^(-1 - m)*b*x^m*Gamma[-1 + m, (2*I)*b*x])/(E^(
(2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int x^{-2+m}\sin^2(a+bx)\,dx &= \int \left(\frac{x^{-2+m}}{2}-\frac{1}{2}x^{-2+m}\cos(2a+2bx)\right)\,dx \\
&= -\frac{x^{-1+m}}{2(1-m)}-\frac{1}{2}\int x^{-2+m}\cos(2a+2bx)\,dx \\
&= -\frac{x^{-1+m}}{2(1-m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{-2+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{-2+m}\,dx \\
&= -\frac{x^{-1+m}}{2(1-m)}-i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\Gamma(-1+m,-2ibx)+i2^{-1-m}be^{-2ia}x^m(ibx
\end{aligned}
$$

**Mathematica** [A]
time = 0.22, size = 117, normalized size = 1.16

$$
\frac{2^{-1-m}x^{-1+m}(b^2x^2)^{-m}\left(2^m(b^2x^2)^m+b(-1+m)x(ibx)^m\Gamma(-1+m,-2ibx)(-i\cos(2a)+\sin(2a))+b(-1+m)x(-ibx)^m\Gamma(-1+m,2ibx)(i\cos(2a)+\sin(2a))\right)}{-1+m}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-1 - m)*x^(-1 + m)*(2^m*(b^2*x^2)^m + b*(-1 + m)*x*(I*b*x)^m*Gamma[-1 +
 m, (-2*I)*b*x]*((-I)*Cos[2*a] + Sin[2*a]) + b*(-1 + m)*x*((-I)*b*x)^m*Gamm
a[-1 + m, (2*I)*b*x]*(I*Cos[2*a] + Sin[2*a])))/((-1 + m)*(b^2*x^2)^m)
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$
\int x^{m-2}\left(\sin^2(bx+a)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-2)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(m-2)*sin(b*x+a)^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m - 1)*x*integrate(x^m*cos(2*b*x + 2*a)/x^2, x) - x^m)/((m - 1)*x)
```

**Fricas [A]**
time = 0.10, size = 77, normalized size = 0.76

$$\frac{4\,bxx^{m-2} + (-i\,m + i)e^{(-(m-2)\log(2i\,b)-2i\,a)}\Gamma(m-1, 2i\,bx) + (i\,m - i)e^{(-(m-2)\log(-2i\,b)+2i\,a)}\Gamma(m-1, -2i\,bx)}{8\,(bm-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 2) + (-I*m + I)*e^(-(m - 2)*log(2*I*b) - 2*I*a)*gamma(m -
 1, 2*I*b*x) + (I*m - I)*e^(-(m - 2)*log(-2*I*b) + 2*I*a)*gamma(m - 1, -2*I
*b*x))/(b*m - b)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-2} \sin^2{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*sin(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 2)*sin(a + b*x)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*sin(b*x + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-2}\,{\sin\left(a+b\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)*sin(a + b*x)^2,x)
```

```
[Out] int(x^(m - 2)*sin(a + b*x)^2, x)
```

# 3.90 $\int x^{-3+m} \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=97

$$-\frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx)-2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\Gamma(-2+m,2ibx)$$

```
[Out] -1/2*x^(-2+m)/(2-m)-b^2*exp(2*I*a)*x^m*GAMMA(-2+m,-2*I*b*x)/(2^m)/((-I*b*x)
^m)-b^2*x^m*GAMMA(-2+m,2*I*b*x)/(2^m)/exp(2*I*a)/((I*b*x)^m)
```

**Rubi** [A]
time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$-e^{2ia}b^2 2^{-m}x^m(-ibx)^{-m}\text{Gamma}(m-2,-2ibx)-e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\text{Gamma}(m-2,2ibx)-\frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Sin[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-2 + m)/(2 - m) - (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) - (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int x^{-3+m}\sin^2(a+bx)\,dx &= \int \left(\frac{x^{-3+m}}{2}-\frac{1}{2}x^{-3+m}\cos(2a+2bx)\right)\,dx \\
&= -\frac{x^{-2+m}}{2(2-m)}-\frac{1}{2}\int x^{-3+m}\cos(2a+2bx)\,dx \\
&= -\frac{x^{-2+m}}{2(2-m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{-3+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{-3+m}\,dx \\
&= -\frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx)-2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}
\end{aligned}
$$

**Mathematica** [A]
time = 0.26, size = 121, normalized size = 1.25

$$
\frac{2^{-1-m}x^{-2+m}(b^2x^2)^{-m}\left(2^m(b^2x^2)^m-2b^2(-2+m)x^2(-ibx)^m\Gamma(-2+m,2ibx)(\cos(a)-i\sin(a))^2+2(-2+m)(ibx)^{2+m}\Gamma(-2+m,-2ibx)(\cos(2a)+i\sin(2a))\right)}{-2+m}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-1 - m)*x^(-2 + m)*(2^m*(b^2*x^2)^m - 2*b^2*(-2 + m)*x^2*((-I)*b*x)^m*G
amma[-2 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + 2*(-2 + m)*(I*b*x)^(2 + m)*
Gamma[-2 + m, (-2*I)*b*x]*(Cos[2*a] + I*Sin[2*a])))/((-2 + m)*(b^2*x^2)^m)
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$
\int x^{m-3}\left(\sin^2(bx+a)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-3)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(m-3)*sin(b*x+a)^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m - 2)*x^2*integrate(x^m*cos(2*b*x + 2*a)/x^3, x) - x^m)/((m - 2)*x^
2)
```

**Fricas [A]**
time = 0.09, size = 77, normalized size = 0.79

$$\frac{4\,bxx^{m-3}+(-i\,m+2i)e^{\left(-(m-3)\log(2i\,b)-2i\,a\right)}\Gamma(m-2,2i\,bx)+(i\,m-2i)e^{\left(-(m-3)\log(-2i\,b)+2i\,a\right)}\Gamma(m-2,-2i\,bx)}{8\,(bm-2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(4*b*x*x^(m - 3) + (-I*m + 2*I)*e^(-(m - 3)*log(2*I*b) - 2*I*a)*gamma(m
 - 2, 2*I*b*x) + (I*m - 2*I)*e^(-(m - 3)*log(-2*I*b) + 2*I*a)*gamma(m - 2,
-2*I*b*x))/(b*m - 2*b)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-3}\sin^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*sin(b*x+a)**2,x)
[Out] Integral(x**(m - 3)*sin(a + b*x)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^(m - 3)*sin(b*x + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-3}\,{\sin\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)*sin(a + b*x)^2,x)
[Out] int(x^(m - 3)*sin(a + b*x)^2, x)
```

# 3.91 $\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)dx$

**Optimal**. Leaf size=42

$$\frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

[Out] 4/9/f^2/csc(f*x+e)^(3/2)-2/3*x*cos(f*x+e)/f/csc(f*x+e)^(1/2)

**Rubi** [A]

time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4272, 4274}

$$\frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[x/Csc[e + f*x]^(3/2) - (x*Sqrt[Csc[e + f*x]])/3,x]

[Out] 4/(9*f^2*Csc[e + f*x]^(3/2)) - (2*x*Cos[e + f*x])/(3*f*Sqrt[Csc[e + f*x]])

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)dx &= -\left(\frac{1}{3}\int x\sqrt{\csc(e+fx)}\,dx\right)+\int \frac{x}{\csc^{\frac{3}{2}}(e+fx)}\,dx\\ &= \frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}+\frac{1}{3}\int x\sqrt{\csc(e+f}\\ &= \frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}\end{aligned}$$

**Mathematica** [A]
time = 0.26, size = 29, normalized size = 0.69

$$-\frac{2(-2+3fx\cot(e+fx))}{9f^2\csc^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csc[e + f*x]^(3/2) - (x*Sqrt[Csc[e + f*x]])/3,x]
[Out] (-2*(-2 + 3*f*x*Cot[e + f*x]))/(9*f^2*Csc[e + f*x]^(3/2))
```

**Maple** [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x}{\csc\left(fx+e\right)^{\frac{3}{2}}}-\frac{x\left(\sqrt{\csc}\left(fx+e\right)\right)}{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x)
[Out] int(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x, algorithm="maxima")
[Out] integrate(-1/3*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(3/2), x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3x}{\csc^{\frac{3}{2}}(e+fx)}\right)dx+\int x\sqrt{\csc\left(e+fx\right)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)**(3/2)-1/3*x*csc(f*x+e)**(1/2),x)
```

```
[Out] -(Integral(-3*x/csc(e + f*x)**(3/2), x) + Integral(x*sqrt(csc(e + f*x)), x)
)/3
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{{\left(\frac{1}{\sin\left(e+f\,x\right)}\right)}^{3/2}}-\frac{x\,\sqrt{\frac{1}{\sin\left(e+f\,x\right)}}}{3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/sin(e + f*x))^(3/2) - (x*(1/sin(e + f*x))^(1/2))/3,x)
```

```
[Out] int(x/(1/sin(e + f*x))^(3/2) - (x*(1/sin(e + f*x))^(1/2))/3, x)
```

# 3.92 $\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)\,dx$

**Optimal**. Leaf size=111

$$\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}-\frac{16\sqrt{\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e}}{27f^3}$$

```
[Out] 8/9*x/f^2/csc(f*x+e)^(3/2)+16/27*cos(f*x+e)/f^3/csc(f*x+e)^(1/2)-2/3*x^2*co
s(f*x+e)/f/csc(f*x+e)^(1/2)+16/27*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1
/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*csc(f*x+e
)^(1/2)*sin(f*x+e)^(1/2)/f^3
```

**Rubi** [A]
time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4273, 4274, 3854, 3856, 2720}

$$\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{16\sqrt{\sin(e+fx)}\,\sqrt{\csc(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{27f^3}+\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Csc[e + f*x]^(3/2) - (x^2*Sqrt[Csc[e + f*x]])/3,x]
```

```
[Out] (8*x)/(9*f^2*Csc[e + f*x]^(3/2)) + (16*Cos[e + f*x])/(27*f^3*Sqrt[Csc[e + f
*x]]) - (2*x^2*Cos[e + f*x])/(3*f*Sqrt[Csc[e + f*x]]) - (16*Sqrt[Csc[e + f*
x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(27*f^3)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4273

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[(c + d*x)^m*Cos[e + f*x]*((b*Csc[e + f*x])^(n + 1)/(b*f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)dx=-\left(\frac{1}{3}\int x^2\sqrt{\csc(e+fx)}\,dx\right)+\int \frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}\,dx$$

$$=\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}+\frac{1}{3}\int x^2\sqrt{\csc(e+$$

$$=\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{2x^2\cos(e+fx}{3f\sqrt{\csc(e+fx}}$$

$$=\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{2x^2\cos(e+fx}{3f\sqrt{\csc(e+fx}}$$

$$=\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{2x^2\cos(e+fx}{3f\sqrt{\csc(e+fx}}$$

**Mathematica** [A]
time = 0.34, size = 87, normalized size = 0.78

$$-\frac{\sqrt{\csc(e+fx)}\left(-12fx+12fx\cos(2(e+fx))-16F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sqrt{\sin(e+fx)}-8\sin(2(e+fx))+9f^2x^2\sin(2(e+fx))\right)}{27f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Csc[e + f*x]^(3/2) - (x^2*Sqrt[Csc[e + f*x]])/3,x]
```

```
[Out] -1/27*(Sqrt[Csc[e + f*x]]*(-12*f*x + 12*f*x*Cos[2*(e + f*x)] - 16*EllipticF
[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*x]] - 8*Sin[2*(e + f*x)] + 9*f^2*
x^2*Sin[2*(e + f*x)]))/f^3
```

**Maple [F]**
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\csc (fx+e)^{\frac{3}{2}}}-\frac{x^2 \left(\sqrt{\csc}\, (fx+e)\right)}{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x)
```

```
[Out] int(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(-1/3*x^2*sqrt(csc(f*x + e)) + x^2/csc(f*x + e)^(3/2), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(-\frac{3x^2}{\csc^{\frac{3}{2}}(e+fx)}\right)dx+\int x^2\sqrt{\csc (e+fx)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csc(f*x+e)**(3/2)-1/3*x**2*csc(f*x+e)**(1/2),x)
```

```
[Out] -(Integral(-3*x**2/csc(e + f*x)**(3/2), x) + Integral(x**2*sqrt(csc(e + f*x
)), x))/3
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(-1/3*x^2*sqrt(csc(f*x + e)) + x^2/csc(f*x + e)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(\frac{1}{\sin(e+f\,x)}\right)^{3/2}}-\frac{x^2\,\sqrt{\frac{1}{\sin\left(e+f\,x\right)}}}{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/sin(e + f*x))^(3/2) - (x^2*(1/sin(e + f*x))^(1/2))/3,x)
```

```
[Out] int(x^2/(1/sin(e + f*x))^(3/2) - (x^2*(1/sin(e + f*x))^(1/2))/3, x)
```

# 3.93 $\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)dx$

**Optimal**. Leaf size=42

$$\frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)}$$

```
[Out] 4/25/f^2/csc(f*x+e)^(5/2)-2/5*x*cos(f*x+e)/f/csc(f*x+e)^(3/2)
```

**Rubi** [A]

time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4272, 4274}

$$\frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csc[e + f*x]^(5/2) - (3*x)/(5*Sqrt[Csc[e + f*x]]),x]
```

```
[Out] 4/(25*f^2*Csc[e + f*x]^(5/2)) - (2*x*Cos[e + f*x])/(5*f*Csc[e + f*x]^(3/2))
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)dx &= -\left(\frac{3}{5}\int \frac{x}{\sqrt{\csc(e+fx)}}\,dx\right)+\int \frac{x}{\csc^{\frac{5}{2}}(e+fx)}\,dx\\ &= \frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)}+\frac{3}{5}\int \frac{x}{\sqrt{\csc(e+fx)}}\,d\\ &= \frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)}\end{aligned}$$

**Mathematica [A]**
time = 0.27, size = 29, normalized size = 0.69

$$-\frac{2(-2+5fx\cot(e+fx))}{25f^2\csc^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csc[e + f*x]^(5/2) - (3*x)/(5*Sqrt[Csc[e + f*x]]),x]
[Out] (-2*(-2 + 5*f*x*Cot[e + f*x]))/(25*f^2*Csc[e + f*x]^(5/2))
```

**Maple [F]**
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x}{\csc(fx+e)^{\frac{5}{2}}}-\frac{3x}{5\sqrt{\csc(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x)
[Out] int(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x, algorithm="maxima")
[Out] integrate(-3/5*x/sqrt(csc(f*x + e)) + x/csc(f*x + e)^(5/2), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{5x}{\csc^{\frac{5}{2}}(e+fx)}\right)dx+\int\frac{3x}{\sqrt{\csc(e+fx)}}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)**(5/2)-3/5*x/csc(f*x+e)**(1/2),x)
```

```
[Out] -(Integral(-5*x/csc(e + f*x)**(5/2), x) + Integral(3*x/sqrt(csc(e + f*x)),
x))/5
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-3/5*x/sqrt(csc(f*x + e)) + x/csc(f*x + e)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(\frac{1}{\sin(e+f\,x)}\right)^{5/2}}-\frac{3\,x}{5\,\sqrt{\frac{1}{\sin\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/sin(e + f*x))^(5/2) - (3*x)/(5*(1/sin(e + f*x))^(1/2)),x)
```

```
[Out] int(x/(1/sin(e + f*x))^(5/2) - (3*x)/(5*(1/sin(e + f*x))^(1/2)), x)
```

# 3.94 $\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)}-\frac{5}{21}x\sqrt{\csc(e+fx)}\right)\,dx$

**Optimal**. Leaf size=83

$$\frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)}+\frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{10x\cos(e+fx)}{21f\sqrt{\csc(e+fx)}}$$

```
[Out] 4/49/f^2/csc(f*x+e)^(7/2)-2/7*x*cos(f*x+e)/f/csc(f*x+e)^(5/2)+20/63/f^2/csc
(f*x+e)^(3/2)-10/21*x*cos(f*x+e)/f/csc(f*x+e)^(1/2)
```

**Rubi** [A]

time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = {4272, 4274}

$$\frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)}-\frac{10x\cos(e+fx)}{21f\sqrt{\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csc[e + f*x]^(7/2) - (5*x*Sqrt[Csc[e + f*x]])/21,x]
```

```
[Out] 4/(49*f^2*Csc[e + f*x]^(7/2)) - (2*x*Cos[e + f*x])/(7*f*Csc[e + f*x]^(5/2))
 + 20/(63*f^2*Csc[e + f*x]^(3/2)) - (10*x*Cos[e + f*x])/(21*f*Sqrt[Csc[e +
f*x]])
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)} - \frac{5}{21}x\sqrt{\csc(e+fx)}\right)dx = -\left(\frac{5}{21}\int x\sqrt{\csc(e+fx)}\,dx\right) + \int \frac{x}{\csc^{\frac{7}{2}}(e+fx)}dx$$
$$= \frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)} + \frac{5}{7}\int \frac{x}{\csc^{\frac{3}{2}}(e+fx)}d$$
$$= \frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)} + \frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)}$$
$$= \frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)} + \frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)}$$

**Mathematica** [A]
time = 1.37, size = 57, normalized size = 0.69

$$\frac{316 - 36\cos(2(e+fx)) - 483fx\cot(e+fx) + 63fx\cos(3(e+fx))\csc(e+fx)}{882f^2\csc^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csc[e + f*x]^(7/2) - (5*x*Sqrt[Csc[e + f*x]])/21,x]
[Out] (316 - 36*Cos[2*(e + f*x)] - 483*f*x*Cot[e + f*x] + 63*f*x*Cos[3*(e + f*x)]
*Csc[e + f*x])/(882*f^2*Csc[e + f*x]^(3/2))
```

**Maple** [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x}{\csc(fx+e)^{\frac{7}{2}}} - \frac{5x\left(\sqrt{\csc}\,(fx+e)\right)}{21}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x)
[Out] int(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x, algorithm="maxima")
[Out] integrate(-5/21*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(7/2), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(-\frac{21x}{\csc^{\frac{7}{2}}(e+fx)}\right)dx+\int 5x\sqrt{\csc(e+fx)}\,dx}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)**(7/2)-5/21*x*csc(f*x+e)**(1/2),x)
[Out] -(Integral(-21*x/csc(e + f*x)**(7/2), x) + Integral(5*x*sqrt(csc(e + f*x)),
 x))/21
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x, algorithm="giac")
[Out] integrate(-5/21*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(7/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(\frac{1}{\sin(e+f\,x)}\right)^{7/2}}-\frac{5\,x\,\sqrt{\frac{1}{\sin(e+f\,x)}}}{21}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/sin(e + f*x))^(7/2) - (5*x*(1/sin(e + f*x))^(1/2))/21,x)
[Out] int(x/(1/sin(e + f*x))^(7/2) - (5*x*(1/sin(e + f*x))^(1/2))/21, x)
```

# 3.95 $\int (c+dx)^3(a+a\sin(e+fx))\,dx$

**Optimal**. Leaf size=90

$$\frac{a(c+dx)^4}{4d}+\frac{6ad^2(c+dx)\cos(e+fx)}{f^3}-\frac{a(c+dx)^3\cos(e+fx)}{f}-\frac{6ad^3\sin(e+fx)}{f^4}+\frac{3ad(c+dx)^2\sin(e+fx)}{f^2}$$

```
[Out] 1/4*a*(d*x+c)^4/d+6*a*d^2*(d*x+c)*cos(f*x+e)/f^3-a*(d*x+c)^3*cos(f*x+e)/f-6
*a*d^3*sin(f*x+e)/f^4+3*a*d*(d*x+c)^2*sin(f*x+e)/f^2
```

**Rubi** [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2717}

$$\frac{6ad^2(c+dx)\cos(e+fx)}{f^3}+\frac{3ad(c+dx)^2\sin(e+fx)}{f^2}-\frac{a(c+dx)^3\cos(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\sin(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + (6*a*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (a*(c + d*x)
^3*Cos[e + f*x])/f - (6*a*d^3*Sin[e + f*x])/f^4 + (3*a*d*(c + d*x)^2*Sin[e
+ f*x])/f^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx = \int \left(a(c+dx)^3 + a(c+dx)^3\sin(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^4}{4d} + a\int (c+dx)^3\sin(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d} - \frac{a(c+dx)^3\cos(e+fx)}{f} + \frac{(3ad)\int (c+dx)^2\cos(e+fx)\,d}{f}$$
$$= \frac{a(c+dx)^4}{4d} - \frac{a(c+dx)^3\cos(e+fx)}{f} + \frac{3ad(c+dx)^2\sin(e+fx)}{f^2} - \frac{(6}{}$$
$$= \frac{a(c+dx)^4}{4d} + \frac{6ad^2(c+dx)\cos(e+fx)}{f^3} - \frac{a(c+dx)^3\cos(e+fx)}{f} + \frac{3a}{}$$
$$= \frac{a(c+dx)^4}{4d} + \frac{6ad^2(c+dx)\cos(e+fx)}{f^3} - \frac{a(c+dx)^3\cos(e+fx)}{f} - \frac{6a}{}$$

**Mathematica [A]**
time = 0.31, size = 123, normalized size = 1.37

$$a\left(\frac{1}{4}x(4c^3+6c^2dx+4cd^2x^2+d^3x^3) - \frac{(c+dx)(c^2f^2+2cdf^2x+d^2(-6+f^2x^2))\cos(e+fx)}{f^3} + \frac{3d(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\sin(e+fx)}{f^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Sin[e + f*x]),x]
[Out] a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - ((c + d*x)*(c^2*f^2
+ 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x])/f^3 + (3*d*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^4)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(88) = 176.
time = 0.08, size = 482, normalized size = 5.36

| method | result |
|---|---|
| risch | $\frac{a\,d^3x^4}{4} + a\,d^2c\,x^3 + \frac{3ad\,c^2x^2}{2} + a\,c^3x + \frac{a\,c^4}{4d} - \frac{a\left(d^3f^2x^3+3c\,d^2f^2x^2+3c^2d\,f^2x+c^3f^2-6d^3x-6c\,d^2\right)\cos(fx}{f^3}$ |
| norman | $\frac{\frac{\left(2a\,c^3f^2-12ac\,d^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^3} + \frac{a\left(c^3f^3-3c^2d\,f^2+6d^3\right)x}{f^3} + \frac{a\,d^2(cf-d)x^3}{f} + \frac{a\left(c^3f^3+3c^2d\,f^2-6d^3\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^3} + \frac{a\,d^2(}{}}{}$ |
| derivativedivides | $\frac{-a\,c^3\cos(fx+e) + \frac{3a\,c^2de\cos(fx+e)}{f} + \frac{3a\,c^2d(\sin(fx+e)-(fx+e)\cos(fx+e))}{f} - \frac{3ac\,d^2e^2\cos(fx+e)}{f^2} - \frac{6ac\,d^2e(\sin(fx+e)-(fx+e)}{f^2}}{}$ |
| default | $\frac{-a\,c^3\cos(fx+e) + \frac{3a\,c^2de\cos(fx+e)}{f} + \frac{3a\,c^2d(\sin(fx+e)-(fx+e)\cos(fx+e))}{f} - \frac{3ac\,d^2e^2\cos(fx+e)}{f^2} - \frac{6ac\,d^2e(\sin(fx+e)-(fx+e)}{f^2}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^3*cos(f*x+e)+3*a/f*c^2*d*e*cos(f*x+e)+3*a/f*c^2*d*(sin(f*x+e)-(f*
x+e)*cos(f*x+e))-3*a/f^2*c*d^2*e^2*cos(f*x+e)-6*a/f^2*c*d^2*e*(sin(f*x+e)-(
f*x+e)*cos(f*x+e))+3*a/f^2*c*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x
+e)*sin(f*x+e))+a/f^3*d^3*e^3*cos(f*x+e)+3*a/f^3*d^3*e^2*(sin(f*x+e)-(f*x+e
)*cos(f*x+e))-3*a/f^3*d^3*e*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*s
in(f*x+e))+a/f^3*d^3*(-(f*x+e)^3*cos(f*x+e)+3*(f*x+e)^2*sin(f*x+e)-6*sin(f*
x+e)+6*(f*x+e)*cos(f*x+e))+a*c^3*(f*x+e)-3*a/f*c^2*d*e*(f*x+e)+3/2*a/f*c^2*
d*(f*x+e)^2+3*a/f^2*c*d^2*e^2*(f*x+e)-3*a/f^2*c*d^2*e*(f*x+e)^2+a/f^2*c*d^2
*(f*x+e)^3-a/f^3*d^3*e^3*(f*x+e)+3/2*a/f^3*d^3*e^2*(f*x+e)^2-a/f^3*d^3*e*(f
*x+e)^3+1/4*a/f^3*d^3*(f*x+e)^4)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(92) = 184$.
time = 0.31, size = 498, normalized size = 5.53

$$\frac{4\,(fx+e)ac^3 + \frac{(fx+e)^4ad^3}{f^3} + \frac{4\,(fx+e)^3acd^2}{f^2} + \frac{6\,(fx+e)^2ac^2d}{f} - 4\,ac^3\cos(fx+e) - \frac{4\,(fx+e)^3ad^3e}{f^3} - \frac{12\,(fx+e)^2acd^2e}{f^2} - \frac{12\,(fx+e)ac^2de}{f} + \frac{12\,ac^2d\cos(fx+e)e}{f} - \frac{12\,((fx+e)\cos(fx+e)-\sin(fx+e))ac^2d}{f} + \frac{6\,(fx+e)^2ad^3e^2}{f^3} + \frac{12\,(fx+e)acd^2e^2}{f^2} - \frac{12\,acd^2\cos(fx+e)e^2}{f^2} + \frac{24\,((fx+e)\cos(fx+e)-\sin(fx+e))acd^2e}{f^2} - \frac{12\left(\left((fx+e)^2-2\right)\cos(fx+e)-2\,(fx+e)\sin(fx+e)\right)acd^2}{f^2} - \frac{4\,(fx+e)ad^3e^3}{f^3} + \frac{4\,ad^3\cos(fx+e)e^3}{f^3} - \frac{12\,((fx+e)\cos(fx+e)-\sin(fx+e))ad^3e^2}{f^3} + \frac{12\left(\left((fx+e)^2-2\right)\cos(fx+e)-2\,(fx+e)\sin(fx+e)\right)ad^3e}{f^3} - \frac{4\left(\left((fx+e)^3-6\,fx-6\,e\right)\cos(fx+e)-3\left((fx+e)^2-2\right)\sin(fx+e)\right)ad^3}{f^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 + 4*(f*x + e)^3*a*c*d^2/f^2
+ 6*(f*x + e)^2*a*c^2*d/f - 4*a*c^3*cos(f*x + e) - 4*(f*x + e)^3*a*d^3*e/f^
3 - 12*(f*x + e)^2*a*c*d^2*e/f^2 - 12*(f*x + e)*a*c^2*d*e/f + 12*a*c^2*d*co
s(f*x + e)*e/f - 12*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*c^2*d/f + 6*(
f*x + e)^2*a*d^3*e^2/f^3 + 12*(f*x + e)*a*c*d^2*e^2/f^2 - 12*a*c*d^2*cos(f*
x + e)*e^2/f^2 + 24*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*c*d^2*e/f^2 -
 12*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a*c*d^2/f^2
 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*a*d^3*cos(f*x + e)*e^3/f^3 - 12*((f*x + e)
*cos(f*x + e) - sin(f*x + e))*a*d^3*e^2/f^3 + 12*(((f*x + e)^2 - 2)*cos(f*x
 + e) - 2*(f*x + e)*sin(f*x + e))*a*d^3*e/f^3 - 4*(((f*x + e)^3 - 6*f*x - 6
*e)*cos(f*x + e) - 3*((f*x + e)^2 - 2)*sin(f*x + e))*a*d^3/f^3)/f
```

**Fricas** [A]
time = 0.34, size = 170, normalized size = 1.89

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x - 4\left(ad^3f^3x^3 + 3\,acd^2f^3x^2 + ac^3f^3 - 6\,acd^2f + 3\left(ac^2df^3 - 2\,ad^3f\right)x\right)\cos(fx+e) + 12\left(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 - 2\,ad^3\right)\sin(fx+e)}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
- 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 - 6*a*c*d^2*f + 3*(a*c^2
*d*f^3 - 2*a*d^3*f)*x)*cos(f*x + e) + 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x +
 a*c^2*d*f^2 - 2*a*d^3)*sin(f*x + e))/f^4
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(88) = 176$.
time = 0.25, size = 264, normalized size = 2.93

$$\begin{cases} ac^3x - \frac{ac^3\cos(e+fx)}{f} + \frac{3ac^2dx^2}{2} - \frac{3ac^2dx\cos(e+fx)}{f} + \frac{3ac^2d\sin(e+fx)}{f^2} + acd^2x^3 - \frac{3acd^2x^2\cos(e+fx)}{f} + \frac{6acd^2x\sin(e+fx)}{f^2} + \frac{6acd^2\cos(e+fx)}{f^3} + \frac{ad^3x^4}{4} - \frac{ad^3x^3\cos(e+fx)}{f} + \frac{3ad^3x^2\sin(e+fx)}{f^2} + \frac{6ad^3x\cos(e+fx)}{f^3} - \frac{6ad^3\sin(e+fx)}{f^4} & \text{for } f \neq 0 \\ (a\sin(e) + a)\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x - a*c**3*cos(e + f*x)/f + 3*a*c**2*d*x**2/2 - 3*a*c**2*
d*x*cos(e + f*x)/f + 3*a*c**2*d*sin(e + f*x)/f**2 + a*c*d**2*x**3 - 3*a*c*d
**2*x**2*cos(e + f*x)/f + 6*a*c*d**2*x*sin(e + f*x)/f**2 + 6*a*c*d**2*cos(e
 + f*x)/f**3 + a*d**3*x**4/4 - a*d**3*x**3*cos(e + f*x)/f + 3*a*d**3*x**2*s
in(e + f*x)/f**2 + 6*a*d**3*x*cos(e + f*x)/f**3 - 6*a*d**3*sin(e + f*x)/f**
4, Ne(f, 0)), ((a*sin(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**
3*x**4/4), True))
```

**Giac** [A]
time = 2.35, size = 157, normalized size = 1.74

$$\frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x - \frac{(ad^3f^3x^3 + 3acd^2f^3x^2 + 3ac^2df^3x + ac^3f^3 - 6ad^3fx - 6acd^2f)\cos(fx+e)}{f^4} + \frac{3(ad^3f^2x^2 + 2acd^2f^2x + ac^2df^2 - 2ad^3)\sin(fx+e)}{f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - (a*d^3*f^3*x^3 +
3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + a*c^3*f^3 - 6*a*d^3*f*x - 6*a*c*d^2*f
)*cos(f*x + e)/f^4 + 3*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a
*d^3)*sin(f*x + e)/f^4
```

**Mupad** [B]
time = 0.26, size = 191, normalized size = 2.12

$$\frac{a\,d^3\,x^4}{4} - \frac{3\,\sin(e+f\,x)\,(2\,a\,d^3 - a\,c^2\,d\,f^2)}{f^4} - \frac{\cos(e+f\,x)\,(a\,c^3\,f^2 - 6\,a\,c\,d^2)}{f^3} + a\,c^3\,x + \frac{3\,x\,\cos(e+f\,x)\,(2\,a\,d^3 - a\,c^2\,d\,f^2)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 - \frac{a\,d^3\,x^3\,\cos(e+f\,x)}{f} + \frac{3\,a\,d^3\,x^2\,\sin(e+f\,x)}{f^2} + \frac{6\,a\,c\,d^2\,x\,\sin(e+f\,x)}{f^2} - \frac{3\,a\,c\,d^2\,x^2\,\cos(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*x)^3,x)
```

```
[Out] (a*d^3*x^4)/4 - (3*sin(e + f*x)*(2*a*d^3 - a*c^2*d*f^2))/f^4 - (cos(e + f*x
)*(a*c^3*f^2 - 6*a*c*d^2))/f^3 + a*c^3*x + (3*x*cos(e + f*x)*(2*a*d^3 - a*c
^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 - (a*d^3*x^3*cos(e + f*x))
/f + (3*a*d^3*x^2*sin(e + f*x))/f^2 + (6*a*c*d^2*x*sin(e + f*x))/f^2 - (3*a
*c*d^2*x^2*cos(e + f*x))/f
```

# 3.96 $\int (c+dx)^2(a+a\sin(e+fx))\,dx$

**Optimal**. Leaf size=68

$$\frac{a(c+dx)^3}{3d}+\frac{2ad^2\cos(e+fx)}{f^3}-\frac{a(c+dx)^2\cos(e+fx)}{f}+\frac{2ad(c+dx)\sin(e+fx)}{f^2}$$

```
[Out] 1/3*a*(d*x+c)^3/d+2*a*d^2*cos(f*x+e)/f^3-a*(d*x+c)^2*cos(f*x+e)/f+2*a*d*(d*
x+c)*sin(f*x+e)/f^2
```

**Rubi** [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2718}

$$\frac{2ad(c+dx)\sin(e+fx)}{f^2}-\frac{a(c+dx)^2\cos(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2ad^2\cos(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*a*d^2*Cos[e + f*x])/f^3 - (a*(c + d*x)^2*Cos[e +
 f*x])/f + (2*a*d*(c + d*x)*Sin[e + f*x])/f^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2(a+a\sin(e+fx))\,dx &= \int \left(a(c+dx)^2+a(c+dx)^2\sin(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^3}{3d}+a\int (c+dx)^2\sin(e+fx)\,dx \\
&= \frac{a(c+dx)^3}{3d}-\frac{a(c+dx)^2\cos(e+fx)}{f}+\frac{(2ad)\int (c+dx)\cos(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^3}{3d}-\frac{a(c+dx)^2\cos(e+fx)}{f}+\frac{2ad(c+dx)\sin(e+fx)}{f^2}-\frac{(2a
\end{aligned}
$$

$$
= \frac{a(c+dx)^3}{3d}+\frac{2ad^2\cos(e+fx)}{f^3}-\frac{a(c+dx)^2\cos(e+fx)}{f}+\frac{2ad(c+d
$$

**Mathematica** [A]
time = 0.24, size = 81, normalized size = 1.19

$$
a\left(c^2x+cdx^2+\frac{d^2x^3}{3}-\frac{(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)}{f^3}+\frac{2d(c+dx)\sin(e+fx)}{f^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Sin[e + f*x]),x]
```

```
[Out] a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 - ((c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*
x^2))*Cos[e + f*x])/f^3 + (2*d*(c + d*x)*Sin[e + f*x])/f^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(66) = 132.
time = 0.05, size = 241, normalized size = 3.54

| method | result |
|---|---|
| risch | $\frac{a\,d^2x^3}{3}+adc\,x^2+a\,c^2x+\frac{a\,c^3}{3d}-\frac{a\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\cos(fx+e)}{f^3}+\frac{2ad(dx+c)\sin(fx+e)}{f^2}$ |
| norman | $\frac{\frac{\left(2a\,c^2f^2-4a\,d^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^3}+\frac{ac(cf-2d)x}{f}+\frac{da(cf-d)x^2}{f}+\frac{ac(cf+2d)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{da(cf+d)x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{a\,d^{\cdots}}{}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$ |
| derivativedivides | $\frac{-a\,c^2\cos(fx+e)+\frac{2acde\cos(fx+e)}{f}+\frac{2acd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}-\frac{a\,d^2e^2\cos(fx+e)}{f^2}-\frac{2a\,d^2e(\sin(fx+e)-(fx+e)\cos(fx\cdots}{f^2}}{}$ |
| default | $\frac{-a\,c^2\cos(fx+e)+\frac{2acde\cos(fx+e)}{f}+\frac{2acd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}-\frac{a\,d^2e^2\cos(fx+e)}{f^2}-\frac{2a\,d^2e(\sin(fx+e)-(fx+e)\cos(fx\cdots}{f^2}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^2*cos(f*x+e)+2*a/f*c*d*e*cos(f*x+e)+2*a/f*c*d*(sin(f*x+e)-(f*x+e)
*cos(f*x+e))-a/f^2*d^2*e^2*cos(f*x+e)-2*a/f^2*d^2*e*(sin(f*x+e)-(f*x+e)*cos
(f*x+e))+a/f^2*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e)
)+a*c^2*(f*x+e)-2*a/f*c*d*e*(f*x+e)+a/f*c*d*(f*x+e)^2+a/f^2*d^2*e^2*(f*x+e)
-a/f^2*d^2*e*(f*x+e)^2+1/3*a/f^2*d^2*(f*x+e)^3)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(69) = 138$.
time = 0.30, size = 260, normalized size = 3.82

$$\frac{3\,(fx+e)ac^2+\frac{(fx+e)^3ad^2}{f^2}+\frac{3\,(fx+e)^2acd}{f}-3\,ac^2\cos\left(fx+e\right)-\frac{3\,(fx+e)^2ad^2e}{f^2}-\frac{6\,(fx+e)acde}{f}+\frac{6\,acd\cos(fx+e)e}{f}-\frac{6\,((fx+e)\cos(fx+e)-\sin(fx+e))acd}{f}+\frac{3\,(fx+e)ad^2e^2}{f^2}-\frac{3\,ad^2\cos(fx+e)e^2}{f^2}+\frac{6\,((fx+e)\cos(fx+e)-\sin(fx+e))ad^2e}{f^2}-\frac{3\left(\left((fx+e)^2-2\right)\cos(fx+e)-2\,(fx+e)\sin(fx+e)\right)ad^2}{f^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 3*
a*c^2*cos(f*x + e) - 3*(f*x + e)^2*a*d^2*e/f^2 - 6*(f*x + e)*a*c*d*e/f + 6*
a*c*d*cos(f*x + e)*e/f - 6*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*c*d/f
+ 3*(f*x + e)*a*d^2*e^2/f^2 - 3*a*d^2*cos(f*x + e)*e^2/f^2 + 6*((f*x + e)*c
os(f*x + e) - sin(f*x + e))*a*d^2*e/f^2 - 3*(((f*x + e)^2 - 2)*cos(f*x + e)
 - 2*(f*x + e)*sin(f*x + e))*a*d^2/f^2)/f
```

**Fricas** [A]
time = 0.36, size = 104, normalized size = 1.53

$$\frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x-3\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2-2\,ad^2\right)\cos\left(fx+e\right)+6\left(ad^2fx+acdf\right)\sin\left(fx+e\right)}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 3*(a*d^2*f^2*x^2 + 2
*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*cos(f*x + e) + 6*(a*d^2*f*x + a*c*d*f)*
sin(f*x + e))/f^3
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.
time = 0.15, size = 151, normalized size = 2.22

$$\begin{cases} ac^2x-\frac{ac^2\cos\left(e+fx\right)}{f}+acdx^2-\frac{2acdx\cos\left(e+fx\right)}{f}+\frac{2acd\sin\left(e+fx\right)}{f^2}+\frac{ad^2x^3}{3}-\frac{ad^2x^2\cos\left(e+fx\right)}{f}+\frac{2ad^2x\sin\left(e+fx\right)}{f^2}+\frac{2ad^2\cos\left(e+fx\right)}{f^3} & \text{for}\ f\neq 0\\ \left(a\sin\left(e\right)+a\right)\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x - a*c**2*cos(e + f*x)/f + a*c*d*x**2 - 2*a*c*d*x*cos(e
+ f*x)/f + 2*a*c*d*sin(e + f*x)/f**2 + a*d**2*x**3/3 - a*d**2*x**2*cos(e +
```

```
f*x)/f + 2*a*d**2*x*sin(e + f*x)/f**2 + 2*a*d**2*cos(e + f*x)/f**3, Ne(f, 0
)), ((a*sin(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

**Giac** [A]
time = 2.63, size = 95, normalized size = 1.40

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x - \frac{(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2)\cos(fx+e)}{f^3} + \frac{2\,(ad^2fx + acdf)\sin(fx+e)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^
2*f^2 - 2*a*d^2)*cos(f*x + e)/f^3 + 2*(a*d^2*f*x + a*c*d*f)*sin(f*x + e)/f^
3
```

**Mupad** [B]
time = 0.15, size = 112, normalized size = 1.65

$$\frac{a\,d^2\,x^3}{3} + \frac{\cos(e+f\,x)\,\left(2\,a\,d^2 - a\,c^2\,f^2\right)}{f^3} + a\,c^2\,x + a\,c\,d\,x^2 + \frac{2\,a\,d^2\,x\,\sin(e+f\,x)}{f^2} - \frac{a\,d^2\,x^2\,\cos(e+f\,x)}{f} + \frac{2\,a\,c\,d\,\sin(e+f\,x)}{f^2} - \frac{2\,a\,c\,d\,x\,\cos(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*x)^2,x)
```

```
[Out] (a*d^2*x^3)/3 + (cos(e + f*x)*(2*a*d^2 - a*c^2*f^2))/f^3 + a*c^2*x + a*c*d*
x^2 + (2*a*d^2*x*sin(e + f*x))/f^2 - (a*d^2*x^2*cos(e + f*x))/f + (2*a*c*d*
sin(e + f*x))/f^2 - (2*a*c*d*x*cos(e + f*x))/f
```

# 3.97 $\int (c+dx)(a+a\sin(e+fx))\,dx$

**Optimal**. Leaf size=45

$$\frac{a(c+dx)^2}{2d}-\frac{a(c+dx)\cos(e+fx)}{f}+\frac{ad\sin(e+fx)}{f^2}$$

```
[Out] 1/2*a*(d*x+c)^2/d-a*(d*x+c)*cos(f*x+e)/f+a*d*sin(f*x+e)/f^2
```

**Rubi** [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {3398, 3377, 2717}

$$-\frac{a(c+dx)\cos(e+fx)}{f}+\frac{a(c+dx)^2}{2d}+\frac{ad\sin(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (a*(c + d*x)*Cos[e + f*x])/f + (a*d*Sin[e + f*x])/f
^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)(a+a\sin(e+fx))\,dx &= \int (a(c+dx)+a(c+dx)\sin(e+fx))\,dx \\
&= \frac{a(c+dx)^2}{2d}+a\int (c+dx)\sin(e+fx)\,dx \\
&= \frac{a(c+dx)^2}{2d}-\frac{a(c+dx)\cos(e+fx)}{f}+\frac{(ad)\int \cos(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^2}{2d}-\frac{a(c+dx)\cos(e+fx)}{f}+\frac{ad\sin(e+fx)}{f^2}
\end{aligned}
$$

**Mathematica** [**A**]
time = 0.19, size = 51, normalized size = 1.13

$$
-\frac{a((e+fx)(de-2cf-dfx)+2f(c+dx)\cos(e+fx)-2d\sin(e+fx))}{2f^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Sin[e + f*x]),x]
```

```
[Out] -1/2*(a*((e + f*x)*(d*e - 2*c*f - d*f*x) + 2*f*(c + d*x)*Cos[e + f*x] - 2*d
*Sin[e + f*x]))/f^2
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(43) = 86$.
time = 0.03, size = 90, normalized size = 2.00

| method | result | size |
|---|---|---|
| risch | $\frac{da\,x^2}{2}+acx-\frac{a(dx+c)\cos(fx+e)}{f}+\frac{ad\sin(fx+e)}{f^2}$ | 42 |
| derivativedivides | $\frac{-ac\cos(fx+e)+\frac{ade\cos(fx+e)}{f}+\frac{ad(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}+ac(fx+e)-\frac{ade(fx+e)}{f}+\frac{ad(fx+e)^2}{2f}}{f}$ | 90 |
| default | $\frac{-ac\cos(fx+e)+\frac{ade\cos(fx+e)}{f}+\frac{ad(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}+ac(fx+e)-\frac{ade(fx+e)}{f}+\frac{ad(fx+e)^2}{2f}}{f}$ | 90 |
| norman | $\frac{\frac{2ac\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{a(cf-d)x}{f}+\frac{a(cf+d)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{da\,x^2}{2}+\frac{2da\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f^2}+\frac{da\,x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$ | 112 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c*cos(f*x+e)+a/f*d*e*cos(f*x+e)+a/f*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e
))+a*c*(f*x+e)-a/f*d*e*(f*x+e)+1/2*a/f*d*(f*x+e)^2)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(45) = 90$.
time = 0.31, size = 103, normalized size = 2.29

$$\frac{2\,(fx+e)ac+\frac{(fx+e)^2ad}{f}-2\,ac\cos\left(fx+e\right)-\frac{2\,(fx+e)ade}{f}+\frac{2\,ad\cos(fx+e)e}{f}-\frac{2\,((fx+e)\cos(fx+e)-\sin(fx+e))ad}{f}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*a*c*cos(f*x + e) - 2*(f*x + e)
*a*d*e/f + 2*a*d*cos(f*x + e)*e/f - 2*((f*x + e)*cos(f*x + e) - sin(f*x + e
))*a*d/f)/f
```

**Fricas** [A]
time = 0.38, size = 53, normalized size = 1.18

$$\frac{adf^2x^2+2\,acf^2x+2\,ad\sin\left(fx+e\right)-2\,(adfx+acf)\cos\left(fx+e\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*a*d*sin(f*x + e) - 2*(a*d*f*x + a*c*f)*c
os(f*x + e))/f^2
```

**Sympy** [A]
time = 0.09, size = 68, normalized size = 1.51

$$\begin{cases} acx-\frac{ac\cos\left(e+fx\right)}{f}+\frac{adx^2}{2}-\frac{adx\cos\left(e+fx\right)}{f}+\frac{ad\sin\left(e+fx\right)}{f^2} & \text{for } f\neq 0 \\ \left(a\sin\left(e\right)+a\right)\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x - a*c*cos(e + f*x)/f + a*d*x**2/2 - a*d*x*cos(e + f*x)/f +
 a*d*sin(e + f*x)/f**2, Ne(f, 0)), ((a*sin(e) + a)*(c*x + d*x**2/2), True))
```

**Giac** [A]
time = 1.51, size = 47, normalized size = 1.04

$$\frac{1}{2}\,adx^2+acx+\frac{ad\sin\left(fx+e\right)}{f^2}-\frac{(adfx+acf)\cos\left(fx+e\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + a*d*sin(f*x + e)/f^2 - (a*d*f*x + a*c*f)*cos(f*x + e)
/f^2
```

**Mupad [B]**
time = 0.10, size = 54, normalized size = 1.20

$$\frac{a\,\left(d\,x^2+2\,c\,x\right)}{2}-\frac{\frac{a\,f\,\left(2\,c\,\cos\left(e+f\,x\right)+2\,d\,x\,\cos\left(e+f\,x\right)\right)}{2}-a\,d\,\sin\left(e+f\,x\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*x),x)
```

```
[Out] (a*(2*c*x + d*x^2))/2 - ((a*f*(2*c*cos(e + f*x) + 2*d*x*cos(e + f*x)))/2 -
a*d*sin(e + f*x))/f^2
```

# 3.98 $\int \frac{a+a\sin(e+fx)}{c+dx}\,dx$

**Optimal**. Leaf size=64

$$\frac{a\log(c+dx)}{d}+\frac{a\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{a\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}$$

```
[Out] a*ln(d*x+c)/d+a*cos(-e+c*f/d)*Si(c*f/d+f*x)/d-a*Ci(c*f/d+f*x)*sin(-e+c*f/d)
/d
```

**Rubi** [A]
time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3398, 3384, 3380, 3383}

$$\frac{a\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{a\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x])/d + (a*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d + (a
*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{a\sin(e+fx)}{c+dx}\right)dx\\ &= \frac{a\log(c+dx)}{d}+a\int \frac{\sin(e+fx)}{c+dx}\,dx\\ &= \frac{a\log(c+dx)}{d}+\left(a\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx+\left(a\sin\left(e-\frac{cf}{d}\right)\right)\int\\ &= \frac{a\log(c+dx)}{d}+\frac{a\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{a\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}\end{aligned}$$

**Mathematica [A]**
time = 0.14, size = 54, normalized size = 0.84

$$\frac{a\left(\log(c+dx)+\text{Ci}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*x),x]
[Out] (a*(Log[c + d*x] + CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + Cos[e - (c*f
)/d]*SinIntegral[f*(c/d + x)]))/d
```

**Maple [A]**
time = 0.05, size = 103, normalized size = 1.61

| method | result |
|---|---|
| derivativedivides | $\frac{af\left(\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{af\ln(cf-de+d(fx+e))}{d}}{f}$ |
| default | $\frac{af\left(\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{af\ln(cf-de+d(fx+e))}{d}}{f}$ |
| risch | $\frac{a\ln(dx+c)}{d}-\frac{ia\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\text{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{2d}+\frac{ia\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\text{expIntegral}\left(1,-ifx-ie-\frac{icf-ide}{d}\right)}{2d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*f*(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*si
n((c*f-d*e)/d)/d)+a*f*ln(c*f-d*e+d*(f*x+e))/d)
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.35, size = 181, normalized size = 2.83

$$\frac{\frac{2\,af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{\left(f\left(-i\,E_1\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+i\,E_1\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f\left(E_1\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+E_1\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a}{d}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d + (f*(-I*exp_integral_e(1, (I*(
f*x + e)*d + I*c*f - I*d*e)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d + I*c*
f - I*d*e)/d))*cos((c*f - d*e)/d) + f*(exp_integral_e(1, (I*(f*x + e)*d + I
*c*f - I*d*e)/d) + exp_integral_e(1, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*s
in((c*f - d*e)/d))*a/d)/f
```

**Fricas** [A]
time = 0.35, size = 94, normalized size = 1.47

$$\frac{2\,a\cos\left(-\frac{cf-de}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+2\,a\log\left(dx+c\right)+\left(a\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+a\,\mathrm{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cos(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) + 2*a*log(d*x +
c) + (a*cos_integral((d*f*x + c*f)/d) + a*cos_integral(-(d*f*x + c*f)/d))*s
in(-(c*f - d*e)/d))/d
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\sin\left(e+fx\right)}{c+dx}\,dx+\int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.86, size = 712, normalized size = 11.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
 a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*
a*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*sin_integral((d*f*x
 + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*real_part(cos_integral(f*x +
 c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*real_part(cos_integral(-f*x - c*
f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) + 2*a*real_part(cos_integral(f*x + c*f/d)
)*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*real_part(cos_integral(-f*x - c*f/d))*t
an(1/2*c*f/d)*tan(1/2*e)^2 - a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*c*f/d)^2 + a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 + 2*a*
log(abs(d*x + c))*tan(1/2*c*f/d)^2 - 2*a*sin_integral((d*f*x + c*f)/d)*tan(
1/2*c*f/d)^2 + 4*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(
1/2*e) - 4*a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e
) + 8*a*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) - a*imag_pa
rt(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 + a*imag_part(cos_integral(-f*x
- c*f/d))*tan(1/2*e)^2 + 2*a*log(abs(d*x + c))*tan(1/2*e)^2 - 2*a*sin_integ
ral((d*f*x + c*f)/d)*tan(1/2*e)^2 - 2*a*real_part(cos_integral(f*x + c*f/d)
)*tan(1/2*c*f/d) - 2*a*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)
 + 2*a*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 2*a*real_part(cos_
integral(-f*x - c*f/d))*tan(1/2*e) + a*imag_part(cos_integral(f*x + c*f/d))
 - a*imag_part(cos_integral(-f*x - c*f/d)) + 2*a*log(abs(d*x + c)) + 2*a*si
n_integral((d*f*x + c*f)/d))/(d*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + d*tan(1/2*c
*f/d)^2 + d*tan(1/2*e)^2 + d)
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c + d*x), x)
```

# 3.99 $\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=88

$$-\frac{a}{d(c+dx)}+\frac{af\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{a\sin(e+fx)}{d(c+dx)}-\frac{af\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2}$$

```
[Out] -a/d/(d*x+c)+a*f*Ci(c*f/d+f*x)*cos(-e+c*f/d)/d^2+a*f*Si(c*f/d+f*x)*sin(-e+c
*f/d)/d^2-a*sin(f*x+e)/d/(d*x+c)
```

**Rubi** [A]
time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3398, 3378, 3384, 3380, 3383}

$$\frac{af\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\sin(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) + (a*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2
- (a*Sin[e + f*x])/(d*(c + d*x)) - (a*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{a\sin(e+fx)}{(c+dx)^2}\right)dx\\ &= -\frac{a}{d(c+dx)}+a\int \frac{\sin(e+fx)}{(c+dx)^2}\,dx\\ &= -\frac{a}{d(c+dx)}-\frac{a\sin(e+fx)}{d(c+dx)}+\frac{(af)\int \frac{\cos(e+fx)}{c+dx}\,dx}{d}\\ &= -\frac{a}{d(c+dx)}-\frac{a\sin(e+fx)}{d(c+dx)}+\frac{\left(af\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}-\frac{\left(af\sin\left(e-\right.\right.}{}\\ &= -\frac{a}{d(c+dx)}+\frac{af\cos\left(e-\frac{cf}{d}\right)\mathrm{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{a\sin(e+fx)}{d(c+dx)}-\frac{af\sin\left(e-\frac{cf}{d}\right)\mathrm{Si}(}{d^2}\end{aligned}$$

**Mathematica** [A]
time = 0.29, size = 110, normalized size = 1.25

$$\frac{a(1+\sin(e+fx))\left(f(c+dx)\cos\left(e-\frac{cf}{d}\right)\mathrm{Ci}\left(f\left(\frac{c}{d}+x\right)\right)-d(1+\sin(e+fx))-f(c+dx)\sin\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d^2(c+dx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(f*(c + d*x)*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)
] - d*(1 + Sin[e + f*x]) - f*(c + d*x)*Sin[e - (c*f)/d]*SinIntegral[f*(c/d
+ x)]))/(d^2*(c + d*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

**Maple** [A]
time = 0.08, size = 141, normalized size = 1.60

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{af^2\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-\frac{af}{(cf-de+d(}}{f}$ |
| default | $\frac{af^2\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-\frac{af}{(cf-de+d(}}{f}$ |
| risch | $-\frac{a}{d(dx+c)}-\frac{fa\,e^{\frac{i(cf-de)}{d}}\text{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^2}-\frac{af\,e^{-\frac{i(cf-de)}{d}}\text{expIntegral}\left(1,-ifx-ie-\frac{icf-ide}{d}\right)}{2d^2}-$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*f^2*(-sin(f*x+e)/(c*f-d*e+d*(f*x+e))/d+(Si(f*x+e+(c*f-d*e)/d)*sin((c
*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d)/d)-a*f^2/(c*f-d*e+d*
(f*x+e))/d)
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.39, size = 208, normalized size = 2.36

$$-\frac{\frac{2\,af^2}{(fx+e)d^2+cdf-d^2e}-\frac{\left(f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+i\,E_2\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f^2\left(E_2\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+E_2\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a}{(fx+e)d^2+cdf-d^2e}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*f^2/((f*x + e)*d^2 + c*d*f - d^2*e) - (f^2*(-I*exp_integral_e(2,
(I*(f*x + e)*d + I*c*f - I*d*e)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d +
I*c*f - I*d*e)/d))*cos((c*f - d*e)/d) + f^2*(exp_integral_e(2, (I*(f*x + e)
*d + I*c*f - I*d*e)/d) + exp_integral_e(2, -(I*(f*x + e)*d + I*c*f - I*d*e)
/d))*sin((c*f - d*e)/d))*a/((f*x + e)*d^2 + c*d*f - d^2*e))/f
```

**Fricas [A]**
time = 0.35, size = 138, normalized size = 1.57

$$-\frac{2\,ad\sin\left(fx+e\right)+2\left(adfx+acf\right)\sin\left(-\frac{cf-de}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)+2\,ad-\left(\left(adfx+acf\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+\left(adfx+acf\right)\operatorname{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\cos\left(-\frac{cf-de}{d}\right)}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*sin(f*x + e) + 2*(a*d*f*x + a*c*f)*sin(-(c*f - d*e)/d)*sin_inte
gral((d*f*x + c*f)/d) + 2*a*d - ((a*d*f*x + a*c*f)*cos_integral((d*f*x + c*
f)/d) + (a*d*f*x + a*c*f)*cos_integral(-(d*f*x + c*f)/d))*cos(-(c*f - d*e)/
d))/(d^3*x + c*d^2)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin{\left(e+fx \right)}}{c^2+2cdx+d^2x^2}\,dx+\int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)**2,x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**
2 + 2*c*d*x + d**2*x**2), x))
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(92) = 184.
time = 1.83, size = 578, normalized size = 6.57

$$\frac{\left((dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)f^2\cos\left(\frac{cf-de}{d}\right)\mathrm{Ci}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)-cf^3\cos\left(\frac{cf-de}{d}\right)\mathrm{Ci}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)+df^2\cos\left(\frac{cf-de}{d}\right)\mathrm{Ci}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)e+(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)f^2\sin\left(\frac{cf-de}{d}\right)\mathrm{Si}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)-cf^3\sin\left(\frac{cf-de}{d}\right)\mathrm{Si}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)+df^2e\sin\left(\frac{cf-de}{d}\right)\mathrm{Si}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)-df^2\sin\left(\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)}{d}\right)\right)ad^2}{\left((dx+c)d^4\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cd^4f+d^5e\right)f}-\frac{a}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos((c*f - d*e)/d)*cos_i
ntegral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - c
*f^3*cos((c*f - d*e)/d)*cos_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(
d*x + c)) - c*f + d*e)/d) + d*f^2*cos((c*f - d*e)/d)*cos_integral(-((d*x +
c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e + (d*x + c)*(c*f/(
d*x + c) - f - d*e/(d*x + c))*f^2*sin((c*f - d*e)/d)*sin_integral(-((d*x +
c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - c*f^3*sin((c*f - d
*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f +
 d*e)/d) + d*f^2*e*sin((c*f - d*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x +
c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - d*f^2*sin((d*x + c)*(c*f/(d*x + c
) - f - d*e/(d*x + c))/d))*a*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - f - d*e/(
d*x + c)) - c*d^4*f + d^5*e)*f) - a/((d*x + c)*d)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(e + f*x))/(c + d*x)^2, x)
```

# 3.100 $\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=123

$$-\frac{a}{2d(c+dx)^2}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{af^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{af^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

```
[Out] -1/2*a/d/(d*x+c)^2-1/2*a*f*cos(f*x+e)/d^2/(d*x+c)-1/2*a*f^2*cos(-e+c*f/d)*S
i(c*f/d+f*x)/d^3+1/2*a*f^2*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^3-1/2*a*sin(f*x+e)
/d/(d*x+c)^2
```

**Rubi** [A]
time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3398, 3378, 3384, 3380, 3383}

$$-\frac{af^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{af^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*a/(d*(c + d*x)^2) - (a*f*Cos[e + f*x])/(2*d^2*(c + d*x)) - (a*f^2*CosI
ntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d^3) - (a*Sin[e + f*x])/(2*d*(c
 + d*x)^2) - (a*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx &= \int \left(\frac{a}{(c+dx)^3}+\frac{a\sin(e+fx)}{(c+dx)^3}\right)dx\\
&= -\frac{a}{2d(c+dx)^2}+a\int \frac{\sin(e+fx)}{(c+dx)^3}\,dx\\
&= -\frac{a}{2d(c+dx)^2}-\frac{a\sin(e+fx)}{2d(c+dx)^2}+\frac{(af)\int \frac{\cos(e+fx)}{(c+dx)^2}\,dx}{2d}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{(af^2)\int \frac{\sin(e+fx)}{c+dx}\,dx}{2d^2}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{\left(af^2\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+f
\right.}{c+dx}
\end{aligned}
$$

$$
= -\frac{a}{2d(c+dx)^2}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{af^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{a\sin(e+fx)}{2d(c+dx)^2}
$$

**Mathematica** [A]
time = 0.37, size = 104, normalized size = 0.85

$$
-\frac{a\left(f^2(c+dx)^2\text{Ci}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+d(f(c+dx)\cos(e+fx)+d(1+\sin(e+fx)))+f^2(c+dx)^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{2d^3(c+dx)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*(a*(f^2*(c + d*x)^2*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + d*(f*(
c + d*x)*Cos[e + f*x] + d*(1 + Sin[e + f*x])) + f^2*(c + d*x)^2*Cos[e - (c*
f)/d]*SinIntegral[f*(c/d + x)]))/(d^3*(c + d*x)^2)
```

**Maple** [A]
time = 0.10, size = 177, normalized size = 1.44

| method | result |
|---|---|
| derivativedivides | $\frac{a f^3\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\operatorname{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\operatorname{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin}{d}}{d}}{2d}\right.}{f}$ |
| default | $\frac{a f^3\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\operatorname{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\operatorname{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin}{d}}{d}}{2d}\right.}{f}$ |
| risch | $-\frac{a}{2d(dx+c)^2}+\frac{i f^2 a\,e^{\frac{i(cf-de)}{d}}\operatorname{expIntegral}\left(1, ifx+ie+\frac{i(cf-de)}{d}\right)}{4d^3}-\frac{i f^2 a\,e^{-\frac{i(cf-de)}{d}}\operatorname{expIntegral}\left(1, -ifx-ie-\frac{icf-ic}{d}\right.}{4d^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*f^3*(-1/2*sin(f*x+e)/(c*f-d*e+d*(f*x+e))^2/d+1/2*(-cos(f*x+e)/(c*f-d
*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e
)/d)*sin((c*f-d*e)/d)/d)/d)/d)-1/2*a*f^3/(c*f-d*e+d*(f*x+e))^2/d)
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.41, size = 279, normalized size = 2.27

$$-\frac{\frac{af^3}{(fx+e)^2d^3+c^2df^2-2\,cd^2fe+d^3e^2+2\,(cd^2f-d^3e)(fx+e)}-\frac{\left(f^3\left(-i\,E_3\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+i\,E_3\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f^3\left(E_3\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+E_3\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a}{(fx+e)^2d^3+c^2df^2-2\,cd^2fe+d^3e^2+2\,(cd^2f-d^3e)(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*f^3/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*(c*d^2
*f - d^3*e)*(f*x + e)) - (f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d + I*c*f
- I*d*e)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*cos(
(c*f - d*e)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d + I*c*f - I*d*e)/d)
+ exp_integral_e(3, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*sin((c*f - d*e)/d)
)*a/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*(c*d^2*f - d^3
*e)*(f*x + e)))/f
```

**Fricas** [A]
time = 0.42, size = 231, normalized size = 1.88

$$-\frac{2\,ad^2\sin\left(fx+e\right)+2\,ad^2+2\,\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\cos\left(-\frac{cf-de}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)+2\,\left(ad^2fx+acdf\right)\cos\left(fx+e\right)+\left(\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\operatorname{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)}{4\,\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^2*sin(f*x + e) + 2*a*d^2 + 2*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a
*c^2*f^2)*cos(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) + 2*(a*d^2*f*x
+ a*c*d*f)*cos(f*x + e) + ((a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2)*cos_
integral((d*f*x + c*f)/d) + (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2)*cos
_integral(-(d*f*x + c*f)/d))*sin(-(c*f - d*e)/d))/(d^5*x^2 + 2*c*d^4*x + c^
2*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{1}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)**3,x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
) + Integral(1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.98, size = 6157, normalized size = 50.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*c*f/d)^2*tan(1/2*e)^2 - a*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*
f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*d^2*f^2*x^2*sin_in
tegral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*
d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f
/d)^2*tan(1/2*e) - 2*a*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 2*a*d^2*f^2*x^2*real_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*d^2*f
^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*
tan(1/2*e)^2 + 2*a*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*c*d*f^2*x*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a*c*d*f^2*x
*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2
 - a*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/
2*c*f/d)^2 + a*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*
x)^2*tan(1/2*c*f/d)^2 - 2*a*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1
/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*a*d^2*f^2*x^2*imag_part(cos_integral(f*x + c
```

```
*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 4*a*d^2*f^2*x^2*imag_part
(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) + 8*a
*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*ta
n(1/2*e) - 4*a*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e) - 4*a*c*d*f^2*x*real_part(cos_integral(-f*x -
 c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - a*d^2*f^2*x^2*imag_pa
rt(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + a*d^2*f^2*x^2*i
mag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*a*d^2*
f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 4*a*c*d
*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*t
an(1/2*e)^2 + 4*a*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + a*d^2*f^2*x^2*imag_part(cos_integral(f*
x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - a*d^2*f^2*x^2*imag_part(cos_int
egral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*d^2*f^2*x^2*sin_in
tegral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a*c^2*f^2*imag_part
(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
a*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/
d)^2*tan(1/2*e)^2 + 2*a*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*d^2*f^2*x^2*real_part(cos_integral(f*
x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*a*d^2*f^2*x^2*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*a*c*d*f^2*x*imag_p
art(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 2*a*c*d*f^
2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 -
 4*a*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^
2 + 2*a*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*e) + 2*a*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x
)^2*tan(1/2*e) + 8*a*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 8*a*c*d*f^2*x*imag_part(cos_integral(-f
*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) + 16*a*c*d*f^2*x*sin_
integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 2*a*d^
2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)
- 2*a*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*ta
n(1/2*e) - 2*a*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*
tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*c^2*f^2*real_part(cos_integral(-f*x - c*f
/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*c*d*f^2*x*imag_part(c
os_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*a*c*d*f^2*x*imag_
part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 4*a*c*d*f^2*
x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*a*d^2*f^2*x
^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*d
^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^
2 + 2*a*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2
*c*f/d)*tan(1/2*e)^2 + 2*a*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*c*d*f^2*x*imag_part(cos_inte
gral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*c*d*f^2*x*imag_part(
cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a*c*d*f^2*x*s
```

```
in_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*d^2*f*x*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a*d^2*f^2*x^2*imag_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*f*x)^2 - a*d^2*f^2*x^2*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*f*x)^2 + 2*a*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/
d)*tan(1/2*f*x)^2 - 4*a*c*d*f^2*x*real_part(cos...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*x)^3,x)
[Out] int((a + a*sin(e + f*x))/(c + d*x)^3, x)
```

# 3.101 $\int (c+dx)^3(a+a\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=237

$$-\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3}-\frac{2a^2(c+dx)^3\cos(e+fx)}{f}-\frac{12a^2d^3\sin(e+}{f^4}$$

```
[Out] -3/4*a^2*c*d^2*x/f^2-3/8*a^2*d^3*x^2/f^2+3/8*a^2*(d*x+c)^4/d+12*a^2*d^2*(d*
x+c)*cos(f*x+e)/f^3-2*a^2*(d*x+c)^3*cos(f*x+e)/f-12*a^2*d^3*sin(f*x+e)/f^4+
6*a^2*d*(d*x+c)^2*sin(f*x+e)/f^2+3/4*a^2*d^2*(d*x+c)*cos(f*x+e)*sin(f*x+e)/
f^3-1/2*a^2*(d*x+c)^3*cos(f*x+e)*sin(f*x+e)/f-3/8*a^2*d^3*sin(f*x+e)^2/f^4+
3/4*a^2*d*(d*x+c)^2*sin(f*x+e)^2/f^2
```

**Rubi** [A]
time = 0.17, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3398, 3377, 2717, 3392, 32, 3391}

$$\frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3}+\frac{3a^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\sin^2(e+fx)}{4f^2}+\frac{6a^2d(c+dx)^2\sin(e+fx)}{f^2}-\frac{2a^2(c+dx)^3\cos(e+fx)}{f}-\frac{a^2(c+dx)^3\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2(c+dx)^4}{8d}-\frac{3a^2d^3\sin^2(e+fx)}{8f^4}-\frac{12a^2d^3\sin(e+fx)}{f^4}-\frac{3a^2d^3x^2}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-3*a^2*c*d^2*x)/(4*f^2) - (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8
*d) + (12*a^2*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (2*a^2*(c + d*x)^3*Cos[e +
f*x])/f - (12*a^2*d^3*Sin[e + f*x])/f^4 + (6*a^2*d*(c + d*x)^2*Sin[e + f*x]
)/f^2 + (3*a^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (a^2*(c +
 d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f) - (3*a^2*d^3*Sin[e + f*x]^2)/(8*f^
4) + (3*a^2*d*(c + d*x)^2*Sin[e + f*x]^2)/(4*f^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^3(a+a\sin(e+fx))^2\,dx = \int \left(a^2(c+dx)^3 + 2a^2(c+dx)^3\sin(e+fx) + a^2(c+dx)^3\sin^2(e+fx\right.$$

$$= \frac{a^2(c+dx)^4}{4d} + a^2\int (c+dx)^3\sin^2(e+fx)\,dx + \left(2a^2\right)\int (c+dx)^3\sin($$

$$= \frac{a^2(c+dx)^4}{4d} - \frac{2a^2(c+dx)^3\cos(e+fx)}{f} - \frac{a^2(c+dx)^3\cos(e+fx)\sin}{2f}$$

$$= \frac{3a^2(c+dx)^4}{8d} - \frac{2a^2(c+dx)^3\cos(e+fx)}{f} + \frac{6a^2d(c+dx)^2\sin(e+fx}{f^2}$$

$$= -\frac{3a^2cd^2x}{4f^2} - \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c+dx)^4}{8d} + \frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3} -$$

$$= -\frac{3a^2cd^2x}{4f^2} - \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c+dx)^4}{8d} + \frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3} -$$

**Mathematica [A]**
time = 0.73, size = 216, normalized size = 0.91

$$\frac{a^2\left(6f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-32f(c+dx)\left(c^2f^2+2cdf^2x+d^2(-6+f^2x^2)\right)\cos(e+fx)-3d\left(2c^2f^2+4cdf^2x+d^2(-1+2f^2x^2)\right)\cos(2(e+fx))+96d\left(c^2f^2+2cdf^2x+d^2(-2+f^2x^2)\right)\sin(e+fx)-2f(c+dx)\left(2c^2f^2+4cdf^2x+d^2(-3+2f^2x^2)\right)\sin(2(e+fx))\right)}{16f^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*(a + a*Sin[e + f*x])^2,x]

[Out] (a^2*(6*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 32*f*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x] - 3*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)] + 96*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x] - 2*f*(c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e + f*x)]))/(16*f^4)

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(223) = 446$.
time = 0.13, size = 1135, normalized size = 4.79

| method | result |
|---|---|
| risch | $\frac{3a^2d^3x^4}{8}+\frac{3a^2c\,d^2x^3}{2}+\frac{9a^2d\,c^2x^2}{4}+\frac{3a^2c^3x}{2}+\frac{3a^2c^4}{8d}-\frac{2a^2\left(d^3f^2x^3+3c\,d^2f^2x^2+3c^2d\,f^2x+c^3f^2-6d^3x-6c\,d^2\right)\,c}{f^3}$ |
| norman | $\frac{\frac{a^2d^3x^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{4a^2c^3f^2-24a^2c\,d^2}{f^3}+\frac{3a^2d^3x^4}{8}-\frac{\left(8a^2c^3f^3-6a^2c^2d\,f^2-48a^2c\,d^2f+3a^2d^3\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f^4}+\frac{3a^2d^3x^4(\mathrm{ta}}{}}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(2*a^2/f^3*d^3*(-(f*x+e)^3*cos(f*x+e)+3*(f*x+e)^2*sin(f*x+e)-6*sin(f*x+e)+6*(f*x+e)*cos(f*x+e))+1/4*a^2/f^3*d^3*(f*x+e)^4+a^2/f^3*d^3*((f*x+e)^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3/4*(f*x+e)^2*cos(f*x+e)^2+3/2*(f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3/8*(f*x+e)^2-3/8*sin(f*x+e)^2-3/8*(f*x+e)^4)+a^2/f^2*c*d^2*(f*x+e)^3+a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a^2*c^3*cos(f*x+e)+a^2*c^3*(f*x+e)+3/2*a^2/f^3*d^3*e^2*(f*x+e)^2-a^2/f^3*d^3*e*(f*x+e)^3+3/2*a^2/f*c^2*d*(f*x+e)^2-a^2/f^3*d^3*e^3*(f*x+e)+2*a^2/f^3*d^3*e^3*cos(f*x+e)-a^2/f^3*d^3*e^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+6*a^2/f*c^2*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e))+6*a^2/f^2*c*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))+3*a^2/f*c^2*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-6*a^2/f^3*d^3*e*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))+3*a^2/f^3*d^3*e^2*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-3*a^2/f^3*d^3*e*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)+6*a^2/f^3*d^3*e^2*(sin(f*x+e)-(f*x+e)*cos(f*x+e))+3*a^2/f^2*c*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)+3*a^2/f^2*c*d^2*e^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-6*a^2/f^2*c*d^2*e*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-

```
1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-12*a^2/f^2*c*d^2*e*(sin(f*x+e)-(f*x+e)*cos(
f*x+e))-3*a^2/f*c^2*d*e*(f*x+e)-3*a^2/f^2*c*d^2*e*(f*x+e)^2-3*a^2/f*c^2*d*e
*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3*a^2/f^2*c*d^2*e^2*(f*x+e)+6*a
^2/f*c^2*d*e*cos(f*x+e)-6*a^2/f^2*c*d^2*e^2*cos(f*x+e))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(233) = 466$.
time = 0.33, size = 1041, normalized size = 4.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^3 + 16*(f*x + e)*a^2*c^3 + 4
*(f*x + e)^4*a^2*d^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 + 24*(f*x + e)^2*a^
2*c^2*d/f - 32*a^2*c^3*cos(f*x + e) - 16*(f*x + e)^3*a^2*d^3*e/f^3 - 48*(f*
x + e)^2*a^2*c*d^2*e/f^2 - 12*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2*d*e/
f - 48*(f*x + e)*a^2*c^2*d*e/f + 96*a^2*c^2*d*cos(f*x + e)*e/f + 6*(2*(f*x
+ e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*c^2*d/f - 96*
((f*x + e)*cos(f*x + e) - sin(f*x + e))*a^2*c^2*d/f + 24*(f*x + e)^2*a^2*d^
3*e^2/f^3 + 12*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d^2*e^2/f^2 + 48*(f*x
 + e)*a^2*c*d^2*e^2/f^2 - 96*a^2*c*d^2*cos(f*x + e)*e^2/f^2 - 12*(2*(f*x +
e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*c*d^2*e/f^2 + 1
92*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a^2*c*d^2*e/f^2 + 2*(4*(f*x + e)
^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))
*a^2*c*d^2/f^2 - 96*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x +
 e))*a^2*c*d^2/f^2 - 4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*d^3*e^3/f^3 - 1
6*(f*x + e)*a^2*d^3*e^3/f^3 + 32*a^2*d^3*cos(f*x + e)*e^3/f^3 + 6*(2*(f*x +
 e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*d^3*e^2/f^3 -
96*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a^2*d^3*e^2/f^3 - 2*(4*(f*x + e)
^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))
*a^2*d^3*e/f^3 + 96*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x +
 e))*a^2*d^3*e/f^3 + (2*(f*x + e)^4 - 3*(2*(f*x + e)^2 - 1)*cos(2*f*x + 2*e
) - 2*(2*(f*x + e)^3 - 3*f*x - 3*e)*sin(2*f*x + 2*e))*a^2*d^3/f^3 - 32*(((f
*x + e)^3 - 6*f*x - 6*e)*cos(f*x + e) - 3*((f*x + e)^2 - 2)*sin(f*x + e))*a
^2*d^3/f^3)/f
```

**Fricas** [A]
time = 0.35, size = 372, normalized size = 1.57

$$\frac{3\,a^2d^3f^4x^4 + 12\,a^2cd^2f^4x^3 + 3\,(6\,a^2c^2df^4 + a^2d^3f^2)x^2 - 3\,(2\,a^2d^3f^2x^2 + 4\,a^2cd^2f^2x + 2\,a^2c^2df^2 - a^2d^3)\cos(fx+e)^2 + 6\,(2\,a^2c^3f^4 + a^2cd^2f^2)x - 16\,(a^2d^3f^3x^3 + 3\,a^2cd^2f^3x^2 + a^2c^3f^3 - 6\,a^2cd^2f + 3\,(a^2c^2df^3 - 2\,a^2d^3f)x)\cos(fx+e) + 2\,(24\,a^2d^3f^2x^2 + 48\,a^2cd^2f^2x + 24\,a^2c^2df^2 - 48\,a^2d^3 - (2\,a^2d^3f^3x^3 + 6\,a^2cd^2f^3x^2 + 2\,a^2c^3f^3 - 3\,a^2cd^2f + 3\,(2\,a^2c^2df^3 - a^2d^3f)x)\cos(fx+e))\sin(fx+e)}{8\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^2*d^3*f^4*x^4 + 12*a^2*c*d^2*f^4*x^3 + 3*(6*a^2*c^2*d*f^4 + a^2*d^
3*f^2)*x^2 - 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a
^2*d^3)*cos(f*x + e)^2 + 6*(2*a^2*c^3*f^4 + a^2*c*d^2*f^2)*x - 16*(a^2*d^3*
f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + a^2*c^3*f^3 - 6*a^2*c*d^2*f + 3*(a^2*c^2*d*
f^3 - 2*a^2*d^3*f)*x)*cos(f*x + e) + 2*(24*a^2*d^3*f^2*x^2 + 48*a^2*c*d^2*f
^2*x + 24*a^2*c^2*d*f^2 - 48*a^2*d^3 - (2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3
*x^2 + 2*a^2*c^3*f^3 - 3*a^2*c*d^2*f + 3*(2*a^2*c^2*d*f^3 - a^2*d^3*f)*x)*c
os(f*x + e))*sin(f*x + e))/f^4
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(243) = 486$.
time = 0.44, size = 779, normalized size = 3.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 +
a**2*c**3*x - a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**3*cos(e
 + f*x)/f + 3*a**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*a**2*c**2*d*x**2*cos(e
 + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 - 3*a**2*c**2*d*x*sin(e + f*x)*cos(e +
f*x)/(2*f) - 6*a**2*c**2*d*x*cos(e + f*x)/f + 3*a**2*c**2*d*sin(e + f*x)**2
/(4*f**2) + 6*a**2*c**2*d*sin(e + f*x)/f**2 + a**2*c*d**2*x**3*sin(e + f*x)
**2/2 + a**2*c*d**2*x**3*cos(e + f*x)**2/2 + a**2*c*d**2*x**3 - 3*a**2*c*d*
*2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*a**2*c*d**2*x**2*cos(e + f*x)/f
 + 3*a**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) + 12*a**2*c*d**2*x*sin(e + f*x)
/f**2 - 3*a**2*c*d**2*x*cos(e + f*x)**2/(4*f**2) + 3*a**2*c*d**2*sin(e + f*
x)*cos(e + f*x)/(4*f**3) + 12*a**2*c*d**2*cos(e + f*x)/f**3 + a**2*d**3*x**
4*sin(e + f*x)**2/8 + a**2*d**3*x**4*cos(e + f*x)**2/8 + a**2*d**3*x**4/4 -
 a**2*d**3*x**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*d**3*x**3*cos(e +
f*x)/f + 3*a**2*d**3*x**2*sin(e + f*x)**2/(8*f**2) + 6*a**2*d**3*x**2*sin(e
 + f*x)/f**2 - 3*a**2*d**3*x**2*cos(e + f*x)**2/(8*f**2) + 3*a**2*d**3*x*si
n(e + f*x)*cos(e + f*x)/(4*f**3) + 12*a**2*d**3*x*cos(e + f*x)/f**3 - 3*a**
2*d**3*sin(e + f*x)**2/(8*f**4) - 12*a**2*d**3*sin(e + f*x)/f**4, Ne(f, 0))
, ((a*sin(e) + a)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)
, True))
```

**Giac** [A]
time = 1.36, size = 339, normalized size = 1.43

$$\frac{3}{8}a^2d^3x^4+\frac{3}{2}a^2cd^2x^3+\frac{9}{4}a^2c^2dx^2+\frac{3}{2}a^2c^3x-\frac{3\,(2\,a^2d^3f^2x^2+4\,a^2cd^2f^2x+2\,a^2c^2df^2-a^2d^3)\cos(2\,fx+2\,e)}{16\,f^4}-\frac{2\,(a^2d^3f^3x^3+3\,a^2cd^2f^3x^2+3\,a^2c^2df^3x+a^2c^3f^3-6\,a^2d^3fx-6\,a^2cd^2f)\cos(fx+e)}{f^4}-\frac{(2\,a^2d^3f^3x^3+6\,a^2cd^2f^3x^2+6\,a^2c^2df^3x+2\,a^2c^3f^3-3\,a^2d^3fx-3\,a^2cd^2f)\sin(2\,fx+2\,e)}{8\,f^4}+\frac{6\,(a^2d^3f^2x^2+2\,a^2cd^2f^2x+a^2c^2df^2-2\,a^2d^3)\sin(fx+e)}{f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x - 3
/16*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a^2*d^3)*cos
(2*f*x + 2*e)/f^4 - 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*
f^3*x + a^2*c^3*f^3 - 6*a^2*d^3*f*x - 6*a^2*c*d^2*f)*cos(f*x + e)/f^4 - 1/8
*(2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*x^2 + 6*a^2*c^2*d*f^3*x + 2*a^2*c^3*f
^3 - 3*a^2*d^3*f*x - 3*a^2*c*d^2*f)*sin(2*f*x + 2*e)/f^4 + 6*(a^2*d^3*f^2*x
^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2 - 2*a^2*d^3)*sin(f*x + e)/f^4
```

**Mupad [B]**
time = 1.32, size = 452, normalized size = 1.91

$$-\frac{96\,a^2\,d^3\,\sin\left(e+f\,x\right)-\frac{3\,a^2\,d^3\,\cos\left(2\,e+2\,f\,x\right)}{2}+16\,a^2\,c^3\,f^3\,\cos\left(e+f\,x\right)-12\,a^2\,c^3\,f^4\,x+2\,a^2\,c^3\,f^3\,\sin\left(2\,e+2\,f\,x\right)-3\,a^2\,d^3\,f^4\,x^4-96\,a^2\,c\,d^2\,f\,\cos\left(e+f\,x\right)-96\,a^2\,d^3\,f\,x\,\cos\left(e+f\,x\right)+3\,a^2\,d^3\,f^2\,x^2\,\cos\left(2\,e+2\,f\,x\right)+2\,a^2\,d^3\,f^3\,x^3\,\sin\left(2\,e+2\,f\,x\right)-3\,a^2\,c\,d^2\,f\,\sin\left(2\,e+2\,f\,x\right)-48\,a^2\,c^2\,d\,f^2\,\sin\left(e+f\,x\right)-3\,a^2\,d^3\,f\,x\,\sin\left(2\,e+2\,f\,x\right)+3\,a^2\,c^2\,d\,f^2\,\cos\left(2\,e+2\,f\,x\right)-18\,a^2\,c^2\,d\,f^4\,x^2-12\,a^2\,c\,d^2\,f^4\,x^3+16\,a^2\,d^3\,f^3\,x^3\,\cos\left(e+f\,x\right)-48\,a^2\,d^3\,f^2\,x^2\,\sin\left(e+f\,x\right)+6\,a^2\,c\,d^2\,f^2\,x\,\cos\left(2\,e+2\,f\,x\right)+48\,a^2\,c\,d^2\,f^3\,x^2\,\cos\left(e+f\,x\right)+6\,a^2\,c^2\,d\,f^3\,x\,\sin\left(2\,e+2\,f\,x\right)+6\,a^2\,c\,d^2\,f^3\,x^2\,\sin\left(2\,e+2\,f\,x\right)+48\,a^2\,c^2\,d\,f^3\,x\,\cos\left(e+f\,x\right)-96\,a^2\,c\,d^2\,f^2\,x\,\sin\left(e+f\,x\right)}{8\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*x)^3,x)
```

```
[Out] -(96*a^2*d^3*sin(e + f*x) - (3*a^2*d^3*cos(2*e + 2*f*x))/2 + 16*a^2*c^3*f^3
*cos(e + f*x) - 12*a^2*c^3*f^4*x + 2*a^2*c^3*f^3*sin(2*e + 2*f*x) - 3*a^2*d
^3*f^4*x^4 - 96*a^2*c*d^2*f*cos(e + f*x) - 96*a^2*d^3*f*x*cos(e + f*x) + 3*
a^2*d^3*f^2*x^2*cos(2*e + 2*f*x) + 2*a^2*d^3*f^3*x^3*sin(2*e + 2*f*x) - 3*a
^2*c*d^2*f*sin(2*e + 2*f*x) - 48*a^2*c^2*d*f^2*sin(e + f*x) - 3*a^2*d^3*f*x
*sin(2*e + 2*f*x) + 3*a^2*c^2*d*f^2*cos(2*e + 2*f*x) - 18*a^2*c^2*d*f^4*x^2
 - 12*a^2*c*d^2*f^4*x^3 + 16*a^2*d^3*f^3*x^3*cos(e + f*x) - 48*a^2*d^3*f^2*
x^2*sin(e + f*x) + 6*a^2*c*d^2*f^2*x*cos(2*e + 2*f*x) + 48*a^2*c*d^2*f^3*x^
2*cos(e + f*x) + 6*a^2*c^2*d*f^3*x*sin(2*e + 2*f*x) + 6*a^2*c*d^2*f^3*x^2*s
in(2*e + 2*f*x) + 48*a^2*c^2*d*f^3*x*cos(e + f*x) - 96*a^2*c*d^2*f^2*x*sin(
e + f*x))/(8*f^4)
```

# 3.102 $\int (c+dx)^2(a+a\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=168

$$-\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d^2\cos(e+fx)}{f^3}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}+\frac{4a^2d(c+dx)\sin(e+fx)}{f^2}+\frac{a^2d^2\cos(e$$

```
[Out] -1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d+4*a^2*d^2*cos(f*x+e)/f^3-2*a^2*(d*x+
c)^2*cos(f*x+e)/f+4*a^2*d*(d*x+c)*sin(f*x+e)/f^2+1/4*a^2*d^2*cos(f*x+e)*sin
(f*x+e)/f^3-1/2*a^2*(d*x+c)^2*cos(f*x+e)*sin(f*x+e)/f+1/2*a^2*d*(d*x+c)*sin
(f*x+e)^2/f^2
```

**Rubi** [A]
time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3398, 3377, 2718, 3392, 32, 2715, 8}

$$\frac{a^2d(c+dx)\sin^2(e+fx)}{2f^2}+\frac{4a^2d(c+dx)\sin(e+fx)}{f^2}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}-\frac{a^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d^2\cos(e+fx)}{f^3}+\frac{a^2d^2\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{a^2d^2x}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(a^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(2*d) + (4*a^2*d^2*Cos[e + f*x])/f
^3 - (2*a^2*(c + d*x)^2*Cos[e + f*x])/f + (4*a^2*d*(c + d*x)*Sin[e + f*x])/
f^2 + (a^2*d^2*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (a^2*(c + d*x)^2*Cos[e
+ f*x]*Sin[e + f*x])/(2*f) + (a^2*d*(c + d*x)*Sin[e + f*x]^2)/(2*f^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+a\sin(e+fx))^2\,dx &= \int \left(a^2(c+dx)^2+2a^2(c+dx)^2\sin(e+fx)+a^2(c+dx)^2\sin^2(e+fx\right. \\
&= \frac{a^2(c+dx)^3}{3d}+a^2\int (c+dx)^2\sin^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^2\sin( \\
&= \frac{a^2(c+dx)^3}{3d}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}-\frac{a^2(c+dx)^2\cos(e+fx)\sin}{2f} \\
&= \frac{a^2(c+dx)^3}{2d}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}+\frac{4a^2d(c+dx)\sin(e+fx)}{f^2} \\
&= -\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d^2\cos(e+fx)}{f^3}-\frac{2a^2(c+dx)^2\cos(e+f}{f}
\end{aligned}$$

**Mathematica [A]**
time = 0.39, size = 182, normalized size = 1.08

$$\frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3-16(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)-2df(c+dx)\cos(2(e+fx))+32cdf\sin(e+fx)+32d^2fx\sin(e+fx)+d^2\sin(2(e+fx))-2c^2f^2\sin(2(e+fx))-4cdf^2x\sin(2(e+fx))-2d^2f^2x^2\sin(2(e+fx)))}{8f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 - 16*(c^2*f^2 + 2*c*d*f
^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x] - 2*d*f*(c + d*x)*Cos[2*(e + f*x)]
+ 32*c*d*f*Sin[e + f*x] + 32*d^2*f*x*Sin[e + f*x] + d^2*Sin[2*(e + f*x)] -
2*c^2*f^2*Sin[2*(e + f*x)] - 4*c*d*f^2*x*Sin[2*(e + f*x)] - 2*d^2*f^2*x^2*S
in[2*(e + f*x)]))/(8*f^3)
```

**<u>Maple</u>** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(158) = 316$.
time = 0.10, size = 567, normalized size = 3.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a^2/f*c*d*e*(-1/2
*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a^2/f*c*d*((f*x+e)*(-1/2*sin(f*x+e)
*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+a^2/f^2*d^2*e^2*
(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a^2/f^2*d^2*e*((f*x+e)*(-1/2*s
in(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+a^2/f^2
*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(
f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)-2*a^2*c^2*c
os(f*x+e)+4*a^2/f*c*d*e*cos(f*x+e)+4*a^2/f*c*d*(sin(f*x+e)-(f*x+e)*cos(f*x+
e))-2*a^2/f^2*d^2*e^2*cos(f*x+e)-4*a^2/f^2*d^2*e*(sin(f*x+e)-(f*x+e)*cos(f*
x+e))+2*a^2/f^2*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e
))+a^2*c^2*(f*x+e)-2*a^2/f*c*d*e*(f*x+e)+a^2/f*c*d*(f*x+e)^2+a^2/f^2*d^2*e^
2*(f*x+e)-a^2/f^2*d^2*e*(f*x+e)^2+1/3*a^2/f^2*d^2*(f*x+e)^3)
```

**<u>Maxima</u>** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(166) = 332$.
time = 0.31, size = 549, normalized size = 3.27

$$\frac{6\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2c^2+24\,(fx+e)a^2c^2+\frac{8\,(fx+e)^3a^2d^2}{f^2}+\frac{24\,(fx+e)^2a^2cd}{f}-48\,a^2c^2\cos(fx+e)-\frac{24\,(fx+e)^2a^2d^2e}{f^2}-\frac{12\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2cde}{f}-\frac{48\,(fx+e)a^2cde}{f}+\frac{96\,a^2cd\cos(fx+e)e}{f}+\frac{6\left(2\,(fx+e)^2-2\,(fx+e)\sin(2\,fx+2\,e)-\cos(2\,fx+2\,e)\right)a^2cd}{f}-\frac{96\,((fx+e)\cos(fx+e)-\sin(fx+e))a^2cd}{f}+\frac{6\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2d^2e^2}{f^2}+\frac{24\,(fx+e)a^2d^2e^2}{f^2}-\frac{48\,a^2d^2\cos(fx+e)e^2}{f^2}-\frac{6\left(2\,(fx+e)^2-2\,(fx+e)\sin(2\,fx+2\,e)-\cos(2\,fx+2\,e)\right)a^2d^2e}{f^2}+\frac{96\,((fx+e)\cos(fx+e)-\sin(fx+e))a^2d^2e}{f^2}+\frac{\left(4\,(fx+e)^3-6\,(fx+e)\cos(2\,fx+2\,e)-3\left(2\,(fx+e)^2-1\right)\sin(2\,fx+2\,e)\right)a^2d^2}{f^2}-\frac{48\left(\left((fx+e)^2-2\right)\cos(fx+e)-2\,(fx+e)\sin(fx+e)\right)a^2d^2}{f^2}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2 + 24*(f*x + e)*a^2*c^2 + 8
*(f*x + e)^3*a^2*d^2/f^2 + 24*(f*x + e)^2*a^2*c*d/f - 48*a^2*c^2*cos(f*x +
e) - 24*(f*x + e)^2*a^2*d^2*e/f^2 - 12*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2
*c*d*e/f - 48*(f*x + e)*a^2*c*d*e/f + 96*a^2*c*d*cos(f*x + e)*e/f + 6*(2*(f
*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*c*d/f - 96
*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a^2*c*d/f + 6*(2*f*x + 2*e - sin(2
*f*x + 2*e))*a^2*d^2*e^2/f^2 + 24*(f*x + e)*a^2*d^2*e^2/f^2 - 48*a^2*d^2*co
s(f*x + e)*e^2/f^2 - 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(
2*f*x + 2*e))*a^2*d^2*e/f^2 + 96*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a^
2*d^2*e/f^2 + (4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e
)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^2/f^2 - 48*(((f*x + e)^2 - 2)*cos(f*x + e)
 - 2*(f*x + e)*sin(f*x + e))*a^2*d^2/f^2)/f
```

**Fricas [A]**
time = 0.39, size = 216, normalized size = 1.29

$$\frac{2\,a^{2}d^{2}f^{3}x^{3}+6\,a^{2}cdf^{3}x^{2}-2\,(a^{2}d^{2}fx+a^{2}cdf)\cos\left(fx+e\right)^{2}+(6\,a^{2}c^{2}f^{3}+a^{2}d^{2}f)x-8\,(a^{2}d^{2}f^{2}x^{2}+2\,a^{2}cdf^{2}x+a^{2}c^{2}f^{2}-2\,a^{2}d^{2})\cos\left(fx+e\right)+(16\,a^{2}d^{2}fx+16\,a^{2}cdf-(2\,a^{2}d^{2}f^{2}x^{2}+4\,a^{2}cdf^{2}x+2\,a^{2}c^{2}f^{2}-a^{2}d^{2})\cos\left(fx+e\right))\sin\left(fx+e\right)}{4\,f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 - 2*(a^2*d^2*f*x + a^2*c*d*f)*co
s(f*x + e)^2 + (6*a^2*c^2*f^3 + a^2*d^2*f)*x - 8*(a^2*d^2*f^2*x^2 + 2*a^2*c
*d*f^2*x + a^2*c^2*f^2 - 2*a^2*d^2)*cos(f*x + e) + (16*a^2*d^2*f*x + 16*a^2
*c*d*f - (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*co
s(f*x + e))*sin(f*x + e))/f^3
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(163) = 326$.
time = 0.29, size = 456, normalized size = 2.71

$$\begin{cases} \frac{a^{2}c^{2}x\sin^{2}{\left(e+fx\right)}}{2}+\frac{a^{2}c^{2}x\cos^{2}{\left(e+fx\right)}}{2}+a^{2}c^{2}x-\frac{a^{2}c^{2}\sin{\left(e+fx\right)}\cos{\left(e+fx\right)}}{2f}-\frac{2a^{2}c^{2}\cos{\left(e+fx\right)}}{f}+\frac{a^{2}cdx^{2}\sin^{2}{\left(e+fx\right)}}{2}+\frac{a^{2}cdx^{2}\cos^{2}{\left(e+fx\right)}}{2}+a^{2}cdx^{2}-\frac{a^{2}cdx\sin{\left(e+fx\right)}\cos{\left(e+fx\right)}}{f}-\frac{4a^{2}cdx\cos{\left(e+fx\right)}}{f}+\frac{a^{2}cd\sin^{2}{\left(e+fx\right)}}{2f^{2}}+\frac{4a^{2}cd\sin{\left(e+fx\right)}}{f^{2}}+\frac{a^{2}d^{2}x^{3}\sin^{2}{\left(e+fx\right)}}{6}+\frac{a^{2}d^{2}x^{3}\cos^{2}{\left(e+fx\right)}}{6}+\frac{a^{2}d^{2}x^{3}}{3}-\frac{a^{2}d^{2}x^{2}\sin{\left(e+fx\right)}\cos{\left(e+fx\right)}}{2f}-\frac{2a^{2}d^{2}x^{2}\cos{\left(e+fx\right)}}{f}+\frac{a^{2}d^{2}x\sin^{2}{\left(e+fx\right)}}{4f^{2}}+\frac{4a^{2}d^{2}x\sin{\left(e+fx\right)}}{f^{2}}-\frac{a^{2}d^{2}x\cos^{2}{\left(e+fx\right)}}{4f^{2}}+\frac{a^{2}d^{2}\sin{\left(e+fx\right)}\cos{\left(e+fx\right)}}{4f^{3}}+\frac{4a^{2}d^{2}\cos{\left(e+fx\right)}}{f^{3}} & \text{for}\: f \neq 0 \\ \left(a\sin{\left(e\right)}+a\right)^{2}\left(c^{2}x+cdx^{2}+\frac{d^{2}x^{3}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
a**2*c**2*x - a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**2*cos(e
 + f*x)/f + a**2*c*d*x**2*sin(e + f*x)**2/2 + a**2*c*d*x**2*cos(e + f*x)**2
/2 + a**2*c*d*x**2 - a**2*c*d*x*sin(e + f*x)*cos(e + f*x)/f - 4*a**2*c*d*x*
cos(e + f*x)/f + a**2*c*d*sin(e + f*x)**2/(2*f**2) + 4*a**2*c*d*sin(e + f*x
)/f**2 + a**2*d**2*x**3*sin(e + f*x)**2/6 + a**2*d**2*x**3*cos(e + f*x)**2/
6 + a**2*d**2*x**3/3 - a**2*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a
**2*d**2*x**2*cos(e + f*x)/f + a**2*d**2*x*sin(e + f*x)**2/(4*f**2) + 4*a**
2*d**2*x*sin(e + f*x)/f**2 - a**2*d**2*x*cos(e + f*x)**2/(4*f**2) + a**2*d*
*2*sin(e + f*x)*cos(e + f*x)/(4*f**3) + 4*a**2*d**2*cos(e + f*x)/f**3, Ne(f
, 0)), ((a*sin(e) + a)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

**Giac [A]**
time = 1.75, size = 207, normalized size = 1.23

$$\frac{1}{2}\,a^{2}d^{2}x^{3}+\frac{3}{2}\,a^{2}cdx^{2}+\frac{3}{2}\,a^{2}c^{2}x-\frac{(a^{2}d^{2}fx+a^{2}cdf)\cos\left(2\,fx+2\,e\right)}{4\,f^{3}}-\frac{2\,(a^{2}d^{2}f^{2}x^{2}+2\,a^{2}cdf^{2}x+a^{2}c^{2}f^{2}-2\,a^{2}d^{2})\cos\left(fx+e\right)}{f^{3}}-\frac{(2\,a^{2}d^{2}f^{2}x^{2}+4\,a^{2}cdf^{2}x+2\,a^{2}c^{2}f^{2}-a^{2}d^{2})\sin\left(2\,fx+2\,e\right)}{8\,f^{3}}+\frac{4\,(a^{2}d^{2}fx+a^{2}cdf)\sin\left(fx+e\right)}{f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x - 1/4*(a^2*d^2*f*x + a^2*
c*d*f)*cos(2*f*x + 2*e)/f^3 - 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^
```

```
2*f^2 - 2*a^2*d^2)*cos(f*x + e)/f^3 - 1/8*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^
2*x + 2*a^2*c^2*f^2 - a^2*d^2)*sin(2*f*x + 2*e)/f^3 + 4*(a^2*d^2*f*x + a^2*
c*d*f)*sin(f*x + e)/f^3
```

**Mupad [B]**
time = 0.97, size = 255, normalized size = 1.52

$$-\frac{8\,a^2\,c^2\,f^2\,\cos\left(e+f\,x\right)-\frac{a^2\,d^2\,\sin\left(2\,e+2\,f\,x\right)}{2}-16\,a^2\,d^2\,\cos\left(e+f\,x\right)-6\,a^2\,c^2\,f^3\,x+a^2\,c^2\,f^2\,\sin\left(2\,e+2\,f\,x\right)-2\,a^2\,d^2\,f^3\,x^3+a^2\,c\,d\,f\,\cos\left(2\,e+2\,f\,x\right)-16\,a^2\,d^2\,f\,x\,\sin\left(e+f\,x\right)+a^2\,d^2\,f^2\,x^2\,\sin\left(2\,e+2\,f\,x\right)-6\,a^2\,c\,d\,f^3\,x^2+a^2\,d^2\,f\,x\,\cos\left(2\,e+2\,f\,x\right)-16\,a^2\,c\,d\,f\,\sin\left(e+f\,x\right)+8\,a^2\,d^2\,f^2\,x^2\,\cos\left(e+f\,x\right)+16\,a^2\,c\,d\,f^2\,x\,\cos\left(e+f\,x\right)+2\,a^2\,c\,d\,f^2\,x\,\sin\left(2\,e+2\,f\,x\right)}{4\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*x)^2,x)
```

```
[Out] -(8*a^2*c^2*f^2*cos(e + f*x) - (a^2*d^2*sin(2*e + 2*f*x))/2 - 16*a^2*d^2*co
s(e + f*x) - 6*a^2*c^2*f^3*x + a^2*c^2*f^2*sin(2*e + 2*f*x) - 2*a^2*d^2*f^3
*x^3 + a^2*c*d*f*cos(2*e + 2*f*x) - 16*a^2*d^2*f*x*sin(e + f*x) + a^2*d^2*f
^2*x^2*sin(2*e + 2*f*x) - 6*a^2*c*d*f^3*x^2 + a^2*d^2*f*x*cos(2*e + 2*f*x)
- 16*a^2*c*d*f*sin(e + f*x) + 8*a^2*d^2*f^2*x^2*cos(e + f*x) + 16*a^2*c*d*f
^2*x*cos(e + f*x) + 2*a^2*c*d*f^2*x*sin(2*e + 2*f*x))/(4*f^3)
```

# 3.103 $\int (c+dx)(a+a\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=118

$$\frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2a^2(c+dx)\cos(e+fx)}{f}+\frac{2a^2d\sin(e+fx)}{f^2}-\frac{a^2(c+dx)\cos(e+fx)\sin(e+$$

```
[Out] 1/2*a^2*c*x+1/4*a^2*d*x^2+1/2*a^2*(d*x+c)^2/d-2*a^2*(d*x+c)*cos(f*x+e)/f+2*
a^2*d*sin(f*x+e)/f^2-1/2*a^2*(d*x+c)*cos(f*x+e)*sin(f*x+e)/f+1/4*a^2*d*sin(
f*x+e)^2/f^2
```

**Rubi** [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3377, 2717, 3391}

$$-\frac{2a^2(c+dx)\cos(e+fx)}{f}-\frac{a^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx+\frac{a^2d\sin^2(e+fx)}{4f^2}+\frac{2a^2d\sin(e+fx)}{f^2}+\frac{1}{4}a^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a^2*(c + d*x)*Co
s[e + f*x])/f + (2*a^2*d*Sin[e + f*x])/f^2 - (a^2*(c + d*x)*Cos[e + f*x]*Si
n[e + f*x])/(2*f) + (a^2*d*Sin[e + f*x]^2)/(4*f^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+a\sin(e+fx))^2\,dx &= \int \left(a^2(c+dx)+2a^2(c+dx)\sin(e+fx)+a^2(c+dx)\sin^2(e+fx)\right)\,dx\\ &= \frac{a^2(c+dx)^2}{2d}+a^2\int (c+dx)\sin^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)\sin(e+\\ &= \frac{a^2(c+dx)^2}{2d}-\frac{2a^2(c+dx)\cos(e+fx)}{f}-\frac{a^2(c+dx)\cos(e+fx)\sin(e+}{2f}\\ &= \frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2a^2(c+dx)\cos(e+fx)}{f}+\frac{2a^2d\sin(e}{f^2}\end{aligned}$$

**Mathematica** [A]
time = 0.47, size = 80, normalized size = 0.68

$$-\frac{a^2(6(e+fx)(-2cf+d(e-fx))+16f(c+dx)\cos(e+fx)+d\cos(2(e+fx))-16d\sin(e+fx)+2f(c+dx)\sin(2(e+fx)))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/8*(a^2*(6*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*f*(c + d*x)*Cos[e + f*x]
 + d*Cos[2*(e + f*x)] - 16*d*Sin[e + f*x] + 2*f*(c + d*x)*Sin[2*(e + f*x)])
)/f^2
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(108) = 216$.
time = 0.06, size = 219, normalized size = 1.86

| method | result |
|---|---|
| risch | $\frac{3a^2dx^2}{4}+\frac{3a^2cx}{2}-\frac{2a^2(dx+c)\cos(fx+e)}{f}+\frac{2a^2d\sin(fx+e)}{f^2}-\frac{a^2d\cos(2fx+2e)}{8f^2}-\frac{a^2(dx+c)\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{a^2c\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2de\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2d\left((fx+e)\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\right.}{f}}{}$ |
| default | $\frac{a^2c\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2de\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2d\left((fx+e)\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\right.}{f}}{}$ |

| | |
|---|---|
| norman | $\frac{\frac{a^2(cf+4d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a^2dx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{4a^2c}{f}+\frac{3a^2dx^2}{4}-\frac{\left(4a^2cf-a^2d\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+3a^2cx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+}{(1}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^2/f*d*e*(-1/2*sin(f
*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2/f*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e
)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-2*a^2*c*cos(f*x+e)+2*a^2/f
*d*e*cos(f*x+e)+2*a^2/f*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e))+a^2*c*(f*x+e)-a^2
/f*d*e*(f*x+e)+1/2*a^2/f*d*(f*x+e)^2)
```

**Maxima [A]**
time = 0.29, size = 224, normalized size = 1.90

$$\frac{2\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))a^2c+8\,(fx+e)a^2c+\frac{4\,(fx+e)^2a^2d}{f}-16\,a^2c\cos\left(fx+e\right)-\frac{2\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2de}{f}-\frac{8\,(fx+e)a^2de}{f}+\frac{16\,a^2d\cos(fx+e)e}{f}+\frac{\left(2\,(fx+e)^2-2\,(fx+e)\sin(2\,fx+2\,e)-\cos(2\,fx+2\,e)\right)a^2d}{f}-\frac{16\,((fx+e)\cos(fx+e)-\sin(fx+e))a^2d}{f}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c + 8*(f*x + e)*a^2*c + 4*(f*x
+ e)^2*a^2*d/f - 16*a^2*c*cos(f*x + e) - 2*(2*f*x + 2*e - sin(2*f*x + 2*e))
*a^2*d*e/f - 8*(f*x + e)*a^2*d*e/f + 16*a^2*d*cos(f*x + e)*e/f + (2*(f*x +
e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*d/f - 16*((f*x
+ e)*cos(f*x + e) - sin(f*x + e))*a^2*d/f)/f
```

**Fricas [A]**
time = 0.35, size = 105, normalized size = 0.89

$$\frac{3\,a^2df^2x^2+6\,a^2cf^2x-a^2d\cos\left(fx+e\right)^2-8\,\left(a^2dfx+a^2cf\right)\cos\left(fx+e\right)+2\,\left(4\,a^2d-\left(a^2dfx+a^2cf\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*d*f^2*x^2 + 6*a^2*c*f^2*x - a^2*d*cos(f*x + e)^2 - 8*(a^2*d*f*x
+ a^2*c*f)*cos(f*x + e) + 2*(4*a^2*d - (a^2*d*f*x + a^2*c*f)*cos(f*x + e))*
sin(f*x + e))/f^2
```

**Sympy [A]**
time = 0.16, size = 219, normalized size = 1.86

$$\begin{cases}\frac{a^2cx\sin^2\left(e+fx\right)}{2}+\frac{a^2cx\cos^2\left(e+fx\right)}{2}+a^2cx-\frac{a^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^2c\cos\left(e+fx\right)}{f}+\frac{a^2dx^2\sin^2\left(e+fx\right)}{4}+\frac{a^2dx^2\cos^2\left(e+fx\right)}{4}+\frac{a^2dx^2}{2}-\frac{a^2dx\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^2dx\cos\left(e+fx\right)}{f}+\frac{a^2d\sin^2\left(e+fx\right)}{4f^2}+\frac{2a^2d\sin\left(e+fx\right)}{f^2} & \text{for}\: f\neq 0\\\left(a\sin\left(e\right)+a\right)^2\left(cx+\frac{dx^2}{2}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*c
*x - a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c*cos(e + f*x)/f + a**
2*d*x**2*sin(e + f*x)**2/4 + a**2*d*x**2*cos(e + f*x)**2/4 + a**2*d*x**2/2
- a**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*d*x*cos(e + f*x)/f + a*
*2*d*sin(e + f*x)**2/(4*f**2) + 2*a**2*d*sin(e + f*x)/f**2, Ne(f, 0)), ((a*
sin(e) + a)**2*(c*x + d*x**2/2), True))
```

**Giac [A]**
time = 1.68, size = 107, normalized size = 0.91

$$\frac{3}{4}\,a^2dx^2+\frac{3}{2}\,a^2cx-\frac{a^2d\cos\left(2\,fx+2\,e\right)}{8\,f^2}+\frac{2\,a^2d\sin\left(fx+e\right)}{f^2}-\frac{2\,\left(a^2dfx+a^2cf\right)\cos\left(fx+e\right)}{f^2}-\frac{\left(a^2dfx+a^2cf\right)\sin\left(2\,fx+2\,e\right)}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*d*x^2 + 3/2*a^2*c*x - 1/8*a^2*d*cos(2*f*x + 2*e)/f^2 + 2*a^2*d*sin(
f*x + e)/f^2 - 2*(a^2*d*f*x + a^2*c*f)*cos(f*x + e)/f^2 - 1/4*(a^2*d*f*x +
a^2*c*f)*sin(2*f*x + 2*e)/f^2
```

**Mupad [B]**
time = 0.73, size = 127, normalized size = 1.08

$$\frac{a^2\,d\sin\left(e+f\,x\right)^2+8\,a^2\,d\,\sin\left(e+f\,x\right)+16\,a^2\,c\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\,d\,f^2\,x^2-a^2\,c\,f\,\sin\left(2\,e+2\,f\,x\right)+6\,a^2\,c\,f^2\,x-a^2\,d\,f\,x\,\sin\left(2\,e+2\,f\,x\right)+8\,a^2\,d\,f\,x\,\left(2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*x),x)
```

```
[Out] (a^2*d*sin(e + f*x)^2 + 8*a^2*d*sin(e + f*x) + 16*a^2*c*f*sin(e/2 + (f*x)/2
)^2 + 3*a^2*d*f^2*x^2 - a^2*c*f*sin(2*e + 2*f*x) + 6*a^2*c*f^2*x - a^2*d*f*
x*sin(2*e + 2*f*x) + 8*a^2*d*f*x*(2*sin(e/2 + (f*x)/2)^2 - 1))/(4*f^2)
```

# 3.104 $\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx$

**Optimal**. Leaf size=145

$$-\frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}+\frac{2a^2\mathrm{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2a^2\cos\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(\frac{cf}{d}+\right.}{d}$$

```
[Out] -1/2*a^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/d+3/2*a^2*ln(d*x+c)/d+2*a^2*co
s(-e+c*f/d)*Si(c*f/d+f*x)/d-1/2*a^2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/d-2
*a^2*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d
```

**Rubi** [A]
time = 0.24, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3399, 3393, 3384, 3380, 3383}

$$\frac{2a^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}-\frac{a^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{a^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{2a^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}+\frac{3a^2\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*x),x]
```

```
[Out] -1/2*(a^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d + (3*a^2*L
og[c + d*x])/(2*d) + (2*a^2*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d
 + (2*a^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d + (a^2*Sin[2*e - (2
*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)}{c+dx}\,dx\\
&= \left(4a^2\right)\int \left(\frac{3}{8(c+dx)}-\frac{\cos(2e+2fx)}{8(c+dx)}+\frac{\sin(e+fx)}{2(c+dx)}\right)dx\\
&= \frac{3a^2\log(c+dx)}{2d}-\frac{1}{2}a^2\int \frac{\cos(2e+2fx)}{c+dx}\,dx+\left(2a^2\right)\int \frac{\sin(e+fx)}{c+dx}\,dx\\
&= \frac{3a^2\log(c+dx)}{2d}-\frac{1}{2}\left(a^2\cos\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx+\left(2a^2\cos\left(\right.\right.\\
&= -\frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}+\frac{2a^2\operatorname{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\right.}{d}\end{aligned}$$

**Mathematica [A]**
time = 0.17, size = 114, normalized size = 0.79

$$\frac{a^2\left(-\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2f(c+dx)}{d}\right)+3\log(c+dx)+4\operatorname{Ci}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+4\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)+\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*x),x]
```

```
[Out] (a^2*(-(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*f*(c + d*x))/d]) + 3*Log[c + d*
x] + 4*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + 4*Cos[e - (c*f)/d]*SinIn
tegral[f*(c/d + x)] + Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d]))
/(2*d)
```

**Maple [A]**
time = 0.08, size = 198, normalized size = 1.37

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{3a^2 f \ln(cf-de+d(fx+e))}{2d} - \frac{a^2 f\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d} + \frac{2\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-}{d}\right.}{d}\right.}{4}}{f}$ |
| default | $\frac{\frac{3a^2 f \ln(cf-de+d(fx+e))}{2d} - \frac{a^2 f\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d} + \frac{2\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-}{d}\right.}{d}\right.}{4}}{f}$ |
| risch | $-\frac{i a^2 e^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1, ifx+ie+\frac{i(cf-de)}{d}\right)}{d} + \frac{3a^2\ln(dx+c)}{2d} + \frac{a^2 e^{\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1, 2ifx+2ie+\frac{2i(cf-}{d}\right.}{4d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(3/2*a^2*f*ln(c*f-d*e+d*(f*x+e))/d-1/4*a^2*f*(2*Si(2*f*x+2*e+2*(c*f-d*e
)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/
d)+2*a^2*f*(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*
sin((c*f-d*e)/d)/d))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.45, size = 359, normalized size = 2.48

$$\frac{\frac{4a^2 f \log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d} + \frac{\left(f\left(E_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+E_1\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\cos\left(\frac{2(cf-de)}{d}\right)+f\left(-iE_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+iE_1\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\sin\left(\frac{2(cf-de)}{d}\right)+2f\log((fx+e)d+cf-de)\right)a^2}{d} + \frac{4\left(f\left(-iE_1\left(\frac{i(fx+e)d+icf-ide}{d}\right)+iE_1\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f\left(E_1\left(\frac{i(fx+e)d+icf-ide}{d}\right)+E_1\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a^2}{d}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d + (f*(exp_integral_e(1, 2*(-I
*(f*x + e)*d - I*c*f + I*d*e)/d) + exp_integral_e(1, -2*(-I*(f*x + e)*d - I
*c*f + I*d*e)/d))*cos(2*(c*f - d*e)/d) + f*(-I*exp_integral_e(1, 2*(-I*(f*x
 + e)*d - I*c*f + I*d*e)/d) + I*exp_integral_e(1, -2*(-I*(f*x + e)*d - I*c*
f + I*d*e)/d))*sin(2*(c*f - d*e)/d) + 2*f*log((f*x + e)*d + c*f - d*e))*a^2
/d + 4*(f*(-I*exp_integral_e(1, (I*(f*x + e)*d + I*c*f - I*d*e)/d) + I*exp_
integral_e(1, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*cos((c*f - d*e)/d) + f*(
exp_integral_e(1, (I*(f*x + e)*d + I*c*f - I*d*e)/d) + exp_integral_e(1, -(
I*(f*x + e)*d + I*c*f - I*d*e)/d))*sin((c*f - d*e)/d))*a^2/d)/f
```

**Fricas** [A]
time = 0.35, size = 191, normalized size = 1.32

$$\frac{2a^2\sin\left(-\frac{2(cf-de)}{d}\right)\mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right)+8a^2\cos\left(-\frac{cf-de}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+6a^2\log(dx+c)-\left(a^2\,\mathrm{Ci}\left(\frac{2(dfx+cf)}{d}\right)+a^2\,\mathrm{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\right)\cos\left(-\frac{2(cf-de)}{d}\right)+4\left(a^2\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+a^2\,\mathrm{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*sin(-2*(c*f - d*e)/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*a^2*co
s(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) + 6*a^2*log(d*x + c) - (a^2
*cos_integral(2*(d*f*x + c*f)/d) + a^2*cos_integral(-2*(d*f*x + c*f)/d))*co
s(-2*(c*f - d*e)/d) + 4*(a^2*cos_integral((d*f*x + c*f)/d) + a^2*cos_integr
al(-(d*f*x + c*f)/d))*sin(-(c*f - d*e)/d))/d
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{2\sin\left(e+fx\right)}{c+dx}\,dx+\int\frac{\sin^{2}\left(e+fx\right)}{c+dx}\,dx+\int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(d*x+c),x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/(c + d*x), x) + Integral(sin(e + f*x)**2/(c +
 d*x), x) + Integral(1/(c + d*x), x))
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.02, size = 7049, normalized size = 48.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 - 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(
c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*real_part(cos_inte
gral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2
- a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 + 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2
*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*a^2*imag_part(cos_integral(2*f*
x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 2*a^2*ima
g_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/
2*e)^2*tan(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*
c*f/d)^2*tan(1/2*e)^2*tan(e) - 8*a^2*real_part(cos_integral(f*x + c*f/d))*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a^2*real_part(cos_inte
gral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 + 8*a
^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2
*e)^2*tan(e)^2 + 8*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*t
an(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 2*a^2*imag_part(cos_integral(2*f*x +
2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*a^2*imag_pa
rt(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2
*tan(e)^2 + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)
```

^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 16*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 16*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 32*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 8*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 8*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 8*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 8*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e) + 2

```
*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*ta
n(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*ta...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\, \sin (e + f\, x))^2}{c + d\, x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*x),x)
[Out] int((a + a*sin(e + f*x))^2/(c + d*x), x)
```

# 3.105 $\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=162

$$\frac{2a^2f\cos\left(e-\frac{cf}{d}\right)\operatorname{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}+\frac{a^2f\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{4a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}-\frac{2a^2f\sin\left(e-\frac{cf}{d}\right)}{d^2}$$

```
[Out] 2*a^2*f*Ci(c*f/d+f*x)*cos(-e+c*f/d)/d^2+a^2*f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+
2*f*x)/d^2-a^2*f*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/d^2+2*a^2*f*Si(c*f/d+f
*x)*sin(-e+c*f/d)/d^2-4*a^2*sin(1/2*e+1/4*Pi+1/2*f*x)^4/d/(d*x+c)
```

**Rubi** [A]
time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3399, 3394, 3384, 3380, 3383}

$$\frac{a^2f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2a^2f\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2a^2f\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(xf+\frac{cf}{d}\right)}{d^2}+\frac{a^2f\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{4a^2\sin^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (2*a^2*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2 + (a^2*f*CosInteg
ral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (4*a^2*Sin[e/2 + Pi/4 +
(f*x)/2]^4)/(d*(c + d*x)) - (2*a^2*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d +
 f*x])/d^2 + (a^2*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^
2
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx &= (4a^2)\int \frac{\sin^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)}{(c+dx)^2}\,dx \\
&= -\frac{4a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{(8a^2f)\int\left(\frac{\cos(e+fx)}{4(c+dx)}+\frac{\sin(2e+2fx)}{8(c+dx)}\right)dx}{d} \\
&= -\frac{4a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{(a^2f)\int\frac{\sin(2e+2fx)}{c+dx}\,dx}{d}+\frac{(2a^2f)\int\frac{\cos(e+fx)}{c+dx}\,dx}{d} \\
&= -\frac{4a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(a^2f\cos\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sin\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{d}+\frac{(2a^2f\cos(e
\\
&= \frac{2a^2f\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}+\frac{a^2f\text{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{4a^2\sin^4\left(\frac{e}{2}\right.}{d(c+}
\end{aligned}$$

**Mathematica [A]**
time = 0.40, size = 206, normalized size = 1.27

$$\frac{a^2\left(-3d+d\cos(2(e+fx))+4f(c+dx)\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(f\left(\frac{c}{d}+x\right)\right)+2f(c+dx)\text{Ci}\left(\frac{2f(c+dx)}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)-4d\sin(e+fx)-4cf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)-4dfx\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+2cf\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)+2dfx\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{2d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (a^2*(-3*d + d*Cos[2*(e + f*x)] + 4*f*(c + d*x)*Cos[e - (c*f)/d]*CosIntegra
l[f*(c/d + x)] + 2*f*(c + d*x)*CosIntegral[(2*f*(c + d*x))/d]*Sin[2*e - (2*
c*f)/d] - 4*d*Sin[e + f*x] - 4*c*f*Sin[e - (c*f)/d]*SinIntegral[f*(c/d + x)
] - 4*d*f*x*Sin[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 2*c*f*Cos[2*e - (2*
c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cos[2*e - (2*c*f)/d]*SinIn
tegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

**Maple [A]**
time = 0.10, size = 274, normalized size = 1.69

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{3a^2f^2}{2(cf-de+d(fx+e))d}-\frac{a^2f^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(2fx \cdots\right.}{}\right.}{d}\right.}{4}}{\cdots}$ |
| default | $\frac{-\frac{3a^2f^2}{2(cf-de+d(fx+e))d}-\frac{a^2f^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(2fx \cdots\right.}{}\right.}{d}\right.}{4}}{\cdots}$ |
| risch | $-\frac{f\,a^2\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{d^2}-\frac{3a^2}{2d(dx+c)}-\frac{ia^2f\,\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,2ifx+2ie+\frac{2i(cf- \cdots}{d}\right.}{2d^2}\cdots$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-3/2*a^2*f^2/(c*f-d*e+d*(f*x+e))/d-1/4*a^2*f^2*(-2*cos(2*f*x+2*e)/(c*f
-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*C
i(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d)+2*a^2*f^2*(-sin(f*x+e)/
(c*f-d*e+d*(f*x+e))/d+(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c
*f-d*e)/d)*cos((c*f-d*e)/d)/d)/d))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.41, size = 396, normalized size = 2.44

$$-\frac{\frac{4\,a^2f^2}{(fx+e)d^2+cdf-d^2e}-\frac{\left(f^2\left(E_2\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+E_2\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\cos\left(\frac{2(cf-de)}{d}\right)-f^2\left(i\,E_2\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)-i\,E_2\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\sin\left(\frac{2(cf-de)}{d}\right)-2f^2\right)a^2}{(fx+e)d^2+cdf-d^2e}-\frac{4\left(f^2\left(-i\,E_2\left(\frac{i(fx+e)d+icf-ide}{d}\right)+i\,E_2\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f^2\left(E_2\left(\frac{i(fx+e)d+icf-ide}{d}\right)+E_2\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a^2}{(fx+e)d^2+cdf-d^2e}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^2*f^2/((f*x + e)*d^2 + c*d*f - d^2*e) - (f^2*(exp_integral_e(2, 2
*(-I*(f*x + e)*d - I*c*f + I*d*e)/d) + exp_integral_e(2, -2*(-I*(f*x + e)*d
 - I*c*f + I*d*e)/d))*cos(2*(c*f - d*e)/d) - f^2*(I*exp_integral_e(2, 2*(-I
*(f*x + e)*d - I*c*f + I*d*e)/d) - I*exp_integral_e(2, -2*(-I*(f*x + e)*d -
 I*c*f + I*d*e)/d))*sin(2*(c*f - d*e)/d) - 2*f^2)*a^2/((f*x + e)*d^2 + c*d*
f - d^2*e) - 4*(f^2*(-I*exp_integral_e(2, (I*(f*x + e)*d + I*c*f - I*d*e)/d
) + I*exp_integral_e(2, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*cos((c*f - d*e
)/d) + f^2*(exp_integral_e(2, (I*(f*x + e)*d + I*c*f - I*d*e)/d) + exp_inte
gral_e(2, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*sin((c*f - d*e)/d))*a^2/((f*
x + e)*d^2 + c*d*f - d^2*e))/f
```

**Fricas** [**A**]
time = 0.37, size = 289, normalized size = 1.78

$$\frac{2\,a^{2} d \cos\left(f x + e\right)^{2} - 4\,a^{2} d \sin\left(f x + e\right) - 4\,a^{2} d + 2\,{\left(a^{2} d f x + a^{2} c f\right)} \cos\left(-\frac{2\,{\left(c f - d e\right)}}{d}\right) \operatorname{Si}\left(\frac{2\,{\left(d f x + c f\right)}}{d}\right) - 4\,{\left(a^{2} d f x + a^{2} c f\right)} \sin\left(-\frac{c f - d e}{d}\right) \operatorname{Si}\left(\frac{d f x + c f}{d}\right) + 2\,{\left({\left(a^{2} d f x + a^{2} c f\right)} \operatorname{Ci}\left(\frac{d f x + c f}{d}\right) + {\left(a^{2} d f x + a^{2} c f\right)} \operatorname{Ci}\left(-\frac{d f x + c f}{d}\right)\right)} \cos\left(-\frac{c f - d e}{d}\right) + {\left({\left(a^{2} d f x + a^{2} c f\right)} \operatorname{Ci}\left(\frac{2\,{\left(d f x + c f\right)}}{d}\right) + {\left(a^{2} d f x + a^{2} c f\right)} \operatorname{Ci}\left(-\frac{2\,{\left(d f x + c f\right)}}{d}\right)\right)} \sin\left(-\frac{2\,{\left(c f - d e\right)}}{d}\right)}{2\,{\left(d^{3} x + c d^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d*cos(f*x + e)^2 - 4*a^2*d*sin(f*x + e) - 4*a^2*d + 2*(a^2*d*f*x
 + a^2*c*f)*cos(-2*(c*f - d*e)/d)*sin_integral(2*(d*f*x + c*f)/d) - 4*(a^2*
d*f*x + a^2*c*f)*sin(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) + 2*((a^
2*d*f*x + a^2*c*f)*cos_integral((d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*co
s_integral(-(d*f*x + c*f)/d))*cos(-(c*f - d*e)/d) + ((a^2*d*f*x + a^2*c*f)*
cos_integral(2*(d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*cos_integral(-2*(d*
f*x + c*f)/d))*sin(-2*(c*f - d*e)/d))/(d^3*x + c*d^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int \frac{2 \sin{\left(e + f x \right)}}{c^{2} + 2 c d x + d^{2} x^{2}}\, dx + \int \frac{\sin^{2}{\left(e + f x \right)}}{c^{2} + 2 c d x + d^{2} x^{2}}\, dx + \int \frac{1}{c^{2} + 2 c d x + d^{2} x^{2}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(s
in(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*d*
x + d**2*x**2), x))
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(164) = 328$.
time = 2.92, size = 1134, normalized size = 7.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos((c*f - d*e
)/d)*cos_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d
*e)/d) - 4*a^2*c*f^3*cos((c*f - d*e)/d)*cos_integral(-((d*x + c)*(c*f/(d*x
+ c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 4*a^2*d*f^2*cos((c*f - d*e)/d)*
cos_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d
)*e - 2*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos_integral(
-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*sin(2*(c*
```

```
f - d*e)/d) + 2*a^2*c*f^3*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d
*e/(d*x + c)) - c*f + d*e)/d)*sin(2*(c*f - d*e)/d) - 2*a^2*d*f^2*cos_integr
al(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e*sin(
2*(c*f - d*e)/d) + 4*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*
sin((c*f - d*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x +
 c)) - c*f + d*e)/d) - 4*a^2*c*f^3*sin((c*f - d*e)/d)*sin_integral(-((d*x +
 c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 4*a^2*d*f^2*e*sin
((c*f - d*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)
) - c*f + d*e)/d) + 2*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2
*cos(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(
d*x + c)) - c*f + d*e)/d) - 2*a^2*c*f^3*cos(2*(c*f - d*e)/d)*sin_integral(-
2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 2*a^2*d*
f^2*cos(2*(c*f - d*e)/d)*e*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f -
d*e/(d*x + c)) - c*f + d*e)/d) - a^2*d*f^2*cos(2*(d*x + c)*(c*f/(d*x + c) -
 f - d*e/(d*x + c))/d) - 4*a^2*d*f^2*sin((d*x + c)*(c*f/(d*x + c) - f - d*e
/(d*x + c))/d) + 3*a^2*d*f^2)*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - f - d*e/
(d*x + c)) - c*d^4*f + d^5*e)*f)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin{(e + f\,x)})^2}{(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^2/(c + d*x)^2, x)
```

# 3.106 $\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=225

$$\frac{a^2f^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{d^3}-\frac{a^2f^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}-\frac{4a^2f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)}$$

```
[Out] a^2*f^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/d^3-a^2*f^2*cos(-e+c*f/d)*Si(c*
f/d+f*x)/d^3+a^2*f^2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/d^3+a^2*f^2*Ci(c*f
/d+f*x)*sin(-e+c*f/d)/d^3-4*a^2*f*cos(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*P
i+1/2*f*x)^3/d^2/(d*x+c)-2*a^2*sin(1/2*e+1/4*Pi+1/2*f*x)^4/d/(d*x+c)^2
```

**Rubi** [A]
time = 0.32, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3399, 3395, 3390, 31, 3384, 3380, 3383, 3393}

$$-\frac{a^2f^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}+\frac{a^2f^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{d^3}-\frac{a^2f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{d^3}-\frac{a^2f^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{4a^2f\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d^2(c+dx)}-\frac{2a^2\sin^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (a^2*f^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d^3 - (a^2*f^
2*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d^3 - (4*a^2*f*Cos[e/2 + Pi/
4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]^3)/(d^2*(c + d*x)) - (2*a^2*Sin[e/2
+ Pi/4 + (f*x)/2]^4)/(d*(c + d*x)^2) - (a^2*f^2*Cos[e - (c*f)/d]*SinIntegra
l[(c*f)/d + f*x])/d^3 - (a^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d
 + 2*f*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx = \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)}{(c+dx)^3}\,dx$$

$$= -\frac{4a^2 f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{2a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} + \frac{\left(6a^2f^2\right)\int}{}$$

$$= -\frac{4a^2 f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{2a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} + \frac{\left(3a^2f^2\right)\int}{d}$$

$$= -\frac{4a^2 f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{2a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} + \frac{\left(a^2f^2\right)\int}{}$$

$$= \frac{3a^2f^2\operatorname{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3} - \frac{4a^2 f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{2a}{}$$

$$= \frac{a^2f^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{d^3} - \frac{a^2f^2\operatorname{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3} - \frac{4a^2 f\cos}{}$$

## Mathematica [A]

time = 0.60, size = 353, normalized size = 1.57

$$-\frac{a^2\left(3d^2+4cdf\cos(e+fx)+4d^2fx\cos(e+fx)-d^2\cos(2(e+fx))-4f^2(c+dx)^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2f(c+dx)}{d}\right)+4f^2(c+dx)^2\operatorname{Ci}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+4d^2\sin(e+fx)+2cdf\sin(2(e+fx))+2d^2fx\sin(2(e+fx))+4c^2f^2\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)+8cdf^2x\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)+4d^2f^2x^2\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)+4c^2f^2\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)+8cdf^2x\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)+4d^2f^2x^2\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{4d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -1/4*(a^2*(3*d^2 + 4*c*d*f*Cos[e + f*x] + 4*d^2*f*x*Cos[e + f*x] - d^2*Cos[
2*(e + f*x)] - 4*f^2*(c + d*x)^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*f*(c +
 d*x))/d] + 4*f^2*(c + d*x)^2*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + 4
*d^2*Sin[e + f*x] + 2*c*d*f*Sin[2*(e + f*x)] + 2*d^2*f*x*Sin[2*(e + f*x)] +
 4*c^2*f^2*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 8*c*d*f^2*x*Cos[e -
(c*f)/d]*SinIntegral[f*(c/d + x)] + 4*d^2*f^2*x^2*Cos[e - (c*f)/d]*SinInteg
ral[f*(c/d + x)] + 4*c^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x
))/d] + 8*c*d*f^2*x*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 4
*d^2*f^2*x^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d]))/(d^3*(c
+ d*x)^2)
```

## Maple [A]

time = 0.15, size = 347, normalized size = 1.54

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{3a^2f^3}{4(cf-de+d(fx+e))^2d}-\frac{a^2f^3\left(-\frac{\cos(2fx+2e)}{(cf-de+d(fx+e))^2d}-\frac{-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{4\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}}{d}\right.}{4}}{}$ |
| default | $\frac{-\frac{3a^2f^3}{4(cf-de+d(fx+e))^2d}-\frac{a^2f^3\left(-\frac{\cos(2fx+2e)}{(cf-de+d(fx+e))^2d}-\frac{-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{4\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}}{d}\right.}{4}}{}$ |
| risch | $\frac{if^2a^2\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^3}-\frac{3a^2}{4d(dx+c)^2}-\frac{a^2f^2\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,2ifx+2ie+\frac{2i(cf-}{d}\right.}{2d^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-3/4*a^2*f^3/(c*f-d*e+d*(f*x+e))^2/d-1/4*a^2*f^3*(-cos(2*f*x+2*e)/(c*f
-d*e+d*(f*x+e))^2/d-(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d+2*(2*Si(2*f*x+
2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2
*(c*f-d*e)/d)/d)/d)/d)+2*a^2*f^3*(-1/2*sin(f*x+e)/(c*f-d*e+d*(f*x+e))^2/d+1
/2*(-cos(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/
d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.59, size = 504, normalized size = 2.24

$$-\frac{\frac{2a^2f^3}{(fx+e)^2d^3+c^2df^2-2cd^2fe+d^3e^2+2(cd^2f-d^3e)(fx+e)}-\frac{\left(f^3\left(E_3\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+E_3\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\cos\left(\frac{2(cf-de)}{d}\right)-f^3\left(iE_3\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)-iE_3\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\sin\left(\frac{2(cf-de)}{d}\right)-f^3\right)a^2}{(fx+e)^2d^3+c^2df^2-2cd^2fe+d^3e^2+2(cd^2f-d^3e)(fx+e)}-\frac{4\left(f^3\left(-iE_3\left(\frac{i(fx+e)d+icf-ide}{d}\right)+iE_3\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f^3\left(E_3\left(\frac{i(fx+e)d+icf-ide}{d}\right)+E_3\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)a^2}{(fx+e)^2d^3+c^2df^2-2cd^2fe+d^3e^2+2(cd^2f-d^3e)(fx+e)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*f^3/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*(c
*d^2*f - d^3*e)*(f*x + e)) - (f^3*(exp_integral_e(3, 2*(-I*(f*x + e)*d - I*
c*f + I*d*e)/d) + exp_integral_e(3, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d))
*cos(2*(c*f - d*e)/d) - f^3*(I*exp_integral_e(3, 2*(-I*(f*x + e)*d - I*c*f
+ I*d*e)/d) - I*exp_integral_e(3, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d))*s
in(2*(c*f - d*e)/d) - f^3)*a^2/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e +
 d^3*e^2 + 2*(c*d^2*f - d^3*e)*(f*x + e)) - 4*(f^3*(-I*exp_integral_e(3, (I
*(f*x + e)*d + I*c*f - I*d*e)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d + I*
c*f - I*d*e)/d))*cos((c*f - d*e)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d
 + I*c*f - I*d*e)/d) + exp_integral_e(3, -(I*(f*x + e)*d + I*c*f - I*d*e)/d
))*sin((c*f - d*e)/d))*a^2/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e + d^3
*e^2 + 2*(c*d^2*f - d^3*e)*(f*x + e)))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(214) = 428$.
time = 0.45, size = 484, normalized size = 2.15

$$\frac{a^2d^2\cos(fx+e)^2 - 2a^2d^2 - 2(a^2d^2f^2x^2 + 2a^2cdf^2x + a^2c^2f^2)\sin\left(-\frac{2(cf-de)}{d}\right)\mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right) - 2(a^2d^2f^2x^2 + 2a^2cdf^2x + a^2c^2f^2)\cos\left(-\frac{cf-de}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right) - 2(a^2d^2fx + a^2cdf)\cos(fx+e) + \left((a^2d^2f^2x^2 + 2a^2cdf^2x + a^2c^2f^2)\mathrm{Ci}\left(\frac{2(dfx+cf)}{d}\right) + (a^2d^2f^2x^2 + 2a^2cdf^2x + a^2c^2f^2)\mathrm{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\right)\cos\left(-\frac{2(cf-de)}{d}\right) - 2(a^2d^2 + (a^2d^2fx + a^2cdf)\cos(fx+e))\sin(fx+e) - \left((a^2d^2f^2x^2 + 2a^2cdf^2x + a^2c^2f^2)\mathrm{Ci}\left(\frac{dfx+cf}{d}\right) + (a^2d^2f^2x^2 + 2a^2cdf^2x + a^2c^2f^2)\mathrm{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)}{2(d^5x^2 + 2cd^4x + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*cos(f*x + e)^2 - 2*a^2*d^2 - 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^
2*x + a^2*c^2*f^2)*sin(-2*(c*f - d*e)/d)*sin_integral(2*(d*f*x + c*f)/d) -
2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*cos(-(c*f - d*e)/d)*sin
_integral((d*f*x + c*f)/d) - 2*(a^2*d^2*f*x + a^2*c*d*f)*cos(f*x + e) + ((a
^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*cos_integral(2*(d*f*x + c*f
)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*cos_integral(-2*(d
*f*x + c*f)/d))*cos(-2*(c*f - d*e)/d) - 2*(a^2*d^2 + (a^2*d^2*f*x + a^2*c*d
*f)*cos(f*x + e))*sin(f*x + e) - ((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*
c^2*f^2)*cos_integral((d*f*x + c*f)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x
 + a^2*c^2*f^2)*cos_integral(-(d*f*x + c*f)/d))*sin(-(c*f - d*e)/d))/(d^5*x
^2 + 2*c*d^4*x + c^2*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\sin(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{\sin^2(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{1}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3), x) + Integral(sin(e + f*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x) + Integral(1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.59, size = 124086, normalized size = 551.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1
/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*d^2*f^2
*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*
```

f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*d^2*f^2*x^2*real_part(c
os_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/
2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*d^2*f^2*x^2*real_part(cos_integral(-
2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*t
an(1/2*e)^2*tan(e)^2 + 2*a^2*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(
f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 -
 2*a^2*d^2*f^2*x^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(
1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 2*a^2*d^2*f^
2*x^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a^2*d^2*f^2*x^2*sin_in
tegral(2*(d*f*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*
f/d)^2*tan(1/2*e)^2*tan(e) - 2*a^2*d^2*f^2*x^2*real_part(cos_integral(f*x +
 c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)
*tan(e)^2 - 2*a^2*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(f*x
)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 + 2*a^
2*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1/2*f*x)^
2*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 2*a^2*d^2*f^2*x^2*rea
l_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*t
an(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 2*a^2*d^2*f^2*x^2*imag_part(cos_integ
ral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2
*tan(1/2*e)^2*tan(e)^2 - 2*a^2*d^2*f^2*x^2*imag_part(cos_integral(-2*f*x -
2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^
2*tan(e)^2 + 4*a^2*d^2*f^2*x^2*sin_integral(2*(d*f*x + c*f)/d)*tan(f*x)^2*t
an(1/2*f*x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 2*a^2*c*d
*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c
*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*a^2*c*d*f^2*x*imag_part(
cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*
c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*a^2*c*d*f^2*x*real_part(cos_integral(2*f
*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(
1/2*e)^2*tan(e)^2 - 2*a^2*c*d*f^2*x*real_part(cos_integral(-2*f*x - 2*c*f/d
))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan
(e)^2 + 4*a^2*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*
x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + a^2*d^2*f^2*x^2*
imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2
*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - a^2*d^2*f^2*x^2*imag_part(cos_integral(-f*
x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2
*e)^2 + a^2*d^2*f^2*x^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2
*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a^2*d^2*f^2*x^
2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c
*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a^2*d^2*f^2*x^2*sin_integral((d*f
*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/
2*e)^2 - 4*a^2*d^2*f^2*x^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x
)^2*tan(1/2*f*x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a^2*
d^2*f^2*x^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*
x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a^2*c*d*f^2*x*imag

```
_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2
*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 4*a^2*c*d*f^2*x*imag_part(cos_integ
ral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d
)^2*tan(1/2*e)^2*tan(e) - 8*a^2*c*d*f^2*x*sin_integral(2*(d*f*x + c*f)/d)*t
an(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)
- a^2*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1/2*f
*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + a^2*d^2*f^2*x^2*imag_part(co
s_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*
f/d)^2*tan(e)^2 - a^2*d^2*f^2*x^2*real_part(cos_integral(2*f*x + 2*c*f/d))*
tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - a^2*d^2*
f^2*x^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2
*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - 2*a^2*d^2*f^2*x^2*sin_integral((d
*f*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(
e)^2 + 4*a^2*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*ta
n(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 4*a^2*d^2*f^
2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c
*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 8*a^2*d^2*f^2*x^2*sin_integral
((d*f*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)*tan
(1/2*e)*tan(e)^2 - 4*a^2*c*d*f^2*x*real_part(co...
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\,\sin(e+f\,x))^2}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^2/(c + d*x)^3, x)
```

# 3.107 $\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=148

$$-\frac{i(c+dx)^3}{af}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{12id^2(c+dx)\mathrm{Li}_2\left(ie^{i(e+fx)}\right)}{af^3}+\frac{12d^3\mathrm{Li}_3\left(ie^{i(e+fx)}\right)}{af^4}$$

```
[Out] -I*(d*x+c)^3/a/f-(d*x+c)^3*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f+6*d*(d*x+c)^2*ln(1
-I*exp(I*(f*x+e)))/a/f^2-12*I*d^2*(d*x+c)*polylog(2,I*exp(I*(f*x+e)))/a/f^3
+12*d^3*polylog(3,I*exp(I*(f*x+e)))/a/f^4
```

**Rubi** [A]
time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3399, 4269, 3798, 2221, 2611, 2320, 6724}

$$-\frac{12id^2(c+dx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{af^4}+\frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}-\frac{i(c+dx)^3}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3)/(a*f) - ((c + d*x)^3*Cot[e/2 + Pi/4 + (f*x)/2])/(a*f) +
(6*d*(c + d*x)^2*Log[1 - I*E^(I*(e + f*x))])/(a*f^2) - ((12*I)*d^2*(c + d*x
)*PolyLog[2, I*E^(I*(e + f*x))])/(a*f^3) + (12*d^3*PolyLog[3, I*E^(I*(e + f
*x))])/(a*f^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx = \frac{\int (c+dx)^3 \csc^2\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx}{2a}$$

$$= -\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(3d)\int (c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\,dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(6d)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1-ie^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2} - \frac{(1}{}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2} - \frac{12}{}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2} - \frac{12}{}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2} - \frac{12}{}$$

**Mathematica** [A]
time = 0.74, size = 126, normalized size = 0.85

$$\frac{-12id^2f(c+dx)\text{Li}_2\left(ie^{i(e+fx)}\right)+12d^3\text{Li}_3\left(ie^{i(e+fx)}\right)+f^2(c+dx)^2\left(-if(c+dx)+6d\log\left(1-ie^{i(e+fx)}\right)+f(c+dx)\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}{af^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((-12*I)*d^2*f*(c + d*x)*PolyLog[2, I*E^(I*(e + f*x))] + 12*d^3*PolyLog[3,
I*E^(I*(e + f*x))] + f^2*(c + d*x)^2*((-I)*f*(c + d*x) + 6*d*Log[1 - I*E^(I
*(e + f*x))] + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]))/(a*f^4)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(131) = 262.
time = 0.12, size = 484, normalized size = 3.27

| method | result |
|---|---|
| risch | $-\frac{2(d^3x^3+3cd^2x^2+3c^2dx+c^3)}{fa(e^{i(fx+e)}+i)} - \frac{6icd^2x^2}{af} - \frac{12icd^2\,\text{polylog}(2,ie^{i(fx+e)})}{af^3} - \frac{12icd^2ex}{af^2} - \frac{6d^3e^2\ln(e^{i(fx+e)})}{af^4} + \frac{6d^3\ln(1-ie^{}}{af}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))+I)-6*I/a/f*c*d^2
*x^2-12*I/a/f^2*c*d^2*e*x+6*I/a/f^3*d^3*e^2*x-6/a/f^4*d^3*e^2*ln(exp(I*(f*x
+e)))+6/a/f^2*d^3*ln(1-I*exp(I*(f*x+e)))*x^2-6/a/f^4*d^3*ln(1-I*exp(I*(f*x+
e)))*e^2-12/a/f^3*c*d^2*e*ln(exp(I*(f*x+e))+I)+12*d^3*polylog(3,I*exp(I*(f*
x+e)))/a/f^4+6/a/f^2*ln(exp(I*(f*x+e))+I)*c^2*d-6*I/a/f^3*c*d^2*e^2-12*I/a/
f^3*d^3*polylog(2,I*exp(I*(f*x+e)))*x+12/a/f^3*c*d^2*e*ln(exp(I*(f*x+e)))-6
/a/f^2*ln(exp(I*(f*x+e)))*c^2*d+6/a/f^4*d^3*e^2*ln(exp(I*(f*x+e))+I)-12*I/a
/f^3*c*d^2*polylog(2,I*exp(I*(f*x+e)))-2*I/a/f*d^3*x^3+4*I/a/f^4*d^3*e^3+12
/a/f^2*c*d^2*ln(1-I*exp(I*(f*x+e)))*x+12/a/f^3*c*d^2*ln(1-I*exp(I*(f*x+e)))
*e
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1054 vs. $2(129) = 258$.
time = 0.39, size = 1054, normalized size = 7.12

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] (6*(2*(f*x + e)*cos(f*x + e) - (cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x
 + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1))*c*d^2
*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 + 2*a*f^2*sin(f*x + e) + a*
f^2) - 3*(2*(f*x + e)*cos(f*x + e) - (cos(f*x + e)^2 + sin(f*x + e)^2 + 2*s
in(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1))
*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*sin(f*x + e) + a*f)
 - 6*c*d^2*e^2/(a*f^2 + a*f^2*sin(f*x + e)/(cos(f*x + e) + 1)) + 6*c^2*d*e/
(a*f + a*f*sin(f*x + e)/(cos(f*x + e) + 1)) - 2*c^3/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1)) + (-2*I*d^3*e^3 + 6*(d^3*cos(f*x + e)*e^2 + I*d^3*e^2*sin(
f*x + e) + I*d^3*e^2)*arctan2(sin(f*x + e) + 1, cos(f*x + e)) - 6*(I*(f*x +
 e)^2*d^3 + 2*(I*c*d^2*f - I*d^3*e)*(f*x + e) + ((f*x + e)^2*d^3 + 2*(c*d^2
*f - d^3*e)*(f*x + e))*cos(f*x + e) + (I*(f*x + e)^2*d^3 + 2*(I*c*d^2*f - I
*d^3*e)*(f*x + e))*sin(f*x + e))*arctan2(cos(f*x + e), sin(f*x + e) + 1) -
2*((f*x + e)^3*d^3 + 3*(f*x + e)*d^3*e^2 + 3*(c*d^2*f - d^3*e)*(f*x + e)^2)
*cos(f*x + e) - 12*(I*(f*x + e)*d^3 + I*c*d^2*f - I*d^3*e + ((f*x + e)*d^3
+ c*d^2*f - d^3*e)*cos(f*x + e) + (I*(f*x + e)*d^3 + I*c*d^2*f - I*d^3*e)*s
in(f*x + e))*dilog(I*e^(I*f*x + I*e)) + 3*((f*x + e)^2*d^3 + d^3*e^2 + 2*(c
*d^2*f - d^3*e)*(f*x + e) - (I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(I*c*d^2*f -
 I*d^3*e)*(f*x + e))*cos(f*x + e) + ((f*x + e)^2*d^3 + d^3*e^2 + 2*(c*d^2*f
 - d^3*e)*(f*x + e))*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*
sin(f*x + e) + 1) - 12*(I*d^3*cos(f*x + e) - d^3*sin(f*x + e) - d^3)*polylo
g(3, I*e^(I*f*x + I*e)) - 2*(I*(f*x + e)^3*d^3 + 3*I*(f*x + e)*d^3*e^2 + 3*
(I*c*d^2*f - I*d^3*e)*(f*x + e)^2)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a
*f^3*sin(f*x + e) + a*f^3))/f
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(129) = 258$.
time = 0.36, size = 951, normalized size = 6.43

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 + (d^3*f^3*x^3 +
3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*cos(f*x + e) + 6*(I*d^3*f*x + I*
c*d^2*f + (I*d^3*f*x + I*c*d^2*f)*cos(f*x + e) + (I*d^3*f*x + I*c*d^2*f)*si
n(f*x + e))*dilog(I*cos(f*x + e) - sin(f*x + e)) + 6*(-I*d^3*f*x - I*c*d^2*
f + (-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e) + (-I*d^3*f*x - I*c*d^2*f)*sin(f*
x + e))*dilog(-I*cos(f*x + e) - sin(f*x + e)) - 3*(c^2*d*f^2 - 2*c*d^2*f*e
+ d^3*e^2 + (c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2)*cos(f*x + e) + (c^2*d*f^2 -
 2*c*d^2*f*e + d^3*e^2)*sin(f*x + e))*log(cos(f*x + e) + I*sin(f*x + e) + I
) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2 + (d^3*f^2*x^2 +
 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x + e) + (d^3*f^2*x^2 + 2*c*d
^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*sin(f*x + e))*log(I*cos(f*x + e) + sin(f*
x + e) + 1) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2 + (d^3
*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x + e) + (d^3*f^2*x
^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*sin(f*x + e))*log(-I*cos(f*x +
e) + sin(f*x + e) + 1) - 3*(c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + (c^2*d*f^2
- 2*c*d^2*f*e + d^3*e^2)*cos(f*x + e) + (c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2)
*sin(f*x + e))*log(-cos(f*x + e) + I*sin(f*x + e) + I) - 6*(d^3*cos(f*x + e
) + d^3*sin(f*x + e) + d^3)*polylog(3, I*cos(f*x + e) - sin(f*x + e)) - 6*(
d^3*cos(f*x + e) + d^3*sin(f*x + e) + d^3)*polylog(3, -I*cos(f*x + e) - sin
(f*x + e)) - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*sin(
f*x + e))/(a*f^4*cos(f*x + e) + a*f^4*sin(f*x + e) + a*f^4)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3}{\sin(e+fx)+1}\,dx + \int \frac{d^3x^3}{\sin(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\sin(e+fx)+1}\,dx + \int \frac{3c^2dx}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c**3/(sin(e + f*x) + 1), x) + Integral(d**3*x**3/(sin(e + f*x) +
1), x) + Integral(3*c*d**2*x**2/(sin(e + f*x) + 1), x) + Integral(3*c**2*d*
x/(sin(e + f*x) + 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*sin(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a + a*sin(e + f*x)), x)
```

# 3.108 $\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=113

$$-\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{4id^2\text{Li}_2\left(ie^{i(e+fx)}\right)}{af^3}$$

```
[Out] -I*(d*x+c)^2/a/f-(d*x+c)^2*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f+4*d*(d*x+c)*ln(1-I
*exp(I*(f*x+e)))/a/f^2-4*I*d^2*polylog(2,I*exp(I*(f*x+e)))/a/f^3
```

**Rubi** [A]
time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3399, 4269, 3798, 2221, 2317, 2438}

$$-\frac{4id^2\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{af^3}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}-\frac{i(c+dx)^2}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) - ((c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2])/(a*f) +
(4*d*(c + d*x)*Log[1 - I*E^(I*(e + f*x))])/(a*f^2) - ((4*I)*d^2*PolyLog[2,
I*E^(I*(e + f*x))])/(a*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx = \frac{\int (c+dx)^2\csc^2\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx}{2a}$$

$$= -\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{(2d)\int (c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\,dx}{af}$$

$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{(4d)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1-ie^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}$$

$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{(4d^2$$

$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}+\frac{(4id$$

$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{4id^2$$

**Mathematica [A]**
time = 0.49, size = 94, normalized size = 0.83

$$\frac{-4id^2\text{Li}_2\left(ie^{i(e+fx)}\right)+f(c+dx)\left(-if(c+dx)+4d\log\left(1-ie^{i(e+fx)}\right)+f(c+dx)\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((-4*I)*d^2*PolyLog[2, I*E^(I*(e + f*x))] + f*(c + d*x)*((-I)*f*(c + d*x) +
 4*d*Log[1 - I*E^(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]))/(
a*f^3)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(99) = 198$.
time = 0.07, size = 254, normalized size = 2.25

| method | result |
|---|---|
| risch | $-\frac{2(d^2x^2+2cdx+c^2)}{fa(e^{i(fx+e)}+i)} - \frac{4\ln(e^{i(fx+e)})cd}{a f^2} + \frac{4\ln(e^{i(fx+e)}+i)cd}{a f^2} - \frac{2id^2x^2}{af} - \frac{4id^2ex}{a f^2} - \frac{2id^2e^2}{a f^3} + \frac{4d^2\ln(1-ie^{i(fx+e)})x}{a f^2} +$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))+I)-4/a/f^2*ln(exp(I*(f*x+e)))*
c*d+4/a/f^2*ln(exp(I*(f*x+e))+I)*c*d-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-2*I/
a/f^3*d^2*e^2+4/a/f^2*d^2*ln(1-I*exp(I*(f*x+e)))*x+4/a/f^3*d^2*ln(1-I*exp(I
*(f*x+e)))*e-4*I*d^2*polylog(2,I*exp(I*(f*x+e)))/a/f^3+4/a/f^3*d^2*e*ln(exp
(I*(f*x+e)))-4/a/f^3*d^2*e*ln(exp(I*(f*x+e))+I)
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(97) = 194$.
time = 0.35, size = 329, normalized size = 2.91

$$-\frac{2\left(-ic^2f^2 - 2\left(cdf\cos(fx+e) + icdf\sin(fx+e) + icdf\right)\arctan\left(\sin(fx+e)+1, \cos(fx+e)\right) + 2\left(d^2fx\cos(fx+e) + id^2fx\sin(fx+e) + id^2fx\right)\arctan\left(\cos(fx+e), \sin(fx+e)+1\right) + \left(d^2f^2x^2 + 2cdf^2x\right)\cos(fx+e) + 2\left(d^2\cos(fx+e) + id^2\sin(fx+e) + id^2\right)\mathrm{Li}_2\left(ie^{(ifx+ie)}\right) - \left(d^2fx + cdf - \left(id^2fx + icdf\right)\cos(fx+e) + \left(d^2fx + cdf\right)\sin(fx+e)\right)\log\left(\cos(fx+e)^2 + \sin(fx+e)^2 + 2\sin(fx+e) + 1\right) + \left(id^2f^2x^2 + 2icdf^2x\right)\sin(fx+e)\right)}{-iaf^3\cos(fx+e) + af^3\sin(fx+e) + af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(-I*c^2*f^2 - 2*(c*d*f*cos(f*x + e) + I*c*d*f*sin(f*x + e) + I*c*d*f)*ar
ctan2(sin(f*x + e) + 1, cos(f*x + e)) + 2*(d^2*f*x*cos(f*x + e) + I*d^2*f*x
*sin(f*x + e) + I*d^2*f*x)*arctan2(cos(f*x + e), sin(f*x + e) + 1) + (d^2*f
^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) + 2*(d^2*cos(f*x + e) + I*d^2*sin(f*x +
e) + I*d^2)*dilog(I*e^(I*f*x + I*e)) - (d^2*f*x + c*d*f - (I*d^2*f*x + I*c*
d*f)*cos(f*x + e) + (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 + si
n(f*x + e)^2 + 2*sin(f*x + e) + 1) + (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin(f*
x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + a*f^3)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(97) = 194$.
time = 0.37, size = 533, normalized size = 4.72

$$-\frac{d^2f^2x^2 + 2cdf^2x + c^2f^2 + \left(d^2f^2x^2 + 2cdf^2x + c^2f^2\right)\cos(fx+e) + 2\left(id^2\cos(fx+e) + id^2\sin(fx+e) + id^2\right)\mathrm{Li}_2\left(i\cos(fx+e) - \sin(fx+e)\right) + 2\left(-id^2\cos(fx+e) - id^2\sin(fx+e) - id^2\right)\mathrm{Li}_2\left(-i\cos(fx+e) - \sin(fx+e)\right) - 2\left(cdf - d^2e + \left(cdf - d^2e\right)\cos(fx+e) + \left(cdf - d^2e\right)\sin(fx+e)\right)\log\left(\cos(fx+e) + i\sin(fx+e) + i\right) - 2\left(d^2fx + d^2e + \left(d^2fx + d^2e\right)\cos(fx+e) + \left(d^2fx + d^2e\right)\sin(fx+e)\right)\log\left(i\cos(fx+e) + \sin(fx+e) + 1\right) - 2\left(d^2fx + d^2e + \left(d^2fx + d^2e\right)\cos(fx+e) + \left(d^2fx + d^2e\right)\sin(fx+e)\right)\log\left(-i\cos(fx+e) + \sin(fx+e) + 1\right) - 2\left(cdf - d^2e + \left(cdf - d^2e\right)\cos(fx+e) + \left(cdf - d^2e\right)\sin(fx+e)\right)\log\left(-\cos(fx+e) + i\sin(fx+e) + i\right) - \left(d^2f^2x^2 + 2cdf^2x + c^2f^2\right)\sin(fx+e)}{af^3\cos(fx+e) + af^3\sin(fx+e) + af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^
2)*cos(f*x + e) + 2*(I*d^2*cos(f*x + e) + I*d^2*sin(f*x + e) + I*d^2)*dilog
(I*cos(f*x + e) - sin(f*x + e)) + 2*(-I*d^2*cos(f*x + e) - I*d^2*sin(f*x +
e) - I*d^2)*dilog(-I*cos(f*x + e) - sin(f*x + e)) - 2*(c*d*f - d^2*e + (c*d
*f - d^2*e)*cos(f*x + e) + (c*d*f - d^2*e)*sin(f*x + e))*log(cos(f*x + e) +
 I*sin(f*x + e) + I) - 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e)
+ (d^2*f*x + d^2*e)*sin(f*x + e))*log(I*cos(f*x + e) + sin(f*x + e) + 1) -
2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e) + (d^2*f*x + d^2*e)*sin
(f*x + e))*log(-I*cos(f*x + e) + sin(f*x + e) + 1) - 2*(c*d*f - d^2*e + (c*
d*f - d^2*e)*cos(f*x + e) + (c*d*f - d^2*e)*sin(f*x + e))*log(-cos(f*x + e)
 + I*sin(f*x + e) + I) - (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*sin(f*x + e)
)/(a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + a*f^3)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2}{\sin(e+fx)+1}\,dx + \int \frac{d^2x^2}{\sin(e+fx)+1}\,dx + \int \frac{2cdx}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c**2/(sin(e + f*x) + 1), x) + Integral(d**2*x**2/(sin(e + f*x) +
1), x) + Integral(2*c*d*x/(sin(e + f*x) + 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{a+a\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*sin(e + f*x)),x)
```

```
[Out] int((c + d*x)^2/(a + a*sin(e + f*x)), x)
```

# 3.109 $\int \frac{c+dx}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=60

$$-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}$$

```
[Out] -(d*x+c)*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f+2*d*ln(sin(1/2*e+1/4*Pi+1/2*f*x))/a/
f^2
```

**Rubi** [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3399, 4269, 3556}

$$\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{af^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/(a*f)) + (2*d*Log[Sin[e/2 + Pi/4 +
(f*x)/2]])/(a*f^2)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+a\sin(e+fx)}\,dx = \frac{\int (c+dx)\csc^2\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx}{2a}$$
$$= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{d\int \cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\,dx}{af}$$
$$= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}$$

**Mathematica** [A]
time = 0.11, size = 51, normalized size = 0.85

$$\frac{2d\log\left(\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)+f(c+dx)\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (2*d*Log[Cos[(2*e - Pi + 2*f*x)/4]] + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]
)/(a*f^2)
```

**Maple** [C] Result contains complex when optimal does not.
time = 0.06, size = 73, normalized size = 1.22

| method | result | size |
|---|---|---|
| risch | $-\frac{2idx}{af}-\frac{2ide}{a\,f^2}-\frac{2(dx+c)}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2d\ln\left(e^{i(fx+e)}+i\right)}{a\,f^2}$ | 73 |
| norman | $\frac{\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}-\frac{dx}{fa}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{a\,f^2}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a\,f^2}$ | 107 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*d/a/f*x-2*I*d/a/f^2*e-2*(d*x+c)/f/a/(exp(I*(f*x+e))+I)+2*d/a/f^2*ln(ex
p(I*(f*x+e))+I)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(50) = 100.
time = 0.30, size = 185, normalized size = 3.08

$$-\frac{\frac{\left(2\,(fx+e)\cos(fx+e)-\left(\cos(fx+e)^2+\sin(fx+e)^2+2\,\sin(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2+2\,\sin(fx+e)+1\right)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2+2\,af\sin(fx+e)+af}-\frac{2\,de}{af+\frac{af\sin(fx+e)}{\cos(fx+e)+1}}+\frac{2\,c}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -((2*(f*x + e)*cos(f*x + e) - (cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x
+ e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1))*d/(a*f
*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*sin(f*x + e) + a*f) - 2*d*e/(a
*f + a*f*sin(f*x + e)/(cos(f*x + e) + 1)) + 2*c/(a + a*sin(f*x + e)/(cos(f*
x + e) + 1)))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(50) = 100$.
time = 0.42, size = 107, normalized size = 1.78

$$-\frac{dfx + cf + (dfx + cf)\cos(fx + e) - (d\cos(fx + e) + d\sin(fx + e) + d)\log(\sin(fx + e) + 1) - (dfx + cf)\sin(fx + e)}{af^2\cos(fx + e) + af^2\sin(fx + e) + af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*f*x + c*f + (d*f*x + c*f)*cos(f*x + e) - (d*cos(f*x + e) + d*sin(f*x +
e) + d)*log(sin(f*x + e) + 1) - (d*f*x + c*f)*sin(f*x + e))/(a*f^2*cos(f*x
+ e) + a*f^2*sin(f*x + e) + a*f^2)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(46) = 92$.
time = 0.45, size = 272, normalized size = 4.53

$$\begin{cases} -\frac{2cf}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} + \frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} - \frac{dfx}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} + \frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} + \frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} - \frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} - \frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} & \text{for } f \neq 0 \\ \frac{cx+\frac{dx^2}{2}}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c*f/(a*f**2*tan(e/2 + f*x/2) + a*f**2) + d*f*x*tan(e/2 + f*x/
2)/(a*f**2*tan(e/2 + f*x/2) + a*f**2) - d*f*x/(a*f**2*tan(e/2 + f*x/2) + a*
f**2) + 2*d*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 + f*
x/2) + a*f**2) + 2*d*log(tan(e/2 + f*x/2) + 1)/(a*f**2*tan(e/2 + f*x/2) + a
*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 +
f*x/2) + a*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)/(a*f**2*tan(e/2 + f*x/2)
+ a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*sin(e) + a), True))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(50) = 100$.
time = 1.57, size = 696, normalized size = 11.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(d*f*x*tan(1/2*f*x)*tan(1/2*e) + d*f*x*tan(1/2*f*x) + d*f*x*tan(1/2*e) + c
*f*tan(1/2*f*x)*tan(1/2*e) - d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1
/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*t
an(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2
 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(ta
n(1/2*e)^2 + 1))*tan(1/2*f*x)*tan(1/2*e) - d*f*x + c*f*tan(1/2*f*x) + d*log
(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f
*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan
(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2
 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x) + c*
f*tan(1/2*e) + d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(
1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*
tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f
*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1
))*tan(1/2*e) - c*f + d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)
^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*
f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*ta
n(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e
)^2 + 1)))/(a*f^2*tan(1/2*f*x)*tan(1/2*e) - a*f^2*tan(1/2*f*x) - a*f^2*tan(
1/2*e) - a*f^2)
```

**Mupad [B]**
time = 1.04, size = 66, normalized size = 1.10

$$\frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{a\,f^2}-\frac{2\,\left(c+d\,x\right)}{a\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}-\frac{d\,x\,2\mathrm{i}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*sin(e + f*x)),x)
```

```
[Out] (2*d*log(exp(e*1i)*exp(f*x*1i) + 1i))/(a*f^2) - (2*(c + d*x))/(a*f*(exp(e*1
i + f*x*1i) + 1i)) - (d*x*2i)/(a*f)
```

# 3.110 $\int \frac{1}{(c+dx)(a+a\sin(e+fx))} dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+a\sin(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Sin[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a + a*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx$$

**Mathematica** [A]
time = 3.33, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])), x]
```

**Maple** [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+a\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f*
x + a*c*d*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x + a*c*d*f)*sin(f*x + e))*integra
te(cos(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*a*c
*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*si
n(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*sin(f*x + e)), x) +
cos(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (a*d*f*
x + a*c*f)*sin(f*x + e)^2 + 2*(a*d*f*x + a*c*f)*sin(f*x + e))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (a*d*x + a*c)*sin(f*x + e)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sin(e+fx)+c+dx\sin(e+fx)+dx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral(1/(c*sin(e + f*x) + c + d*x*sin(e + f*x) + d*x), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*sin(f*x + e) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+a\,\sin\left(e+f\,x\right))\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c + d*x)), x)
```

# 3.111 $\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+a\sin(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+a*sin(f*x+e)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Sin[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$$

**Mathematica** [A]
time = 3.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])), x]
```

**Maple** [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+a\sin(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*sin(f*x+e)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*f
*x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*
sin(f*x + e)^2 + 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*sin(f*x + e))*
integrate(cos(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c
^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x +
e)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f*x +
e)^2 + 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f*x
+ e)), x) + cos(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x
^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a
*c^2*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*sin(f*x +
e))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*sin(f*x + e)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2 \sin{(e+fx)}+c^2+2cdx\sin{(e+fx)}+2cdx+d^2x^2\sin{(e+fx)}+d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral(1/(c**2*sin(e + f*x) + c**2 + 2*c*d*x*sin(e + f*x) + 2*c*d*x + d**
2*x**2*sin(e + f*x) + d**2*x**2), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*sin(f*x + e) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c + d*x)^2), x)
```

# 3.112 $\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=309

$$-\frac{i(c+dx)^3}{3a^2f}-\frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f}-\frac{d(c+dx)^2\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2}-(c$$

```
[Out] -1/3*I*(d*x+c)^3/a^2/f-2*d^2*(d*x+c)*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f^3-1/3*
(d*x+c)^3*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f-1/2*d*(d*x+c)^2*csc(1/2*e+1/4*Pi+
1/2*f*x)^2/a^2/f^2-1/6*(d*x+c)^3*cot(1/2*e+1/4*Pi+1/2*f*x)*csc(1/2*e+1/4*Pi
+1/2*f*x)^2/a^2/f+2*d*(d*x+c)^2*ln(1-I*exp(I*(f*x+e)))/a^2/f^2+4*d^3*ln(sin
(1/2*e+1/4*Pi+1/2*f*x))/a^2/f^4-4*I*d^2*(d*x+c)*polylog(2,I*exp(I*(f*x+e)))
/a^2/f^3+4*d^3*polylog(3,I*exp(I*(f*x+e)))/a^2/f^4
```

**Rubi** [A]
time = 0.25, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3399, 4271, 4269, 3556, 3798, 2221, 2611, 2320, 6724}

$$-\frac{4id^2(c+dx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{a^2f^3}+\frac{4d^3\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{a^2f^4}-\frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{a^2f^3}+\frac{2d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{a^2f^2}-\frac{d(c+dx)^2\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{2a^2f^2}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{6a^2f}-\frac{i(c+dx)^3}{3a^2f}+\frac{4d^3\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{a^2f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((-1/3*I)*(c + d*x)^3)/(a^2*f) - (2*d^2*(c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2]
)/(a^2*f^3) - ((c + d*x)^3*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f) - (d*(c + d
*x)^2*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(2*a^2*f^2) - ((c + d*x)^3*Cot[e/2 + Pi/
4 + (f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(6*a^2*f) + (2*d*(c + d*x)^2*Log[
1 - I*E^(I*(e + f*x))])/(a^2*f^2) + (4*d^3*Log[Sin[e/2 + Pi/4 + (f*x)/2]])/
(a^2*f^4) - ((4*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(e + f*x))])/(a^2*f^3) +
 (4*d^3*PolyLog[3, I*E^(I*(e + f*x))])/(a^2*f^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx = \frac{\int (c+dx)^3 \csc^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx}{4a^2}$$

$$= -\frac{d(c+dx)^2\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right.}{6a^2f}$$

$$= -\frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)^2\csc}{2a}$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} -$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} -$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} -$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} -$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} -$$

## Mathematica [A]

time = 1.34, size = 257, normalized size = 0.83

$$\frac{-2if(c+dx)^3+12d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)+\frac{24d^3\log\left(\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}{f^2}+\frac{24d^2\left(-if(c+dx)\text{Li}_2\left(ie^{i(e+fx)}\right)+d\text{Li}_3\left(ie^{i(e+fx)}\right)\right)}{f^2}-3d(c+dx)^2\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)+\frac{12d^2(c+dx)\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{f}+2f(c+dx)^3\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)+f(c+dx)^3\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{6a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((-2*I)*f*(c + d*x)^3 + 12*d*(c + d*x)^2*Log[1 - I*E^(I*(e + f*x))] + (24*d
^3*Log[Cos[(2*e - Pi + 2*f*x)/4]])/f^2 + (24*d^2*((-I)*f*(c + d*x)*PolyLog[
2, I*E^(I*(e + f*x))] + d*PolyLog[3, I*E^(I*(e + f*x))]))/f^2 - 3*d*(c + d*
x)^2*Sec[(2*e - Pi + 2*f*x)/4]^2 + (12*d^2*(c + d*x)*Tan[(2*e - Pi + 2*f*x)
```

```
/4])/f + 2*f*(c + d*x)^3*Tan[(2*e - Pi + 2*f*x)/4] + f*(c + d*x)^3*Sec[(2*e
 - Pi + 2*f*x)/4]^2*Tan[(2*e - Pi + 2*f*x)/4])/(6*a^2*f^2)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(254) = 508.
time = 0.94, size = 807, normalized size = 2.61

| method | result |
|---|---|
| risch | $-\frac{2i\left(3ic^2d f^2x+3d^3f^2x^3e^{i(fx+e)}+6ifcd^2x\,e^{i(fx+e)}+3ifc^2d\,e^{i(fx+e)}+9cd^2f^2x^2e^{i(fx+e)}+3fd^3x^2e^{2i(fx+e)}-6id^3x\,e^{2i(fx+e)}+ic^3f\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*(3*I*c^2*d*f^2*x+3*d^3*f^2*x^3*exp(I*(f*x+e))+6*I*f*c*d^2*x*exp(I*(f
*x+e))+3*I*f*c^2*d*exp(I*(f*x+e))+9*c*d^2*f^2*x^2*exp(I*(f*x+e))+3*f*d^3*x^
2*exp(2*I*(f*x+e))-6*I*d^3*x*exp(2*I*(f*x+e))+I*c^3*f^2+6*I*c*d^2+9*c^2*d*f
^2*x*exp(I*(f*x+e))+6*f*c*d^2*x*exp(2*I*(f*x+e))-6*I*c*d^2*exp(2*I*(f*x+e))
+6*I*d^3*x+3*I*c*d^2*f^2*x^2+3*c^3*f^2*exp(I*(f*x+e))+3*f*c^2*d*exp(2*I*(f*
x+e))+I*d^3*f^2*x^3+12*d^3*x*exp(I*(f*x+e))+3*I*f*d^3*x^2*exp(I*(f*x+e))+12
*c*d^2*exp(I*(f*x+e)))/(exp(I*(f*x+e))+I)^3/f^3/a^2+4/a^2/f^4*ln(exp(I*(f*x
+e))+I)*d^3-4/a^2/f^4*ln(exp(I*(f*x+e)))*d^3-2/3*I/a^2/f*d^3*x^3+2/a^2/f^2*
ln(1-I*exp(I*(f*x+e)))*d^3*x^2+2*I/a^2/f^3*d^3*e^2*x+4/3*I/a^2/f^4*d^3*e^3-
2*I/a^2/f*c*d^2*x^2-4*I/a^2/f^3*polylog(2,I*exp(I*(f*x+e)))*d^3*x-2*I/a^2/f
^3*c*d^2*e^2-2/a^2/f^4*ln(1-I*exp(I*(f*x+e)))*d^3*e^2+2/a^2/f^4*ln(exp(I*(f
*x+e))+I)*d^3*e^2-2/a^2/f^4*ln(exp(I*(f*x+e)))*d^3*e^2-2/a^2/f^2*ln(exp(I*(
f*x+e)))*c^2*d+2/a^2/f^2*ln(exp(I*(f*x+e))+I)*c^2*d-4*I/a^2/f^2*c*d^2*e*x-4
*I/a^2/f^3*c*d^2*polylog(2,I*exp(I*(f*x+e)))+4/a^2/f^2*ln(1-I*exp(I*(f*x+e)
))*c*d^2*x+4/a^2/f^3*ln(1-I*exp(I*(f*x+e)))*c*d^2*e-4/a^2/f^3*ln(exp(I*(f*x
+e))+I)*c*d^2*e+4/a^2/f^3*ln(exp(I*(f*x+e)))*c*d^2*e+4*d^3*polylog(3,I*exp(
I*(f*x+e)))/a^2/f^4
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3779 vs. 2(257) = 514.
time = 0.94, size = 3779, normalized size = 12.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*c*d^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 2)*e^2/(a^2*f^2 + 3*a^2*f^2*sin(f*x + e)/(cos(f*x + e) + 1)
 + 3*a^2*f^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*f^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3) + 6*(2*(f*x + 3*(f*x + e)*sin(f*x + e) + cos(f*x + e)
```

+ e + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 2*(9*(f*x + e)*cos(f*x + e) - 6*
sin(f*x + e) - 1)*cos(2*f*x + 2*e) - 6*cos(2*f*x + 2*e)^2 - 6*cos(f*x + e)^
2 - (6*(cos(f*x + e) + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - cos(3*f*x + 3*e
)^2 + 6*(3*sin(f*x + e) + 1)*cos(2*f*x + 2*e) - 9*cos(2*f*x + 2*e)^2 - 9*co
s(f*x + e)^2 - 2*(3*cos(2*f*x + 2*e) - 3*sin(f*x + e) - 1)*sin(3*f*x + 3*e)
 - sin(3*f*x + 3*e)^2 - 18*cos(f*x + e)*sin(2*f*x + 2*e) - 9*sin(2*f*x + 2*
e)^2 - 9*sin(f*x + e)^2 - 6*sin(f*x + e) - 1)*log(cos(f*x + e)^2 + sin(f*x
+ e)^2 + 2*sin(f*x + e) + 1) - 2*(3*(f*x + e)*cos(f*x + e) + cos(2*f*x + 2*
e) - sin(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*sin(f*x + e) + 2
*cos(f*x + e) + e)*sin(2*f*x + 2*e) - 6*sin(2*f*x + 2*e)^2 - 6*sin(f*x + e)
^2 - 2*sin(f*x + e))*c*d^2*e/(a^2*f^2*cos(3*f*x + 3*e)^2 + 9*a^2*f^2*cos(2*
f*x + 2*e)^2 + 9*a^2*f^2*cos(f*x + e)^2 + a^2*f^2*sin(3*f*x + 3*e)^2 + 18*a
^2*f^2*cos(f*x + e)*sin(2*f*x + 2*e) + 9*a^2*f^2*sin(2*f*x + 2*e)^2 + 9*a^2
*f^2*sin(f*x + e)^2 + 6*a^2*f^2*sin(f*x + e) + a^2*f^2 - 6*(a^2*f^2*cos(f*x
 + e) + a^2*f^2*sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(3*a^2*f^2*sin(f*x +
 e) + a^2*f^2)*cos(2*f*x + 2*e) + 2*(3*a^2*f^2*cos(2*f*x + 2*e) - 3*a^2*f^2
*sin(f*x + e) - a^2*f^2)*sin(3*f*x + 3*e)) - 6*c^2*d*(3*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)*e/(a^2*f + 3*a^2*
f*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*f*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + a^2*f*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 3*(2*(f*x + 3*(f*x + e)
*sin(f*x + e) + cos(f*x + e) + e + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 2*(
9*(f*x + e)*cos(f*x + e) - 6*sin(f*x + e) - 1)*cos(2*f*x + 2*e) - 6*cos(2*f
*x + 2*e)^2 - 6*cos(f*x + e)^2 - (6*(cos(f*x + e) + sin(2*f*x + 2*e))*cos(3
*f*x + 3*e) - cos(3*f*x + 3*e)^2 + 6*(3*sin(f*x + e) + 1)*cos(2*f*x + 2*e)
- 9*cos(2*f*x + 2*e)^2 - 9*cos(f*x + e)^2 - 2*(3*cos(2*f*x + 2*e) - 3*sin(f
*x + e) - 1)*sin(3*f*x + 3*e) - sin(3*f*x + 3*e)^2 - 18*cos(f*x + e)*sin(2*
f*x + 2*e) - 9*sin(2*f*x + 2*e)^2 - 9*sin(f*x + e)^2 - 6*sin(f*x + e) - 1)*
log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 2*(3*(f*x + e)*
cos(f*x + e) + cos(2*f*x + 2*e) - sin(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x +
 3*(f*x + e)*sin(f*x + e) + 2*cos(f*x + e) + e)*sin(2*f*x + 2*e) - 6*sin(2*
f*x + 2*e)^2 - 6*sin(f*x + e)^2 - 2*sin(f*x + e))*c^2*d/(a^2*f*cos(3*f*x +
3*e)^2 + 9*a^2*f*cos(2*f*x + 2*e)^2 + 9*a^2*f*cos(f*x + e)^2 + a^2*f*sin(3*
f*x + 3*e)^2 + 18*a^2*f*cos(f*x + e)*sin(2*f*x + 2*e) + 9*a^2*f*sin(2*f*x +
 2*e)^2 + 9*a^2*f*sin(f*x + e)^2 + 6*a^2*f*sin(f*x + e) + a^2*f - 6*(a^2*f*
cos(f*x + e) + a^2*f*sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(3*a^2*f*sin(f*
x + e) + a^2*f)*cos(2*f*x + 2*e) + 2*(3*a^2*f*cos(2*f*x + 2*e) - 3*a^2*f*si
n(f*x + e) - a^2*f)*sin(3*f*x + 3*e)) + 2*c^3*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e
)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3) - 3*(-12*I*c*d^2*f - 2*d^3*(-I*e^3 - 6*I*e
) + 6*(d^3*(e^2 + 2)*cos(3*f*x + 3*e) - 3*d^3*(-I*e^2 - 2*I)*cos(2*f*x + 2*
e) - 3*d^3*(e^2 + 2)*cos(f*x + e) - d^3*(-I*e^2 - 2*I)*sin(3*f*x + 3*e) - 3
*d^3*(e^2 + 2)*sin(2*f*x + 2*e) - 3*d^3*(I*e^2 + 2*I)*sin(f*x + e) - d^3*(I
*e^2 + 2*I))*arctan2(sin(f*x + e) + 1, cos(f*x + e)) - 6*(-I*(f*x + e)^2*d^
3 + 2*(-I*c*d^2*f + I*d^3*e)*(f*x + e) + ((f*x + e)^2*d^3 + 2*(c*d^2*f - d^

```
3*e)*(f*x + e))*cos(3*f*x + 3*e) + 3*(I*(f*x + e)^2*d^3 + 2*(I*c*d^2*f - I*
d^3*e)*(f*x + e))*cos(2*f*x + 2*e) - 3*((f*x + e)^2*d^3 + 2*(c*d^2*f - d^3*
e)*(f*x + e))*cos(f*x + e) + (I*(f*x + e)^2*d^3 + 2*(I*c*d^2*f - I*d^3*e)*(
f*x + e))*sin(3*f*x + 3*e) - 3*((f*x + e)^2*d^3 + 2*(c*d^2*f - d^3*e)*(f*x
+ e))*sin(2*f*x + 2*e) + 3*(-I*(f*x + e)^2*d^3 + 2*(-I*c*d^2*f + I*d^3*e)*(
f*x + e))*sin(f*x + e))*arctan2(cos(f*x + e), sin(f*x + e) + 1) - 2*((f*x +
 e)^3*d^3 + 3*(f*x + e)*d^3*(e^2 + 2) + 3*(c*d^2*f - d^3*e)*(f*x + e)^2)*co
s(3*f*x + 3*e) - 6*(I*(f*x + e)^3*d^3 - 2*I*c*d^2*f + d^3*(e^2 + 2*I*e) + (
3*I*c*d^2*f + d^3*(-3*I*e + 1))*(f*x + e)^2 + (2*c*d^2*f + d^3*(3*I*e^2 - 2
*e + 4*I))*(f*x + e))*cos(2*f*x + 2*e) - 6*(I*(f*x + e)^2*d^3 + 4*c*d^2*f -
 d^3*(e^3 - I*e^2 + 4*e) + 2*(I*c*d^2*f + d^3*(-I*e - 1))*(f*x + e))*cos(f*
x + e) - 12*(-I*(f*x + e)*d^3 - I*c*d^2*f + I*d^3*e + ((f*x + e)*d^3 + c*d^
2*f - d^3*e)*cos(3*f*x + 3*e) + 3*(I*(f*x + e)*d^3 + I*c*d^2*f - I*d^3*e)*c
os(2*f*x + 2*e) - 3*((f*x + e)*d^3 + c*d^2*f - d^3*e)*cos(f*x + e) + (I*(f*
x + e)*d^3 + I*c*d^2*f - I*d^3*e)*sin(3*f*x + 3*e) - 3*((f*x + e)*d^3 + c*d
^2*f - d^3*e)*sin(2*f*x + 2*e) + 3*(-I*(f*x + e)*d^3 - I*c*d^2*f + I*d^3*e)
*sin(f*x + e))*dilog(I*e^(I*f*x + I*e)) - 3*((f*x + e)^2*d^3 + d^3*(e^2 + 2
) + 2*(c*d^2*f - d^3*e)*(f*x + e) + (I*(f*x + e...
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1764 vs. $2(257) = 514$.
time = 0.46, size = 1764, normalized size = 5.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(d^3*f^3*x^3 + c^3*f^3 + 3*c^2*d*f^2 + 3*(c*d^2*f^3 + d^3*f^2)*x^2 + (d
^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + c^3*f^3 + 6*c*d^2*f + 3*(c^2*d*f^3 + 2*d^3*f
)*x)*cos(f*x + e)^2 + 3*(c^2*d*f^3 + 2*c*d^2*f^2)*x + (2*d^3*f^3*x^3 + 2*c^
3*f^3 + 3*c^2*d*f^2 + 6*c*d^2*f + 3*(2*c*d^2*f^3 + d^3*f^2)*x^2 + 6*(c^2*d*
f^3 + c*d^2*f^2 + d^3*f)*x)*cos(f*x + e) + 6*(2*I*d^3*f*x + 2*I*c*d^2*f + (
-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e)^2 + (I*d^3*f*x + I*c*d^2*f)*cos(f*x +
e) + (2*I*d^3*f*x + 2*I*c*d^2*f + (I*d^3*f*x + I*c*d^2*f)*cos(f*x + e))*sin
(f*x + e))*dilog(I*cos(f*x + e) - sin(f*x + e)) + 6*(-2*I*d^3*f*x - 2*I*c*d
^2*f + (I*d^3*f*x + I*c*d^2*f)*cos(f*x + e)^2 + (-I*d^3*f*x - I*c*d^2*f)*co
s(f*x + e) + (-2*I*d^3*f*x - 2*I*c*d^2*f + (-I*d^3*f*x - I*c*d^2*f)*cos(f*x
 + e))*sin(f*x + e))*dilog(-I*cos(f*x + e) - sin(f*x + e)) - 3*(2*c^2*d*f^2
 - 4*c*d^2*f*e + 2*d^3*e^2 + 4*d^3 - (c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2
*d^3)*cos(f*x + e)^2 + (c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*d^3)*cos(f*x
+ e) + (2*c^2*d*f^2 - 4*c*d^2*f*e + 2*d^3*e^2 + 4*d^3 + (c^2*d*f^2 - 2*c*d^
2*f*e + d^3*e^2 + 2*d^3)*cos(f*x + e))*sin(f*x + e))*log(cos(f*x + e) + I*s
in(f*x + e) + I) - 3*(2*d^3*f^2*x^2 + 4*c*d^2*f^2*x + 4*c*d^2*f*e - 2*d^3*e
^2 - (d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x + e)^2 +
```

```
 (d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x + e) + (2*d^
3*f^2*x^2 + 4*c*d^2*f^2*x + 4*c*d^2*f*e - 2*d^3*e^2 + (d^3*f^2*x^2 + 2*c*d^
2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x + e))*sin(f*x + e))*log(I*cos(f*x
+ e) + sin(f*x + e) + 1) - 3*(2*d^3*f^2*x^2 + 4*c*d^2*f^2*x + 4*c*d^2*f*e -
 2*d^3*e^2 - (d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x
+ e)^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x + e)
 + (2*d^3*f^2*x^2 + 4*c*d^2*f^2*x + 4*c*d^2*f*e - 2*d^3*e^2 + (d^3*f^2*x^2
+ 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x + e))*sin(f*x + e))*log(-I
*cos(f*x + e) + sin(f*x + e) + 1) - 3*(2*c^2*d*f^2 - 4*c*d^2*f*e + 2*d^3*e^
2 + 4*d^3 - (c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*d^3)*cos(f*x + e)^2 + (c
^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*d^3)*cos(f*x + e) + (2*c^2*d*f^2 - 4*c
*d^2*f*e + 2*d^3*e^2 + 4*d^3 + (c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*d^3)*
cos(f*x + e))*sin(f*x + e))*log(-cos(f*x + e) + I*sin(f*x + e) + I) + 6*(d^
3*cos(f*x + e)^2 - d^3*cos(f*x + e) - 2*d^3 - (d^3*cos(f*x + e) + 2*d^3)*si
n(f*x + e))*polylog(3, I*cos(f*x + e) - sin(f*x + e)) + 6*(d^3*cos(f*x + e)
^2 - d^3*cos(f*x + e) - 2*d^3 - (d^3*cos(f*x + e) + 2*d^3)*sin(f*x + e))*po
lylog(3, -I*cos(f*x + e) - sin(f*x + e)) - (d^3*f^3*x^3 + c^3*f^3 - 3*c^2*d
*f^2 + 3*(c*d^2*f^3 - d^3*f^2)*x^2 + 3*(c^2*d*f^3 - 2*c*d^2*f^2)*x - (d^3*f
^3*x^3 + 3*c*d^2*f^3*x^2 + c^3*f^3 + 6*c*d^2*f + 3*(c^2*d*f^3 + 2*d^3*f)*x)
*cos(f*x + e))*sin(f*x + e))/(a^2*f^4*cos(f*x + e)^2 - a^2*f^4*cos(f*x + e)
 - 2*a^2*f^4 - (a^2*f^4*cos(f*x + e) + 2*a^2*f^4)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{d^3x^3}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{3c^2dx}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(c**3/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(d**3*x
**3/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(si
n(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(3*c**2*d*x/(sin(e + f*x)
**2 + 2*sin(e + f*x) + 1), x))/a**2
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*sin(e + f*x))^2,x)
[Out] \text{Hanged}
```

# 3.113 $\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=243

$$-\frac{i(c+dx)^2}{3a^2f}-\frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f}-\frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2}-\frac{(c+dx)^2\,\mathrm{c}}{}$$

```
[Out] -1/3*I*(d*x+c)^2/a^2/f-2/3*d^2*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f^3-1/3*(d*x+c
)^2*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f-1/3*d*(d*x+c)*csc(1/2*e+1/4*Pi+1/2*f*x)
^2/a^2/f^2-1/6*(d*x+c)^2*cot(1/2*e+1/4*Pi+1/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x
)^2/a^2/f+4/3*d*(d*x+c)*ln(1-I*exp(I*(f*x+e)))/a^2/f^2-4/3*I*d^2*polylog(2,
I*exp(I*(f*x+e)))/a^2/f^3
```

**Rubi** [A]
time = 0.18, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3399, 4271, 3852, 8, 4269, 3798, 2221, 2317, 2438}

$$-\frac{4id^2\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{3a^2f^3}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{3a^2f^2}-\frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f^2}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{6a^2f}-\frac{i(c+dx)^2}{3a^2f}-\frac{2d^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((-1/3*I)*(c + d*x)^2)/(a^2*f) - (2*d^2*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f
^3) - ((c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f) - (d*(c + d*x)*Csc[
e/2 + Pi/4 + (f*x)/2]^2)/(3*a^2*f^2) - ((c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/
2]*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(6*a^2*f) + (4*d*(c + d*x)*Log[1 - I*E^(I*(
e + f*x))])/(3*a^2*f^2) - (((4*I)/3)*d^2*PolyLog[2, I*E^(I*(e + f*x))])/(a^
2*f^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx = \frac{\int (c+dx)^2 \csc^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx}{4a^2}$$

$$= -\frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f}$$

$$= -\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{(c+dx)^2\cot\left(\right.}{}$$

$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx}{}$$

$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx}{}$$

$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx}{}$$

$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx}{}$$

**Mathematica** [A]
time = 1.53, size = 175, normalized size = 0.72

$$\frac{-2if(c+dx)\left(f(c+dx)+4id\log\left(1-ie^{i(e+fx)}\right)\right)-8id^2\operatorname{Li}_2\left(ie^{i(e+fx)}\right)+2(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)+f(c+dx)\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\left(-2d+f(c+dx)\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}{6a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((-2*I)*f*(c + d*x)*(f*(c + d*x) + (4*I)*d*Log[1 - I*E^(I*(e + f*x))]) - (8
*I)*d^2*PolyLog[2, I*E^(I*(e + f*x))] + 2*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 +
 f^2*x^2))*Tan[(2*e - Pi + 2*f*x)/4] + f*(c + d*x)*Sec[(2*e - Pi + 2*f*x)/4
]^2*(-2*d + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]))/(6*a^2*f^3)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(191) = 382.
time = 0.88, size = 421, normalized size = 1.73

| method | result |
|---|---|
| risch | $-\frac{2i\left(id^2x^2f^2+3d^2f^2x^2e^{i(fx+e)}+2icd\,f^2x+2if\,d^2x\,e^{i(fx+e)}+6cd\,f^2x\,e^{i(fx+e)}+2f\,d^2x\,e^{2i(fx+e)}+ic^2f^2+2ifcd\,e^{i(fx+e)}-2id^2e^{2i(f}\right.}{3\left(e^{i(fx+e)}+i\right)^3f^3a^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*(I*d^2*x^2*f^2+3*d^2*f^2*x^2*exp(I*(f*x+e))+2*I*c*d*f^2*x+2*I*f*d^2*
x*exp(I*(f*x+e))+6*c*d*f^2*x*exp(I*(f*x+e))+2*f*d^2*x*exp(2*I*(f*x+e))+I*c^
2*f^2+2*I*f*c*d*exp(I*(f*x+e))-2*I*d^2*exp(2*I*(f*x+e))+3*c^2*f^2*exp(I*(f*
x+e))+2*f*c*d*exp(2*I*(f*x+e))+2*I*d^2+4*d^2*exp(I*(f*x+e)))/(exp(I*(f*x+e)
)+I)^3/f^3/a^2-4/3/a^2/f^2*ln(exp(I*(f*x+e)))*c*d+4/3/a^2/f^2*ln(exp(I*(f*x
+e))+I)*c*d-2/3*I/a^2/f*d^2*x^2-4/3*I/a^2/f^2*d^2*e*x-2/3*I/a^2/f^3*d^2*e^2
+4/3/a^2/f^2*d^2*ln(1-I*exp(I*(f*x+e)))*x+4/3/a^2/f^3*d^2*ln(1-I*exp(I*(f*x
+e)))*e-4/3*I*d^2*polylog(2,I*exp(I*(f*x+e)))/a^2/f^3+4/3/a^2/f^3*d^2*e*ln(
exp(I*(f*x+e)))-4/3/a^2/f^3*d^2*e*ln(exp(I*(f*x+e))+I)
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(193) = 386$.
time = 0.60, size = 871, normalized size = 3.58

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2*(I*c^2*f^2 + 2*I*d^2 - 2*(c*d*f*cos(3*f*x + 3*e) + 3*I*c*d*f*cos(2*f*x +
 2*e) - 3*c*d*f*cos(f*x + e) + I*c*d*f*sin(3*f*x + 3*e) - 3*c*d*f*sin(2*f*x
 + 2*e) - 3*I*c*d*f*sin(f*x + e) - I*c*d*f)*arctan2(sin(f*x + e) + 1, cos(f
*x + e)) + 2*(d^2*f*x*cos(3*f*x + 3*e) + 3*I*d^2*f*x*cos(2*f*x + 2*e) - 3*d
^2*f*x*cos(f*x + e) + I*d^2*f*x*sin(3*f*x + 3*e) - 3*d^2*f*x*sin(2*f*x + 2*
e) - 3*I*d^2*f*x*sin(f*x + e) - I*d^2*f*x)*arctan2(cos(f*x + e), sin(f*x +
e) + 1) + (d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(3*f*x + 3*e) + (3*I*d^2*f^2*x^2 +
 2*c*d*f - 2*I*d^2 + 2*(3*I*c*d*f^2 + d^2*f)*x)*cos(2*f*x + 2*e) + (3*c^2*f
^2 + 2*I*d^2*f*x + 2*I*c*d*f + 4*d^2)*cos(f*x + e) + 2*(d^2*cos(3*f*x + 3*e
) + 3*I*d^2*cos(2*f*x + 2*e) - 3*d^2*cos(f*x + e) + I*d^2*sin(3*f*x + 3*e)
- 3*d^2*sin(2*f*x + 2*e) - 3*I*d^2*sin(f*x + e) - I*d^2)*dilog(I*e^(I*f*x +
 I*e)) + (d^2*f*x + c*d*f + (I*d^2*f*x + I*c*d*f)*cos(3*f*x + 3*e) - 3*(d^2
*f*x + c*d*f)*cos(2*f*x + 2*e) + 3*(-I*d^2*f*x - I*c*d*f)*cos(f*x + e) - (d
^2*f*x + c*d*f)*sin(3*f*x + 3*e) + 3*(-I*d^2*f*x - I*c*d*f)*sin(2*f*x + 2*e
) + 3*(d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 +
 2*sin(f*x + e) + 1) + (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin(3*f*x + 3*e) - (
3*d^2*f^2*x^2 - 2*I*c*d*f - 2*d^2 + 2*(3*c*d*f^2 - I*d^2*f)*x)*sin(2*f*x +
2*e) + (3*I*c^2*f^2 - 2*d^2*f*x - 2*c*d*f + 4*I*d^2)*sin(f*x + e))/(-3*I*a^
2*f^3*cos(3*f*x + 3*e) + 9*a^2*f^3*cos(2*f*x + 2*e) + 9*I*a^2*f^3*cos(f*x +
 e) + 3*a^2*f^3*sin(3*f*x + 3*e) + 9*I*a^2*f^3*sin(2*f*x + 2*e) - 9*a^2*f^3
*sin(f*x + e) - 3*a^2*f^3)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(193) = 386$.
time = 0.38, size = 940, normalized size = 3.87

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(d^2*f^2*x^2 + c^2*f^2 + 2*c*d*f + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2
 + 2*d^2)*cos(f*x + e)^2 + 2*(c*d*f^2 + d^2*f)*x + 2*(d^2*f^2*x^2 + c^2*f^2
 + c*d*f + d^2 + (2*c*d*f^2 + d^2*f)*x)*cos(f*x + e) + 2*(-I*d^2*cos(f*x +
e)^2 + I*d^2*cos(f*x + e) + 2*I*d^2 + (I*d^2*cos(f*x + e) + 2*I*d^2)*sin(f*
x + e))*dilog(I*cos(f*x + e) - sin(f*x + e)) + 2*(I*d^2*cos(f*x + e)^2 - I*
d^2*cos(f*x + e) - 2*I*d^2 + (-I*d^2*cos(f*x + e) - 2*I*d^2)*sin(f*x + e))*
dilog(-I*cos(f*x + e) - sin(f*x + e)) - 2*(2*c*d*f - (c*d*f - d^2*e)*cos(f*
x + e)^2 - 2*d^2*e + (c*d*f - d^2*e)*cos(f*x + e) + (2*c*d*f - 2*d^2*e + (c
*d*f - d^2*e)*cos(f*x + e))*sin(f*x + e))*log(cos(f*x + e) + I*sin(f*x + e)
 + I) - 2*(2*d^2*f*x - (d^2*f*x + d^2*e)*cos(f*x + e)^2 + 2*d^2*e + (d^2*f*
x + d^2*e)*cos(f*x + e) + (2*d^2*f*x + 2*d^2*e + (d^2*f*x + d^2*e)*cos(f*x
+ e))*sin(f*x + e))*log(I*cos(f*x + e) + sin(f*x + e) + 1) - 2*(2*d^2*f*x -
 (d^2*f*x + d^2*e)*cos(f*x + e)^2 + 2*d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e
) + (2*d^2*f*x + 2*d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e))*sin(f*x + e))*lo
g(-I*cos(f*x + e) + sin(f*x + e) + 1) - 2*(2*c*d*f - (c*d*f - d^2*e)*cos(f*
x + e)^2 - 2*d^2*e + (c*d*f - d^2*e)*cos(f*x + e) + (2*c*d*f - 2*d^2*e + (c
*d*f - d^2*e)*cos(f*x + e))*sin(f*x + e))*log(-cos(f*x + e) + I*sin(f*x + e
) + I) - (d^2*f^2*x^2 + c^2*f^2 - 2*c*d*f + 2*(c*d*f^2 - d^2*f)*x - (d^2*f^
2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2)*cos(f*x + e))*sin(f*x + e))/(a^2*f^3
*cos(f*x + e)^2 - a^2*f^3*cos(f*x + e) - 2*a^2*f^3 - (a^2*f^3*cos(f*x + e)
+ 2*a^2*f^3)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx+\int \frac{d^2x^2}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx+\int \frac{2cdx}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(c**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(d**2*x
**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(2*c*d*x/(sin(e +
f*x)**2 + 2*sin(e + f*x) + 1), x))/a**2
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F(-1)**]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*sin(e + f*x))^2,x)
[Out] \text{Hanged}
```

# 3.114 $\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=148

$$-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f}-\frac{d\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f}+\frac{2d\log\left(\sin\right.}{}$$

```
[Out] -1/3*(d*x+c)*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f-1/6*d*csc(1/2*e+1/4*Pi+1/2*f*x
)^2/a^2/f^2-1/6*(d*x+c)*cot(1/2*e+1/4*Pi+1/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x)
^2/a^2/f+2/3*d*ln(sin(1/2*e+1/4*Pi+1/2*f*x))/a^2/f^2
```

**Rubi** [A]
time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3399, 4270, 4269, 3556}

$$-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{6a^2f}-\frac{d\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{6a^2f^2}+\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{3a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*((c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/(a^2*f) - (d*Csc[e/2 + Pi/4 + (f
*x)/2]^2)/(6*a^2*f^2) - ((c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2]*Csc[e/2 + Pi/4
 + (f*x)/2]^2)/(6*a^2*f) + (2*d*Log[Sin[e/2 + Pi/4 + (f*x)/2]])/(3*a^2*f^2)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
```

```
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int (c+dx)\csc^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx}{4a^2}\\ &= -\frac{d\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f}+\frac{\int (c+d\ldots}{}\\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f}-\frac{d\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\ldots}{6a^2f}\\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f}-\frac{d\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\ldots}{6a^2f}\end{aligned}$$

**Mathematica [A]**
time = 0.62, size = 225, normalized size = 1.52

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(d\cos\left(\frac{1}{2}(e+fx)\right)\left(2+3e+3fx-6\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+\cos\left(\frac{3}{2}(e+fx)\right)\left(-de+2cf+dfx+2d\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+2\left(d+2de-3cf-dfx+d\cos(e+fx)\left(e+fx-2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-4d\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{6a^2f^2(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/6*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*Cos[(e + f*x)/2]*(2 + 3*e +
3*f*x - 6*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Cos[(3*(e + f*x))/2]*
(-(d*e) + 2*c*f + d*f*x + 2*d*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2
*(d + 2*d*e - 3*c*f - d*f*x + d*Cos[e + f*x]*(e + f*x - 2*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]]) - 4*d*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Si
n[(e + f*x)/2]))/(a^2*f^2*(1 + Sin[e + f*x])^2)
```

**Maple [A]**
time = 0.39, size = 169, normalized size = 1.14

| method | result |
|---|---|
| risch | $-\frac{2idx}{3a^2f}-\frac{2ide}{3a^2f^2}-\frac{2i\left(idfx+3dfx\,e^{i(fx+e)}+icf+id\,e^{i(fx+e)}+3cf\,e^{i(fx+e)}+d\,e^{2i(fx+e)}\right)}{3f^2\left(e^{i(fx+e)}+i\right)^3a^2}+\frac{2d\ln\left(e^{i(fx+e)}+i\right)}{3a^2f^2}$ |
| norman | $\frac{-\frac{4c}{3fa}-\frac{2dx}{3fa}+\frac{(-6cf+2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3a\,f^2}+\frac{(-6cf+2d)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a\,f^2}+\frac{2dx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3fa}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{3a^2f^2}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}\ldots\right.\right.}{3a^2f^2}$ |

| | |
|---|---|
| default | $-\frac{2\left(-\frac{c\left(\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{2f}+\frac{\frac{dx}{3f}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f^{2}}-\frac{d\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f^{2}}-\frac{dx\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{d\ln}{}\right.}{a^{2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2*(-1/2*c/f*(2/(tan(1/2*f*x+1/2*e)+1)^2-4/3/(tan(1/2*f*x+1/2*e)+1)^3-2
/(tan(1/2*f*x+1/2*e)+1))+(1/3/f*d*x-1/3*d/f^2*tan(1/2*f*x+1/2*e)-1/3*d/f^2*
tan(1/2*f*x+1/2*e)^2-1/3/f*d*x*tan(1/2*f*x+1/2*e)^3)/(tan(1/2*f*x+1/2*e)+1)
^3-1/3*d/f^2*ln(tan(1/2*f*x+1/2*e)+1)+1/6*d/f^2*ln(1+tan(1/2*f*x+1/2*e)^2))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 993 vs. $2(115) = 230$.
time = 0.33, size = 993, normalized size = 6.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(2*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 2)*e/(a^2*f + 3*a^2*f*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*f*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*f*sin(f*x + e)^3/(cos(f*x + e) +
1)^3) + (2*(f*x + 3*(f*x + e)*sin(f*x + e) + cos(f*x + e) + e + sin(2*f*x +
 2*e))*cos(3*f*x + 3*e) - 2*(9*(f*x + e)*cos(f*x + e) - 6*sin(f*x + e) - 1)
*cos(2*f*x + 2*e) - 6*cos(2*f*x + 2*e)^2 - 6*cos(f*x + e)^2 - (6*(cos(f*x +
 e) + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - cos(3*f*x + 3*e)^2 + 6*(3*sin(f*
x + e) + 1)*cos(2*f*x + 2*e) - 9*cos(2*f*x + 2*e)^2 - 9*cos(f*x + e)^2 - 2*
(3*cos(2*f*x + 2*e) - 3*sin(f*x + e) - 1)*sin(3*f*x + 3*e) - sin(3*f*x + 3*
e)^2 - 18*cos(f*x + e)*sin(2*f*x + 2*e) - 9*sin(2*f*x + 2*e)^2 - 9*sin(f*x
+ e)^2 - 6*sin(f*x + e) - 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*
x + e) + 1) - 2*(3*(f*x + e)*cos(f*x + e) + cos(2*f*x + 2*e) - sin(f*x + e)
)*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*sin(f*x + e) + 2*cos(f*x + e) + e
)*sin(2*f*x + 2*e) - 6*sin(2*f*x + 2*e)^2 - 6*sin(f*x + e)^2 - 2*sin(f*x +
e))*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f*x + 2*e)^2 + 9*a^2*f*cos(
f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 18*a^2*f*cos(f*x + e)*sin(2*f*x + 2
*e) + 9*a^2*f*sin(2*f*x + 2*e)^2 + 9*a^2*f*sin(f*x + e)^2 + 6*a^2*f*sin(f*x
 + e) + a^2*f - 6*(a^2*f*cos(f*x + e) + a^2*f*sin(2*f*x + 2*e))*cos(3*f*x +
 3*e) - 6*(3*a^2*f*sin(f*x + e) + a^2*f)*cos(2*f*x + 2*e) + 2*(3*a^2*f*cos(
2*f*x + 2*e) - 3*a^2*f*sin(f*x + e) - a^2*f)*sin(3*f*x + 3*e)) - 2*c*(3*sin
(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(
a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

**Fricas [A]**
time = 0.34, size = 217, normalized size = 1.47

$$\frac{dfx + (dfx + cf)\cos(fx+e)^2 + cf + (2\,dfx + 2\,cf + d)\cos(fx+e) + (d\cos(fx+e)^2 - d\cos(fx+e) - (d\cos(fx+e) + 2\,d)\sin(fx+e) - 2\,d)\log(\sin(fx+e)+1) - (dfx + cf - (dfx+cf)\cos(fx+e) - d)\sin(fx+e) + d}{3\,(a^2f^2\cos(fx+e)^2 - a^2f^2\cos(fx+e) - 2\,a^2f^2 - (a^2f^2\cos(fx+e) + 2\,a^2f^2)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(d*f*x + (d*f*x + c*f)*cos(f*x + e)^2 + c*f + (2*d*f*x + 2*c*f + d)*cos
(f*x + e) + (d*cos(f*x + e)^2 - d*cos(f*x + e) - (d*cos(f*x + e) + 2*d)*sin
(f*x + e) - 2*d)*log(sin(f*x + e) + 1) - (d*f*x + c*f - (d*f*x + c*f)*cos(f
*x + e) - d)*sin(f*x + e) + d)/(a^2*f^2*cos(f*x + e)^2 - a^2*f^2*cos(f*x +
e) - 2*a^2*f^2 - (a^2*f^2*cos(f*x + e) + 2*a^2*f^2)*sin(f*x + e))
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(122) = 244$.
time = 0.86, size = 1336, normalized size = 9.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c*f*tan(e/2 + f*x/2)**2/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*
a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2)
 - 6*c*f*tan(e/2 + f*x/2)/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*ta
n(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) - 4*c*f/(3*
a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f*
*2*tan(e/2 + f*x/2) + 3*a**2*f**2) + 2*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*
*2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(
e/2 + f*x/2) + 3*a**2*f**2) - 2*d*f*x/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*
a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2)
 + 2*d*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)**3/(3*a**2*f**2*tan(e/2 +
 f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2)
 + 3*a**2*f**2) + 6*d*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)**2/(3*a**2
*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*t
an(e/2 + f*x/2) + 3*a**2*f**2) + 6*d*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*
x/2)/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9
*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) + 2*d*log(tan(e/2 + f*x/2) + 1)/
(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2
*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)*tan(
e/2 + f*x/2)**3/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*
x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) - 3*d*log(tan(e/2 + f
*x/2)**2 + 1)*tan(e/2 + f*x/2)**2/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2
*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) - 3
```

```
*d*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)/(3*a**2*f**2*tan(e/2 + f*x
/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3
*a**2*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)/(3*a**2*f**2*tan(e/2 + f*x/2)*
*3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**
2*f**2) + 2*d*tan(e/2 + f*x/2)**2/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2
*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) + 2
*d*tan(e/2 + f*x/2)/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2
+ f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2), Ne(f, 0)), ((c*x
 + d*x**2/2)/(a*sin(e) + a)**2, True))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 3094 vs. $2(115) = 230$.
time = 2.43, size = 3094, normalized size = 20.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*d*f*x*tan(1/2*f*x)^3*tan(1/2*e)^3 + 2*c*f*tan(1/2*f*x)^3*tan(1/2*e)
^3 - d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2
*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)
^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + ta
n(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2
*f*x)^3*tan(1/2*e)^3 - 6*d*f*x*tan(1/2*f*x)^2*tan(1/2*e)^2 + 3*d*log(2*(tan
(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*t
an(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*
x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*ta
n(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)^3*tan(1/2*e
)^2 + 3*d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e)
- 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2
*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 +
 tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(
1/2*f*x)^2*tan(1/2*e)^3 + d*tan(1/2*f*x)^3*tan(1/2*e)^3 + 2*d*f*x*tan(1/2*f
*x)^3 + 6*d*f*x*tan(1/2*f*x)^2*tan(1/2*e) - 3*d*log(2*(tan(1/2*f*x)^4*tan(1
/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan
(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*
f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*ta
n(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)^3*tan(1/2*e) + 6*d*f*x*tan(1
/2*f*x)*tan(1/2*e)^2 - 6*c*f*tan(1/2*f*x)^2*tan(1/2*e)^2 - 3*d*log(2*(tan(1
/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan
(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)
^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(
1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)^2*tan(1/2*e)^
2 - d*tan(1/2*f*x)^3*tan(1/2*e)^2 + 2*d*f*x*tan(1/2*e)^3 - 3*d*log(2*(tan(1
```

```
/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan
(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)
^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(
1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)*tan(1/2*e)^3
- d*tan(1/2*f*x)^2*tan(1/2*e)^3 + 2*c*f*tan(1/2*f*x)^3 + d*log(2*(tan(1/2*f
*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2
*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 -
 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*
f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)^3 + 6*d*f*x*tan(1
/2*f*x)*tan(1/2*e) + 6*c*f*tan(1/2*f*x)^2*tan(1/2*e) - 3*d*log(2*(tan(1/2*f
*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2
*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 -
 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*
f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)^2*tan(1/2*e) + d*
tan(1/2*f*x)^3*tan(1/2*e) + 6*c*f*tan(1/2*f*x)*tan(1/2*e)^2 - 3*d*log(2*(ta
n(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*
tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f
*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*t
an(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x)*tan(1/2*e)
^2 - d*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*c*f*tan(1/2*e)^3 + d*log(2*(tan(1/2*
f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/
2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3
- 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2
*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*e)^3 + d*tan(1/2*f*x)
*tan(1/2*e)^3 + 3*d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*t
an(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)
^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/
2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2
+ 1))*tan(1/2*f*x)^2 - d*tan(1/2*f*x)^3 + 6*c*f*tan(1/2*f*x)*tan(1/2*e) + 3
*d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan
(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 +
 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/
2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x
)*tan(1/2*e) - d*tan(1/2*f*x)^2*tan(1/2*e) + 3*d*log(2*(tan(1/2*f*x)^4*tan(
1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + ta
n(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2
*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*t
an(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*e)^2 - d*tan(1/2*f*x)*tan(1/2*e)
^2 - d*tan(1/2*e)^3 - 2*d*f*x + 3*d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*
tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4
+ 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x...
```

**Mupad [B]**

time = 4.81, size = 183, normalized size = 1.24

$$\frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{3\,a^2\,f^2}-\frac{\left(c\,f+d\,f\,x-d\,1\mathrm{i}\right)\,2\mathrm{i}}{3\,a^2\,f^2\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2\mathrm{i}\right)}-\frac{d\,x\,2\mathrm{i}}{3\,a^2\,f}-\frac{d\,2\mathrm{i}}{3\,a^2\,f^2\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(c+d\,x\right)\,4\mathrm{i}}{3\,a^2\,f\,\left(3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,3\mathrm{i}-\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (2*d*log(exp(e*1i)*exp(f*x*1i) + 1i))/(3*a^2*f^2) - ((c*f - d*1i + d*f*x)*2
i)/(3*a^2*f^2*(exp(e*1i + f*x*1i)*2i + exp(e*2i + f*x*2i) - 1)) - (d*x*2i)/
(3*a^2*f) - (d*2i)/(3*a^2*f^2*(exp(e*1i + f*x*1i) + 1i)) + (exp(e*1i + f*x*
1i)*(c + d*x)*4i)/(3*a^2*f*(3*exp(e*1i + f*x*1i) - exp(e*2i + f*x*2i)*3i -
exp(e*3i + f*x*3i) + 1i))
```

# 3.115 $\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=23

$$\mathrm{Int}\left(\frac{1}{(c+dx)(a+a\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+a*sin(f*x+e))^2,x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Sin[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)*(a + a*Sin[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$$

**Mathematica** [A]
time = 9.40, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])^2), x]
```

**Maple** [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)/(a+a*sin(f*x+e))^2,x)

[Out] int(1/(d*x+c)/(a+a*sin(f*x+e))^2,x)

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] 1/3*(6*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 - 4*d^2*cos(f*x + e) + 6*(d^2*f*x + c*d*f)*cos(f*x + e)^2 + 6*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 6*(d^2*f*x + c*d*f)*sin(f*x + e)^2 + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 - 2*d^2*cos(2*f*x + 2*e) + 2*d^2 - (d^2*f*x + c*d*f)*cos(f*x + e) - (d^2*f*x + c*d*f)*sin(2*f*x + 2*e) + (3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) - 2*(d^2*f*x + c*d*f + 3*(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 2*d^2)*cos(f*x + e) + 6*(d^2*f*x + c*d*f)*sin(f*x + e))*cos(2*f*x + 2*e) - 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)^2 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(3*f*x + 3*e)^2 + 18*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)*sin(2*f*x + 2*e) + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e)^2 - 6*((a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e) + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))*cos(2*f*x + 2*e) - 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 - 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))*sin(3*f*x + 3*e) + 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))*integrate(2/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 6*d^3)*cos(f*x + e)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d

```
*f^3*x + a^2*c^4*f^3)*sin(f*x + e)), x) - 2*(2*d^2*sin(2*f*x + 2*e) - (d^2*
f*x + c*d*f)*cos(2*f*x + 2*e) + (3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 +
4*d^2)*cos(f*x + e) + (d^2*f*x + c*d*f)*sin(f*x + e))*sin(3*f*x + 3*e) - 2*
(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 4*d^2 - 6*(d^2*f*x + c*d*f)*cos(
f*x + e) + 3*(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 2*d^2)*sin(f*x + e)
)*sin(2*f*x + 2*e) + 2*(d^2*f*x + c*d*f)*sin(f*x + e))/(a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 +
9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)
*cos(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*
d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)^2 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*
x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(3*f*x + 3*e)^2 + 18*(a^2*d^3*f^3
*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)*
sin(2*f*x + 2*e) + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f
^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f
^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e)^2 - 6*((a^2*d^3*f^3*
x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e) +
 (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*
sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x
^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3
*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))*cos(2*f*x + 2*e) - 2*
(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 -
3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)
*cos(2*f*x + 2*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*
f^3*x + a^2*c^3*f^3)*sin(f*x + e))*sin(3*f*x + 3*e) + 6*(a^2*d^3*f^3*x^3 +
3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(2*a^2*d*x + 2*a^2*c - (a^2*d*x + a^2*c)*cos(f*x + e)^2 + 2*(a^2
*d*x + a^2*c)*sin(f*x + e)), x)
```

**Sympy** [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sin^2(e+fx)+2c\sin(e+fx)+c+dx\sin^2(e+fx)+2dx\sin(e+fx)+dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(1/(c*sin(e + f*x)**2 + 2*c*sin(e + f*x) + c + d*x*sin(e + f*x)**2
+ 2*d*x*sin(e + f*x) + d*x), x)/a**2
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*sin(f*x + e) + a)^2), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*x)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c + d*x)), x)
```

# 3.116 $\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+a\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Sin[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$$

**Mathematica** [A]
time = 9.80, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2), x]
```

**Maple** [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(a+a\sin(fx+e)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x)

[Out] int(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x)

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] 1/3*(12*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 - 12*d^2*cos(f*x + e) + 12*(d^2*f*x + c*d*f)*cos(f*x + e)^2 + 12*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 12*(d^2*f*x + c*d*f)*sin(f*x + e)^2 + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 - 6*d^2*cos(2*f*x + 2*e) + 6*d^2 - 2*(d^2*f*x + c*d*f)*cos(f*x + e) - 2*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e) + 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) - 2*(2*d^2*f*x + 2*c*d*f + 9*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2)*cos(f*x + e) + 12*(d^2*f*x + c*d*f)*sin(f*x + e))*cos(2*f*x + 2*e) - 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x + 3*e)^2 + 18*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e)*sin(2*f*x + 2*e) + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 - 6*((a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e) + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e))*cos(2*f*x + 2*e) - 2*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 - 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e))*sin(3*f*x + 3*e) + 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e))*integrate(4/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 12*d^3)*cos(f*x + e)/(a^2*d^5*f^3*x

```
^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2
+ 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4
+ 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2
*c^5*f^3)*cos(f*x + e)^2 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*
c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)
*sin(f*x + e)^2 + 2*(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3
*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*sin(f*
x + e)), x) - 2*(6*d^2*sin(2*f*x + 2*e) - 2*(d^2*f*x + c*d*f)*cos(2*f*x + 2
*e) + 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*cos(f*x + e) + 2*(d^2
*f*x + c*d*f)*sin(f*x + e))*sin(3*f*x + 3*e) - 6*(d^2*f^2*x^2 + 2*c*d*f^2*x
 + c^2*f^2 + 4*d^2 - 4*(d^2*f*x + c*d*f)*cos(f*x + e) + 3*(d^2*f^2*x^2 + 2*
c*d*f^2*x + c^2*f^2 + 2*d^2)*sin(f*x + e))*sin(2*f*x + 2*e) + 4*(d^2*f*x +
c*d*f)*sin(f*x + e))/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2
*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3
*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(3*f
*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*
x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*
x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2
*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x + 3*e)^2 + 18*(
a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d
*f^3*x + a^2*c^4*f^3)*cos(f*x + e)*sin(2*f*x + 2*e) + 9*(a^2*d^4*f^3*x^4 +
4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f
^3)*sin(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 - 6*((a^2*
d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3
*x + a^2*c^4*f^3)*cos(f*x + e) + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6
*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e))*c
os(3*f*x + 3*e) - 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*
f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^
3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*
x + e))*cos(2*f*x + 2*e) - 2*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2
*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*...
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(2*a^2*d^2*x^2 + 4*a^2*c*d*x + 2*a^2*c^2 - (a^2*d^2*x^2 + 2*a^2*
c*d*x + a^2*c^2)*cos(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*s
in(f*x + e)), x)
```

**Sympy** [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\sin^2(e+fx)+2c^2\sin(e+fx)+c^2+2cdx\sin^2(e+fx)+4cdx\sin(e+fx)+2cdx+d^2x^2\sin^2(e+fx)+2d^2x^2\sin(e+fx)+d^2x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(1/(c**2*sin(e + f*x)**2 + 2*c**2*sin(e + f*x) + c**2 + 2*c*d*x*sin
(e + f*x)**2 + 4*c*d*x*sin(e + f*x) + 2*c*d*x + d**2*x**2*sin(e + f*x)**2 +
 2*d**2*x**2*sin(e + f*x) + d**2*x**2), x)/a**2
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*sin(f*x + e) + a)^2), x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c + d*x)^2), x)
```

# 3.117 $\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=147

$$-\frac{i(c+dx)^3}{af}+\frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2}-\frac{12id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3}+\frac{12d^3\mathrm{Li}_3\left(-ie^{i(e+fx)}\right)}{af^4}+\frac{(c+dx)^3}{\phantom{x}}$$

```
[Out] -I*(d*x+c)^3/a/f+6*d*(d*x+c)^2*ln(1+I*exp(I*(f*x+e)))/a/f^2-12*I*d^2*(d*x+c
)*polylog(2,-I*exp(I*(f*x+e)))/a/f^3+12*d^3*polylog(3,-I*exp(I*(f*x+e)))/a/
f^4+(d*x+c)^3*tan(1/2*e+1/4*Pi+1/2*f*x)/a/f
```

**Rubi** [A]
time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3399, 4269, 3798, 2221, 2611, 2320, 6724}

$$-\frac{12id^2(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{af^3}+\frac{12d^3\mathrm{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{af^4}+\frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}-\frac{i(c+dx)^3}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a - a*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3)/(a*f) + (6*d*(c + d*x)^2*Log[1 + I*E^(I*(e + f*x))])/(a*
f^2) - ((12*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(e + f*x))])/(a*f^3) + (1
2*d^3*PolyLog[3, (-I)*E^(I*(e + f*x))])/(a*f^4) + ((c + d*x)^3*Tan[e/2 + Pi
/4 + (f*x)/2])/(a*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = \frac{\int (c+dx)^3\csc^2\left(\frac{1}{2}\left(e-\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx}{2a}$$

$$= \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(3d)\int (c+dx)^2\cot\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)\,dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(6d)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+ie^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(12}{}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\text{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3} + \frac{(}{}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\text{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3} + \frac{(}{}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\text{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3} + \frac{1}{}$$

**Mathematica** [A]
time = 0.81, size = 124, normalized size = 0.84

$$\frac{-12id^2f(c+dx)\text{Li}_2\left(-ie^{i(e+fx)}\right)+12d^3\text{Li}_3\left(-ie^{i(e+fx)}\right)+f^2(c+dx)^2\left(-if(c+dx)+6d\log\left(1+ie^{i(e+fx)}\right)+f(c+dx)\tan\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)}{af^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a - a*Sin[e + f*x]),x]
```

```
[Out] ((-12*I)*d^2*f*(c + d*x)*PolyLog[2, (-I)*E^(I*(e + f*x))] + 12*d^3*PolyLog[
3, (-I)*E^(I*(e + f*x))] + f^2*(c + d*x)^2*((-I)*f*(c + d*x) + 6*d*Log[1 +
I*E^(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e + Pi + 2*f*x)/4]))/(a*f^4)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(130) = 260.
time = 0.13, size = 484, normalized size = 3.29

| method | result |
|---|---|
| risch | $\frac{2d^3x^3+6c\,d^2x^2+6c^2dx+2c^3}{fa\left(e^{i(fx+e)}-i\right)}+\frac{12d^3\,\mathrm{polylog}\left(3,-ie^{i(fx+e)}\right)}{a\,f^4}+\frac{6\ln\left(e^{i(fx+e)}-i\right)c^2d}{a\,f^2}-\frac{6d^3e^2\ln\left(e^{i(fx+e)}\right)}{a\,f^4}-\frac{6ic\,d^2e^2}{a\,f^3}-\frac{12ic\,d^2e}{a\,f^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a-a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))-I)+12*d^3*polylog
(3,-I*exp(I*(f*x+e)))/a/f^4+6/a/f^2*ln(exp(I*(f*x+e))-I)*c^2*d-6/a/f^4*d^3*
e^2*ln(exp(I*(f*x+e)))-6*I/a/f^3*c*d^2*e^2-12*I/a/f^2*c*d^2*e*x+6*I/a/f^3*d
^3*e^2*x-6/a/f^2*ln(exp(I*(f*x+e)))*c^2*d-12*I/a/f^3*d^3*polylog(2,-I*exp(I
*(f*x+e)))*x-6*I/a/f*c*d^2*x^2+6/a/f^4*d^3*e^2*ln(exp(I*(f*x+e))-I)+6/a/f^2
*d^3*ln(1+I*exp(I*(f*x+e)))*x^2-6/a/f^4*d^3*ln(1+I*exp(I*(f*x+e)))*e^2-12/a
/f^3*c*d^2*e*ln(exp(I*(f*x+e))-I)+12/a/f^3*c*d^2*e*ln(exp(I*(f*x+e)))+4*I/a
/f^4*d^3*e^3-2*I/a/f*d^3*x^3-12*I/a/f^3*c*d^2*polylog(2,-I*exp(I*(f*x+e)))+
12/a/f^2*c*d^2*ln(1+I*exp(I*(f*x+e)))*x+12/a/f^3*c*d^2*ln(1+I*exp(I*(f*x+e)
))*e
```

**Maxima** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(128) = 256$.
time = 0.48, size = 1059, normalized size = 7.20

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*(2*(f*x + e)*cos(f*x + e) + (cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*
x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1))*c*d^
2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 - 2*a*f^2*sin(f*x + e) + a
*f^2) - 3*(2*(f*x + e)*cos(f*x + e) + (cos(f*x + e)^2 + sin(f*x + e)^2 - 2*
sin(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1)
)*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*sin(f*x + e) + a*f
) - 6*c*d^2*e^2/(a*f^2 - a*f^2*sin(f*x + e)/(cos(f*x + e) + 1)) + 6*c^2*d*e
/(a*f - a*f*sin(f*x + e)/(cos(f*x + e) + 1)) - 2*c^3/(a - a*sin(f*x + e)/(c
os(f*x + e) + 1)) - (2*I*d^3*e^3 + 6*(d^3*cos(f*x + e)*e^2 + I*d^3*e^2*sin(
f*x + e) - I*d^3*e^2)*arctan2(sin(f*x + e) - 1, cos(f*x + e)) - 6*(I*(f*x +
 e)^2*d^3 + 2*(I*c*d^2*f - I*d^3*e)*(f*x + e) - ((f*x + e)^2*d^3 + 2*(c*d^2
*f - d^3*e)*(f*x + e))*cos(f*x + e) + (-I*(f*x + e)^2*d^3 + 2*(-I*c*d^2*f +
 I*d^3*e)*(f*x + e))*sin(f*x + e))*arctan2(cos(f*x + e), -sin(f*x + e) + 1)
 - 2*((f*x + e)^3*d^3 + 3*(f*x + e)*d^3*e^2 + 3*(c*d^2*f - d^3*e)*(f*x + e)
^2)*cos(f*x + e) - 12*(-I*(f*x + e)*d^3 - I*c*d^2*f + I*d^3*e + ((f*x + e)*
d^3 + c*d^2*f - d^3*e)*cos(f*x + e) + (I*(f*x + e)*d^3 + I*c*d^2*f - I*d^3*
e)*sin(f*x + e))*dilog(-I*e^(I*f*x + I*e)) - 3*((f*x + e)^2*d^3 + d^3*e^2 +
 2*(c*d^2*f - d^3*e)*(f*x + e) + (I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(I*c*d^
2*f - I*d^3*e)*(f*x + e))*cos(f*x + e) - ((f*x + e)^2*d^3 + d^3*e^2 + 2*(c*
d^2*f - d^3*e)*(f*x + e))*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2
 - 2*sin(f*x + e) + 1) - 12*(I*d^3*cos(f*x + e) - d^3*sin(f*x + e) + d^3)*p
olylog(3, -I*e^(I*f*x + I*e)) - 2*(I*(f*x + e)^3*d^3 + 3*I*(f*x + e)*d^3*e^
2 + 3*(I*c*d^2*f - I*d^3*e)*(f*x + e)^2)*sin(f*x + e))/(-I*a*f^3*cos(f*x +
e) + a*f^3*sin(f*x + e) - a*f^3))/f
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(128) = 256$.
time = 0.38, size = 952, normalized size = 6.48

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 + (d^3*f^3*x^3 + 3
*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*cos(f*x + e) + 6*(I*d^3*f*x + I*c
*d^2*f + (I*d^3*f*x + I*c*d^2*f)*cos(f*x + e) + (-I*d^3*f*x - I*c*d^2*f)*si
n(f*x + e))*dilog(I*cos(f*x + e) + sin(f*x + e)) + 6*(-I*d^3*f*x - I*c*d^2*
f + (-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e) + (I*d^3*f*x + I*c*d^2*f)*sin(f*x
 + e))*dilog(-I*cos(f*x + e) + sin(f*x + e)) + 3*(c^2*d*f^2 - 2*c*d^2*f*e +
 d^3*e^2 + (c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2)*cos(f*x + e) - (c^2*d*f^2 -
2*c*d^2*f*e + d^3*e^2)*sin(f*x + e))*log(cos(f*x + e) - I*sin(f*x + e) + I)
 + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2 + (d^3*f^2*x^2 +
2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x + e) - (d^3*f^2*x^2 + 2*c*d^
2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*sin(f*x + e))*log(I*cos(f*x + e) - sin(f*x
 + e) + 1) + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2 + (d^3*
f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*cos(f*x + e) - (d^3*f^2*x^
2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*sin(f*x + e))*log(-I*cos(f*x + e
) - sin(f*x + e) + 1) + 3*(c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + (c^2*d*f^2 -
 2*c*d^2*f*e + d^3*e^2)*cos(f*x + e) - (c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2)*
sin(f*x + e))*log(-cos(f*x + e) - I*sin(f*x + e) + I) + 6*(d^3*cos(f*x + e)
 - d^3*sin(f*x + e) + d^3)*polylog(3, I*cos(f*x + e) + sin(f*x + e)) + 6*(d
^3*cos(f*x + e) - d^3*sin(f*x + e) + d^3)*polylog(3, -I*cos(f*x + e) + sin(
f*x + e)) + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*sin(f
*x + e))/(a*f^4*cos(f*x + e) - a*f^4*sin(f*x + e) + a*f^4)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{c^3}{\sin(e+fx)-1}\,dx + \int \frac{d^3x^3}{\sin(e+fx)-1}\,dx + \int \frac{3cd^2x^2}{\sin(e+fx)-1}\,dx + \int \frac{3c^2dx}{\sin(e+fx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a-a*sin(f*x+e)),x)
```

```
[Out] -(Integral(c**3/(sin(e + f*x) - 1), x) + Integral(d**3*x**3/(sin(e + f*x) -
 1), x) + Integral(3*c*d**2*x**2/(sin(e + f*x) - 1), x) + Integral(3*c**2*d
*x/(sin(e + f*x) - 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^3/(a*sin(f*x + e) - a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{a-a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a - a*sin(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a - a*sin(e + f*x)), x)
```

## 3.118 $\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=112

$$-\frac{i(c+dx)^2}{af}+\frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2}-\frac{4id^2\text{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}$$

```
[Out] -I*(d*x+c)^2/a/f+4*d*(d*x+c)*ln(1+I*exp(I*(f*x+e)))/a/f^2-4*I*d^2*polylog(2
,-I*exp(I*(f*x+e)))/a/f^3+(d*x+c)^2*tan(1/2*e+1/4*Pi+1/2*f*x)/a/f
```

**Rubi** [A]
time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3399, 4269, 3798, 2221, 2317, 2438}

$$-\frac{4id^2\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{af^3}+\frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}-\frac{i(c+dx)^2}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a - a*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) + (4*d*(c + d*x)*Log[1 + I*E^(I*(e + f*x))])/(a*f^
2) - ((4*I)*d^2*PolyLog[2, (-I)*E^(I*(e + f*x))])/(a*f^3) + ((c + d*x)^2*Ta
n[e/2 + Pi/4 + (f*x)/2])/(a*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx = \frac{\int (c+dx)^2\csc^2\left(\frac{1}{2}\left(e-\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx}{2a}$$

$$= \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(2d)\int (c+dx)\cot\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)\,dx}{af}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(4d)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+ie^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(4d}{}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(4i}{}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{4id^2\text{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^2 t}{}$$

**Mathematica** [A]
time = 0.52, size = 92, normalized size = 0.82

$$\frac{-4id^2\text{Li}_2\left(-ie^{i(e+fx)}\right)+f(c+dx)\left(-if(c+dx)+4d\log\left(1+ie^{i(e+fx)}\right)+f(c+dx)\tan\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a - a*Sin[e + f*x]),x]
```

```
[Out] ((-4*I)*d^2*PolyLog[2, (-I)*E^(I*(e + f*x))] + f*(c + d*x)*((-I)*f*(c + d*x
) + 4*d*Log[1 + I*E^(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e + Pi + 2*f*x)/4])
)/(a*f^3)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(98) = 196$.
time = 0.09, size = 254, normalized size = 2.27

| method | result |
|---|---|
| risch | $\frac{2d^2x^2+4cdx+2c^2}{fa\left(e^{i(fx+e)}-i\right)}-\frac{4\ln\left(e^{i(fx+e)}\right)cd}{a\,f^2}+\frac{4\ln\left(e^{i(fx+e)}-i\right)cd}{a\,f^2}-\frac{2id^2x^2}{af}-\frac{4id^2ex}{a\,f^2}-\frac{2id^2e^2}{a\,f^3}+\frac{4d^2\ln\left(1+ie^{i(fx+e)}\right)x}{a\,f^2}+\frac{4d^2\ln}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a-a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))-I)-4/a/f^2*ln(exp(I*(f*x+e)))*c
*d+4/a/f^2*ln(exp(I*(f*x+e))-I)*c*d-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-2*I/a
/f^3*d^2*e^2+4/a/f^2*d^2*ln(1+I*exp(I*(f*x+e)))*x+4/a/f^3*d^2*ln(1+I*exp(I*
(f*x+e)))*e-4*I*d^2*polylog(2,-I*exp(I*(f*x+e)))/a/f^3+4/a/f^3*d^2*e*ln(exp
(I*(f*x+e)))-4/a/f^3*d^2*e*ln(exp(I*(f*x+e))-I)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(96) = 192$.
time = 0.38, size = 331, normalized size = 2.96

$$-\frac{2\left(ic^2f^2-2\left(cdf\cos\left(fx+e\right)+icdf\sin\left(fx+e\right)-icdf\right)\arctan\left(\sin\left(fx+e\right)-1,\cos\left(fx+e\right)\right)-2\left(d^2fx\cos\left(fx+e\right)+id^2fx\sin\left(fx+e\right)-id^2fx\right)\arctan\left(\cos\left(fx+e\right),-\sin\left(fx+e\right)+1\right)+\left(d^2f^2x^2+2cdf^2x\right)\cos\left(fx+e\right)+2\left(d^2\cos\left(fx+e\right)+id^2\sin\left(fx+e\right)-id^2\right){\rm Li}_2\left(-ie^{(ifx+ie)}\right)+\left(d^2fx+cdf+\left(id^2fx+icdf\right)\cos\left(fx+e\right)-\left(d^2fx+cdf\right)\sin\left(fx+e\right)\right)\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2-2\,\sin\left(fx+e\right)+1\right)+\left(id^2f^2x^2+2i\,cdf^2x\right)\sin\left(fx+e\right)\right)}{-i\,af^3\cos\left(fx+e\right)+af^3\sin\left(fx+e\right)-af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(I*c^2*f^2 - 2*(c*d*f*cos(f*x + e) + I*c*d*f*sin(f*x + e) - I*c*d*f)*arc
tan2(sin(f*x + e) - 1, cos(f*x + e)) - 2*(d^2*f*x*cos(f*x + e) + I*d^2*f*x*
sin(f*x + e) - I*d^2*f*x)*arctan2(cos(f*x + e), -sin(f*x + e) + 1) + (d^2*f
^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) + 2*(d^2*cos(f*x + e) + I*d^2*sin(f*x +
e) - I*d^2)*dilog(-I*e^(I*f*x + I*e)) + (d^2*f*x + c*d*f + (I*d^2*f*x + I*c
*d*f)*cos(f*x + e) - (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 + s
in(f*x + e)^2 - 2*sin(f*x + e) + 1) + (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin(f
*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) - a*f^3)
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(96) = 192$.
time = 0.35, size = 536, normalized size = 4.79

$$\frac{d^2f^2x^2+2\,cdf^2x+c^2f^2+\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)\cos\left(fx+e\right)+2\left(id^2\cos\left(fx+e\right)-id^2\sin\left(fx+e\right)+id^2\right){\rm Li}_2\left(i\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)+2\left(-id^2\cos\left(fx+e\right)+id^2\sin\left(fx+e\right)-id^2\right){\rm Li}_2\left(-i\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)+2\left(cdf-d^2e+\left(cdf-d^2e\right)\cos\left(fx+e\right)-\left(cdf-d^2e\right)\sin\left(fx+e\right)\right)\log\left(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)+i\right)+2\left(d^2fx+d^2e+\left(d^2fx+d^2e\right)\cos\left(fx+e\right)-\left(d^2fx+d^2e\right)\sin\left(fx+e\right)\right)\log\left(i\,\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)+2\left(d^2fx+d^2e+\left(d^2fx+d^2e\right)\cos\left(fx+e\right)-\left(d^2fx+d^2e\right)\sin\left(fx+e\right)\right)\log\left(-i\,\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)+2\left(cdf-d^2e+\left(cdf-d^2e\right)\cos\left(fx+e\right)-\left(cdf-d^2e\right)\sin\left(fx+e\right)\right)\log\left(-\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)+i\right)+\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)\sin\left(fx+e\right)}{af^3\cos\left(fx+e\right)-af^3\sin\left(fx+e\right)+af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2
)*cos(f*x + e) + 2*(I*d^2*cos(f*x + e) - I*d^2*sin(f*x + e) + I*d^2)*dilog(
I*cos(f*x + e) + sin(f*x + e)) + 2*(-I*d^2*cos(f*x + e) + I*d^2*sin(f*x + e
) - I*d^2)*dilog(-I*cos(f*x + e) + sin(f*x + e)) + 2*(c*d*f - d^2*e + (c*d*
f - d^2*e)*cos(f*x + e) - (c*d*f - d^2*e)*sin(f*x + e))*log(cos(f*x + e) -
I*sin(f*x + e) + I) + 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e) -
 (d^2*f*x + d^2*e)*sin(f*x + e))*log(I*cos(f*x + e) - sin(f*x + e) + 1) + 2
*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e) - (d^2*f*x + d^2*e)*sin(
f*x + e))*log(-I*cos(f*x + e) - sin(f*x + e) + 1) + 2*(c*d*f - d^2*e + (c*d
*f - d^2*e)*cos(f*x + e) - (c*d*f - d^2*e)*sin(f*x + e))*log(-cos(f*x + e)
- I*sin(f*x + e) + I) + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*sin(f*x + e))
/(a*f^3*cos(f*x + e) - a*f^3*sin(f*x + e) + a*f^3)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{c^2}{\sin(e+fx)-1}\,dx + \int \frac{d^2x^2}{\sin(e+fx)-1}\,dx + \int \frac{2cdx}{\sin(e+fx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a-a*sin(f*x+e)),x)
```

```
[Out] -(Integral(c**2/(sin(e + f*x) - 1), x) + Integral(d**2*x**2/(sin(e + f*x) -
 1), x) + Integral(2*c*d*x/(sin(e + f*x) - 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^2/(a*sin(f*x + e) - a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{a-a\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a - a*sin(e + f*x)),x)
```

```
[Out] int((c + d*x)^2/(a - a*sin(e + f*x)), x)
```

# 3.119 $\int \frac{c+dx}{a-a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=59

$$\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}$$

```
[Out] 2*d*ln(cos(1/2*e+1/4*Pi+1/2*f*x))/a/f^2+(d*x+c)*tan(1/2*e+1/4*Pi+1/2*f*x)/a
/f
```

**Rubi** [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3399, 4269, 3556}

$$\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}+\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - a*Sin[e + f*x]),x]
```

```
[Out] (2*d*Log[Cos[e/2 + Pi/4 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tan[e/2 + Pi/4 +
(f*x)/2])/(a*f)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{a-a\sin(e+fx)}\,dx &= \frac{\int (c+dx)\csc^2\left(\frac{1}{2}\left(e-\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx}{2a} \\
&= \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{d\int \cot\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)\,dx}{af} \\
&= \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}
\end{aligned}
$$

**Mathematica [A]**
time = 0.11, size = 47, normalized size = 0.80

$$
\frac{2d\log\left(\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)+f(c+dx)\tan\left(\frac{1}{4}(2e+\pi+2fx)\right)}{af^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - a*Sin[e + f*x]),x]
```

```
[Out] (2*d*Log[Cos[(2*e + Pi + 2*f*x)/4]] + f*(c + d*x)*Tan[(2*e + Pi + 2*f*x)/4]
)/(a*f^2)
```

**Maple [C]** Result contains complex when optimal does not.
time = 0.08, size = 73, normalized size = 1.24

| method | result | size |
|---|---|---|
| risch | $-\frac{2idx}{af}-\frac{2ide}{a\,f^2}+\frac{2dx+2c}{fa\left(e^{i(fx+e)}-i\right)}+\frac{2d\ln\left(e^{i(fx+e)}-i\right)}{a\,f^2}$ | 73 |
| norman | $\frac{-\frac{2c}{fa}-\frac{dx}{fa}-\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{a\,f^2}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a\,f^2}$ | 99 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a-a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*d/a/f*x-2*I*d/a/f^2*e+2*(d*x+c)/f/a/(exp(I*(f*x+e))-I)+2*d/a/f^2*ln(ex
p(I*(f*x+e))-I)
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(49) = 98.
time = 0.36, size = 185, normalized size = 3.14

$$
\frac{\frac{\left(2\,(fx+e)\cos(fx+e)+\left(\cos(fx+e)^2+\sin(fx+e)^2-2\,\sin(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2-2\,\sin(fx+e)+1\right)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2-2\,af\sin(fx+e)+af}-\frac{2\,de}{af-\frac{af\sin(fx+e)}{\cos(fx+e)+1}}+\frac{2\,c}{a-\frac{a\sin(fx+e)}{\cos(fx+e)+1}}}{f}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] ((2*(f*x + e)*cos(f*x + e) + (cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x +
 e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1))*d/(a*f*
cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*sin(f*x + e) + a*f) - 2*d*e/(a*
f - a*f*sin(f*x + e)/(cos(f*x + e) + 1)) + 2*c/(a - a*sin(f*x + e)/(cos(f*x
 + e) + 1)))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(49) = 98$.
time = 0.34, size = 108, normalized size = 1.83

$$\frac{dfx + cf + (dfx + cf)\cos(fx + e) + (d\cos(fx + e) - d\sin(fx + e) + d)\log(-\sin(fx + e) + 1) + (dfx + cf)\sin(fx + e)}{af^2\cos(fx + e) - af^2\sin(fx + e) + af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (d*f*x + c*f + (d*f*x + c*f)*cos(f*x + e) + (d*cos(f*x + e) - d*sin(f*x + e
) + d)*log(-sin(f*x + e) + 1) + (d*f*x + c*f)*sin(f*x + e))/(a*f^2*cos(f*x
+ e) - a*f^2*sin(f*x + e) + a*f^2)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(44) = 88$.
time = 0.45, size = 272, normalized size = 4.61

$$\begin{cases} -\frac{2cf}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} - \frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} - \frac{dfx}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} + \frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} - \frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} - \frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} + \frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} & \text{for}\ f \neq 0 \\ \frac{cx+\frac{dx^2}{2}}{-a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c*f/(a*f**2*tan(e/2 + f*x/2) - a*f**2) - d*f*x*tan(e/2 + f*x/
2)/(a*f**2*tan(e/2 + f*x/2) - a*f**2) - d*f*x/(a*f**2*tan(e/2 + f*x/2) - a*
f**2) + 2*d*log(tan(e/2 + f*x/2) - 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 + f*
x/2) - a*f**2) - 2*d*log(tan(e/2 + f*x/2) - 1)/(a*f**2*tan(e/2 + f*x/2) - a
*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 +
f*x/2) - a*f**2) + d*log(tan(e/2 + f*x/2)**2 + 1)/(a*f**2*tan(e/2 + f*x/2)
- a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(-a*sin(e) + a), True))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(49) = 98$.
time = 2.03, size = 697, normalized size = 11.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*f*x*tan(1/2*f*x)*tan(1/2*e) - d*f*x*tan(1/2*f*x) - d*f*x*tan(1/2*e) + c*
f*tan(1/2*f*x)*tan(1/2*e) + d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 + 2*tan(1/
2*f*x)^4*tan(1/2*e) + 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*ta
n(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3 + 2*tan(1/2*f*x)*tan(1/2*e)^2
+ 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 2*tan(1/2*e) + 1)/(tan
(1/2*e)^2 + 1))*tan(1/2*f*x)*tan(1/2*e) - d*f*x - c*f*tan(1/2*f*x) + d*log(
2*(tan(1/2*f*x)^4*tan(1/2*e)^2 + 2*tan(1/2*f*x)^4*tan(1/2*e) + 2*tan(1/2*f*
x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(
1/2*f*x)^3 + 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2
- 2*tan(1/2*f*x) - 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1))*tan(1/2*f*x) - c*f
*tan(1/2*e) + d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 + 2*tan(1/2*f*x)^4*tan(1
/2*e) + 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*t
an(1/2*e)^2 - 2*tan(1/2*f*x)^3 + 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*
x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 2*tan(1/2*e) + 1)/(tan(1/2*e)^2 + 1)
)*tan(1/2*e) - c*f - d*log(2*(tan(1/2*f*x)^4*tan(1/2*e)^2 + 2*tan(1/2*f*x)^
4*tan(1/2*e) + 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f
*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3 + 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan
(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 2*tan(1/2*e) + 1)/(tan(1/2*e)
^2 + 1)))/(a*f^2*tan(1/2*f*x)*tan(1/2*e) + a*f^2*tan(1/2*f*x) + a*f^2*tan(1
/2*e) - a*f^2)
```

**Mupad** [B]
time = 0.89, size = 66, normalized size = 1.12

$$\frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}-\mathrm{i}\right)}{a\,f^2}+\frac{2\,\left(c+d\,x\right)}{a\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)}-\frac{d\,x\,2\mathrm{i}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - a*sin(e + f*x)),x)
```

```
[Out] (2*d*log(exp(e*1i)*exp(f*x*1i) - 1i))/(a*f^2) + (2*(c + d*x))/(a*f*(exp(e*1
i + f*x*1i) - 1i)) - (d*x*2i)/(a*f)
```

# 3.120 $\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$

**Optimal**. Leaf size=24

$$\text{Int}\left(\frac{1}{(c+dx)(a-a\sin(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a - a*Sin[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a - a*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$$

**Mathematica** [A]
time = 3.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a - a*Sin[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a - a*Sin[e + f*x])), x]
```

**Maple** [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a-a\sin(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f*x
 + a*c*d*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x + a*c*d*f)*sin(f*x + e))*integrat
e(cos(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*a*c*
d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*sin
(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*sin(f*x + e)), x) + c
os(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (a*d*f*x
 + a*c*f)*sin(f*x + e)^2 - 2*(a*d*f*x + a*c*f)*sin(f*x + e))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c - (a*d*x + a*c)*sin(f*x + e)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c\sin{\left(e+fx \right)}-c+dx\sin{\left(e+fx \right)}-dx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

```
[Out] -Integral(1/(c*sin(e + f*x) - c + d*x*sin(e + f*x) - d*x), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate(-1/((d*x + c)*(a*sin(f*x + e) - a)), x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a - a\,\sin(e + f\,x))\,(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*sin(e + f*x))*(c + d*x)),x)
[Out] int(1/((a - a*sin(e + f*x))*(c + d*x)), x)
```

# 3.121 $\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$

**Optimal**. Leaf size=24

$$\mathrm{Int}\left(\frac{1}{(c+dx)^2(a-a\sin(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a-a*sin(f*x+e)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a - a*Sin[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)^2*(a - a*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$$

**Mathematica** [A]
time = 3.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a - a*Sin[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a - a*Sin[e + f*x])), x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a-a\sin(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a-a*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a-a*sin(f*x+e)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*f*
x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*s
in(f*x + e)^2 - 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*sin(f*x + e))*i
ntegrate(cos(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^
3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x + e
)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f*x + e
)^2 - 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f*x +
 e)), x) + cos(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^
2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*
c^2*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*sin(f*x + e
))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*sin(f*x + e)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c^2\sin{(e+fx)}-c^2+2cdx\sin{(e+fx)}-2cdx+d^2x^2\sin{(e+fx)}-d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a-a*sin(f*x+e)),x)
```

```
[Out] -Integral(1/(c**2*sin(e + f*x) - c**2 + 2*c*d*x*sin(e + f*x) - 2*c*d*x + d*
*2*x**2*sin(e + f*x) - d**2*x**2), x)/a
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate(-1/((d*x + c)^2*(a*sin(f*x + e) - a)), x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a-a\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*sin(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a - a*sin(e + f*x))*(c + d*x)^2), x)
```

# 3.122 $\int x^3 \sqrt{a+a\sin(c+dx)}\, dx$

**Optimal**. Leaf size=120

$$-\frac{96\sqrt{a+a\sin(c+dx)}}{d^4}+\frac{12x^2\sqrt{a+a\sin(c+dx)}}{d^2}+\frac{48x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d^3}-\frac{2x^3\cot\left(\frac{c}{2}\right.}{}$$

```
[Out] -96*(a+a*sin(d*x+c))^(1/2)/d^4+12*x^2*(a+a*sin(d*x+c))^(1/2)/d^2+48*x*cot(1
/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*x+c))^(1/2)/d^3-2*x^3*cot(1/2*c+1/4*Pi+1/2*
d*x)*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3400, 3377, 2718}

$$-\frac{96\sqrt{a\sin(c+dx)+a}}{d^4}+\frac{48x\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d^3}+\frac{12x^2\sqrt{a\sin(c+dx)+a}}{d^2}-\frac{2x^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-96*Sqrt[a + a*Sin[c + d*x]])/d^4 + (12*x^2*Sqrt[a + a*Sin[c + d*x]])/d^2
+ (48*x*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]])/d^3 - (2*x^3*Co
t[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx = \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int x^3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx$$

$$= -\frac{2x^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d} + \frac{\left(6\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin\right.}{}$$

$$= \frac{12x^2\sqrt{a+a\sin(c+dx)}}{d^2} - \frac{2x^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d} - \frac{(2}{}$$

$$= \frac{12x^2\sqrt{a+a\sin(c+dx)}}{d^2} + \frac{48x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d^3} - \frac{2x}{}$$

$$= -\frac{96\sqrt{a+a\sin(c+dx)}}{d^4} + \frac{12x^2\sqrt{a+a\sin(c+dx)}}{d^2} + \frac{48x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right.}{}$$

**Mathematica** [A]
time = 0.16, size = 108, normalized size = 0.90

$$-\frac{2\left((48-24dx-6d^2x^2+d^3x^3)\cos\left(\frac{1}{2}(c+dx)\right)+(48+24dx-6d^2x^2-d^3x^3)\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d^4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*((48 - 24*d*x - 6*d^2*x^2 + d^3*x^3)*Cos[(c + d*x)/2] + (48 + 24*d*x -
6*d^2*x^2 - d^3*x^3)*Sin[(c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d^4*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple** [C] Result contains complex when optimal does not.
time = 0.08, size = 145, normalized size = 1.21

| method | result |
|---|---|
| risch | $-\frac{i\sqrt{2}\,\sqrt{-a\left(-2-2\sin\left(dx+c\right)\right)}\,\left(-ix^3d^3+d^3x^3\mathrm{e}^{i(dx+c)}+6id^2x^2\mathrm{e}^{i(dx+c)}-6d^2x^2+24idx-24dx\,\mathrm{e}^{i(dx+c)}-48i\mathrm{e}^{i(dx+c)}\right.}{\left(\mathrm{e}^{2i(dx+c)}+2i\mathrm{e}^{i(dx+c)}-1\right)d^4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(-a*(-2-2*sin(d*x+c)))^(1/2)/(exp(2*I*(d*x+c))+2*I*exp(I*(d*x+c)
)-1)*(-I*x^3*d^3+d^3*x^3*exp(I*(d*x+c))+6*I*d^2*x^2*exp(I*(d*x+c))-6*d^2*x^
2+24*I*d*x-24*d*x*exp(I*(d*x+c))-48*I*exp(I*(d*x+c))+48)*(exp(I*(d*x+c))+I)
/d^4
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*x^3, x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac [A]**
time = 1.74, size = 117, normalized size = 0.98

$$2\sqrt{2}\sqrt{a}\left(\frac{6\left(d^{2}x^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right)\cos\left(\frac{1}{4}\pi-\frac{1}{2}dx-\frac{1}{2}c\right)}{d^{4}}-\frac{\left(d^{3}x^{3}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-24dx\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right)\sin\left(\frac{1}{4}\pi-\frac{1}{2}dx-\frac{1}{2}c\right)}{d^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*(6*(d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 8*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c)))*cos(1/4*pi - 1/2*d*x - 1/2*c)/d^4 - (d^3*x^
3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 24*d*x*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c)))*sin(1/4*pi - 1/2*d*x - 1/2*c)/d^4)
```

**Mupad [B]**
time = 0.96, size = 82, normalized size = 0.68

$$-\frac{2\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(48\,\sin\left(c+d\,x\right)-6\,d^2\,x^2+d^3\,x^3\,\cos\left(c+d\,x\right)-6\,d^2\,x^2\,\sin\left(c+d\,x\right)-24\,d\,x\,\cos\left(c+d\,x\right)+48\right)}{d^4\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] -(2*(a*(sin(c + d*x) + 1))^(1/2)*(48*sin(c + d*x) - 6*d^2*x^2 + d^3*x^3*cos
(c + d*x) - 6*d^2*x^2*sin(c + d*x) - 24*d*x*cos(c + d*x) + 48))/(d^4*(sin(c
 + d*x) + 1))
```

# 3.123 $\int x^2 \sqrt{a+a\sin(c+dx)}\, dx$

**Optimal**. Leaf size=98

$$\frac{8x\sqrt{a+a\sin(c+dx)}}{d^2}+\frac{16\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d^3}-\frac{2x^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}$$

```
[Out] 8*x*(a+a*sin(d*x+c))^(1/2)/d^2+16*cot(1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*x+c)
)^(1/2)/d^3-2*x^2*cot(1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3400, 3377, 2718}

$$\frac{16\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d^3}+\frac{8x\sqrt{a\sin(c+dx)+a}}{d^2}-\frac{2x^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*x*Sqrt[a + a*Sin[c + d*x]])/d^2 + (16*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a +
 a*Sin[c + d*x]])/d^3 - (2*x^2*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c +
 d*x]])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+a\sin(c+dx)}\,dx &= \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int x^2\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx\\ &= -\frac{2x^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}+\frac{\left(4\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\,s\right.}}{}\\ &= \frac{8x\sqrt{a+a\sin(c+dx)}}{d^2}-\frac{2x^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}-\frac{\left(8\,\mathrm{c}\right.}{}\\ &= \frac{8x\sqrt{a+a\sin(c+dx)}}{d^2}+\frac{16\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d^3}-\frac{2x^2\,\mathrm{c}}{}\end{aligned}$$

**Mathematica [A]**
time = 0.12, size = 92, normalized size = 0.94

$$-\frac{2\left(\left(-8-4dx+d^2x^2\right)\cos\left(\frac{1}{2}(c+dx)\right)-\left(-8+4dx+d^2x^2\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*((-8 - 4*d*x + d^2*x^2)*Cos[(c + d*x)/2] - (-8 + 4*d*x + d^2*x^2)*Sin[(
c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d^3*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]))
```

**Maple [C]** Result contains complex when optimal does not.
time = 0.04, size = 119, normalized size = 1.21

| method | result | size |
|---|---|---|
| risch | $-\frac{i\sqrt{2}\,\sqrt{-a\left(-2-2\sin\left(dx+c\right)\right)}\,\left(-id^2x^2+d^2x^2\,\mathrm{e}^{i(dx+c)}+4idx\,\mathrm{e}^{i(dx+c)}-4dx+8i-8\,\mathrm{e}^{i(dx+c)}\right)\left(\mathrm{e}^{i(dx+c)}+i\right)}{\left(\mathrm{e}^{2i(dx+c)}+2i\mathrm{e}^{i(dx+c)}-1\right)d^3}$ | 119 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(-a*(-2-2*sin(d*x+c)))^(1/2)/(exp(2*I*(d*x+c))+2*I*exp(I*(d*x+c)
)-1)*(-I*d^2*x^2+d^2*x^2*exp(I*(d*x+c))+4*I*d*x*exp(I*(d*x+c))-4*d*x+8*I-8*
exp(I*(d*x+c)))*(exp(I*(d*x+c))+I)/d^3
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*x^2, x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a (\sin (c + dx) + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac** [A]
time = 1.72, size = 93, normalized size = 0.95

$$2 \sqrt{2} \sqrt{a} \left(\frac{4 x \cos \left(\frac{1}{4} \pi - \frac{1}{2} dx - \frac{1}{2} c\right) \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)}{d^2} - \frac{\left(d^2 x^2 \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) - 8 \,\mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)\right) \sin \left(\frac{1}{4} \pi - \frac{1}{2} dx - \frac{1}{2} c\right)}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*(4*x*cos(1/4*pi - 1/2*d*x - 1/2*c)*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))/d^2 - (d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 8*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c)))*sin(1/4*pi - 1/2*d*x - 1/2*c)/d^3)
```

**Mupad** [B]
time = 0.84, size = 64, normalized size = 0.65

$$\frac{2 \sqrt{a\, (\sin (c + d\, x) + 1)}\, (8 \cos (c + d\, x) + 4\, d\, x - d^2\, x^2\, \cos (c + d\, x) + 4\, d\, x\, \sin (c + d\, x))}{d^3\, (\sin (c + d\, x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(1/2)*(8*cos(c + d*x) + 4*d*x - d^2*x^2*cos(c + d
*x) + 4*d*x*sin(c + d*x)))/(d^3*(sin(c + d*x) + 1))
```

# 3.124 $\int x\sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=58

$$\frac{4\sqrt{a+a\sin(c+dx)}}{d^2}-\frac{2x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}$$

```
[Out] 4*(a+a*sin(d*x+c))^(1/2)/d^2-2*x*cot(1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*x+c))
^(1/2)/d
```

**Rubi** [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {3400, 3377, 2718}

$$\frac{4\sqrt{a\sin(c+dx)+a}}{d^2}-\frac{2x\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + a*Sin[c + d*x]])/d^2 - (2*x*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a +
a*Sin[c + d*x]])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x\sqrt{a+a\sin(c+dx)}\,dx = \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int x\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx$$
$$= -\frac{2x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d} + \frac{\left(2\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(\right.}{d}$$
$$= \frac{4\sqrt{a+a\sin(c+dx)}}{d^2} - \frac{2x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}$$

**Mathematica** [A]
time = 0.10, size = 76, normalized size = 1.31

$$-\frac{2\left((-2+dx)\cos\left(\frac{1}{2}(c+dx)\right)-(2+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Sin[c + d*x]],x]
[Out] (-2*((-2 + d*x)*Cos[(c + d*x)/2] - (2 + d*x)*Sin[(c + d*x)/2])*Sqrt[a*(1 +
Sin[c + d*x]])])/(d^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple** [C] Result contains complex when optimal does not.
time = 0.04, size = 93, normalized size = 1.60

| method | result | size |
|---|---|---|
| risch | $-\frac{i\sqrt{2}\,\sqrt{-a\left(-2-2\sin\left(dx+c\right)\right)}\,\left(-idx+dx\,e^{i(dx+c)}+2ie^{i(dx+c)}-2\right)\left(e^{i(dx+c)}+i\right)}{\left(e^{2i(dx+c)}+2ie^{i(dx+c)}-1\right)d^2}$ | 93 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -I*2^(1/2)*(-a*(-2-2*sin(d*x+c)))^(1/2)/(exp(2*I*(d*x+c))+2*I*exp(I*(d*x+c)
)-1)*(-I*d*x+d*x*exp(I*(d*x+c))+2*I*exp(I*(d*x+c))-2)*(exp(I*(d*x+c))+I)/d^
2
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*x, x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac [A]**
time = 1.52, size = 69, normalized size = 1.19

$$-2\sqrt{2}\left(\frac{x\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{4}\pi-\frac{1}{2}dx-\frac{1}{2}c\right)}{d}-\frac{2\cos\left(\frac{1}{4}\pi-\frac{1}{2}dx-\frac{1}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d^2}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*(x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi - 1/2*d*x - 1/
2*c)/d - 2*cos(1/4*pi - 1/2*d*x - 1/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)/d^2)*sqrt(a)
```

**Mupad [B]**
time = 0.23, size = 47, normalized size = 0.81

$$\frac{2\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(2\,\sin\left(c+d\,x\right)-d\,x\,\cos\left(c+d\,x\right)+2\right)}{d^2\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(1/2)*(2*sin(c + d*x) - d*x*cos(c + d*x) + 2))/(d
^2*(sin(c + d*x) + 1))
```

# 3.125 $\int \frac{\sqrt{a+a\sin(c+dx)}}{x}\,dx$

**Optimal**. Leaf size=101

$$\text{Ci}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c+\pi)\right)\sqrt{a+a\sin(c+dx)}+\cos\left(\frac{1}{4}(2c+\pi)\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{}$$

```
[Out] cos(1/2*c+1/4*Pi)*csc(1/2*c+1/4*Pi+1/2*d*x)*Si(1/2*d*x)*(a+a*sin(d*x+c))^(1
/2)+Ci(1/2*d*x)*csc(1/2*c+1/4*Pi+1/2*d*x)*sin(1/2*c+1/4*Pi)*(a+a*sin(d*x+c)
)^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3400, 3384, 3380, 3383}

$$\sin\left(\frac{1}{4}(2c+\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}+\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/x,x]
```

```
[Out] CosIntegral[(d*x)/2]*Csc[c/2 + Pi/4 + (d*x)/2]*Sin[(2*c + Pi)/4]*Sqrt[a + a
*Sin[c + d*x]] + Cos[(2*c + Pi)/4]*Csc[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin
[c + d*x]]*SinIntegral[(d*x)/2]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x}\,dx = \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{x}\,dx$$
$$= \left(\cos\left(\frac{1}{4}(2c+\pi)\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sin\left(\frac{dx}{2}\right)}{x}\,dx +$$
$$= \mathrm{Ci}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c+\pi)\right)\sqrt{a+a\sin(c+dx)} + \cos\left(\frac{1}{4}(\right.$$

**Mathematica [A]**
time = 0.09, size = 83, normalized size = 0.82

$$\frac{\sqrt{a(1+\sin(c+dx))}\left(\mathrm{Ci}\left(\frac{dx}{2}\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)+\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\mathrm{Si}\left(\frac{dx}{2}\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/x,x]
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(CosIntegral[(d*x)/2]*(Cos[c/2] + Sin[c/2]) + (
Cos[c/2] - Sin[c/2])*SinIntegral[(d*x)/2]))/(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])
```

**Maple [F]**
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\sin\left(dx+c\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/x,x)
[Out] int((a+a*sin(d*x+c))^(1/2)/x,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(a*sin(d*x + c) + a)/x, x)

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(1/2)/x,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (has polynomial part)

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(c + dx\right) + 1\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**(1/2)/x,x)

[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/x, x)

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.54, size = 383, normalized size = 3.79

$$-\frac{\sqrt{2}\left(\Im\left(\mathrm{Ci}\left(\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}c\right)^{2}-\Im\left(\mathrm{Ci}\left(-\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}c\right)^{2}+\Re\left(\mathrm{Ci}\left(\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}c\right)^{2}+\Re\left(\mathrm{Ci}\left(-\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}c\right)^{2}+2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\mathrm{Si}\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{4}c\right)^{2}+2\,\Im\left(\mathrm{Ci}\left(\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}c\right)-2\,\Im\left(\mathrm{Ci}\left(-\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}c\right)-2\,\Re\left(\mathrm{Ci}\left(\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}c\right)-2\,\Re\left(\mathrm{Ci}\left(-\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}c\right)+4\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\mathrm{Si}\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{4}c\right)-\Im\left(\mathrm{Ci}\left(\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\Im\left(\mathrm{Ci}\left(-\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\Re\left(\mathrm{Ci}\left(\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\Re\left(\mathrm{Ci}\left(-\frac{1}{2}dx\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\mathrm{Si}\left(\frac{1}{2}dx\right)\right)\sqrt{a}}{2\left(\sqrt{2}\tan\left(\frac{1}{4}c\right)^{2}+\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(1/2)/x,x, algorithm="giac")

[Out] -1/2*sqrt(2)*(imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/4*c)^2 - imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))*tan(1/4*c)^2 + real_part(cos_integral(1/2*d*x))*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + real_part(cos_integral(-1/2*d*x))
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 2*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*c)^2 + 2*imag_part(cos_integra
l(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - 2*imag_part(co
s_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - 2*re
al_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*
c) - 2*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*tan(1/4*c) + 4*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*

```
tan(1/4*c) - imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c)) + imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c)) - real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
- real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 2
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x))*sqrt(a)/(sqrt(2
)*tan(1/4*c)^2 + sqrt(2))
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/x,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/x, x)
```

# 3.126 $\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx$

**Optimal**. Leaf size=130

$$-\frac{\sqrt{a+a\sin(c+dx)}}{x}-\frac{1}{2}d\text{Ci}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c-\pi)\right)\sqrt{a+a\sin(c+dx)}-\frac{1}{2}d\csc\left(\frac{c}{2}+\right.$$

```
[Out] -(a+a*sin(d*x+c))^(1/2)/x+1/2*d*Ci(1/2*d*x)*csc(1/2*c+1/4*Pi+1/2*d*x)*cos(1
/2*c+1/4*Pi)*(a+a*sin(d*x+c))^(1/2)-1/2*d*csc(1/2*c+1/4*Pi+1/2*d*x)*Si(1/2*
d*x)*sin(1/2*c+1/4*Pi)*(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}d\sin\left(\frac{1}{4}(2c-\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}-\frac{1}{2}d\sin\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}-\frac{\sqrt{a\sin(c+dx)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Sin[c + d*x]]/x) - (d*CosIntegral[(d*x)/2]*Csc[c/2 + Pi/4 + (d
*x)/2]*Sin[(2*c - Pi)/4]*Sqrt[a + a*Sin[c + d*x]])/2 - (d*Csc[c/2 + Pi/4 +
(d*x)/2]*Sin[(2*c + Pi)/4]*Sqrt[a + a*Sin[c + d*x]]*SinIntegral[(d*x)/2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx &= \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{x^2}\,dx\\ &= -\frac{\sqrt{a+a\sin(c+dx)}}{x}+\frac{1}{2}\left(d\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\text{c}}{}\\ &= -\frac{\sqrt{a+a\sin(c+dx)}}{x}-\frac{1}{2}\left(d\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c-\pi)\right)\sqrt{a+a\,\text{si}}\right.\\ &= -\frac{\sqrt{a+a\sin(c+dx)}}{x}-\frac{1}{2}d\text{Ci}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c-\pi)\right)\sqrt{}\end{aligned}$$

**Mathematica [A]**
time = 0.17, size = 117, normalized size = 0.90

$$\frac{\sqrt{a(1+\sin(c+dx))}\left(dx\text{Ci}\left(\frac{dx}{2}\right)\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)-2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-dx\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\text{Si}\left(\frac{dx}{2}\right)\right)}{2x\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/x^2,x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(d*x*CosIntegral[(d*x)/2]*(Cos[c/2] - Sin[c/2])
 - 2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - d*x*(Cos[c/2] + Sin[c/2])*SinI
ntegral[(d*x)/2]))/(2*x*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple [F]**
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\sin(dx+c)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/x^2,x)
```

```
[Out] int((a+a*sin(d*x+c))^(1/2)/x^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/x^2, x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/x**2, x)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.20, size = 1140, normalized size = 8.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 - d*x*imag_part(cos_integral(-1/2*d*
```

```
x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 - d*x*r
eal_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4
*d*x)^2*tan(1/4*c)^2 - d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d*x*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c)^2 - 2*d*
x*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(
1/4*d*x)^2*tan(1/4*c) + 2*d*x*imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 2*d*x*real_part(cos_in
tegral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4
*c) - 2*d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 4*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin_integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c) - d*x*imag_part(cos_integ
ral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 + d*x*imag
_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d
*x)^2 + d*x*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/4*d*x)^2 + d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - 2*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2 + d*x*imag_part(cos_integral(1/2
*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 - d*x*imag_part(cos
_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 - d*x
*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1
/4*c)^2 - d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*tan(1/4*c)^2 + 2*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integ
ral(1/2*d*x)*tan(1/4*c)^2 - 2*d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) + 2*d*x*imag_part(cos_integral(-1/2*
d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - 2*d*x*real_part(cos_
integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - 2*d*x*r
eal_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
4*c) - 4*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(
1/4*c) - 4*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2
- d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
+ d*x*imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
 + d*x*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
 + d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
) - 2*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x) + 8*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) + 8*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/4*c)^2 + 4*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))*tan(1/4*d*x)^2 + 16*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(
1/4*d*x)*tan(1/4*c) + 4*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 -
8*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x) - 8*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))*tan(1/4*c) - 4*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))*sqrt(a)
/(sqrt(2)*x*tan(1/4*d*x)^2*tan(1/4*c)^2 + sqrt(2)*x*tan(1/4*d*x)^2 + sqrt(2
)*x*tan(1/4*c)^2 + sqrt(2)*x)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a\,\sin\left(c + d\,x\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/x^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/x^2, x)
```

# 3.127 $\int \frac{\sqrt{a+a\sin(c+dx)}}{x^3}\,dx$

**Optimal**. Leaf size=174

$$-\frac{\sqrt{a+a\sin(c+dx)}}{2x^2}-\frac{d\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{4x}-\frac{1}{8}d^2\text{Ci}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2\right.$$

```
[Out] -1/2*(a+a*sin(d*x+c))^(1/2)/x^2-1/4*d*cot(1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*
x+c))^(1/2)/x-1/8*d^2*cos(1/2*c+1/4*Pi)*csc(1/2*c+1/4*Pi+1/2*d*x)*Si(1/2*d*
x)*(a+a*sin(d*x+c))^(1/2)-1/8*d^2*Ci(1/2*d*x)*csc(1/2*c+1/4*Pi+1/2*d*x)*sin
(1/2*c+1/4*Pi)*(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3378, 3384, 3380, 3383}

$$-\frac{1}{8}d^2\sin\left(\frac{1}{4}(2c+\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}-\frac{1}{8}d^2\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}-\frac{\sqrt{a\sin(c+dx)+a}}{2x^2}-\frac{d\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + a*Sin[c + d*x]]/x^2 - (d*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a
*Sin[c + d*x]])/(4*x) - (d^2*CosIntegral[(d*x)/2]*Csc[c/2 + Pi/4 + (d*x)/2]
*Sin[(2*c + Pi)/4]*Sqrt[a + a*Sin[c + d*x]])/8 - (d^2*Cos[(2*c + Pi)/4]*Csc
[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]]*SinIntegral[(d*x)/2])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\sin(c+dx)}}{x^3}\,dx &= \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{x^3}\,dx \\
&= -\frac{\sqrt{a+a\sin(c+dx)}}{2x^2}+\frac{1}{4}\left(d\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\cos
\\
&= -\frac{\sqrt{a+a\sin(c+dx)}}{2x^2}-\frac{d\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{4x}-\frac{1}{8}\left(d^2\csc\left(
\\
&= -\frac{\sqrt{a+a\sin(c+dx)}}{2x^2}-\frac{d\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{4x}-\frac{1}{8}\left(d^2\cos\left(
\\
&= -\frac{\sqrt{a+a\sin(c+dx)}}{2x^2}-\frac{d\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{4x}-\frac{1}{8}d^2\mathrm{Ci}\left(\frac{dx}{2}
\end{aligned}
$$

**Mathematica** [A]
time = 0.18, size = 153, normalized size = 0.88

$$
-\frac{\sqrt{a(1+\sin(c+dx))}\left(4\cos\left(\frac{1}{2}(c+dx)\right)+2dx\cos\left(\frac{1}{2}(c+dx)\right)+d^2x^2\mathrm{Ci}\left(\frac{dx}{2}\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)+4\sin\left(\frac{1}{2}(c+dx)\right)-2dx\sin\left(\frac{1}{2}(c+dx)\right)+d^2x^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\mathrm{Si}\left(\frac{dx}{2}\right)\right)}{8x^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/x^3,x]
```

```
[Out] -1/8*(Sqrt[a*(1 + Sin[c + d*x])]*(4*Cos[(c + d*x)/2] + 2*d*x*Cos[(c + d*x)/
2] + d^2*x^2*CosIntegral[(d*x)/2]*(Cos[c/2] + Sin[c/2]) + 4*Sin[(c + d*x)/2
] - 2*d*x*Sin[(c + d*x)/2] + d^2*x^2*(Cos[c/2] - Sin[c/2])*SinIntegral[(d*x
)/2]))/(x^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple** [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\sin\left(dx+c\right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/x^3,x)
```

```
[Out] int((a+a*sin(d*x+c))^(1/2)/x^3,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/x^3, x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/x**3, x)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.64, size = 1487, normalized size = 8.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(1/2)/x^3,x, algorithm="giac")

[Out] 1/16*sqrt(2)*(d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 - d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 2*d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 2*d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 2*d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) + 4*d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c) - d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 + d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - 2*d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2 + d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 - d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 2*d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*c)^2 + 2*d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - 2*d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - 2*d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - 2*d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) + 4*d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*c) - 4*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 - d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) + d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 2*d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x) - 8*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 8*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/4*c)^2 + 4*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)

```
)*tan(1/4*d*x)^2 + 16*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)*
tan(1/4*c) + 4*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 - 8*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 8*d*x*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x) + 8*d*x*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))*tan(1/4*c) + 16*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x
)^2*tan(1/4*c) + 16*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/
4*c)^2 - 4*d*x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) + 8*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))*tan(1/4*d*x)^2 + 32*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan
(1/4*d*x)*tan(1/4*c) + 8*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 -
 16*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x) - 16*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))*tan(1/4*c) - 8*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))*sqrt
(a)/(sqrt(2)*x^2*tan(1/4*d*x)^2*tan(1/4*c)^2 + sqrt(2)*x^2*tan(1/4*d*x)^2 +
 sqrt(2)*x^2*tan(1/4*c)^2 + sqrt(2)*x^2)
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/x^3,x)
[Out] int((a + a*sin(c + d*x))^(1/2)/x^3, x)
```

# 3.128 $\int x^3(a+a\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=337

$$-\frac{1280a\sqrt{a+a\sin(e+fx)}}{9f^4}+\frac{16ax^2\sqrt{a+a\sin(e+fx)}}{f^2}+\frac{640ax\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3}-\frac{8a$$

```
[Out] -1280/9*a*(a+a*sin(f*x+e))^(1/2)/f^4+16*a*x^2*(a+a*sin(f*x+e))^(1/2)/f^2+64
0/9*a*x*cot(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/f^3-8/3*a*x^3*cot(
1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/f+32/9*a*x*cos(1/2*e+1/4*Pi+1/
2*f*x)*sin(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/f^3-4/3*a*x^3*cos(1
/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/f-64/
27*a*sin(1/2*e+1/4*Pi+1/2*f*x)^2*(a+a*sin(f*x+e))^(1/2)/f^4+8/3*a*x^2*sin(1
/2*e+1/4*Pi+1/2*f*x)^2*(a+a*sin(f*x+e))^(1/2)/f^2
```

**Rubi** [A]
time = 0.15, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3392, 3377, 2718, 3391}

$$-\frac{64a\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{27f^4}-\frac{1280a\sqrt{a\sin(e+fx)+a}}{9f^4}+\frac{32ax\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^3}+\frac{640ax\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^3}+\frac{8ax^2\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{3f^2}+\frac{16ax^2\sqrt{a\sin(e+fx)+a}}{f^2}-\frac{4ax^3\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{8ax^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-1280*a*Sqrt[a + a*Sin[e + f*x]])/(9*f^4) + (16*a*x^2*Sqrt[a + a*Sin[e + f
*x]])/f^2 + (640*a*x*Cot[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(9
*f^3) - (8*a*x^3*Cot[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
+ (32*a*x*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Si
n[e + f*x]])/(9*f^3) - (4*a*x^3*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 +
(f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f) - (64*a*Sin[e/2 + Pi/4 + (f*x)/2]^
2*Sqrt[a + a*Sin[e + f*x]])/(27*f^4) + (8*a*x^2*Sin[e/2 + Pi/4 + (f*x)/2]^2
*Sqrt[a + a*Sin[e + f*x]])/(3*f^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^3(a+a\sin(e+fx))^{3/2}\,dx = \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int x^3\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)$$

$$= -\frac{4ax^3\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{8ax^2\sin^2}{}$$

$$= -\frac{8ax^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{32ax\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin}{}$$

$$= \frac{16ax^2\sqrt{a+a\sin(e+fx)}}{f^2} + \frac{64ax\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3}$$

$$= -\frac{128a\sqrt{a+a\sin(e+fx)}}{9f^4} + \frac{16ax^2\sqrt{a+a\sin(e+fx)}}{f^2} + \frac{640ax\cot\left(\frac{e}{2}\right.}{}$$

$$= -\frac{1280a\sqrt{a+a\sin(e+fx)}}{9f^4} + \frac{16ax^2\sqrt{a+a\sin(e+fx)}}{f^2} + \frac{640ax\cot\left(\frac{e}{2}\right.}{}$$

**Mathematica** [A]

time = 0.84, size = 231, normalized size = 0.69

$$\frac{2a\left(-\frac{2\left(\left(968-480fx-117f^2x^2+18f^3x^3\right)\cos\left(\frac{e}{2}\right)+\left(968+480fx-117f^2x^2-18f^3x^3\right)\sin\left(\frac{e}{2}\right)\right)}{\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)}-\cos(fx)\left(3fx(-8+3f^2x^2)\cos(e)+2(8-9f^2x^2)\sin(e)\right)+\left(2(-8+9f^2x^2)\cos(e)+3fx(-8+3f^2x^2)\sin(e)\right)\sin(fx)+\frac{24fx(-80+3f^2x^2)\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)\sqrt{a(1+\sin(e+fx))}}{27f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a*((-2*((968 - 480*f*x - 117*f^2*x^2 + 18*f^3*x^3)*Cos[e/2] + (968 + 480
*f*x - 117*f^2*x^2 - 18*f^3*x^3)*Sin[e/2]))/(Cos[e/2] + Sin[e/2]) - Cos[f*x
]*(3*f*x*(-8 + 3*f^2*x^2)*Cos[e] + 2*(8 - 9*f^2*x^2)*Sin[e]) + (2*(-8 + 9*f
^2*x^2)*Cos[e] + 3*f*x*(-8 + 3*f^2*x^2)*Sin[e])*Sin[f*x] + (24*f*x*(-80 + 3
*f^2*x^2)*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]))))*Sqrt[a*(1 + Sin[e + f*x])])/(27*f^4)
```

**Maple [F]**
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3(a+a\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^3*(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*x^3, x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a(\sin(e+fx)+1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3*(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 1210 vs. $2(275) = 550$.
time = 3.07, size = 1210, normalized size = 3.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/216*sqrt(2)*(972*(4*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))/f^3 + (pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f
*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)/f^3)*e^2 + 36*(4*a*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))/f^3 + 3*(pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2
*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x +
3/2*e)/f^3)*e^2 - 486*(8*(pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi -
2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x +
 1/2*e)/f^3 + (pi^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*
x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)^2*a*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)*sin(-1/4*pi + 1/2*f*x + 1/2*e)/f^3)*e - 6*(24*(pi*a*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*
cos(-3/4*pi + 3/2*f*x + 3/2*e)/f^3 + (9*pi^2*a*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) - 18*pi*(pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
9*(pi - 2*f*x - 2*e)^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e)/f^3)*e - 648*a
*e^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)/f^3
 - 72*a*e^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3/2*f*x + 3/2
*e)/f^3 + 972*(pi^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*
x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)^2*a*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)*cos(-1/4*pi + 1/2*f*x + 1/2*e)/f^3 + 4*(9*pi^2*a*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) - 18*pi*(pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 9*(pi - 2*f*x - 2*e)^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*sgn
```

```
(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e)/f^3 + 81*(
pi^3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*pi^2*(pi - 2*f*x - 2*e)*a*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*pi*(pi - 2*f*x - 2*e)^2*a*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)^3*a*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) - 96*pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 96*(pi - 2*f*x -
 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
/f^3 + 3*(3*pi^3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 9*pi^2*(pi - 2*f*x
 - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*pi*(pi - 2*f*x - 2*e)^2*a
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*(pi - 2*f*x - 2*e)^3*a*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) - 32*pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 32
*(pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/
2*f*x + 3/2*e)/f^3)*sqrt(a)/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(a+a\,\sin(e+f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*sin(e + f*x))^(3/2),x)
[Out] int(x^3*(a + a*sin(e + f*x))^(3/2), x)
```

# 3.129 $\int x^2(a+a\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=271

$$\frac{32ax\sqrt{a+a\sin(e+fx)}}{3f^2}+\frac{224a\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3}-\frac{8ax^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin}}{3f}$$

```
[Out] 32/3*a*x*(a+a*sin(f*x+e))^(1/2)/f^2+224/9*a*cot(1/2*e+1/4*Pi+1/2*f*x)*(a+a*
sin(f*x+e))^(1/2)/f^3-8/3*a*x^2*cot(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^
(1/2)/f-32/27*a*cos(1/2*e+1/4*Pi+1/2*f*x)^2*cot(1/2*e+1/4*Pi+1/2*f*x)*(a+a*
sin(f*x+e))^(1/2)/f^3-4/3*a*x^2*cos(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi+
1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/f+16/9*a*x*sin(1/2*e+1/4*Pi+1/2*f*x)^2*(a+a
*sin(f*x+e))^(1/2)/f^2
```

**Rubi** [A]
time = 0.11, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3392, 3377, 2718, 2713}

$$\frac{224a\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^3}-\frac{32a\cos^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{27f^3}+\frac{16ax\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^2}+\frac{32ax\sqrt{a\sin(e+fx)+a}}{3f^2}-\frac{4ax^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{8ax^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (32*a*x*Sqrt[a + a*Sin[e + f*x]])/(3*f^2) + (224*a*Cot[e/2 + Pi/4 + (f*x)/2
]*Sqrt[a + a*Sin[e + f*x]])/(9*f^3) - (8*a*x^2*Cot[e/2 + Pi/4 + (f*x)/2]*Sq
rt[a + a*Sin[e + f*x]])/(3*f) - (32*a*Cos[e/2 + Pi/4 + (f*x)/2]^2*Cot[e/2 +
 Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(27*f^3) - (4*a*x^2*Cos[e/2 + Pi
/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (
16*a*x*Sin[e/2 + Pi/4 + (f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/(9*f^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^2(a+a\sin(e+fx))^{3/2}\,dx = \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int x^2\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\,dx$$

$$= -\frac{4ax^2\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{16ax\sin^2\left(\right.}{}$$

$$= -\frac{8ax^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}-\frac{4ax^2\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\right.}{}$$

$$= \frac{32ax\sqrt{a+a\sin(e+fx)}}{3f^2}+\frac{32a\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3}-\frac{8}{}$$

$$= \frac{32ax\sqrt{a+a\sin(e+fx)}}{3f^2}+\frac{224a\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3}-$$

**Mathematica** [A]
time = 0.62, size = 191, normalized size = 0.70

$$\frac{2a\left(-\frac{4\left(\left(-80-39fx+9f^2x^2\right)\cos\left(\frac{e}{2}\right)+\left(80-39fx-9f^2x^2\right)\sin\left(\frac{e}{2}\right)\right)}{\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)}-\cos(fx)\left(\left(-8+9f^2x^2\right)\cos(e)-12fx\sin(e)\right)+\left(12fx\cos(e)+\left(-8+9f^2x^2\right)\sin(e)\right)\sin(fx)+\frac{8\left(-80+9f^2x^2\right)\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)\sqrt{a(1+\sin(e+fx))}}{27f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a*((-4*((-80 - 39*f*x + 9*f^2*x^2)*Cos[e/2] + (80 - 39*f*x - 9*f^2*x^2)*
Sin[e/2]))/(Cos[e/2] + Sin[e/2]) - Cos[f*x]*((-8 + 9*f^2*x^2)*Cos[e] - 12*f
*x*Sin[e]) + (12*f*x*Cos[e] + (-8 + 9*f^2*x^2)*Sin[e])*Sin[f*x] + (8*(-80 +
 9*f^2*x^2)*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])))*Sqrt[a*(1 + Sin[e + f*x])])/(27*f^3)
```

**Maple [F]**
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{2}(a+a\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^2*(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*x^2, x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}(a(\sin(e+fx)+1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**2*(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(218) = 436.
time = 3.17, size = 633, normalized size = 2.34

$$-\frac{\sqrt{2}\left(324\left(\frac{4a\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f^2}+\frac{\left(\pi a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-(\pi-2fx-2e)a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{f^2}\right)e+12\left(\frac{4a\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f^2}+\frac{3\left(\pi a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-(\pi-2fx-2e)a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)}{f^2}\right)e-\frac{324ae^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{f^2}-\frac{36ae^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)}{f^2}-\frac{648\left(\pi a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-(\pi-2fx-2e)a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{f^2}-\frac{24\left(\pi a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-(\pi-2fx-2e)a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)}{f^2}-\frac{81\left(\pi^2a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\pi(\pi-2fx-2e)a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+(\pi-2fx-2e)^2a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-32a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{f^2}-\frac{\left(9\pi^2a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-18\pi(\pi-2fx-2e)a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+9(\pi-2fx-2e)^2a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-32a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)}{f^2}\right)\sqrt{a}}{108f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/108*sqrt(2)*(324*(4*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))/f^2 + (pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*
f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)/f^2)*e + 12*(4*a*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))/f^2 + 3*(pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*
f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3
/2*e)/f^2)*e - 324*a*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)/f^2 - 36*a*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/
4*pi + 3/2*f*x + 3/2*e)/f^2 - 648*(pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 - (pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi +
1/2*f*x + 1/2*e)/f^2 - 24*(pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi -
 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x
+ 3/2*e)/f^2 - 81*(pi^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi -
2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)^2*a
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)/f^2 - (9*pi^2*a*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) - 18*pi*(pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 9*(pi - 2*f*x - 2*e)^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e)/f^2)*sqr
t(a)/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(x^2*(a + a*sin(e + f*x))^(3/2), x)
```

# 3.130 $\int x(a + a\sin(e + fx))^{3/2}\, dx$

**Optimal**. Leaf size=165

$$\frac{16a\sqrt{a + a\sin(e + fx)}}{3f^2} - \frac{8ax\cot\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a + a\sin(e + fx)}}{3f} - \frac{4ax\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{f}{}\right.}{3f}$$

```
[Out] 16/3*a*(a+a*sin(f*x+e))^(1/2)/f^2-8/3*a*x*cot(1/2*e+1/4*Pi+1/2*f*x)*(a+a*si
n(f*x+e))^(1/2)/f-4/3*a*x*cos(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi+1/2*f*
x)*(a+a*sin(f*x+e))^(1/2)/f+8/9*a*sin(1/2*e+1/4*Pi+1/2*f*x)^2*(a+a*sin(f*x+
e))^(1/2)/f^2
```

**Rubi** [A]
time = 0.06, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3400, 3391, 3377, 2718}

$$\frac{8a\sin^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{\pi}{4}\right)\sqrt{a\sin(e + fx) + a}}{9f^2} + \frac{16a\sqrt{a\sin(e + fx) + a}}{3f^2} - \frac{4ax\sin\left(\frac{e}{2} + \frac{fx}{2} + \frac{\pi}{4}\right)\cos\left(\frac{e}{2} + \frac{fx}{2} + \frac{\pi}{4}\right)\sqrt{a\sin(e + fx) + a}}{3f} - \frac{8ax\cot\left(\frac{e}{2} + \frac{fx}{2} + \frac{\pi}{4}\right)\sqrt{a\sin(e + fx) + a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (16*a*Sqrt[a + a*Sin[e + f*x]])/(3*f^2) - (8*a*x*Cot[e/2 + Pi/4 + (f*x)/2]*
Sqrt[a + a*Sin[e + f*x]])/(3*f) - (4*a*x*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2
+ Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (8*a*Sin[e/2 + Pi/4 + (
f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/(9*f^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x(a+a\sin(e+fx))^{3/2}\,dx = \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int x\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)dx$$

$$= -\frac{4ax\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{8a\sin^2\left(\frac{e}{2}+\right.}{}$$

$$= -\frac{8ax\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}-\frac{4ax\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\right.}{3}$$

$$= \frac{16a\sqrt{a+a\sin(e+fx)}}{3f^2}-\frac{8ax\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}-\frac{4ax}{}$$

**Mathematica [A]**
time = 0.33, size = 113, normalized size = 0.68

$$-\frac{\left(27(-2+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(2+3fx)\cos\left(\frac{3}{2}(e+fx)\right)+2(-4(7+3fx)+(-2+3fx)\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}{9f^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + a*Sin[e + f*x])^(3/2),x]
[Out] -1/9*((27*(-2 + f*x)*Cos[(e + f*x)/2] + (2 + 3*f*x)*Cos[(3*(e + f*x))/2] +
2*(-4*(7 + 3*f*x) + (-2 + 3*f*x)*Cos[e + f*x])*Sin[(e + f*x)/2])*(a*(1 + Si
n[e + f*x]))^(3/2))/(f^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple [F]**
time = 0.01, size = 0, normalized size = 0.00

$$\int x(a+a\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*sin(f*x+e))^(3/2),x)
[Out] int(x*(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*x, x)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a(\sin(e+fx)+1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x*(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(133) = 266.
time = 2.56, size = 274, normalized size = 1.66

$$-\frac{\sqrt{2}\left(\frac{54\,a e\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{f}+\frac{6\,a e\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)}{f}-\frac{108\,a\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}-\frac{4\,a\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}-\frac{27\left(\pi a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-(\pi-2fx-2e)a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{f}-\frac{3\left(\pi a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-(\pi-2fx-2e)a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)}{f}\right)\sqrt{a}}{18\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(2)*(54*a*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)/f + 6*a*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3
/2*f*x + 3/2*e)/f - 108*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))/f - 4*a*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))/f - 27*(pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi -
```

```
2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x +
 1/2*e)/f - 3*(pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e
)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e)/f)*
sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+a\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(x*(a + a*sin(e + f*x))^(3/2), x)
```

# 3.131 $\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx$

**Optimal**. Leaf size=221

$$\frac{1}{2}a\cos\left(\frac{3}{4}(2e-\pi)\right)\operatorname{Ci}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}+\frac{3}{2}a\operatorname{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\mathrm{s}$$

```
[Out] -1/2*a*Ci(3/2*f*x)*cos(3/2*e+1/4*Pi)*csc(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x
+e))^(1/2)+3/2*a*cos(1/2*e+1/4*Pi)*csc(1/2*e+1/4*Pi+1/2*f*x)*Si(1/2*f*x)*(a
+a*sin(f*x+e))^(1/2)+1/2*a*csc(1/2*e+1/4*Pi+1/2*f*x)*Si(3/2*f*x)*sin(3/2*e+
1/4*Pi)*(a+a*sin(f*x+e))^(1/2)+3/2*a*Ci(1/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x)*
sin(1/2*e+1/4*Pi)*(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.18, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3393, 3384, 3380, 3383}

$$\frac{3}{2}a\sin\left(\frac{1}{4}(2e+\pi)\right)\operatorname{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}+\frac{1}{2}a\cos\left(\frac{3}{4}(2e-\pi)\right)\operatorname{CosIntegral}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}-\frac{1}{2}a\sin\left(\frac{3}{4}(2e-\pi)\right)\operatorname{Si}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}+\frac{3}{2}a\cos\left(\frac{1}{4}(2e+\pi)\right)\operatorname{Si}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/x,x]
```

```
[Out] (a*Cos[(3*(2*e - Pi))/4]*CosIntegral[(3*f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]*S
qrt[a + a*Sin[e + f*x]])/2 + (3*a*CosIntegral[(f*x)/2]*Csc[e/2 + Pi/4 + (f*
x)/2]*Sin[(2*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]])/2 + (3*a*Cos[(2*e + Pi)/4
]*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]*SinIntegral[(f*x)/2])/
2 - (a*Csc[e/2 + Pi/4 + (f*x)/2]*Sin[(3*(2*e - Pi))/4]*Sqrt[a + a*Sin[e + f
*x]]*SinIntegral[(3*f*x)/2])/2
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx &= \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{x}\,dx\\ &= \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \left(\frac{3\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{4x}+\frac{\sin}{}\right.\\ &= \frac{1}{2}\left(a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\sin\left(\frac{3e}{2}-\frac{\pi}{4}+\frac{3fx}{2}\right)}{x}\,dx+\frac{1}{2}\\ &= \frac{1}{2}\left(a\cos\left(\frac{3}{4}(2e-\pi)\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\cos\left(\frac{3fx}{2}\right)}{x}\\ &= \frac{1}{2}a\cos\left(\frac{3}{4}(2e-\pi)\right)\operatorname{Ci}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}+\frac{3}{2}a\end{aligned}$$

**Mathematica [A]**
time = 0.31, size = 127, normalized size = 0.57

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(3\operatorname{Ci}\left(\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)+\operatorname{Ci}\left(\frac{3fx}{2}\right)\left(-\cos\left(\frac{3e}{2}\right)+\sin\left(\frac{3e}{2}\right)\right)+\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(3\operatorname{Si}\left(\frac{fx}{2}\right)+(1+2\sin(e))\operatorname{Si}\left(\frac{3fx}{2}\right)\right)\right)}{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/x,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(3*CosIntegral[(f*x)/2]*(Cos[e/2] + Sin[e/2])
 + CosIntegral[(3*f*x)/2]*(-Cos[(3*e)/2] + Sin[(3*e)/2]) + (Cos[e/2] - Sin[
```

```
e/2])*(3*SinIntegral[(f*x)/2] + (1 + 2*Sin[e])*SinIntegral[(3*f*x)/2])))/(2
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple** [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx + e))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/x,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)/x,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/x, x)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/x,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/x, x)
```

**Giac [A]**
time = 2.97, size = 138, normalized size = 0.62

$$\frac{\sqrt{2}\left(af\cos\left(\frac{3}{4}\pi-\frac{3}{2}e\right)\mathrm{Ci}\left(\frac{3}{2}fx\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3af\cos\left(\frac{1}{4}\pi-\frac{1}{2}e\right)\mathrm{Ci}\left(\frac{1}{2}fx\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+af\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(\frac{3}{4}\pi-\frac{3}{2}e\right)\mathrm{Si}\left(\frac{3}{2}fx\right)+3af\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(\frac{1}{4}\pi-\frac{1}{2}e\right)\mathrm{Si}\left(\frac{1}{2}fx\right)\right)\sqrt{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*f*cos(3/4*pi - 3/2*e)*cos_integral(3/2*f*x)*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 3*a*f*cos(1/4*pi - 1/2*e)*cos_integral(1/2*f*x)*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + a*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*si
n(3/4*pi - 3/2*e)*sin_integral(3/2*f*x) + 3*a*f*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(1/4*pi - 1/2*e)*sin_integral(1/2*f*x))*sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/x,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/x, x)
```

# 3.132 $\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx$

**Optimal**. Leaf size=263

$$-\frac{3}{4}af\mathrm{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e-\pi)\right)\sqrt{a+a\sin(e+fx)}+\frac{3}{4}af\mathrm{Ci}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f}{2}\right.$$

```
[Out] 3/4*a*f*cos(3/2*e+1/4*Pi)*csc(1/2*e+1/4*Pi+1/2*f*x)*Si(3/2*f*x)*(a+a*sin(f*
x+e))^(1/2)+3/4*a*f*Ci(1/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x)*cos(1/2*e+1/4*Pi)
*(a+a*sin(f*x+e))^(1/2)-3/4*a*f*csc(1/2*e+1/4*Pi+1/2*f*x)*Si(1/2*f*x)*sin(1
/2*e+1/4*Pi)*(a+a*sin(f*x+e))^(1/2)+3/4*a*f*Ci(3/2*f*x)*csc(1/2*e+1/4*Pi+1/
2*f*x)*sin(3/2*e+1/4*Pi)*(a+a*sin(f*x+e))^(1/2)-2*a*sin(1/2*e+1/4*Pi+1/2*f*
x)^2*(a+a*sin(f*x+e))^(1/2)/x
```

**Rubi** [A]

time = 0.18, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3394, 3384, 3380, 3383}

$$-\frac{3}{4}af\sin\left(\frac{1}{4}(2e-\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}+\frac{3}{4}af\sin\left(\frac{1}{4}(6e+\pi)\right)\text{CosIntegral}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}-\frac{3}{4}af\sin\left(\frac{1}{4}(2e+\pi)\right)\mathrm{Si}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}+\frac{3}{4}af\cos\left(\frac{1}{4}(6e+\pi)\right)\mathrm{Si}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}-\frac{2a\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/x^2,x]
```

```
[Out] (-3*a*f*CosIntegral[(f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]*Sin[(2*e - Pi)/4]*Sq
rt[a + a*Sin[e + f*x]])/4 + (3*a*f*CosIntegral[(3*f*x)/2]*Csc[e/2 + Pi/4 +
(f*x)/2]*Sin[(6*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]])/4 - (2*a*Sin[e/2 + Pi/
4 + (f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/x - (3*a*f*Csc[e/2 + Pi/4 + (f*x)/
2]*Sin[(2*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]]*SinIntegral[(f*x)/2])/4 + (3*
a*f*Cos[(6*e + Pi)/4]*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]*Si
nIntegral[(3*f*x)/2])/4
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx &= \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{x^2}\,dx \\
&= -\frac{2a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{x}+\left(3af\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a-}\right. \\
&= -\frac{2a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{x}+\frac{1}{4}\left(3af\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a}\right. \\
&= -\frac{2a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{x}+\frac{1}{4}\left(3af\cos\left(\frac{1}{4}(6e+\pi)\right)\csc\left(\right.\right. \\
&= -\frac{3}{4}af\,\mathrm{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e-\pi)\right)\sqrt{a+a\sin(e+fx)}+\frac{3}{4}
\end{aligned}
$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.54, size = 226, normalized size = 0.86

$$
\frac{i\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^{3/2}\left(2-6ie^{i(e+fx)}-6e^{2i(e+fx)}+2ie^{3i(e+fx)}+3e^{ie+\frac{3ifx}{2}}fx\mathrm{Ei}\left(-\frac{1}{2}ifx\right)+3ie^{2ie+\frac{3ifx}{2}}fx\mathrm{Ei}\left(\frac{ifx}{2}\right)+3ie^{\frac{3ifx}{2}}fx\mathrm{Ei}\left(-\frac{3}{2}ifx\right)+3e^{\frac{3}{2}i(2e+fx)}fx\mathrm{Ei}\left(\frac{3ifx}{2}\right)\right)}{4\sqrt{2}\left(i+e^{i(e+fx)}\right)^3x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/x^2,x]
```

```
[Out] ((I/4)*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*(2 - (6*I)*
E^(I*(e + f*x)) - 6*E^((2*I)*(e + f*x)) + (2*I)*E^((3*I)*(e + f*x)) + 3*E^(
I*e + ((3*I)/2)*f*x)*f*x*ExpIntegralEi[(-1/2*I)*f*x] + (3*I)*E^((2*I)*e + (
(3*I)/2)*f*x)*f*x*ExpIntegralEi[(I/2)*f*x] + (3*I)*E^(((3*I)/2)*f*x)*f*x*Ex
pIntegralEi[((-3*I)/2)*f*x] + 3*E^(((3*I)/2)*(2*e + f*x))*f*x*ExpIntegralEi
[((3*I)/2)*f*x]))/(Sqrt[2]*(I + E^(I*(e + f*x)))^3*x)
```

**Maple [F]**
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin (fx + e))^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/x^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)/x^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/x^2, x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin (e + fx) + 1))^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/x**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/x**2, x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(211) = 422.
time = 4.00, size = 541, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*pi*a*f^2*cos_integral(3/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(3/4*pi - 3/2*e) - 3*(pi - 2*f*x - 2*e)*a*f^2*cos_integral(3/2*f*
x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi - 3/2*e) - 6*a*f^2*cos_in
tegral(3/2*f*x)*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi - 3/2*e) +
 3*pi*a*f^2*cos_integral(1/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(1
/4*pi - 1/2*e) - 3*(pi - 2*f*x - 2*e)*a*f^2*cos_integral(1/2*f*x)*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(1/4*pi - 1/2*e) - 6*a*f^2*cos_integral(1/2*f
*x)*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(1/4*pi - 1/2*e) - 3*pi*a*f^2*
cos(3/4*pi - 3/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(3/2*f*
x) + 3*(pi - 2*f*x - 2*e)*a*f^2*cos(3/4*pi - 3/2*e)*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sin_integral(3/2*f*x) + 6*a*f^2*cos(3/4*pi - 3/2*e)*e*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(3/2*f*x) - 3*pi*a*f^2*cos(1/4*pi -
 1/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(1/2*f*x) + 3*(pi -
 2*f*x - 2*e)*a*f^2*cos(1/4*pi - 1/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sin_integral(1/2*f*x) + 6*a*f^2*cos(1/4*pi - 1/2*e)*e*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sin_integral(1/2*f*x) - 12*a*f^2*cos(-1/4*pi + 1/2*f*x + 1/
2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*a*f^2*cos(-3/4*pi + 3/2*f*x +
3/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(f^2*x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\,\sin\left(e + f\,x\right))^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/x^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/x^2, x)
```

# 3.133 $\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx$

**Optimal**. Leaf size=332

$$-\frac{9}{16}af^2\cos\left(\frac{3}{4}(2e-\pi)\right)\mathrm{Ci}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}-\frac{3}{16}af^2\mathrm{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}\right.$$

```
[Out] 9/16*a*f^2*Ci(3/2*f*x)*cos(3/2*e+1/4*Pi)*csc(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin
(f*x+e))^(1/2)-3/16*a*f^2*cos(1/2*e+1/4*Pi)*csc(1/2*e+1/4*Pi+1/2*f*x)*Si(1/
2*f*x)*(a+a*sin(f*x+e))^(1/2)-9/16*a*f^2*csc(1/2*e+1/4*Pi+1/2*f*x)*Si(3/2*f
*x)*sin(3/2*e+1/4*Pi)*(a+a*sin(f*x+e))^(1/2)-3/16*a*f^2*Ci(1/2*f*x)*csc(1/2
*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi)*(a+a*sin(f*x+e))^(1/2)-3/2*a*f*cos(1/2
*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/x-a*sin
(1/2*e+1/4*Pi+1/2*f*x)^2*(a+a*sin(f*x+e))^(1/2)/x^2
```

**Rubi** [A]
time = 0.23, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3400, 3395, 3384, 3380, 3383, 3393}

$$-\frac{3}{16}af^2\sin\left(\frac{1}{4}(2e+\pi)\right)\mathrm{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}-\frac{9}{16}af^2\cos\left(\frac{3}{4}(2e-\pi)\right)\mathrm{CosIntegral}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}+\frac{9}{16}af^2\sin\left(\frac{3}{4}(2e-\pi)\right)\mathrm{Si}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}-\frac{3}{16}af^2\cos\left(\frac{1}{4}(2e+\pi)\right)\mathrm{Si}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}-\frac{a\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{x^2}-\frac{3af\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/x^3,x]
```

```
[Out] (-9*a*f^2*Cos[(3*(2*e - Pi))/4]*CosIntegral[(3*f*x)/2]*Csc[e/2 + Pi/4 + (f*
x)/2]*Sqrt[a + a*Sin[e + f*x]])/16 - (3*a*f^2*CosIntegral[(f*x)/2]*Csc[e/2
+ Pi/4 + (f*x)/2]*Sin[(2*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]])/16 - (3*a*f*C
os[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]
)/(2*x) - (a*Sin[e/2 + Pi/4 + (f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/x^2 - (3
*a*f^2*Cos[(2*e + Pi)/4]*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]
*SinIntegral[(f*x)/2])/16 + (9*a*f^2*Csc[e/2 + Pi/4 + (f*x)/2]*Sin[(3*(2*e
- Pi))/4]*Sqrt[a + a*Sin[e + f*x]]*SinIntegral[(3*f*x)/2])/16
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx &= \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{x^3}\,dx \\ &= -\frac{3af\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{2x} - \frac{a\sin^2\left(\frac{e}{2}+\right.}{} \\ &= -\frac{3af\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{2x} - \frac{a\sin^2\left(\frac{e}{2}+\right.}{} \\ &= \frac{3}{2}af^2\text{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e+\pi)\right)\sqrt{a+a\sin(e+fx)} - \\ &= \frac{3}{2}af^2\text{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e+\pi)\right)\sqrt{a+a\sin(e+fx)} - \\ &= -\frac{9}{16}af^2\cos\left(\frac{3}{4}(2e-\pi)\right)\text{Ci}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx}\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.46, size = 295, normalized size = 0.89

$$-\frac{i\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^{3/2}\left(-4+12ie^{i(e+fx)}+12e^{2i(e+fx)}-4ie^{3i(e+fx)}+6ifx+6e^{i(e+fx)}fx+6ie^{2i(e+fx)}fx+6e^{3i(e+fx)}fx+3ie^{ie+\frac{3ifx}{2}}f^2x^2\text{Ei}\left(-\frac{1}{2}ifx\right)+3e^{2ie+\frac{3ifx}{2}}f^2x^2\text{Ei}\left(\frac{ifx}{2}\right)-9e^{\frac{3ifx}{2}}f^2x^2\text{Ei}\left(-\frac{3}{2}ifx\right)-9ie^{\frac{3}{2}i(2e+fx)}f^2x^2\text{Ei}\left(\frac{3ifx}{2}\right)\right)}{16\sqrt{2}\left(i+e^{i(e+fx)}\right)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/x^3,x]
```

```
[Out] ((-1/16*I)*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*(-4 + (
12*I)*E^(I*(e + f*x)) + 12*E^((2*I)*(e + f*x)) - (4*I)*E^((3*I)*(e + f*x))
+ (6*I)*f*x + 6*E^(I*(e + f*x))*f*x + (6*I)*E^((2*I)*(e + f*x))*f*x + 6*E^(
(3*I)*(e + f*x))*f*x + (3*I)*E^(I*e + ((3*I)/2)*f*x)*f^2*x^2*ExpIntegralEi[
(-1/2*I)*f*x] + 3*E^((2*I)*e + ((3*I)/2)*f*x)*f^2*x^2*ExpIntegralEi[(I/2)*f
*x] - 9*E^(((3*I)/2)*f*x)*f^2*x^2*ExpIntegralEi[((-3*I)/2)*f*x] - (9*I)*E^(
((3*I)/2)*(2*e + f*x))*f^2*x^2*ExpIntegralEi[((3*I)/2)*f*x]))/(Sqrt[2]*(I +
 E^(I*(e + f*x)))^3*x^2)
```

**Maple** [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/x^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)/x^3,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/x^3, x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/x**3,x)
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/x**3, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(265) = 530$.
time = 2.81, size = 1344, normalized size = 4.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="giac")
[Out] -1/16*sqrt(2)*(9*pi^2*a*f^3*cos(3/4*pi - 3/2*e)*cos_integral(3/2*f*x)*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) - 18*pi*(pi - 2*f*x - 2*e)*a*f^3*cos(3/4*pi
- 3/2*e)*cos_integral(3/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*(pi
```

```
- 2*f*x - 2*e)^2*a*f^3*cos(3/4*pi - 3/2*e)*cos_integral(3/2*f*x)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 3*pi^2*a*f^3*cos(1/4*pi - 1/2*e)*cos_integral(1
/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*pi*(pi - 2*f*x - 2*e)*a*f^3
*cos(1/4*pi - 1/2*e)*cos_integral(1/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 3*(pi - 2*f*x - 2*e)^2*a*f^3*cos(1/4*pi - 1/2*e)*cos_integral(1/2*f*x
)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 36*pi*a*f^3*cos(3/4*pi - 3/2*e)*cos
_integral(3/2*f*x)*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*(pi - 2*f*x -
 2*e)*a*f^3*cos(3/4*pi - 3/2*e)*cos_integral(3/2*f*x)*e*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) - 12*pi*a*f^3*cos(1/4*pi - 1/2*e)*cos_integral(1/2*f*x)*e*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*(pi - 2*f*x - 2*e)*a*f^3*cos(1/4*p
i - 1/2*e)*cos_integral(1/2*f*x)*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*
pi^2*a*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi - 3/2*e)*sin_inte
gral(3/2*f*x) - 18*pi*(pi - 2*f*x - 2*e)*a*f^3*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sin(3/4*pi - 3/2*e)*sin_integral(3/2*f*x) + 9*(pi - 2*f*x - 2*e)^2*
a*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi - 3/2*e)*sin_integral(
3/2*f*x) - 36*pi*a*f^3*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi - 3
/2*e)*sin_integral(3/2*f*x) + 36*(pi - 2*f*x - 2*e)*a*f^3*e*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(3/4*pi - 3/2*e)*sin_integral(3/2*f*x) + 3*pi^2*a*f
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(1/4*pi - 1/2*e)*sin_integral(1/2
*f*x) - 6*pi*(pi - 2*f*x - 2*e)*a*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(1/4*pi - 1/2*e)*sin_integral(1/2*f*x) + 3*(pi - 2*f*x - 2*e)^2*a*f^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(1/4*pi - 1/2*e)*sin_integral(1/2*f*x)
- 12*pi*a*f^3*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(1/4*pi - 1/2*e)*sin
_integral(1/2*f*x) + 12*(pi - 2*f*x - 2*e)*a*f^3*e*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(1/4*pi - 1/2*e)*sin_integral(1/2*f*x) + 36*a*f^3*cos(3/4*pi
 - 3/2*e)*cos_integral(3/2*f*x)*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1
2*a*f^3*cos(1/4*pi - 1/2*e)*cos_integral(1/2*f*x)*e^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 36*a*f^3*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*p
i - 3/2*e)*sin_integral(3/2*f*x) + 12*a*f^3*e^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(1/4*pi - 1/2*e)*sin_integral(1/2*f*x) - 12*pi*a*f^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 12*(pi - 2*f*x -
 2*e)*a*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2
*e) + 24*a*f^3*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e) - 12*pi*a*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3/
2*f*x + 3/2*e) + 12*(pi - 2*f*x - 2*e)*a*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 24*a*f^3*e*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 48*a*f^3*cos(-1/4*pi + 1/2*f*x
+ 1/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 16*a*f^3*cos(-3/4*pi + 3/2*f
*x + 3/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((pi^2 - 2*pi*(pi
- 2*f*x - 2*e) + (pi - 2*f*x - 2*e)^2 - 4*pi*e + 4*(pi - 2*f*x - 2*e)*e + 4
*e^2)*f)
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\,\sin{(e + f\,x)})^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/x^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/x^3, x)
```

# 3.134 $\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=417

$$-\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}}+\frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}}-\frac{12ix^2\text{Li}_2\left(e^{\frac{1}{4}i(2c}\right.}{d^2\sqrt{a}}$$

```
[Out] -4*x^3*arctanh(exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d/(a+a*
sin(d*x+c))^(1/2)+12*I*x^2*polylog(2,-exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+
1/4*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-12*I*x^2*polylog(2,exp(1/4*I*(2*
d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-48*x*pol
ylog(3,-exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^3/(a+a*sin(d
*x+c))^(1/2)+48*x*polylog(3,exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2
*d*x)/d^3/(a+a*sin(d*x+c))^(1/2)-96*I*polylog(4,-exp(1/4*I*(2*d*x+Pi+2*c)))
*sin(1/2*c+1/4*Pi+1/2*d*x)/d^4/(a+a*sin(d*x+c))^(1/2)+96*I*polylog(4,exp(1/
4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^4/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.15, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3400, 4268, 2611, 6744, 2320, 6724}

$$-\frac{96i\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(4,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^4\sqrt{a\sin(c+dx)+a}}+\frac{96i\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(4,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^4\sqrt{a\sin(c+dx)+a}}-\frac{48x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(3,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^3\sqrt{a\sin(c+dx)+a}}+\frac{48x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(3,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^3\sqrt{a\sin(c+dx)+a}}+\frac{12ix^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{12ix^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{4x^3\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\tanh^{-1}\left(e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*x^3*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d
*Sqrt[a + a*Sin[c + d*x]]) + ((12*I)*x^2*PolyLog[2, -E^((I/4)*(2*c + Pi + 2
*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a + a*Sin[c + d*x]]) - ((12*I)
*x^2*PolyLog[2, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d
^2*Sqrt[a + a*Sin[c + d*x]]) - (48*x*PolyLog[3, -E^((I/4)*(2*c + Pi + 2*d*x
))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a + a*Sin[c + d*x]]) + (48*x*PolyL
og[3, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a
+ a*Sin[c + d*x]]) - ((96*I)*PolyLog[4, -E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[
c/2 + Pi/4 + (d*x)/2])/(d^4*Sqrt[a + a*Sin[c + d*x]]) + ((96*I)*PolyLog[4,
E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^4*Sqrt[a + a*Si
n[c + d*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^3\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\left(6\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2\log}{d\sqrt{a+a\sin(c}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin}{d^2\sqrt{a+a\sin(c+}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin}{d^2\sqrt{a+a\sin(c+}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin}{d^2\sqrt{a+a\sin(c+}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin}{d^2\sqrt{a+a\sin(c+}}$$

**Mathematica** [A]
time = 0.52, size = 306, normalized size = 0.73

$$\frac{\sqrt[4]{-1}\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\left(-id^3x^3\log\left(1-\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)+id^3x^3\log\left(1+\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)+6d^2x^2\text{Li}_2\left(-\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)-6d^2x^2\text{Li}_2\left(\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)+24idx\text{Li}_3\left(-\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)-24idx\text{Li}_3\left(\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)-48\text{Li}_4\left(-\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)+48\text{Li}_4\left(\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)\right)}{d^4\sqrt{-iae^{-i(c+dx)}\left(i+e^{i(c+dx)}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[2]*(I + E^(I*(c + d*x)))*((-I)*d^3*x^3*Log[1 - (-1)^(1/4)*
E^((I/2)*(c + d*x))] + I*d^3*x^3*Log[1 + (-1)^(1/4)*E^((I/2)*(c + d*x))] +
6*d^2*x^2*PolyLog[2, -((-1)^(1/4)*E^((I/2)*(c + d*x)))] - 6*d^2*x^2*PolyLog
[2, (-1)^(1/4)*E^((I/2)*(c + d*x))] + (24*I)*d*x*PolyLog[3, -((-1)^(1/4)*E^
((I/2)*(c + d*x)))] - (24*I)*d*x*PolyLog[3, (-1)^(1/4)*E^((I/2)*(c + d*x))]
 - 48*PolyLog[4, -((-1)^(1/4)*E^((I/2)*(c + d*x)))] + 48*PolyLog[4, (-1)^(1
/4)*E^((I/2)*(c + d*x))]))/(d^4*E^((I/2)*(c + d*x))*Sqrt[((-I)*a*(I + E^(I*
(c + d*x)))^2)/E^(I*(c + d*x))])
```

**Maple** [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+a\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(x^3/(a+a*sin(d*x+c))^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(a*sin(d*x + c) + a), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(a*sin(d*x + c) + a), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a*sin(d*x + c) + a), x)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a + a\,\sin\left(c + d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + a*sin(c + d*x))^(1/2),x)
[Out] int(x^3/(a + a*sin(c + d*x))^(1/2), x)
```

# 3.135 $\int \frac{x^2}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=293

$$-\frac{4x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}}+\frac{8ix\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}}-\frac{8ix\text{Li}_2\left(e^{\frac{1}{4}i(2c+\pi+2d}\right.}{d^2\sqrt{a+a}}$$

```
[Out] -4*x^2*arctanh(exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d/(a+a*
sin(d*x+c))^(1/2)+8*I*x*polylog(2,-exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4
*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-8*I*x*polylog(2,exp(1/4*I*(2*d*x+Pi
+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-16*polylog(3,-
exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^3/(a+a*sin(d*x+c))^(
1/2)+16*polylog(3,exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^3/
(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 4268, 2611, 2320, 6724}

$$-\frac{16\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(3,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^3\sqrt{a\sin(c+dx)+a}}+\frac{16\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(3,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^3\sqrt{a\sin(c+dx)+a}}+\frac{8ix\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{8ix\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{4x^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\tanh^{-1}\left(e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*x^2*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d
*Sqrt[a + a*Sin[c + d*x]]) + ((8*I)*x*PolyLog[2, -E^((I/4)*(2*c + Pi + 2*d*
x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a + a*Sin[c + d*x]]) - ((8*I)*x*P
olyLog[2, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqr
t[a + a*Sin[c + d*x]]) - (16*PolyLog[3, -E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[
c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a + a*Sin[c + d*x]]) + (16*PolyLog[3, E^((
I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a + a*Sin[c
+ d*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^2\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\left(4\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\log\left(\right.}{d\sqrt{a+a\sin(c+}}$$

$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{8ix\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}\right.}{d^2\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{8ix\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}\right.}{d^2\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{8ix\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}\right.}{d^2\sqrt{a+a\sin(c+dx)}}$$

**Mathematica [A]**
time = 0.38, size = 245, normalized size = 0.84

$$\frac{\sqrt[4]{-1}\,\sqrt{2}\,e^{-\frac{1}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\left(4dx\text{Li}_2\left(-\sqrt[4]{-1}\,e^{\frac{1}{2}i(c+dx)}\right)-i\left(d^2x^2\log\left(1-\sqrt[4]{-1}\,e^{\frac{1}{2}i(c+dx)}\right)-d^2x^2\log\left(1+\sqrt[4]{-1}\,e^{\frac{1}{2}i(c+dx)}\right)-4idx\text{Li}_2\left(\sqrt[4]{-1}\,e^{\frac{1}{2}i(c+dx)}\right)-8\text{Li}_3\left(-\sqrt[4]{-1}\,e^{\frac{1}{2}i(c+dx)}\right)+8\text{Li}_3\left(\sqrt[4]{-1}\,e^{\frac{1}{2}i(c+dx)}\right)\right)\right)}{d^3\sqrt{-iae^{-i(c+dx)}\left(i+e^{i(c+dx)}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[2]*(I + E^(I*(c + d*x)))*(4*d*x*PolyLog[2, -((-1)^(1/4)*E^
((I/2)*(c + d*x)))] - I*(d^2*x^2*Log[1 - (-1)^(1/4)*E^((I/2)*(c + d*x))] -
d^2*x^2*Log[1 + (-1)^(1/4)*E^((I/2)*(c + d*x))] - (4*I)*d*x*PolyLog[2, (-1)
^(1/4)*E^((I/2)*(c + d*x))] - 8*PolyLog[3, -((-1)^(1/4)*E^((I/2)*(c + d*x))
)] + 8*PolyLog[3, (-1)^(1/4)*E^((I/2)*(c + d*x))])))/(d^3*E^((I/2)*(c + d*x
))*Sqrt[((-I)*a*(I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))])
```

**Maple [F]**
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+a\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(x^2/(a+a*sin(d*x+c))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(a*sin(d*x + c) + a), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(x^2/sqrt(a*sin(d*x + c) + a), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(x**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(a*sin(d*x + c) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + a*sin(c + d*x))^(1/2),x)
[Out] int(x^2/(a + a*sin(c + d*x))^(1/2), x)
```

# 3.136 $\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=175

$$-\frac{4x\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}}+\frac{4i\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}}-\frac{4i\text{Li}_2\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)}{d^2\sqrt{a+a\sin}}$$

```
[Out] -4*x*arctanh(exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d/(a+a*si
n(d*x+c))^(1/2)+4*I*polylog(2,-exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+
1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-4*I*polylog(2,exp(1/4*I*(2*d*x+Pi+2*c))
)*sin(1/2*c+1/4*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3400, 4268, 2317, 2438}

$$\frac{4i\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{4i\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{4x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\tanh^{-1}\left(e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*x*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d*S
qrt[a + a*Sin[c + d*x]]) + ((4*I)*PolyLog[2, -E^((I/4)*(2*c + Pi + 2*d*x))]
*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a + a*Sin[c + d*x]]) - ((4*I)*PolyLog
[2, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a +
a*Sin[c + d*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
```

```
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\int x\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\left(2\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\log\left(1\right.}{d\sqrt{a+a\sin(c+}}$$

$$= -\frac{4x\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\left(4i\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\text{Subst}\left(\right.}{d^2\sqrt{a+a\,\text{si}}}$$

$$= -\frac{4x\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{4i\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}\right.}{d^2\sqrt{a+a\sin(c+dx)}}$$

## Mathematica [A]

time = 1.02, size = 231, normalized size = 1.32

$$\frac{2\left(\frac{\left(-\pi\tanh^{-1}\left(\frac{-1+\tan\left(\frac{1}{4}(c+dx)\right)}{\sqrt{2}}\right)+\frac{1}{2}(2c+\pi+2dx)\left(\log\left(1-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)-\log\left(1+e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\right)+2i\left(\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)-\text{Li}_2\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{2}}+\frac{c\sin^{-1}\left(\csc\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)\sin\left(\frac{1}{4}(2c-\pi+2dx)\right)}{\sqrt{\frac{-1+\sin(c+dx)}{1+\sin(c+dx)}}}\right)}{d^2\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(((-(Pi*ArcTanh[(-1 + Tan[(c + d*x)/4])/Sqrt[2]]) + ((2*c + Pi + 2*d*x)*
(Log[1 - E^((I/4)*(2*c + Pi + 2*d*x))] - Log[1 + E^((I/4)*(2*c + Pi + 2*d*x
))]))/2 + (2*I)*(PolyLog[2, -E^((I/4)*(2*c + Pi + 2*d*x))] - PolyLog[2, E^(
(I/4)*(2*c + Pi + 2*d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/Sqrt[2]
 + (c*ArcSin[Csc[(2*c + Pi + 2*d*x)/4]]*Sin[(2*c - Pi + 2*d*x)/4])/Sqrt[(-1
 + Sin[c + d*x])/(1 + Sin[c + d*x])]))/(d^2*Sqrt[a*(1 + Sin[c + d*x])])
```

**Maple** [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + a\sin(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(x/(a+a*sin(d*x+c))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*sin(d*x + c) + a), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(a*sin(d*x + c) + a), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a(\sin(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(a*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(x/(a + a*sin(c + d*x))^(1/2), x)
```

# 3.137 $\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=21

$$\operatorname{Int}\left(\frac{1}{x\sqrt{a+a\sin(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/x/(a+a*sin(d*x+c))^(1/2),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[a + a*Sin[c + d*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[a + a*Sin[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$$

**Mathematica** [A]
time = 2.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a + a*Sin[c + d*x]]),x]
[Out] Integrate[1/(x*Sqrt[a + a*Sin[c + d*x]]), x]
```

**Maple** [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(1/x/(a+a*sin(d*x+c))^(1/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(d*x + c) + a)*x), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(d*x + c) + a)/(a*x*sin(d*x + c) + a*x), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*(sin(c + d*x) + 1))), x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(d*x + c) + a)*x), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(1/(x*(a + a*sin(c + d*x))^(1/2)), x)
```

# 3.138 $\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2\sqrt{a+a\sin(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+a*sin(d*x+c))^(1/2),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[a + a*Sin[c + d*x]]),x]
[Out] Defer[Int][1/(x^2*Sqrt[a + a*Sin[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$$

**Mathematica** [A]
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[a + a*Sin[c + d*x]]),x]
[Out] Integrate[1/(x^2*Sqrt[a + a*Sin[c + d*x]]), x]
```

**Maple** [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(1/x^2/(a+a*sin(d*x+c))^(1/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(d*x + c) + a)*x^2), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(d*x + c) + a)/(a*x^2*sin(d*x + c) + a*x^2), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a \left(\sin \left(c + dx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*(sin(c + d*x) + 1))), x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(d*x + c) + a)*x^2), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(1/(x^2*(a + a*sin(c + d*x))^(1/2)), x)
```

# 3.139 $\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=691

$$-\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}}-\frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}}-\frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+a\sin(e+fx)}}-\frac{x^3\tanh}{}$$

```
[Out] -3*x^2/a/f^2/(a+a*sin(f*x+e))^(1/2)-1/2*x^3*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f/(
a+a*sin(f*x+e))^(1/2)-24*x*arctanh(exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4
*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)-x^3*arctanh(exp(1/4*I*(2*f*x+Pi+2
*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f/(a+a*sin(f*x+e))^(1/2)+24*I*polylog(2,-
exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^4/(a+a*sin(f*x+e))
^(1/2)+3*I*x^2*polylog(2,-exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f
*x)/a/f^2/(a+a*sin(f*x+e))^(1/2)-24*I*polylog(2,exp(1/4*I*(2*f*x+Pi+2*e)))*
sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^4/(a+a*sin(f*x+e))^(1/2)-3*I*x^2*polylog(2,ex
p(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^2/(a+a*sin(f*x+e))^(
1/2)-12*x*polylog(3,-exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a
/f^3/(a+a*sin(f*x+e))^(1/2)+12*x*polylog(3,exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1
/2*e+1/4*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)-24*I*polylog(4,-exp(1/4*I
*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^4/(a+a*sin(f*x+e))^(1/2)+24
*I*polylog(4,exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^4/(a+
a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.24, antiderivative size = 691, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3400, 4271, 4268, 2317, 2438, 2611, 6744, 2320, 6724}

$$\frac{24i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^4\sqrt{a\sin(e+fx)+a}}-\frac{24i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^4\sqrt{a\sin(e+fx)+a}}-\frac{24i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(4,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^4\sqrt{a\sin(e+fx)+a}}+\frac{24i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(4,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^4\sqrt{a\sin(e+fx)+a}}-\frac{12x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(3,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^3\sqrt{a\sin(e+fx)+a}}+\frac{12x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(3,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^3\sqrt{a\sin(e+fx)+a}}+\frac{3ix^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{3ix^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{24x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\tanh^{-1}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^3\sqrt{a\sin(e+fx)+a}}-\frac{3x^2}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{x^3\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\tanh^{-1}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af\sqrt{a\sin(e+fx)+a}}-\frac{x^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*x^2)/(a*f^2*Sqrt[a + a*Sin[e + f*x]]) - (x^3*Cot[e/2 + Pi/4 + (f*x)/2])
/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (24*x*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*
x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]]) - (x^3*Arc
Tanh[E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f*Sqrt[a +
 a*Sin[e + f*x]]) + ((24*I)*PolyLog[2, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e
/2 + Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin[e + f*x]]) + ((3*I)*x^2*PolyLog
[2, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a
 + a*Sin[e + f*x]]) - ((24*I)*PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[
e/2 + Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin[e + f*x]]) - ((3*I)*x^2*PolyLo
g[2, E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a
 + a*Sin[e + f*x]]) - (12*x*PolyLog[3, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e
```

```
/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]]) + (12*x*PolyLog[3, E
^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*S
in[e + f*x]]) - ((24*I)*PolyLog[4, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 +
 Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin[e + f*x]]) + ((24*I)*PolyLog[4, E^(
(I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin
[e + f*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x^3\csc^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\,dx}{2a\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} + \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int}{4a\sqrt{a+}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e}\right.}{af^3\sqrt{a-}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e}\right.}{af^3\sqrt{a-}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e}\right.}{af^3\sqrt{a-}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e}\right.}{af^3\sqrt{a-}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e}\right.}{af^3\sqrt{a-}}$$

**Mathematica [A]**
time = 1.78, size = 455, normalized size = 0.66

$$-\frac{(-1)^{3/4}e^{-\frac{3}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)^3\left(6(8+f^2x^2)\operatorname{Li}_2\left(-\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-6(8+f^2x^2)\operatorname{Li}_2\left(\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-i\left(24fx\log\left(1-\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)+f^3x^3\log\left(1-\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-24fx\log\left(1+\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-f^3x^3\log\left(1+\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-24fx\operatorname{Li}_3\left(-\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)+24fx\operatorname{Li}_3\left(\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-48i\operatorname{Li}_4\left(-\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)+48i\operatorname{Li}_4\left(\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)\right)\right)}{2\sqrt{2}\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^{3/2}f^4}-\frac{x^2\left((6+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(6-fx)\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}{2a^2f^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((-1)^(3/4)*(I + E^(I*(e + f*x)))^3*(6*(8 + f^2*x^2)*PolyLog[2, -((-1)
^(1/4)*E^((I/2)*(e + f*x)))] - 6*(8 + f^2*x^2)*PolyLog[2, (-1)^(1/4)*E^((I/
2)*(e + f*x))] - I*(24*f*x*Log[1 - (-1)^(1/4)*E^((I/2)*(e + f*x))] + f^3*x^
3*Log[1 - (-1)^(1/4)*E^((I/2)*(e + f*x))] - 24*f*x*Log[1 + (-1)^(1/4)*E^((I
/2)*(e + f*x))] - f^3*x^3*Log[1 + (-1)^(1/4)*E^((I/2)*(e + f*x))] - 24*f*x*
PolyLog[3, -((-1)^(1/4)*E^((I/2)*(e + f*x)))] + 24*f*x*PolyLog[3, (-1)^(1/4
)*E^((I/2)*(e + f*x))] - (48*I)*PolyLog[4, -((-1)^(1/4)*E^((I/2)*(e + f*x))
)] + (48*I)*PolyLog[4, (-1)^(1/4)*E^((I/2)*(e + f*x))])))/(Sqrt[2]*E^(((3*I
)/2)*(e + f*x))*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*f^
4) - (x^2*((6 + f*x)*Cos[(e + f*x)/2] + (6 - f*x)*Sin[(e + f*x)/2])*Sqrt[a*
(1 + Sin[e + f*x])])/(2*a^2*f^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple** [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + a\sin(fx + e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^3/(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*x^3/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x
+ e) - 2*a^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(x^3/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.140 $\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=435

$$-\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}}-\frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}}-\frac{x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+a\sin(e+fx)}}-4\tanh^{-1}$$

```
[Out] -2*x/a/f^2/(a+a*sin(f*x+e))^(1/2)-1/2*x^2*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f/(a+
a*sin(f*x+e))^(1/2)-x^2*arctanh(exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi
+1/2*f*x)/a/f/(a+a*sin(f*x+e))^(1/2)-4*arctanh(cos(1/2*e+1/4*Pi+1/2*f*x))*s
in(1/2*e+1/4*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)+2*I*x*polylog(2,-exp(
1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^2/(a+a*sin(f*x+e))^(1/
2)-2*I*x*polylog(2,exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f
^2/(a+a*sin(f*x+e))^(1/2)-4*polylog(3,-exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e
+1/4*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)+4*polylog(3,exp(1/4*I*(2*f*x+
Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.17, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3400, 4271, 3855, 4268, 2611, 2320, 6724}

$$-\frac{4\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(3,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^3\sqrt{a\sin(e+fx)+a}}+\frac{4\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(3,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^3\sqrt{a\sin(e+fx)+a}}+\frac{2ix\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{2ix\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{4\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\tanh^{-1}\left(\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{af^3\sqrt{a\sin(e+fx)+a}}-\frac{2x}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\tanh^{-1}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af\sqrt{a\sin(e+fx)+a}}-\frac{x^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*x)/(a*f^2*Sqrt[a + a*Sin[e + f*x]]) - (x^2*Cot[e/2 + Pi/4 + (f*x)/2])/(
2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (x^2*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*x))
]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - (4*ArcTanh[Co
s[e/2 + Pi/4 + (f*x)/2]]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e
 + f*x]]) + ((2*I)*x*PolyLog[2, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi
/4 + (f*x)/2])/(a*f^2*Sqrt[a + a*Sin[e + f*x]]) - ((2*I)*x*PolyLog[2, E^((I
/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a + a*Sin[e
 + f*x]]) - (4*PolyLog[3, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (
f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]]) + (4*PolyLog[3, E^((I/4)*(2*e + P
i + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x^2\csc^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\,dx}{2a\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} + \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x}{4a\sqrt{a+a}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+}\right.}{af\sqrt{a+a}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+}\right.}{af\sqrt{a+a}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+}\right.}{af\sqrt{a+a}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+}\right.}{af\sqrt{a+a}}$$

**Mathematica [A]**
time = 1.21, size = 352, normalized size = 0.81

$$\frac{\sqrt[4]{-1}e^{-\frac{3}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)^3\left(16\tanh^{-1}\left(\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-f^2x^2\log\left(1-\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)+f^2x^2\log\left(1+\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-4ifx\text{Li}_2\left(-\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)+4ifx\text{Li}_2\left(\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)+8\text{Li}_3\left(-\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-8\text{Li}_3\left(\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)\right)}{2\sqrt{2}\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^{3/2}f^3}-\frac{x\left((4+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(4-fx)\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}{2a^2f^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((-1)^(1/4)*(I + E^(I*(e + f*x)))^3*(16*ArcTanh[(-1)^(1/4)*E^((I/2)*(e + f*
x))] - f^2*x^2*Log[1 - (-1)^(1/4)*E^((I/2)*(e + f*x))] + f^2*x^2*Log[1 + (-
1)^(1/4)*E^((I/2)*(e + f*x))] - (4*I)*f*x*PolyLog[2, -((-1)^(1/4)*E^((I/2)*
(e + f*x)))] + (4*I)*f*x*PolyLog[2, (-1)^(1/4)*E^((I/2)*(e + f*x))] + 8*Pol
yLog[3, -((-1)^(1/4)*E^((I/2)*(e + f*x)))] - 8*PolyLog[3, (-1)^(1/4)*E^((I/
2)*(e + f*x))]))/(2*Sqrt[2]*E^(((3*I)/2)*(e + f*x))*(((-I)*a*(I + E^(I*(e +
 f*x)))^2)/E^(I*(e + f*x)))^(3/2)*f^3) - (x*((4 + f*x)*Cos[(e + f*x)/2] + (
4 - f*x)*Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])/(2*a^2*f^2*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple [F]**
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*sin(f*x+e))^(3/2),x)
[Out] int(x^2/(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(-sqrt(a*sin(f*x + e) + a)*x^2/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x
+ e) - 2*a^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral(x**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + a*sin(e + f*x))^(3/2),x)
[Out] int(x^2/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.141 $\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=249

$$-\frac{1}{af^2\sqrt{a+a\sin(e+fx)}}-\frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}}-\frac{x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+a\sin(e+fx)}}+\frac{i\mathrm{Li}_2\left(-e\right.}{a}$$

```
[Out] -1/a/f^2/(a+a*sin(f*x+e))^(1/2)-1/2*x*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f/(a+a*si
n(f*x+e))^(1/2)-x*arctanh(exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f
*x)/a/f/(a+a*sin(f*x+e))^(1/2)+I*polylog(2,-exp(1/4*I*(2*f*x+Pi+2*e)))*sin(
1/2*e+1/4*Pi+1/2*f*x)/a/f^2/(a+a*sin(f*x+e))^(1/2)-I*polylog(2,exp(1/4*I*(2
*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^2/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3400, 4270, 4268, 2317, 2438}

$$\frac{i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\mathrm{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\mathrm{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{1}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\tanh^{-1}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af\sqrt{a\sin(e+fx)+a}}-\frac{x\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(1/(a*f^2*Sqrt[a + a*Sin[e + f*x]])) - (x*Cot[e/2 + Pi/4 + (f*x)/2])/(2*a*
f*Sqrt[a + a*Sin[e + f*x]]) - (x*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[
e/2 + Pi/4 + (f*x)/2])/(a*f*Sqrt[a + a*Sin[e + f*x]]) + (I*PolyLog[2, -E^((
I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a + a*Sin[
e + f*x]]) - (I*PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (
f*x)/2])/(a*f^2*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
```

```
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx &= \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x\csc^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\,dx}{2a\sqrt{a+a\sin(e+fx)}} \\
&= -\frac{1}{af^2\sqrt{a+a\sin(e+fx)}}-\frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}}+\frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x}{4a\sqrt{a+a}} \\
&= -\frac{1}{af^2\sqrt{a+a\sin(e+fx)}}-\frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}}-\frac{x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2}\right.}{af\sqrt{a+a}} \\
&= -\frac{1}{af^2\sqrt{a+a\sin(e+fx)}}-\frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}}-\frac{x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2}\right.}{af\sqrt{a+a}} \\
&= -\frac{1}{af^2\sqrt{a+a\sin(e+fx)}}-\frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}}-\frac{x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2}\right.}{af\sqrt{a+a}}
\end{aligned}
$$

**Mathematica** [A]
time = 1.66, size = 308, normalized size = 1.24

$$
\frac{2fx\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-(2+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+\frac{\left(-\pi\tanh^{-1}\left(\frac{-1+\tan\left(\frac{1}{4}(e+fx)\right)}{\sqrt{2}}\right)+\frac{1}{2}(2e+\pi+2fx)\left(\log\left(1-e^{\frac{1}{4}i(2e+\pi+2fx)}\right)-\log\left(1+e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\right)+2i\left(\mathrm{Li}_2\left(-e^{\frac{1}{4}i(2e+\pi+2fx)}\right)-\mathrm{Li}_2\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{\sqrt{2}}+\frac{e\sin^{-1}\left(\csc\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)(1+\sin(e+fx))\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}}}{2f^2(a(1+\sin(e+fx)))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*f*x*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (2 + f*x)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + ((-(Pi*ArcTanh[(-1 + Tan[(e + f*x)
/4])/Sqrt[2]]) + ((2*e + Pi + 2*f*x)*(Log[1 - E^((I/4)*(2*e + Pi + 2*f*x))]
 - Log[1 + E^((I/4)*(2*e + Pi + 2*f*x))]))/2 + (2*I)*(PolyLog[2, -E^((I/4)*
(2*e + Pi + 2*f*x))] - PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))]))*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[2] + (e*ArcSin[Csc[(2*e + Pi + 2*f*x)/
4]]*(1 + Sin[e + f*x])*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[(-1 + Sin[e + f*x])/
(1 + Sin[e + f*x])])/(2*f^2*(a*(1 + Sin[e + f*x]))^(3/2))
```

**Maple [F]**
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x/(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*x/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x +
e) - 2*a^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(a*sin(f*x + e) + a)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(x/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.142 $\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=21

$$\text{Int}\left(\frac{1}{x(a+a\sin(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x/(a+a*sin(f*x+e))^(3/2),x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + a*Sin[e + f*x])^(3/2)),x]
[Out] Defer[Int][1/(x*(a + a*Sin[e + f*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$$

**Mathematica** [A]
time = 20.59, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + a*Sin[e + f*x])^(3/2)),x]
[Out] Integrate[1/(x*(a + a*Sin[e + f*x])^(3/2)), x]
```

**Maple** [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(1/x/(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*x), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)/(a^2*x*cos(f*x + e)^2 - 2*a^2*x*sin(f*x
+ e) - 2*a^2*x), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(x*(a*(sin(e + f*x) + 1))**(3/2)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,(a+a\,\sin\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + a*sin(e + f*x))^(3/2)),x)
[Out] int(1/(x*(a + a*sin(e + f*x))^(3/2)), x)
```

# 3.143 $\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2(a+a\sin(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+a*sin(f*x+e))^(3/2),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(a + a*Sin[e + f*x])^(3/2)),x]
[Out] Defer[Int][1/(x^2*(a + a*Sin[e + f*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$$

**Mathematica** [A]
time = 10.46, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(a + a*Sin[e + f*x])^(3/2)),x]
[Out] Integrate[1/(x^2*(a + a*Sin[e + f*x])^(3/2)), x]
```

**Maple** [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(1/x^2/(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*x^2), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)/(a^2*x^2*cos(f*x + e)^2 - 2*a^2*x^2*sin(
f*x + e) - 2*a^2*x^2), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a \left(\sin \left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*(sin(e + f*x) + 1))**(3/2)), x)
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\,(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + a*sin(e + f*x))^(3/2)),x)
[Out] int(1/(x^2*(a + a*sin(e + f*x))^(3/2)), x)
```

# 3.144 $\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$

**Optimal**. Leaf size=21

$$\mathrm{Int}\left(\frac{\sqrt[3]{a+a\sin(c+dx)}}{x},x\right)$$

```
[Out] Unintegrable((a+a*sin(d*x+c))^(1/3)/x,x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + a*Sin[c + d*x])^(1/3)/x,x]
```

```
[Out] Defer[Int][(a + a*Sin[c + d*x])^(1/3)/x, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$$

**Mathematica** [A]
time = 2.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[c + d*x])^(1/3)/x,x]
```

```
[Out] Integrate[(a + a*Sin[c + d*x])^(1/3)/x, x]
```

**Maple** [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(dx+c))^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/3)/x,x)
```

```
[Out] int((a+a*sin(d*x+c))^(1/3)/x,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(1/3)/x, x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a\left(\sin\left(c+dx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/3)/x,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(1/3)/x, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(1/3)/x, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a+a\sin(c+dx))^{1/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/3)/x,x)
[Out] int((a + a*sin(c + d*x))^(1/3)/x, x)
```

# 3.145 $\int (c+dx)^m(a+a\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=23

$$\mathrm{Int}((c+dx)^m(a+a\sin(e+fx))^n,x)$$

```
[Out] Unintegrable((d*x+c)^m*(a+a*sin(f*x+e))^n,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m(a+a\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + a*Sin[e + f*x])^n,x]
[Out] Defer[Int][(c + d*x)^m*(a + a*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+a\sin(e+fx))^n\,dx = \int (c+dx)^m(a+a\sin(e+fx))^n\,dx$$

**Mathematica** [A]
time = 0.75, size = 0, normalized size = 0.00

$$\int (c+dx)^m(a+a\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^n,x]
[Out] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^n, x]
```

**Maple** [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(a+a\sin(fx+e)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*sin(f*x+e))^n,x)
[Out] int((d*x+c)^m*(a+a*sin(f*x+e))^n,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(a*sin(f*x + e) + a)^n, x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(a*sin(f*x + e) + a)^n, x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*sin(f*x+e))**n,x)
[Out] Timed out
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(a*sin(f*x + e) + a)^n, x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + a\,\sin(e + f\,x))^n\,(c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^n*(c + d*x)^m,x)
[Out] int((a + a*sin(e + f*x))^n*(c + d*x)^m, x)
```

# 3.146 $\int (c+dx)^m (a+a\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=449

$$\frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{15a^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{8f}-\frac{15a^3e^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+}{d}\right.}{8f}$$

```
[Out] 5/2*a^3*(d*x+c)^(1+m)/d/(1+m)-15/8*a^3*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m
,-I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-15/8*a^3*(d*x+c)^m*GAMMA(1+m,I*f*(d
*x+c)/d)/exp(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+3*I*2^(-3-m)*a^3*exp(2*I*(e
-c*f/d))*(d*x+c)^m*GAMMA(1+m,-2*I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-3*I*2
^(-3-m)*a^3*(d*x+c)^m*GAMMA(1+m,2*I*f*(d*x+c)/d)/exp(2*I*(e-c*f/d))/f/((I*f
*(d*x+c)/d)^m)+1/8*3^(-1-m)*a^3*exp(3*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-3*I
*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)+1/8*3^(-1-m)*a^3*(d*x+c)^m*GAMMA(1+m,3
*I*f*(d*x+c)/d)/exp(3*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

**Rubi** [A]
time = 0.41, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3399, 3393, 3388, 2212, 3389}

$$-\frac{15a^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{8f}+\frac{3ia^32^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{f}+\frac{a^33^{-m-1}e^{3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3if(c+dx)}{d}\right)}{8f}-\frac{15a^3e^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{8f}-\frac{3ia^32^{-m-3}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2if(c+dx)}{d}\right)}{f}+\frac{a^33^{-m-1}e^{-3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3if(c+dx)}{d}\right)}{8f}+\frac{5a^3(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) - (15*a^3*E^(I*(e - (c*f)/d))*(c +
d*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) -
 (15*a^3*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(8*E^(I*(e - (c*f)/d)
)*f*((I*f*(c + d*x))/d)^m) + ((3*I)*2^(-3 - m)*a^3*E^((2*I)*(e - (c*f)/d))*
(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)
^m) - ((3*I)*2^(-3 - m)*a^3*(c + d*x)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d]
)/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (3^(-1 - m)*a^3*E^((3
*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*f*(c + d*x))/d])/(8*f*(
((-I)*f*(c + d*x))/d)^m) + (3^(-1 - m)*a^3*(c + d*x)^m*Gamma[1 + m, ((3*I)*
f*(c + d*x))/d])/(8*E^((3*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m(a+a\sin(e+fx))^3\,dx &= (8a^3)\int (c+dx)^m\sin^6\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx\\ &= (8a^3)\int\left(\frac{5}{16}(c+dx)^m-\frac{3}{16}(c+dx)^m\cos(2e+2fx)+\frac{15}{32}(c+dx)^m\right.\\ &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{1}{4}a^3\int (c+dx)^m\sin(3e+3fx)\,dx-\frac{1}{2}(3a^3)\int (c\\ &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{1}{8}(ia^3)\int e^{-i(3e+3fx)}(c+dx)^m\,dx+\frac{1}{8}(ia^3)\int e^{i(3e}\\ &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{15a^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\right.}{8f}\end{aligned}$$

**Mathematica [A]**
time = 0.57, size = 376, normalized size = 0.84

$$\frac{1}{24}a^3(c+dx)^m\left(\frac{60(c+dx)}{d(1+m)}-\frac{45e^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f}-\frac{45e^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{f}+\frac{9i2^{-m}e^{2i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{f}-\frac{9i2^{-m}e^{-2i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{f}+\frac{3^{-m}e^{3i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3if(c+dx)}{d}\right)}{f}+\frac{3^{-m}e^{-3i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3if(c+dx)}{d}\right)}{f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(c + d*x)^m*((60*(c + d*x))/(d*(1 + m)) - (45*E^(I*(e - (c*f)/d))*Gamm
a[1 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (45*Gamma[1
+ m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + ((
9*I)*E^((2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m*f*(
((-I)*f*(c + d*x))/d)^m) - ((9*I)*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(2^m
*E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (E^((3*I)*(e - (c*f)/d)
)*Gamma[1 + m, ((-3*I)*f*(c + d*x))/d])/(3^m*f*(((-I)*f*(c + d*x))/d)^m) +
Gamma[1 + m, ((3*I)*f*(c + d*x))/d]/(3^m*E^((3*I)*(e - (c*f)/d))*f*((I*f*(c
 + d*x))/d)^m)))/24
```

**Maple [F]**
time = 0.13, size = 0, normalized size = 0.00

$$\int (dx+c)^m (a+a\sin(fx+e))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+a*sin(f*x+e))^3,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(6*a^3*e^(m*log(d*x + c) + log(d*x
+ c)) - 6*(a^3*d*m + a^3*d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) - (a
^3*d*m + a^3*d)*integrate((d*x + c)^m*sin(3*f*x + 3*e), x) + 15*(a^3*d*m +
a^3*d)*integrate((d*x + c)^m*sin(f*x + e), x))/(d*m + d)
```

**Fricas [A]**
time = 0.13, size = 392, normalized size = 0.87

$$-\frac{45\,(a^3dm+a^3d)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)-icf+ide}{d}\right)}\Gamma\left(m+1,\frac{idfx+icf}{d}\right)+9\,(-i\,a^3dm-i\,a^3d)e^{\left(-\frac{dm\log\left(-\frac{2if}{d}\right)+2icf-2ide}{d}\right)}\Gamma\left(m+1,-\frac{2\,(idfx+icf)}{d}\right)-(a^3dm+a^3d)e^{\left(-\frac{dm\log\left(-\frac{3if}{d}\right)+3icf-3ide}{d}\right)}\Gamma\left(m+1,-\frac{3\,(idfx+icf)}{d}\right)+45\,(a^3dm+a^3d)e^{\left(-\frac{dm\log\left(-\frac{if}{d}\right)+icf-ide}{d}\right)}\Gamma\left(m+1,\frac{-idfx-icf}{d}\right)+9\,(i\,a^3dm+i\,a^3d)e^{\left(-\frac{dm\log\left(\frac{2if}{d}\right)-2icf+2ide}{d}\right)}\Gamma\left(m+1,-\frac{2\,(-idfx-icf)}{d}\right)-(a^3dm+a^3d)e^{\left(-\frac{dm\log\left(\frac{3if}{d}\right)-3icf+3ide}{d}\right)}\Gamma\left(m+1,-\frac{3\,(-idfx-icf)}{d}\right)-60\,(a^3dfx+a^3cf)(dx+c)^m}{24\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/24*(45*(a^3*d*m + a^3*d)*e^(-(d*m*log(I*f/d) - I*c*f + I*d*e)/d)*gamma(m
 + 1, (I*d*f*x + I*c*f)/d) + 9*(-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(-2*I*f/d
```

```
) + 2*I*c*f - 2*I*d*e)/d)*gamma(m + 1, -2*(I*d*f*x + I*c*f)/d) - (a^3*d*m +
 a^3*d)*e^(-(d*m*log(-3*I*f/d) + 3*I*c*f - 3*I*d*e)/d)*gamma(m + 1, -3*(I*d
*f*x + I*c*f)/d) + 45*(a^3*d*m + a^3*d)*e^(-(d*m*log(-I*f/d) + I*c*f - I*d*
e)/d)*gamma(m + 1, (-I*d*f*x - I*c*f)/d) + 9*(I*a^3*d*m + I*a^3*d)*e^(-(d*m
*log(2*I*f/d) - 2*I*c*f + 2*I*d*e)/d)*gamma(m + 1, -2*(-I*d*f*x - I*c*f)/d)
 - (a^3*d*m + a^3*d)*e^(-(d*m*log(3*I*f/d) - 3*I*c*f + 3*I*d*e)/d)*gamma(m
+ 1, -3*(-I*d*f*x - I*c*f)/d) - 60*(a^3*d*f*x + a^3*c*f)*(d*x + c)^m)/(d*f*
m + d*f)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3(c+dx)^m \sin(e+fx)\,dx + \int 3(c+dx)^m \sin^2(e+fx)\,dx + \int (c+dx)^m \sin^3(e+fx)\,dx + \int (c+dx)^m\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*sin(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(3*(c + d*x)**m*sin(e + f*x), x) + Integral(3*(c + d*x)**m*si
n(e + f*x)**2, x) + Integral((c + d*x)**m*sin(e + f*x)**3, x) + Integral((c
 + d*x)**m, x))
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*x + c)^m, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\,\sin(e+f\,x))^3\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*x)^m,x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c + d*x)^m, x)
```

# 3.147 $\int (c+dx)^m (a+a\sin(e+fx))^2 \, dx$

**Optimal**. Leaf size=299

$$\frac{3a^2(c+dx)^{1+m}}{2d(1+m)}-\frac{a^2e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f}-\frac{a^2e^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)}{f}$$

```
[Out] 3/2*a^2*(d*x+c)^(1+m)/d/(1+m)-a^2*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*f
*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-a^2*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)/e
xp(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+I*2^(-3-m)*a^2*exp(2*I*(e-c*f/d))*(d*
x+c)^m*GAMMA(1+m,-2*I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-I*2^(-3-m)*a^2*(d
*x+c)^m*GAMMA(1+m,2*I*f*(d*x+c)/d)/exp(2*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

**Rubi** [A]
time = 0.25, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3399, 3393, 3388, 2212, 3389}

$$-\frac{a^2e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{f}+\frac{ia^22^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{f}-\frac{a^2e^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{f}-\frac{ia^22^{-m-3}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2if(c+dx)}{d}\right)}{f}+\frac{3a^2(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*(c + d*x)^(1 + m))/(2*d*(1 + m)) - (a^2*E^(I*(e - (c*f)/d))*(c + d*x
)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (a^2
*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*
(c + d*x))/d)^m) + (I*2^(-3 - m)*a^2*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (I*2^(-3
 - m)*a^2*(c + d*x)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(E^((2*I)*(e - (
c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m(a+a\sin(e+fx))^2\,dx &= \left(4a^2\right)\int (c+dx)^m\sin^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)\,dx\\ &= \left(4a^2\right)\int \left(\frac{3}{8}(c+dx)^m-\frac{1}{8}(c+dx)^m\cos(2e+2fx)+\frac{1}{2}(c+dx)^m\sin\right.\\ &= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}-\frac{1}{2}a^2\int (c+dx)^m\cos(2e+2fx)\,dx+\left(2a^2\right)\int (c+\\ &= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\left(ia^2\right)\int e^{-i(e+fx)}(c+dx)^m\,dx-\left(ia^2\right)\int e^{i(e+fx)}(c\\ &= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}-\frac{a^2e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if}{}\right.}{f}\end{aligned}$$

**Mathematica [A]**
time = 0.19, size = 260, normalized size = 0.87

$$\frac{1}{8}a^2(c+dx)^m\left(\frac{12(c+dx)}{d(1+m)}-\frac{8e^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f}-\frac{8e^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{f}+\frac{i2^{-m}e^{2i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{f}-\frac{i2^{-m}e^{-2i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^2,x]
[Out] (a^2*(c + d*x)^m*((12*(c + d*x))/(d*(1 + m)) - (8*E^(I*(e - (c*f)/d))*Gamma
[1 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (8*Gamma[1 +
```

```
m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (I*E
^((2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m*f*(((-I)*
f*(c + d*x))/d)^m) - (I*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(2^m*E^((2*I)*
(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)))/8
```

**Maple [F]**
time = 0.16, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + a \sin (fx + e))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+a*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^2/(d*(m + 1)) + 1/2*(a^2*e^(m*log(d*x + c) + log(d*x +
c)) - (a^2*d*m + a^2*d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) + 4*(a^2
*d*m + a^2*d)*integrate((d*x + c)^m*sin(f*x + e), x))/(d*m + d)
```

**Fricas [A]**
time = 0.11, size = 274, normalized size = 0.92

$$-\frac{8 \left(a^{2} d m + a^{2} d\right) e^{\left(-\frac{d m \log\left(\frac{i f}{d}\right) - i c f + i d e}{d}\right)} \Gamma\left(m + 1, \frac{i d f x + i c f}{d}\right) - \left(i a^{2} d m + i a^{2} d\right) e^{\left(-\frac{d m \log\left(-\frac{2 i f}{d}\right) + 2 i c f - 2 i d e}{d}\right)} \Gamma\left(m + 1, -\frac{2 \left(i d f x + i c f\right)}{d}\right) + 8 \left(a^{2} d m + a^{2} d\right) e^{\left(-\frac{d m \log\left(-\frac{i f}{d}\right) + i c f - i d e}{d}\right)} \Gamma\left(m + 1, \frac{-i d f x - i c f}{d}\right) - \left(-i a^{2} d m - i a^{2} d\right) e^{\left(-\frac{d m \log\left(\frac{2 i f}{d}\right) - 2 i c f + 2 i d e}{d}\right)} \Gamma\left(m + 1, -\frac{2 \left(-i d f x - i c f\right)}{d}\right) - 12 \left(a^{2} d f x + a^{2} c f\right) (d x + c)^{m}}{8 \left(d f m + d f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*(a^2*d*m + a^2*d)*e^(-(d*m*log(I*f/d) - I*c*f + I*d*e)/d)*gamma(m +
 1, (I*d*f*x + I*c*f)/d) - (I*a^2*d*m + I*a^2*d)*e^(-(d*m*log(-2*I*f/d) + 2
*I*c*f - 2*I*d*e)/d)*gamma(m + 1, -2*(I*d*f*x + I*c*f)/d) + 8*(a^2*d*m + a^
2*d)*e^(-(d*m*log(-I*f/d) + I*c*f - I*d*e)/d)*gamma(m + 1, (-I*d*f*x - I*c*
f)/d) - (-I*a^2*d*m - I*a^2*d)*e^(-(d*m*log(2*I*f/d) - 2*I*c*f + 2*I*d*e)/d
)*gamma(m + 1, -2*(-I*d*f*x - I*c*f)/d) - 12*(a^2*d*f*x + a^2*c*f)*(d*x + c
)^m)/(d*f*m + d*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int 2 (c + dx)^{m} \sin (e + fx) \, dx + \int (c + dx)^{m} \sin^{2} (e + fx) \, dx + \int (c + dx)^{m} \, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*sin(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(2*(c + d*x)**m*sin(e + f*x), x) + Integral((c + d*x)**m*sin(
e + f*x)**2, x) + Integral((c + d*x)**m, x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*x + c)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\,\sin\left(e + f\,x\right))^2\,\left(c + d\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*x)^m,x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c + d*x)^m, x)
```

# 3.148 $\int (c+dx)^m(a+a\sin(e+fx))\,dx$

**Optimal**. Leaf size=148

$$\frac{a(c+dx)^{1+m}}{d(1+m)}-\frac{ae^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{ae^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma}{2f}$$

```
[Out] a*(d*x+c)^(1+m)/d/(1+m)-1/2*a*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*f*(d*
x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-1/2*a*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)/exp
(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

**Rubi** [A]
time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3389, 2212}

$$-\frac{ae^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{ae^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{2f}+\frac{a(c+dx)^{m+1}}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) - (a*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamm
a[1 + m, ((-I)*f*(c + d*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (a*(c + d*
x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d))*f*((I*f*(c + d
*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^m(a+a\sin(e+fx))\,dx &= \int \left(a(c+dx)^m + a(c+dx)^m\sin(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^{1+m}}{d(1+m)} + a\int (c+dx)^m \sin(e+fx)\,dx \\
&= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}(ia)\int e^{-i(e+fx)}(c+dx)^m\,dx - \frac{1}{2}(ia)\int e^{i(e+fx)}(c+ \\
&= \frac{a(c+dx)^{1+m}}{d(1+m)} - \frac{ae^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right.}{2f}
\end{aligned}
$$

**Mathematica [A]**
time = 1.79, size = 199, normalized size = 1.34

$$
-\frac{a(c+dx)^m\left(2de-2cf-2d(e+fx)+d(1+m)\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)\left(\cos\left(e-\frac{cf}{d}\right)-i\sin\left(e-\frac{cf}{d}\right)\right)+d(1+m)\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)\left(\cos\left(e-\frac{cf}{d}\right)+i\sin\left(e-\frac{cf}{d}\right)\right)\right)(1+\sin(e+fx))}{2df(1+m)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x]),x]
```

```
[Out] -1/2*(a*(c + d*x)^m*(2*d*e - 2*c*f - 2*d*(e + f*x) + (d*(1 + m)*Gamma[1 + m
, (I*f*(c + d*x))/d]*(Cos[e - (c*f)/d] - I*Sin[e - (c*f)/d]))/((I*f*(c + d*
x))/d)^m + (d*(1 + m)*Gamma[1 + m, ((-I)*f*(c + d*x))/d]*(Cos[e - (c*f)/d]
+ I*Sin[e - (c*f)/d]))/(((-I)*f*(c + d*x))/d)^m)*(1 + Sin[e + f*x]))/(d*f*(
1 + m)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

**Maple [F]**
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\left(a+a\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+a*sin(f*x+e)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] a*integrate((d*x + c)^m*sin(f*x + e), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

**Fricas [A]**
time = 0.11, size = 138, normalized size = 0.93

$$-\frac{(adm+ad)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)-icf+ide}{d}\right)}\Gamma\left(m+1,\frac{idfx+icf}{d}\right)+(adm+ad)e^{\left(-\frac{dm\log\left(-\frac{if}{d}\right)+icf-ide}{d}\right)}\Gamma\left(m+1,\frac{-idfx-icf}{d}\right)-2\,(adfx+acf)(dx+c)^m}{2\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((a*d*m + a*d)*e^(-(d*m*log(I*f/d) - I*c*f + I*d*e)/d)*gamma(m + 1, (I
*d*f*x + I*c*f)/d) + (a*d*m + a*d)*e^(-(d*m*log(-I*f/d) + I*c*f - I*d*e)/d)
*gamma(m + 1, (-I*d*f*x - I*c*f)/d) - 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(d*f
*m + d*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (c+dx)^m \sin(e+fx)\,dx+\int (c+dx)^m\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*sin(f*x+e)),x)
```

```
[Out] a*(Integral((c + d*x)**m*sin(e + f*x), x) + Integral((c + d*x)**m, x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*x + c)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin(e+f\,x))\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*x)^m,x)
```

```
[Out] int((a + a*sin(e + f*x))*(c + d*x)^m, x)
```

# 3.149 $\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=23

$$\mathrm{Int}\left(\frac{(c+dx)^m}{a+a\sin(e+fx)},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+a*sin(f*x+e)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + a*Sin[e + f*x]),x]
[Out] Defer[Int][(c + d*x)^m/(a + a*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx=\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$$

**Mathematica** [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + a*Sin[e + f*x]),x]
[Out] Integrate[(c + d*x)^m/(a + a*Sin[e + f*x]), x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*sin(f*x+e)),x)
[Out] int((d*x+c)^m/(a+a*sin(f*x+e)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(a*sin(f*x + e) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((d*x + c)^m/(a*sin(f*x + e) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c+dx)^m}{\sin (e+fx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*sin(f*x+e)),x)
[Out] Integral((c + d*x)**m/(sin(e + f*x) + 1), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^m/(a*sin(f*x + e) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{a+a\,\sin\left(e+f\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*sin(e + f*x)),x)
[Out] int((c + d*x)^m/(a + a*sin(e + f*x)), x)
```

# 3.150 $\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{(a+a\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+a*sin(f*x+e))^2,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + a*Sin[e + f*x])^2,x]
[Out] Defer[Int][(c + d*x)^m/(a + a*Sin[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$$

**Mathematica** [A]
time = 3.60, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + a*Sin[e + f*x])^2,x]
[Out] Integrate[(c + d*x)^m/(a + a*Sin[e + f*x])^2, x]
```

**Maple** [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{(a+a\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*sin(f*x+e))^2,x)
[Out] int((d*x+c)^m/(a+a*sin(f*x+e))^2,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(a*sin(f*x + e) + a)^2, x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(d*x + c)^m/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c+dx)^m}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*sin(f*x+e))**2,x)
[Out] Integral((c + d*x)**m/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**2
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m/(a*sin(f*x + e) + a)^2, x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{(a+a\,\sin(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*sin(e + f*x))^2,x)
[Out] int((c + d*x)^m/(a + a*sin(e + f*x))^2, x)
```

# 3.151 $\int (c+dx)^3(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=90

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3}-\frac{b(c+dx)^3\cos(e+fx)}{f}-\frac{6bd^3\sin(e+fx)}{f^4}+\frac{3bd(c+dx)^2\sin(e+fx)}{f^2}$$

```
[Out] 1/4*a*(d*x+c)^4/d+6*b*d^2*(d*x+c)*cos(f*x+e)/f^3-b*(d*x+c)^3*cos(f*x+e)/f-6
*b*d^3*sin(f*x+e)/f^4+3*b*d*(d*x+c)^2*sin(f*x+e)/f^2
```

**Rubi** [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2717}

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3}+\frac{3bd(c+dx)^2\sin(e+fx)}{f^2}-\frac{b(c+dx)^3\cos(e+fx)}{f}-\frac{6bd^3\sin(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + (6*b*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (b*(c + d*x)
^3*Cos[e + f*x])/f - (6*b*d^3*Sin[e + f*x])/f^4 + (3*b*d*(c + d*x)^2*Sin[e
+ f*x])/f^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx = \int \left(a(c+dx)^3+b(c+dx)^3\sin(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^4}{4d}+b\int (c+dx)^3\sin(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d}-\frac{b(c+dx)^3\cos(e+fx)}{f}+\frac{(3bd)\int (c+dx)^2\cos(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^4}{4d}-\frac{b(c+dx)^3\cos(e+fx)}{f}+\frac{3bd(c+dx)^2\sin(e+fx)}{f^2}-\frac{(6bd$$
$$= \frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3}-\frac{b(c+dx)^3\cos(e+fx)}{f}+\frac{3bd($$
$$= \frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3}-\frac{b(c+dx)^3\cos(e+fx)}{f}-\frac{6bd^3$$

**Mathematica** [A]
time = 0.24, size = 124, normalized size = 1.38

$$\frac{1}{4}ax\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)-\frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2(-6+f^2x^2)\right)\cos(e+fx)}{f^3}+\frac{3bd(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\sin(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (b*(c + d*x)*(c^2*f^2
 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x])/f^3 + (3*b*d*(c^2*f^2 +
2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^4
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(88) = 176.
time = 0.06, size = 482, normalized size = 5.36

| method | result |
|---|---|
| risch | $\frac{a\,d^3x^4}{4}+a\,d^2c\,x^3+\frac{3ad\,c^2x^2}{2}+a\,c^3x+\frac{a\,c^4}{4d}-\frac{b(d^3f^2x^3+3c\,d^2f^2x^2+3c^2d\,f^2x+c^3f^2-6d^3x-6c\,d^2)\cos(fx+e}{f^3}$ |
| norman | $\frac{\frac{(a\,c^3f^3-3b\,c^2d\,f^2+6b\,d^3)x}{f^3}+\frac{(2b\,c^3f^2-12c\,d^2b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^3}+\frac{d^2(acf-bd)x^3}{f}+\frac{(a\,c^3f^3+3b\,c^2d\,f^2-6b\,d^3)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^3}+}{}$ |
| derivativedivides | $\frac{a\,c^3(fx+e)-\frac{3a\,c^2de(fx+e)}{f}+\frac{3a\,c^2d(fx+e)^2}{2f}+\frac{3ac\,d^2e^2(fx+e)}{f^2}-\frac{3ac\,d^2e(fx+e)^2}{f^2}+\frac{ac\,d^2(fx+e)^3}{f^2}-\frac{a\,d^3e^3(fx+e)}{f^3}+\frac{3a\,d^3e^2(fx+e)}{2f^3}}{}$ |
| default | $\frac{a\,c^3(fx+e)-\frac{3a\,c^2de(fx+e)}{f}+\frac{3a\,c^2d(fx+e)^2}{2f}+\frac{3ac\,d^2e^2(fx+e)}{f^2}-\frac{3ac\,d^2e(fx+e)^2}{f^2}+\frac{ac\,d^2(fx+e)^3}{f^2}-\frac{a\,d^3e^3(fx+e)}{f^3}+\frac{3a\,d^3e^2(fx+e)}{2f^3}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^3*(f*x+e)-3/f*a*c^2*d*e*(f*x+e)+3/2/f*a*c^2*d*(f*x+e)^2+3/f^2*a*c*
d^2*e^2*(f*x+e)-3/f^2*a*c*d^2*e*(f*x+e)^2+1/f^2*a*c*d^2*(f*x+e)^3-1/f^3*a*d
^3*e^3*(f*x+e)+3/2/f^3*a*d^3*e^2*(f*x+e)^2-1/f^3*a*d^3*e*(f*x+e)^3+1/4/f^3*
a*d^3*(f*x+e)^4-b*c^3*cos(f*x+e)+3/f*b*c^2*d*e*cos(f*x+e)+3/f*b*c^2*d*(sin(
f*x+e)-(f*x+e)*cos(f*x+e))-3/f^2*b*c*d^2*e^2*cos(f*x+e)-6/f^2*b*c*d^2*e*(si
n(f*x+e)-(f*x+e)*cos(f*x+e))+3/f^2*b*c*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x
+e)+2*(f*x+e)*sin(f*x+e))+1/f^3*b*d^3*e^3*cos(f*x+e)+3/f^3*b*d^3*e^2*(sin(f
*x+e)-(f*x+e)*cos(f*x+e))-3/f^3*b*d^3*e*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)
+2*(f*x+e)*sin(f*x+e))+1/f^3*b*d^3*(-(f*x+e)^3*cos(f*x+e)+3*(f*x+e)^2*sin(f
*x+e)-6*sin(f*x+e)+6*(f*x+e)*cos(f*x+e)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(92) = 184$.
time = 0.30, size = 498, normalized size = 5.53

$$\frac{4\,(fx+e)ac^3+\frac{(fx+e)^4ad^3}{f^3}+\frac{4\,(fx+e)^3acd^2}{f^2}+\frac{6\,(fx+e)^2ac^2d}{f}-4\,bc^3\cos(fx+e)-\frac{4\,(fx+e)^3ad^3e}{f^3}-\frac{12\,(fx+e)^2acd^2e}{f^2}-\frac{12\,(fx+e)ac^2de}{f}+\frac{12\,bc^2d\cos(fx+e)e}{f}-\frac{12\,((fx+e)\cos(fx+e)-\sin(fx+e))bc^2d}{f}+\frac{6\,(fx+e)^2ad^3e^2}{f^3}+\frac{12\,(fx+e)acd^2e^2}{f^2}-\frac{12\,bcd^2\cos(fx+e)e^2}{f^2}+\frac{24\,((fx+e)\cos(fx+e)-\sin(fx+e))bcd^2e}{f^2}-\frac{12\left(\left((fx+e)^2-2\right)\cos(fx+e)-2\,(fx+e)\sin(fx+e)\right)bcd^2}{f^2}-\frac{4\,(fx+e)ad^3e^3}{f^3}+\frac{4\,bd^3\cos(fx+e)e^3}{f^3}-\frac{12\,((fx+e)\cos(fx+e)-\sin(fx+e))bd^3e^2}{f^3}+\frac{12\left(\left((fx+e)^2-2\right)\cos(fx+e)-2\,(fx+e)\sin(fx+e)\right)bd^3e}{f^3}-\frac{4\left(\left((fx+e)^3-6\,fx-6\,e\right)\cos(fx+e)-3\left((fx+e)^2-2\right)\sin(fx+e)\right)bd^3}{f^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 + 4*(f*x + e)^3*a*c*d^2/f^2
+ 6*(f*x + e)^2*a*c^2*d/f - 4*b*c^3*cos(f*x + e) - 4*(f*x + e)^3*a*d^3*e/f^
3 - 12*(f*x + e)^2*a*c*d^2*e/f^2 - 12*(f*x + e)*a*c^2*d*e/f + 12*b*c^2*d*co
s(f*x + e)*e/f - 12*((f*x + e)*cos(f*x + e) - sin(f*x + e))*b*c^2*d/f + 6*(
f*x + e)^2*a*d^3*e^2/f^3 + 12*(f*x + e)*a*c*d^2*e^2/f^2 - 12*b*c*d^2*cos(f*
x + e)*e^2/f^2 + 24*((f*x + e)*cos(f*x + e) - sin(f*x + e))*b*c*d^2*e/f^2 -
 12*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*b*c*d^2/f^2
 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*b*d^3*cos(f*x + e)*e^3/f^3 - 12*((f*x + e)
*cos(f*x + e) - sin(f*x + e))*b*d^3*e^2/f^3 + 12*(((f*x + e)^2 - 2)*cos(f*x
 + e) - 2*(f*x + e)*sin(f*x + e))*b*d^3*e/f^3 - 4*(((f*x + e)^3 - 6*f*x - 6
*e)*cos(f*x + e) - 3*((f*x + e)^2 - 2)*sin(f*x + e))*b*d^3/f^3)/f
```

**Fricas** [A]
time = 0.35, size = 170, normalized size = 1.89

$$\frac{ad^3f^4x^4+4\,acd^2f^4x^3+6\,ac^2df^4x^2+4\,ac^3f^4x-4\,(bd^3f^3x^3+3\,bcd^2f^3x^2+bc^3f^3-6\,bcd^2f+3\,(bc^2df^3-2\,bd^3f)x)\cos(fx+e)+12\,(bd^3f^2x^2+2\,bcd^2f^2x+bc^2df^2-2\,bd^3)\sin(fx+e)}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
- 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 - 6*b*c*d^2*f + 3*(b*c^2
*d*f^3 - 2*b*d^3*f)*x)*cos(f*x + e) + 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x +
 b*c^2*d*f^2 - 2*b*d^3)*sin(f*x + e))/f^4
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(88) = 176$.
time = 0.25, size = 264, normalized size = 2.93

$$\begin{cases} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} - \frac{bc^3\cos(e+fx)}{f} - \frac{3bc^2dx\cos(e+fx)}{f} + \frac{3bc^2d\sin(e+fx)}{f^2} - \frac{3bcd^2x^2\cos(e+fx)}{f} + \frac{6bcd^2x\sin(e+fx)}{f^2} + \frac{6bcd^2\cos(e+fx)}{f^3} - \frac{bd^3x^3\cos(e+fx)}{f} + \frac{3bd^3x^2\sin(e+fx)}{f^2} + \frac{6bd^3x\cos(e+fx)}{f^3} - \frac{6bd^3\sin(e+fx)}{f^4} & \text{for } f \neq 0 \\ (a + b\sin(e))\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 - b
*c**3*cos(e + f*x)/f - 3*b*c**2*d*x*cos(e + f*x)/f + 3*b*c**2*d*sin(e + f*x
)/f**2 - 3*b*c*d**2*x**2*cos(e + f*x)/f + 6*b*c*d**2*x*sin(e + f*x)/f**2 +
6*b*c*d**2*cos(e + f*x)/f**3 - b*d**3*x**3*cos(e + f*x)/f + 3*b*d**3*x**2*s
in(e + f*x)/f**2 + 6*b*d**3*x*cos(e + f*x)/f**3 - 6*b*d**3*sin(e + f*x)/f**
4, Ne(f, 0)), ((a + b*sin(e))*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**
3*x**4/4), True))
```

**Giac** [A]
time = 5.80, size = 157, normalized size = 1.74

$$\frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x - \frac{(bd^3f^3x^3 + 3bcd^2f^3x^2 + 3bc^2df^3x + bc^3f^3 - 6bd^3fx - 6bcd^2f)\cos(fx+e)}{f^4} + \frac{3(bd^3f^2x^2 + 2bcd^2f^2x + bc^2df^2 - 2bd^3)\sin(fx+e)}{f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - (b*d^3*f^3*x^3 +
3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*c^3*f^3 - 6*b*d^3*f*x - 6*b*c*d^2*f
)*cos(f*x + e)/f^4 + 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2 - 2*b
*d^3)*sin(f*x + e)/f^4
```

**Mupad** [B]
time = 0.80, size = 191, normalized size = 2.12

$$\frac{ad^3x^4}{4} - \frac{3\sin(e+fx)(2bd^3 - bc^2df^2)}{f^4} - \frac{\cos(e+fx)(bc^3f^2 - 6bcd^2)}{f^3} + ac^3x + \frac{3x\cos(e+fx)(2bd^3 - bc^2df^2)}{f^3} + \frac{3ac^2dx^2}{2} + acd^2x^3 - \frac{bd^3x^3\cos(e+fx)}{f} + \frac{3bd^3x^2\sin(e+fx)}{f^2} + \frac{6bcd^2x\sin(e+fx)}{f^2} - \frac{3bcd^2x^2\cos(e+fx)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*x)^3,x)
```

```
[Out] (a*d^3*x^4)/4 - (3*sin(e + f*x)*(2*b*d^3 - b*c^2*d*f^2))/f^4 - (cos(e + f*x
)*(b*c^3*f^2 - 6*b*c*d^2))/f^3 + a*c^3*x + (3*x*cos(e + f*x)*(2*b*d^3 - b*c
^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 - (b*d^3*x^3*cos(e + f*x))
/f + (3*b*d^3*x^2*sin(e + f*x))/f^2 + (6*b*c*d^2*x*sin(e + f*x))/f^2 - (3*b
*c*d^2*x^2*cos(e + f*x))/f
```

# 3.152 $\int (c+dx)^2(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=68

$$\frac{a(c+dx)^3}{3d}+\frac{2bd^2\cos(e+fx)}{f^3}-\frac{b(c+dx)^2\cos(e+fx)}{f}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}$$

```
[Out] 1/3*a*(d*x+c)^3/d+2*b*d^2*cos(f*x+e)/f^3-b*(d*x+c)^2*cos(f*x+e)/f+2*b*d*(d*
x+c)*sin(f*x+e)/f^2
```

**Rubi** [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2718}

$$\frac{a(c+dx)^3}{3d}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}-\frac{b(c+dx)^2\cos(e+fx)}{f}+\frac{2bd^2\cos(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*b*d^2*Cos[e + f*x])/f^3 - (b*(c + d*x)^2*Cos[e +
 f*x])/f + (2*b*d*(c + d*x)*Sin[e + f*x])/f^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx = \int \left(a(c+dx)^2+b(c+dx)^2\sin(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^3}{3d} + b\int (c+dx)^2\sin(e+fx)\,dx$$
$$= \frac{a(c+dx)^3}{3d} - \frac{b(c+dx)^2\cos(e+fx)}{f} + \frac{(2bd)\int (c+dx)\cos(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^3}{3d} - \frac{b(c+dx)^2\cos(e+fx)}{f} + \frac{2bd(c+dx)\sin(e+fx)}{f^2} - \frac{(2bd^2}{}$$
$$= \frac{a(c+dx)^3}{3d} + \frac{2bd^2\cos(e+fx)}{f^3} - \frac{b(c+dx)^2\cos(e+fx)}{f} + \frac{2bd(c+dx)}{f}$$

**Mathematica** [A]
time = 0.20, size = 84, normalized size = 1.24

$$\frac{1}{3}ax\left(3c^2+3cdx+d^2x^2\right) - \frac{b(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)}{f^3} + \frac{2bd(c+dx)\sin(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 - (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 +
 f^2*x^2))*Cos[e + f*x])/f^3 + (2*b*d*(c + d*x)*Sin[e + f*x])/f^2
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(66) = 132$.
time = 0.05, size = 241, normalized size = 3.54

| method | result |
|---|---|
| risch | $\frac{a\,d^2x^3}{3} + adc\,x^2 + a\,c^2x + \frac{a\,c^3}{3d} - \frac{b\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\cos(fx+e)}{f^3} + \frac{2bd(dx+c)\sin(fx+e)}{f^2}$ |
| norman | $\frac{\frac{\left(2b\,c^2f^2-4b\,d^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^3} + \frac{c(acf-2bd)x}{f} + \frac{d(acf-bd)x^2}{f} + \frac{c(acf+2bd)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f} + \frac{d(acf+bd)x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f} + a}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$ |
| derivativedivides | $a\,c^2(fx+e) - \frac{2acde(fx+e)}{f} + \frac{acd(fx+e)^2}{f} + \frac{a\,d^2e^2(fx+e)}{f^2} - \frac{a\,d^2e(fx+e)^2}{f^2} + \frac{a\,d^2(fx+e)^3}{3f^2} - b\,c^2\cos(fx+e) + \frac{2bcde\cos(fx+e)}{f} + \frac{2bc}{}$ |
| default | $a\,c^2(fx+e) - \frac{2acde(fx+e)}{f} + \frac{acd(fx+e)^2}{f} + \frac{a\,d^2e^2(fx+e)}{f^2} - \frac{a\,d^2e(fx+e)^2}{f^2} + \frac{a\,d^2(fx+e)^3}{3f^2} - b\,c^2\cos(fx+e) + \frac{2bcde\cos(fx+e)}{f} + \frac{2bc}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^2*(f*x+e)-2*a/f*c*d*e*(f*x+e)+a/f*c*d*(f*x+e)^2+a/f^2*d^2*e^2*(f*x
+e)-a/f^2*d^2*e*(f*x+e)^2+1/3*a/f^2*d^2*(f*x+e)^3-b*c^2*cos(f*x+e)+2/f*b*c*
d*e*cos(f*x+e)+2/f*b*c*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e))-1/f^2*b*d^2*e^2*co
s(f*x+e)-2/f^2*b*d^2*e*(sin(f*x+e)-(f*x+e)*cos(f*x+e))+1/f^2*b*d^2*(-(f*x+e
)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(69) = 138$.
time = 0.37, size = 260, normalized size = 3.82

$$\frac{3\,(fx+e)ac^2 + \frac{(fx+e)^3ad^2}{f^2} + \frac{3\,(fx+e)^2acd}{f} - 3\,bc^2\cos\left(fx+e\right) - \frac{3\,(fx+e)^2ad^2e}{f^2} - \frac{6\,(fx+e)acde}{f} + \frac{6\,bcd\cos(fx+e)e}{f} - \frac{6\,((fx+e)\cos(fx+e)-\sin(fx+e))bcd}{f} + \frac{3\,(fx+e)ad^2e^2}{f^2} - \frac{3\,bd^2\cos(fx+e)e^2}{f^2} + \frac{6\,((fx+e)\cos(fx+e)-\sin(fx+e))bd^2e}{f^2} - \frac{3\left(\left((fx+e)^2-2\right)\cos(fx+e)-2\,(fx+e)\sin(fx+e)\right)bd^2}{f^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 3*
b*c^2*cos(f*x + e) - 3*(f*x + e)^2*a*d^2*e/f^2 - 6*(f*x + e)*a*c*d*e/f + 6*
b*c*d*cos(f*x + e)*e/f - 6*((f*x + e)*cos(f*x + e) - sin(f*x + e))*b*c*d/f
+ 3*(f*x + e)*a*d^2*e^2/f^2 - 3*b*d^2*cos(f*x + e)*e^2/f^2 + 6*((f*x + e)*c
os(f*x + e) - sin(f*x + e))*b*d^2*e/f^2 - 3*(((f*x + e)^2 - 2)*cos(f*x + e)
 - 2*(f*x + e)*sin(f*x + e))*b*d^2/f^2)/f
```

**Fricas** [A]
time = 0.37, size = 104, normalized size = 1.53

$$\frac{ad^2f^3x^3 + 3\,acdf^3x^2 + 3\,ac^2f^3x - 3\,(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2)\cos\left(fx+e\right) + 6\,(bd^2fx + bcdf)\sin\left(fx+e\right)}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 3*(b*d^2*f^2*x^2 + 2
*b*c*d*f^2*x + b*c^2*f^2 - 2*b*d^2)*cos(f*x + e) + 6*(b*d^2*f*x + b*c*d*f)*
sin(f*x + e))/f^3
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.
time = 0.15, size = 151, normalized size = 2.22

$$\begin{cases} ac^2x + acdx^2 + \frac{ad^2x^3}{3} - \frac{bc^2\cos\left(e+fx\right)}{f} - \frac{2bcdx\cos\left(e+fx\right)}{f} + \frac{2bcd\sin\left(e+fx\right)}{f^2} - \frac{bd^2x^2\cos\left(e+fx\right)}{f} + \frac{2bd^2x\sin\left(e+fx\right)}{f^2} + \frac{2bd^2\cos\left(e+fx\right)}{f^3} & \text{for}\: f \neq 0 \\ \left(a + b\sin\left(e\right)\right)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 - b*c**2*cos(e + f*x)/f -
2*b*c*d*x*cos(e + f*x)/f + 2*b*c*d*sin(e + f*x)/f**2 - b*d**2*x**2*cos(e +
```

```
f*x)/f + 2*b*d**2*x*sin(e + f*x)/f**2 + 2*b*d**2*cos(e + f*x)/f**3, Ne(f, 0
)), ((a + b*sin(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

**Giac** [A]
time = 5.39, size = 95, normalized size = 1.40

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x - \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2)\cos{(fx+e)}}{f^3} + \frac{2\,(bd^2fx + bcdf)\sin{(fx+e)}}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^
2*f^2 - 2*b*d^2)*cos(f*x + e)/f^3 + 2*(b*d^2*f*x + b*c*d*f)*sin(f*x + e)/f^
3
```

**Mupad** [B]
time = 0.67, size = 112, normalized size = 1.65

$$\frac{a\,d^2\,x^3}{3} + \frac{\cos{(e+f\,x)}\,\left(2\,b\,d^2 - b\,c^2\,f^2\right)}{f^3} + a\,c^2\,x + a\,c\,d\,x^2 + \frac{2\,b\,d^2\,x\,\sin{(e+f\,x)}}{f^2} - \frac{b\,d^2\,x^2\,\cos{(e+f\,x)}}{f} + \frac{2\,b\,c\,d\,\sin{(e+f\,x)}}{f^2} - \frac{2\,b\,c\,d\,x\,\cos{(e+f\,x)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*x)^2,x)
```

```
[Out] (a*d^2*x^3)/3 + (cos(e + f*x)*(2*b*d^2 - b*c^2*f^2))/f^3 + a*c^2*x + a*c*d*
x^2 + (2*b*d^2*x*sin(e + f*x))/f^2 - (b*d^2*x^2*cos(e + f*x))/f + (2*b*c*d*
sin(e + f*x))/f^2 - (2*b*c*d*x*cos(e + f*x))/f
```

# 3.153 $\int (c+dx)(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=45

$$\frac{a(c+dx)^2}{2d}-\frac{b(c+dx)\cos(e+fx)}{f}+\frac{bd\sin(e+fx)}{f^2}$$

```
[Out] 1/2*a*(d*x+c)^2/d-b*(d*x+c)*cos(f*x+e)/f+b*d*sin(f*x+e)/f^2
```

**Rubi** [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3398, 3377, 2717}

$$\frac{a(c+dx)^2}{2d}-\frac{b(c+dx)\cos(e+fx)}{f}+\frac{bd\sin(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (b*(c + d*x)*Cos[e + f*x])/f + (b*d*Sin[e + f*x])/f
^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)(a+b\sin(e+fx))\,dx &= \int (a(c+dx)+b(c+dx)\sin(e+fx))\,dx \\
&= \frac{a(c+dx)^2}{2d} + b\int (c+dx)\sin(e+fx)\,dx \\
&= \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)\cos(e+fx)}{f} + \frac{(bd)\int \cos(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)\cos(e+fx)}{f} + \frac{bd\sin(e+fx)}{f^2}
\end{aligned}
$$

**Mathematica** [A]
time = 0.08, size = 43, normalized size = 0.96

$$
\frac{1}{2}ax(2c+dx) - \frac{b(c+dx)\cos(e+fx)}{f} + \frac{bd\sin(e+fx)}{f^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*x*(2*c + d*x))/2 - (b*(c + d*x)*Cos[e + f*x])/f + (b*d*Sin[e + f*x])/f^2
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(43) = 86$.
time = 0.03, size = 90, normalized size = 2.00

| method | result | size |
|---|---|---|
| risch | $\frac{da\,x^2}{2} + acx - \frac{b(dx+c)\cos(fx+e)}{f} + \frac{bd\sin(fx+e)}{f^2}$ | 42 |
| derivativedivides | $\frac{ac(fx+e) - \frac{ade(fx+e)}{f} + \frac{ad(fx+e)^2}{2f} - cb\cos(fx+e) + \frac{bde\cos(fx+e)}{f} + \frac{bd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}}{f}$ | 90 |
| default | $\frac{ac(fx+e) - \frac{ade(fx+e)}{f} + \frac{ad(fx+e)^2}{2f} - cb\cos(fx+e) + \frac{bde\cos(fx+e)}{f} + \frac{bd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}}{f}$ | 90 |
| norman | $\frac{\frac{2cb\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f} + \frac{(acf-bd)x}{f} + \frac{(acf+bd)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f} + \frac{da\,x^2}{2} + \frac{2bd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f^2} + \frac{da\,x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$ | 115 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c*(f*x+e)-a/f*d*e*(f*x+e)+1/2*a/f*d*(f*x+e)^2-c*b*cos(f*x+e)+1/f*b*d
*e*cos(f*x+e)+1/f*b*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(45) = 90$.
time = 0.29, size = 103, normalized size = 2.29

$$\frac{2(fx+e)ac + \frac{(fx+e)^2ad}{f} - 2bc\cos(fx+e) - \frac{2(fx+e)ade}{f} + \frac{2bd\cos(fx+e)e}{f} - \frac{2((fx+e)\cos(fx+e)-\sin(fx+e))bd}{f}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*b*c*cos(f*x + e) - 2*(f*x + e)
*a*d*e/f + 2*b*d*cos(f*x + e)*e/f - 2*((f*x + e)*cos(f*x + e) - sin(f*x + e
))*b*d/f)/f
```

**Fricas** [A]
time = 0.34, size = 53, normalized size = 1.18

$$\frac{adf^2x^2 + 2\,acf^2x + 2\,bd\sin(fx+e) - 2\,(bdfx + bcf)\cos(fx+e)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*b*d*sin(f*x + e) - 2*(b*d*f*x + b*c*f)*c
os(f*x + e))/f^2
```

**Sympy** [A]
time = 0.09, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{adx^2}{2} - \frac{bc\cos(e+fx)}{f} - \frac{bdx\cos(e+fx)}{f} + \frac{bd\sin(e+fx)}{f^2} & \text{for } f \neq 0 \\ (a+b\sin(e))\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*d*x**2/2 - b*c*cos(e + f*x)/f - b*d*x*cos(e + f*x)/f +
 b*d*sin(e + f*x)/f**2, Ne(f, 0)), ((a + b*sin(e))*(c*x + d*x**2/2), True))
```

**Giac** [A]
time = 4.54, size = 47, normalized size = 1.04

$$\frac{1}{2}adx^2 + acx + \frac{bd\sin(fx+e)}{f^2} - \frac{(bdfx+bcf)\cos(fx+e)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + b*d*sin(f*x + e)/f^2 - (b*d*f*x + b*c*f)*cos(f*x + e)
/f^2
```

**Mupad [B]**
time = 0.63, size = 50, normalized size = 1.11

$$a\,c\,x-\frac{f\,\left(b\,c\,\cos\left(e+f\,x\right)+b\,d\,x\,\cos\left(e+f\,x\right)\right)-b\,d\,\sin\left(e+f\,x\right)}{f^2}+\frac{a\,d\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*x),x)
```

```
[Out] a*c*x - (f*(b*c*cos(e + f*x) + b*d*x*cos(e + f*x)) - b*d*sin(e + f*x))/f^2
+ (a*d*x^2)/2
```

# 3.154 $\int \frac{a+b\sin(e+fx)}{c+dx}\,dx$

**Optimal**. Leaf size=64

$$\frac{a\log(c+dx)}{d}+\frac{b\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}$$

```
[Out] a*ln(d*x+c)/d+b*cos(-e+c*f/d)*Si(c*f/d+f*x)/d-b*Ci(c*f/d+f*x)*sin(-e+c*f/d)
/d
```

**Rubi** [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3384, 3380, 3383}

$$\frac{a\log(c+dx)}{d}+\frac{b\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x])/d + (b*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d + (b
*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{b\sin(e+fx)}{c+dx}\right)dx\\ &= \frac{a\log(c+dx)}{d}+b\int \frac{\sin(e+fx)}{c+dx}\,dx\\ &= \frac{a\log(c+dx)}{d}+\left(b\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx+\left(b\sin\left(e-\frac{cf}{d}\right)\right)\int \frac{\mathrm{co}}{}\\ &= \frac{a\log(c+dx)}{d}+\frac{b\mathrm{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cos\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(\frac{cf}{d}+fx\right)}{d}\end{aligned}$$

**Mathematica** [A]
time = 0.08, size = 57, normalized size = 0.89

$$\frac{a\log(c+dx)+b\mathrm{Ci}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+b\cos\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*x),x]
[Out] (a*Log[c + d*x] + b*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + b*Cos[e - (
c*f)/d]*SinIntegral[f*(c/d + x)])/d
```

**Maple** [A]
time = 0.05, size = 103, normalized size = 1.61

| method | result | s |
|---|---|---|
| derivativedivides | $\frac{\frac{af\ln(cf-de+d(fx+e))}{d}+fb\left(\frac{\mathrm{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)}{f}$ | 1 |
| default | $\frac{\frac{af\ln(cf-de+d(fx+e))}{d}+fb\left(\frac{\mathrm{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)}{f}$ | 1 |
| risch | $\frac{a\ln(dx+c)}{d}-\frac{ib\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{2d}+\frac{ib\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,-ifx-ie-\frac{icf-ide}{d}\right)}{2d}$ | 1 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*f*ln(c*f-d*e+d*(f*x+e))/d+f*b*(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d
)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d))
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.34, size = 181, normalized size = 2.83

$$\frac{\frac{2\,af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{\left(f\left(-i\,E_1\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+i\,E_1\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f\left(E_1\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+E_1\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)b}{d}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d + (f*(-I*exp_integral_e(1, (I*(
f*x + e)*d + I*c*f - I*d*e)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d + I*c*
f - I*d*e)/d))*cos((c*f - d*e)/d) + f*(exp_integral_e(1, (I*(f*x + e)*d + I
*c*f - I*d*e)/d) + exp_integral_e(1, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*s
in((c*f - d*e)/d))*b/d)/f
```

**Fricas** [A]
time = 0.36, size = 94, normalized size = 1.47

$$\frac{2\,b\cos\left(-\frac{cf-de}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+2\,a\log\left(dx+c\right)+\left(b\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+b\,\mathrm{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*cos(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) + 2*a*log(d*x +
c) + (b*cos_integral((d*f*x + c*f)/d) + b*cos_integral(-(d*f*x + c*f)/d))*s
in(-(c*f - d*e)/d))/d
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\sin\left(e+fx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*sin(e + f*x))/(c + d*x), x)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.93, size = 712, normalized size = 11.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
 b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*
a*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*sin_integral((d*f*x
 + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*real_part(cos_integral(f*x +
 c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*real_part(cos_integral(-f*x - c*
f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) + 2*b*real_part(cos_integral(f*x + c*f/d)
)*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*real_part(cos_integral(-f*x - c*f/d))*t
an(1/2*c*f/d)*tan(1/2*e)^2 - b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*c*f/d)^2 + b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 + 2*a*
log(abs(d*x + c))*tan(1/2*c*f/d)^2 - 2*b*sin_integral((d*f*x + c*f)/d)*tan(
1/2*c*f/d)^2 + 4*b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(
1/2*e) - 4*b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e
) + 8*b*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) - b*imag_pa
rt(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 + b*imag_part(cos_integral(-f*x
- c*f/d))*tan(1/2*e)^2 + 2*a*log(abs(d*x + c))*tan(1/2*e)^2 - 2*b*sin_integ
ral((d*f*x + c*f)/d)*tan(1/2*e)^2 - 2*b*real_part(cos_integral(f*x + c*f/d)
)*tan(1/2*c*f/d) - 2*b*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)
 + 2*b*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 2*b*real_part(cos_
integral(-f*x - c*f/d))*tan(1/2*e) + b*imag_part(cos_integral(f*x + c*f/d))
 - b*imag_part(cos_integral(-f*x - c*f/d)) + 2*a*log(abs(d*x + c)) + 2*b*si
n_integral((d*f*x + c*f)/d))/(d*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + d*tan(1/2*c
*f/d)^2 + d*tan(1/2*e)^2 + d)
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\sin\left(e+f\,x\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + b*sin(e + f*x))/(c + d*x), x)
```

# 3.155 $\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=88

$$-\frac{a}{d(c+dx)}+\frac{bf\cos\left(e-\frac{cf}{d}\right)\operatorname{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{b\sin(e+fx)}{d(c+dx)}-\frac{bf\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(\frac{cf}{d}+fx\right)}{d^2}$$

```
[Out] -a/d/(d*x+c)+b*f*Ci(c*f/d+f*x)*cos(-e+c*f/d)/d^2+b*f*Si(c*f/d+f*x)*sin(-e+c
*f/d)/d^2-b*sin(f*x+e)/d/(d*x+c)
```

**Rubi** [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3398, 3378, 3384, 3380, 3383}

$$-\frac{a}{d(c+dx)}+\frac{bf\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{bf\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\sin(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) + (b*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2
- (b*Sin[e + f*x])/(d*(c + d*x)) - (b*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{b\sin(e+fx)}{(c+dx)^2}\right)dx\\
&= -\frac{a}{d(c+dx)}+b\int \frac{\sin(e+fx)}{(c+dx)^2}\,dx\\
&= -\frac{a}{d(c+dx)}-\frac{b\sin(e+fx)}{d(c+dx)}+\frac{(bf)\int \frac{\cos(e+fx)}{c+dx}\,dx}{d}\\
&= -\frac{a}{d(c+dx)}-\frac{b\sin(e+fx)}{d(c+dx)}+\frac{\left(bf\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}-\frac{\left(bf\sin\left(e-\frac{cf}{d}\right.\right.}{}\\
&= -\frac{a}{d(c+dx)}+\frac{bf\cos\left(e-\frac{cf}{d}\right)\operatorname{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{b\sin(e+fx)}{d(c+dx)}-\frac{bf\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(\frac{cf}{d}\right.}{d^2}
\end{aligned}$$

**Mathematica [A]**
time = 0.23, size = 72, normalized size = 0.82

$$\frac{bf\cos\left(e-\frac{cf}{d}\right)\operatorname{Ci}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(a+b\sin(e+fx))}{c+dx}-bf\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (b*f*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] - (d*(a + b*Sin[e + f*x]))/(
c + d*x) - b*f*Sin[e - (c*f)/d]*SinIntegral[f*(c/d + x)])/d^2
```

**Maple [A]**
time = 0.18, size = 141, normalized size = 1.60

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{a f^2}{(cf-de+d(fx+e))d}+f^2 b\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos}{d}}{d}\right.}{f}$ |
| default | $\frac{-\frac{a f^2}{(cf-de+d(fx+e))d}+f^2 b\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos}{d}}{d}\right.}{f}$ |
| risch | $-\frac{a}{d(dx+c)}-\frac{fb\,e^{\frac{i(cf-de)}{d}}\text{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^2}-\frac{fb\,e^{-\frac{i(cf-de)}{d}}\text{expIntegral}\left(1,-ifx-ie-\frac{icf-ide}{d}\right)}{2d^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*f^2/(c*f-d*e+d*(f*x+e))/d+f^2*b*(-sin(f*x+e)/(c*f-d*e+d*(f*x+e))/d+
(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*cos((c*f-d*
e)/d)/d)/d))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.37, size = 208, normalized size = 2.36

$$-\frac{\frac{2\,af^2}{(fx+e)d^2+cdf-d^2e}-\frac{\left(f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+i\,E_2\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f^2\left(E_2\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+E_2\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)b}{(fx+e)d^2+cdf-d^2e}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*f^2/((f*x + e)*d^2 + c*d*f - d^2*e) - (f^2*(-I*exp_integral_e(2,
(I*(f*x + e)*d + I*c*f - I*d*e)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d +
I*c*f - I*d*e)/d))*cos((c*f - d*e)/d) + f^2*(exp_integral_e(2, (I*(f*x + e)
*d + I*c*f - I*d*e)/d) + exp_integral_e(2, -(I*(f*x + e)*d + I*c*f - I*d*e)
/d))*sin((c*f - d*e)/d))*b/((f*x + e)*d^2 + c*d*f - d^2*e))/f
```

**Fricas [A]**
time = 0.37, size = 138, normalized size = 1.57

$$-\frac{2\,bd\sin\left(fx+e\right)+2\left(bdfx+bcf\right)\sin\left(-\frac{cf-de}{d}\right)\text{Si}\left(\frac{dfx+cf}{d}\right)+2\,ad-\left(\left(bdfx+bcf\right)\text{Ci}\left(\frac{dfx+cf}{d}\right)+\left(bdfx+bcf\right)\text{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\cos\left(-\frac{cf-de}{d}\right)}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*sin(f*x + e) + 2*(b*d*f*x + b*c*f)*sin(-(c*f - d*e)/d)*sin_inte
gral((d*f*x + c*f)/d) + 2*a*d - ((b*d*f*x + b*c*f)*cos_integral((d*f*x + c*
f)/d) + (b*d*f*x + b*c*f)*cos_integral(-(d*f*x + c*f)/d))*cos(-(c*f - d*e)/
d))/(d^3*x + c*d^2)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin{(e+fx)}}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))/(c + d*x)**2, x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(92) = 184.
time = 2.77, size = 578, normalized size = 6.57

$$\frac{\left((dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)f^2\cos\left(\frac{cf-de}{d}\right)\operatorname{Ci}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)-cf^3\cos\left(\frac{cf-de}{d}\right)\operatorname{Ci}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)+df^2\cos\left(\frac{cf-de}{d}\right)\operatorname{Ci}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)e+(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)f^2\sin\left(\frac{cf-de}{d}\right)\operatorname{Si}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)-cf^3\sin\left(\frac{cf-de}{d}\right)\operatorname{Si}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)+df^2e\sin\left(\frac{cf-de}{d}\right)\operatorname{Si}\left(-\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de}{d}\right)-df^2\sin\left(\frac{(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)}{d}\right)\right)bd^2}{\left((dx+c)d^4\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cd^4f+d^5e\right)f}-\frac{a}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos((c*f - d*e)/d)*cos_i
ntegral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - c
*f^3*cos((c*f - d*e)/d)*cos_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(
d*x + c)) - c*f + d*e)/d) + d*f^2*cos((c*f - d*e)/d)*cos_integral(-((d*x +
c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e + (d*x + c)*(c*f/(
d*x + c) - f - d*e/(d*x + c))*f^2*sin((c*f - d*e)/d)*sin_integral(-((d*x +
c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - c*f^3*sin((c*f - d
*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f +
 d*e)/d) + d*f^2*e*sin((c*f - d*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x +
c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - d*f^2*sin((d*x + c)*(c*f/(d*x + c
) - f - d*e/(d*x + c))/d))*b*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - f - d*e/(
d*x + c)) - c*d^4*f + d^5*e)*f) - a/((d*x + c)*d)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(e+f\,x\right)}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x))/(c + d*x)^2, x)
```

# 3.156 $\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=123

$$-\frac{a}{2d(c+dx)^2}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{bf^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{b\sin(e+fx)}{2d(c+dx)^2}-\frac{bf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

```
[Out] -1/2*a/d/(d*x+c)^2-1/2*b*f*cos(f*x+e)/d^2/(d*x+c)-1/2*b*f^2*cos(-e+c*f/d)*S
i(c*f/d+f*x)/d^3+1/2*b*f^2*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^3-1/2*b*sin(f*x+e)
/d/(d*x+c)^2
```

**Rubi** [A]

time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3398, 3378, 3384, 3380, 3383}

$$-\frac{a}{2d(c+dx)^2}-\frac{bf^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{bf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{b\sin(e+fx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*a/(d*(c + d*x)^2) - (b*f*Cos[e + f*x])/(2*d^2*(c + d*x)) - (b*f^2*CosI
ntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d^3) - (b*Sin[e + f*x])/(2*d*(c
 + d*x)^2) - (b*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx &= \int \left(\frac{a}{(c+dx)^3}+\frac{b\sin(e+fx)}{(c+dx)^3}\right)dx\\ &= -\frac{a}{2d(c+dx)^2}+b\int \frac{\sin(e+fx)}{(c+dx)^3}\,dx\\ &= -\frac{a}{2d(c+dx)^2}-\frac{b\sin(e+fx)}{2d(c+dx)^2}+\frac{(bf)\int \frac{\cos(e+fx)}{(c+dx)^2}\,dx}{2d}\\ &= -\frac{a}{2d(c+dx)^2}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{b\sin(e+fx)}{2d(c+dx)^2}-\frac{(bf^2)\int \frac{\sin(e+fx)}{c+dx}\,dx}{2d^2}\\ &= -\frac{a}{2d(c+dx)^2}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{b\sin(e+fx)}{2d(c+dx)^2}-\frac{\left(bf^2\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\\ &= -\frac{a}{2d(c+dx)^2}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{bf^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{b\sin(e+fx)}{2d(c+dx)^2}-\end{aligned}$$

**Mathematica [A]**
time = 0.51, size = 94, normalized size = 0.76

$$-\frac{bf^2\text{Ci}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+\frac{d(bf(c+dx)\cos(e+fx)+d(a+b\sin(e+fx)))}{(c+dx)^2}+bf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*(b*f^2*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + (d*(b*f*(c + d*x)*C
os[e + f*x] + d*(a + b*Sin[e + f*x])))/(c + d*x)^2 + b*f^2*Cos[e - (c*f)/d]
*SinIntegral[f*(c/d + x)])/d^3
```

**Maple [A]**
time = 0.11, size = 177, normalized size = 1.44

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{a f^3}{2(cf-de+d(fx+e))^2 d}+f^3 b\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\text{cosin}}{d}}{d}}{2d}\right.}{f}$ |
| default | $\frac{-\frac{a f^3}{2(cf-de+d(fx+e))^2 d}+f^3 b\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\text{cosin}}{d}}{d}}{2d}\right.}{f}$ |
| risch | $-\frac{a}{2d(dx+c)^2}+\frac{i f^2 b\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\text{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{4d^3}-\frac{i f^2 b\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\text{expIntegral}\left(1,-ifx-ie-\frac{icf-}{d}\right.}{4d^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*a*f^3/(c*f-d*e+d*(f*x+e))^2/d+f^3*b*(-1/2*sin(f*x+e)/(c*f-d*e+d*(
f*x+e))^2/d+1/2*(-cos(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*c
os((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.49, size = 279, normalized size = 2.27

$$-\frac{\frac{af^3}{(fx+e)^2d^3+c^2df^2-2\,cd^2fe+d^3e^2+2\,(cd^2f-d^3e)(fx+e)}-\frac{\left(f^3\left(-i\,E_3\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+i\,E_3\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f^3\left(E_3\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+E_3\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)b}{(fx+e)^2d^3+c^2df^2-2\,cd^2fe+d^3e^2+2\,(cd^2f-d^3e)(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*f^3/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*(c*d^2
*f - d^3*e)*(f*x + e)) - (f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d + I*c*f
- I*d*e)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*cos(
(c*f - d*e)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d + I*c*f - I*d*e)/d)
+ exp_integral_e(3, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*sin((c*f - d*e)/d)
)*b/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*(c*d^2*f - d^3
*e)*(f*x + e)))/f
```

**Fricas [A]**
time = 0.37, size = 231, normalized size = 1.88

$$-\frac{2\,bd^2\sin\left(fx+e\right)+2\,ad^2+2\,(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2)\cos\left(-\frac{cf-de}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+2\,(bd^2fx+bcdf)\cos\left(fx+e\right)+\left((bd^2f^2x^2+2\,bcdf^2x+bc^2f^2)\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2)\,\mathrm{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)}{4\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*d^2*sin(f*x + e) + 2*a*d^2 + 2*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b
*c^2*f^2)*cos(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) + 2*(b*d^2*f*x
+ b*c*d*f)*cos(f*x + e) + ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*cos_
integral((d*f*x + c*f)/d) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*cos
_integral(-(d*f*x + c*f)/d))*sin(-(c*f - d*e)/d))/(d^5*x^2 + 2*c*d^4*x + c^
2*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sin(e + fx)}{(c + dx)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))/(c + d*x)**3, x)
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.85, size = 6157, normalized size = 50.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*c*f/d)^2*tan(1/2*e)^2 - b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*
f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*d^2*f^2*x^2*sin_in
tegral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*
d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f
/d)^2*tan(1/2*e) - 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 2*b*d^2*f^2*x^2*real_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*d^2*f
^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*
tan(1/2*e)^2 + 2*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*c*d*f^2*x*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*b*c*d*f^2*x
*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2
 - b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/
2*c*f/d)^2 + b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*
x)^2*tan(1/2*c*f/d)^2 - 2*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1
/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c
*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 4*b*d^2*f^2*x^2*imag_part
```

(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) + 8*b
*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*ta
n(1/2*e) - 4*b*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e) - 4*b*c*d*f^2*x*real_part(cos_integral(-f*x -
 c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - b*d^2*f^2*x^2*imag_pa
rt(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + b*d^2*f^2*x^2*i
mag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*b*d^2*
f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 4*b*c*d
*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*t
an(1/2*e)^2 + 4*b*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + b*d^2*f^2*x^2*imag_part(cos_integral(f*
x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - b*d^2*f^2*x^2*imag_part(cos_int
egral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*d^2*f^2*x^2*sin_in
tegral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b*c^2*f^2*imag_part
(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/
d)^2*tan(1/2*e)^2 + 2*b*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*d^2*f^2*x^2*real_part(cos_integral(f*
x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*b*d^2*f^2*x^2*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*b*c*d*f^2*x*imag_p
art(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 2*b*c*d*f^
2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 -
 4*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^
2 + 2*b*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*e) + 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x
)^2*tan(1/2*e) + 8*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 8*b*c*d*f^2*x*imag_part(cos_integral(-f
*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) + 16*b*c*d*f^2*x*sin_
integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 2*b*d^
2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)
- 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*ta
n(1/2*e) - 2*b*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*
tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*c^2*f^2*real_part(cos_integral(-f*x - c*f
/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*c*d*f^2*x*imag_part(c
os_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*b*c*d*f^2*x*imag_
part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 4*b*c*d*f^2*
x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*b*d^2*f^2*x
^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*d
^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^
2 + 2*b*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2
*c*f/d)*tan(1/2*e)^2 + 2*b*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*c*d*f^2*x*imag_part(cos_inte
gral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*c*d*f^2*x*imag_part(
cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*b*c*d*f^2*x*s
in_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*d^2*f*x*ta

```
n(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b*d^2*f^2*x^2*imag_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*f*x)^2 - b*d^2*f^2*x^2*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*f*x)^2 + 2*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/
d)*tan(1/2*f*x)^2 - 4*b*c*d*f^2*x*real_part(cos...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(e+f\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x))/(c + d*x)^3, x)
```

# 3.157 $\int (c+dx)^3(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=250

$$-\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx)\cos(e+fx)}{f^3}-\frac{2ab(c+dx)^3\cos(e+fx)}{f}-1$$

```
[Out] -3/4*b^2*c*d^2*x/f^2-3/8*b^2*d^3*x^2/f^2+1/4*a^2*(d*x+c)^4/d+1/8*b^2*(d*x+c
)^4/d+12*a*b*d^2*(d*x+c)*cos(f*x+e)/f^3-2*a*b*(d*x+c)^3*cos(f*x+e)/f-12*a*b
*d^3*sin(f*x+e)/f^4+6*a*b*d*(d*x+c)^2*sin(f*x+e)/f^2+3/4*b^2*d^2*(d*x+c)*co
s(f*x+e)*sin(f*x+e)/f^3-1/2*b^2*(d*x+c)^3*cos(f*x+e)*sin(f*x+e)/f-3/8*b^2*d
^3*sin(f*x+e)^2/f^4+3/4*b^2*d*(d*x+c)^2*sin(f*x+e)^2/f^2
```

**Rubi** [A]
time = 0.17, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3398, 3377, 2717, 3392, 32, 3391}

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\cos(e+fx)}{f^3}+\frac{6abd(c+dx)^2\sin(e+fx)}{f^2}-\frac{2ab(c+dx)^3\cos(e+fx)}{f}-\frac{12abd^3\sin(e+fx)}{f^4}+\frac{3b^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{3b^2cd^2x}{4f^2}+\frac{3b^2d(c+dx)^2\sin^2(e+fx)}{4f^2}-\frac{b^2(c+dx)^3\sin(e+fx)\cos(e+fx)}{2f}+\frac{b^2(c+dx)^4}{8d}-\frac{3b^2d^3\sin^2(e+fx)}{8f^4}-\frac{3b^2d^3x^2}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-3*b^2*c*d^2*x)/(4*f^2) - (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d
) + (b^2*(c + d*x)^4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (2*
a*b*(c + d*x)^3*Cos[e + f*x])/f - (12*a*b*d^3*Sin[e + f*x])/f^4 + (6*a*b*d*
(c + d*x)^2*Sin[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f
*x])/(4*f^3) - (b^2*(c + d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f) - (3*b^2*d
^3*Sin[e + f*x]^2)/(8*f^4) + (3*b^2*d*(c + d*x)^2*Sin[e + f*x]^2)/(4*f^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3(a+b\sin(e+fx))^2\,dx &= \int \left(a^2(c+dx)^3+2ab(c+dx)^3\sin(e+fx)+b^2(c+dx)^3\sin^2(e+fx)\right) \\
&= \frac{a^2(c+dx)^4}{4d}+(2ab)\int (c+dx)^3\sin(e+fx)\,dx+b^2\int (c+dx)^3\sin^2(e \\
&= \frac{a^2(c+dx)^4}{4d}-\frac{2ab(c+dx)^3\cos(e+fx)}{f}-\frac{b^2(c+dx)^3\cos(e+fx)\sin(}{2f} \\
&= \frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}-\frac{2ab(c+dx)^3\cos(e+fx)}{f}+\frac{6abd(c+dx)^{}}{f} \\
&= -\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx)\,\mathrm{co}}{f^3} \\
&= -\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx)\,\mathrm{co}}{f^3}
\end{aligned}$$

**Mathematica [A]**
time = 0.71, size = 232, normalized size = 0.93

$$\frac{2(2a^2+b^2)\,f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-32abf(c+dx)\,(c^2f^2+2cdf^2x+d^2(-6+f^2x^2))\cos(e+fx)-3b^2d(2c^2f^2+4cdf^2x+d^2(-1+2f^2x^2))\cos(2(e+fx))+96abd(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\sin(e+fx)-2b^2f(c+dx)\,(2c^2f^2+4cdf^2x+d^2(-3+2f^2x^2))\sin(2(e+fx))}{16f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2)*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 32*a*b
*f*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x] - 3*
b^2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)] + 9
6*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x] - 2*b^2*f
*(c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e + f*x)
])/(16*f^4)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(234) = 468$.
time = 0.12, size = 1125, normalized size = 4.50

| method | result |
|---|---|
| risch | $\frac{a^2 d^3 x^4}{4} + \frac{d^3 b^2 x^4}{8} + a^2 c\, d^2 x^3 + \frac{d^2 b^2 c\, x^3}{2} + \frac{3a^2 d\, c^2 x^2}{2} + \frac{3d\, b^2 c^2 x^2}{4} + a^2 c^3 x + \frac{b^2 c^3 x}{2} + \frac{a^2 c^4}{4d} + \frac{b^2 c^4}{8d} -$ |
| norman | $\frac{\left(\frac{1}{4}a^2 d^3+\frac{1}{8}b^2 d^3\right)x^4+\left(\frac{1}{2}a^2 d^3+\frac{1}{4}b^2 d^3\right)x^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{1}{4}a^2 d^3+\frac{1}{8}b^2 d^3\right)x^4\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{b^2 d^3 x^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*a*b*c^3*cos(f*x+e)+1/f^3*b^2*d^3*((f*x+e)^3*(-1/2*sin(f*x+e)*cos(f*
x+e)+1/2*f*x+1/2*e)-3/4*(f*x+e)^2*cos(f*x+e)^2+3/2*(f*x+e)*(1/2*sin(f*x+e)*
cos(f*x+e)+1/2*f*x+1/2*e)-3/8*(f*x+e)^2-3/8*sin(f*x+e)^2-3/8*(f*x+e)^4)+1/4
*a^2/f^3*d^3*(f*x+e)^4+a^2/f^2*c*d^2*(f*x+e)^3+6/f^3*a*b*d^3*e^2*(sin(f*x+e
)-(f*x+e)*cos(f*x+e))-6/f^3*a*b*d^3*e*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2
*(f*x+e)*sin(f*x+e))+6/f*a*b*c^2*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e))+a^2*c^3*
(f*x+e)-6/f^2*a*b*c*d^2*e^2*cos(f*x+e)-12/f^2*a*b*c*d^2*e*(sin(f*x+e)-(f*x+
e)*cos(f*x+e))+6/f*a*b*c^2*d*e*cos(f*x+e)+3/2*a^2/f^3*d^3*e^2*(f*x+e)^2-a^2
/f^3*d^3*e*(f*x+e)^3+3/2*a^2/f*c^2*d*(f*x+e)^2-a^2/f^3*d^3*e^3*(f*x+e)+2/f^
3*a*b*d^3*(-(f*x+e)^3*cos(f*x+e)+3*(f*x+e)^2*sin(f*x+e)-6*sin(f*x+e)+6*(f*x
+e)*cos(f*x+e))+3/f^3*b^2*d^3*e^2*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*
f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+3/f^2*b^2*c*d^2*((f*x+e)^2*(-1/2
*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+
e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)-3/f^3*b^2*d^3*e*((f*x+e)^2*(-1/2
*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+
e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)-1/f^3*b^2*d^3*e^3*(-1/2*sin(f*x+
e)*cos(f*x+e)+1/2*f*x+1/2*e)+3/f*b^2*c^2*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*
x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-3*a^2/f*c^2*d*e*(f*x+e)
-3*a^2/f^2*c*d^2*e*(f*x+e)^2+3*a^2/f^2*c*d^2*e^2*(f*x+e)+3/f^2*b^2*c*d^2*e^
```

```
2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2/f^3*a*b*d^3*e^3*cos(f*x+e)+6
/f^2*a*b*c*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))-6/
f^2*b^2*c*d^2*e*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*
x+e)^2+1/4*sin(f*x+e)^2)-3/f*b^2*c^2*d*e*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*
x+1/2*e)+b^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(244) = 488$.
time = 0.39, size = 1031, normalized size = 4.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(f*x + e)*a^2*c^3 + 4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^3 + 4
*(f*x + e)^4*a^2*d^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 + 24*(f*x + e)^2*a^
2*c^2*d/f - 32*a*b*c^3*cos(f*x + e) - 16*(f*x + e)^3*a^2*d^3*e/f^3 - 48*(f*
x + e)^2*a^2*c*d^2*e/f^2 - 48*(f*x + e)*a^2*c^2*d*e/f - 12*(2*f*x + 2*e - s
in(2*f*x + 2*e))*b^2*c^2*d*e/f + 96*a*b*c^2*d*cos(f*x + e)*e/f - 96*((f*x +
 e)*cos(f*x + e) - sin(f*x + e))*a*b*c^2*d/f + 6*(2*(f*x + e)^2 - 2*(f*x +
e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*b^2*c^2*d/f + 24*(f*x + e)^2*a^2*d^
3*e^2/f^3 + 48*(f*x + e)*a^2*c*d^2*e^2/f^2 + 12*(2*f*x + 2*e - sin(2*f*x +
2*e))*b^2*c*d^2*e^2/f^2 - 96*a*b*c*d^2*cos(f*x + e)*e^2/f^2 + 192*((f*x + e
)*cos(f*x + e) - sin(f*x + e))*a*b*c*d^2*e/f^2 - 12*(2*(f*x + e)^2 - 2*(f*x
 + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*b^2*c*d^2*e/f^2 - 96*(((f*x + e)
^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a*b*c*d^2/f^2 + 2*(4*(f*x
+ e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2
*e))*b^2*c*d^2/f^2 - 16*(f*x + e)*a^2*d^3*e^3/f^3 - 4*(2*f*x + 2*e - sin(2*
f*x + 2*e))*b^2*d^3*e^3/f^3 + 32*a*b*d^3*cos(f*x + e)*e^3/f^3 - 96*((f*x +
e)*cos(f*x + e) - sin(f*x + e))*a*b*d^3*e^2/f^3 + 6*(2*(f*x + e)^2 - 2*(f*x
 + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*b^2*d^3*e^2/f^3 + 96*(((f*x + e)
^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a*b*d^3*e/f^3 - 2*(4*(f*x
+ e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2
*e))*b^2*d^3*e/f^3 - 32*(((f*x + e)^3 - 6*f*x - 6*e)*cos(f*x + e) - 3*((f*x
 + e)^2 - 2)*sin(f*x + e))*a*b*d^3/f^3 + (2*(f*x + e)^4 - 3*(2*(f*x + e)^2
- 1)*cos(2*f*x + 2*e) - 2*(2*(f*x + e)^3 - 3*f*x - 3*e)*sin(2*f*x + 2*e))*b
^2*d^3/f^3)/f
```

**Fricas** [A]
time = 0.36, size = 386, normalized size = 1.54

$$\frac{(2a^2+b^2)d^3f^4x^4+4(2a^2+b^2)cd^2f^4x^3+3(2(2a^2+b^2)c^2df^4+b^2d^3f^2)x^2-3(2b^2d^3f^2x^2+4b^2cd^2f^2x+2b^2c^2df^2-b^2d^3)\cos(fx+e)^2+2(2(2a^2+b^2)c^3f^4+3b^2cd^2f^2)x-16(abd^3f^3x^3+3abcd^2f^3x^2+abc^3f^3-6abcd^2f+3(abc^2df^3-2abd^3f)x)\cos(fx+e)+2(24abd^3f^2x^2+48abcd^2f^2x+24abc^2df^2-48abd^3-(2b^2d^3f^3x^3+6b^2cd^2f^3x^2+2b^2c^3f^3-3b^2cd^2f+3(2b^2c^2df^3-b^2d^3f)x)\cos(fx+e))\sin(fx+e)}{8f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^2 + b^2)*d^3*f^4*x^4 + 4*(2*a^2 + b^2)*c*d^2*f^4*x^3 + 3*(2*(2*a^
2 + b^2)*c^2*d*f^4 + b^2*d^3*f^2)*x^2 - 3*(2*b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*
f^2*x + 2*b^2*c^2*d*f^2 - b^2*d^3)*cos(f*x + e)^2 + 2*(2*(2*a^2 + b^2)*c^3*
f^4 + 3*b^2*c*d^2*f^2)*x - 16*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + a*b*
c^3*f^3 - 6*a*b*c*d^2*f + 3*(a*b*c^2*d*f^3 - 2*a*b*d^3*f)*x)*cos(f*x + e) +
 2*(24*a*b*d^3*f^2*x^2 + 48*a*b*c*d^2*f^2*x + 24*a*b*c^2*d*f^2 - 48*a*b*d^3
 - (2*b^2*d^3*f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 2*b^2*c^3*f^3 - 3*b^2*c*d^2*f
 + 3*(2*b^2*c^2*d*f^3 - b^2*d^3*f)*x)*cos(f*x + e))*sin(f*x + e))/f^4
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(255) = 510$.
time = 0.45, size = 779, normalized size = 3.12

$$\begin{cases} a^{2}c^{3}x + \frac{3a^{2}c^{2}dx^{2}}{2} + a^{2}cd^{2}x^{3} + \frac{a^{2}d^{3}x^{4}}{4} - \frac{2abc^{3}\cos(e+fx)}{f} - \frac{6abc^{2}dx\cos(e+fx)}{f} + \frac{6abc^{2}d\sin(e+fx)}{f^{2}} - \frac{6abcd^{2}x^{2}\cos(e+fx)}{f} + \frac{12abcd^{2}x\sin(e+fx)}{f^{2}} + \frac{12abcd^{2}\cos(e+fx)}{f^{3}} - \frac{2abd^{3}x^{3}\cos(e+fx)}{f} + \frac{6abd^{3}x^{2}\sin(e+fx)}{f^{2}} + \frac{12abd^{3}x\cos(e+fx)}{f^{3}} - \frac{12abd^{3}\sin(e+fx)}{f^{4}} + \frac{b^{2}c^{3}x\sin^{2}(e+fx)}{2} + \frac{b^{2}c^{3}x\cos^{2}(e+fx)}{2} - \frac{b^{2}c^{3}\sin(e+fx)\cos(e+fx)}{2f} + \frac{3b^{2}c^{2}dx^{2}\sin^{2}(e+fx)}{4} + \frac{3b^{2}c^{2}dx^{2}\cos^{2}(e+fx)}{4} - \frac{3b^{2}c^{2}dx\sin(e+fx)\cos(e+fx)}{2f} + \frac{3b^{2}c^{2}d\sin^{2}(e+fx)}{4f^{2}} + \frac{b^{2}cd^{2}x^{3}\sin^{2}(e+fx)}{2} + \frac{b^{2}cd^{2}x^{3}\cos^{2}(e+fx)}{2} - \frac{3b^{2}cd^{2}x^{2}\sin(e+fx)\cos(e+fx)}{2f} + \frac{3b^{2}cd^{2}x\sin^{2}(e+fx)}{4f^{2}} - \frac{3b^{2}cd^{2}x\cos^{2}(e+fx)}{4f^{2}} + \frac{3b^{2}cd^{2}\sin(e+fx)\cos(e+fx)}{4f^{3}} + \frac{b^{2}d^{3}x^{4}\sin^{2}(e+fx)}{8} + \frac{b^{2}d^{3}x^{4}\cos^{2}(e+fx)}{8} - \frac{b^{2}d^{3}x^{3}\sin(e+fx)\cos(e+fx)}{2f} + \frac{3b^{2}d^{3}x^{2}\sin^{2}(e+fx)}{8f^{2}} - \frac{3b^{2}d^{3}x^{2}\cos^{2}(e+fx)}{8f^{2}} + \frac{3b^{2}d^{3}x\sin(e+fx)\cos(e+fx)}{4f^{3}} - \frac{3b^{2}d^{3}\sin^{2}(e+fx)}{8f^{4}} & \text{for}\: f \neq 0 \\ (a + b\sin(e))^{2}\left(c^{3}x + \frac{3c^{2}dx^{2}}{2} + cd^{2}x^{3} + \frac{d^{3}x^{4}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d**
3*x**4/4 - 2*a*b*c**3*cos(e + f*x)/f - 6*a*b*c**2*d*x*cos(e + f*x)/f + 6*a*
b*c**2*d*sin(e + f*x)/f**2 - 6*a*b*c*d**2*x**2*cos(e + f*x)/f + 12*a*b*c*d*
*2*x*sin(e + f*x)/f**2 + 12*a*b*c*d**2*cos(e + f*x)/f**3 - 2*a*b*d**3*x**3*
cos(e + f*x)/f + 6*a*b*d**3*x**2*sin(e + f*x)/f**2 + 12*a*b*d**3*x*cos(e +
f*x)/f**3 - 12*a*b*d**3*sin(e + f*x)/f**4 + b**2*c**3*x*sin(e + f*x)**2/2 +
 b**2*c**3*x*cos(e + f*x)**2/2 - b**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f)
+ 3*b**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*b**2*c**2*d*x**2*cos(e + f*x)**2
/4 - 3*b**2*c**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) + 3*b**2*c**2*d*sin(e
+ f*x)**2/(4*f**2) + b**2*c*d**2*x**3*sin(e + f*x)**2/2 + b**2*c*d**2*x**3*
cos(e + f*x)**2/2 - 3*b**2*c*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 3*
b**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) - 3*b**2*c*d**2*x*cos(e + f*x)**2/(4
*f**2) + 3*b**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(4*f**3) + b**2*d**3*x**4*
sin(e + f*x)**2/8 + b**2*d**3*x**4*cos(e + f*x)**2/8 - b**2*d**3*x**3*sin(e
 + f*x)*cos(e + f*x)/(2*f) + 3*b**2*d**3*x**2*sin(e + f*x)**2/(8*f**2) - 3*
b**2*d**3*x**2*cos(e + f*x)**2/(8*f**2) + 3*b**2*d**3*x*sin(e + f*x)*cos(e
+ f*x)/(4*f**3) - 3*b**2*d**3*sin(e + f*x)**2/(8*f**4), Ne(f, 0)), ((a + b*
sin(e))**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), True))
```

**Giac** [**A**]
time = 6.01, size = 371, normalized size = 1.48

$$\frac{1}{4}a^{2}d^{3}x^{4} + \frac{1}{8}b^{2}d^{3}x^{4} + a^{2}cd^{2}x^{3} + \frac{1}{2}b^{2}cd^{2}x^{3} + \frac{3}{2}a^{2}c^{2}dx^{2} + \frac{3}{4}b^{2}c^{2}dx^{2} + a^{2}c^{3}x + \frac{1}{2}b^{2}c^{3}x - \frac{3\left(2b^{2}d^{3}f^{2}x^{2} + 4b^{2}cd^{2}f^{2}x + 2b^{2}c^{2}df^{2} - b^{2}d^{3}\right)\cos\left(2fx + 2e\right)}{16f^{4}} - \frac{2\left(abd^{3}f^{3}x^{3} + 3abcd^{2}f^{3}x^{2} + 3abc^{2}df^{3}x + abc^{3}f^{3} - 6abd^{3}fx - 6abcd^{2}f\right)\cos\left(fx + e\right)}{f^{4}} - \frac{\left(2b^{2}d^{3}f^{3}x^{3} + 6b^{2}cd^{2}f^{3}x^{2} + 6b^{2}c^{2}df^{3}x + 2b^{2}c^{3}f^{3} - 3b^{2}d^{3}fx - 3b^{2}cd^{2}f\right)\sin\left(2fx + 2e\right)}{8f^{4}} + \frac{6\left(abd^{3}f^{2}x^{2} + 2abcd^{2}f^{2}x + abc^{2}df^{2} - 2abd^{3}\right)\sin\left(fx + e\right)}{f^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

[Out] 1/4*a^2*d^3*x^4 + 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 + 1/2*b^2*c*d^2*x^3 + 3/2
*a^2*c^2*d*x^2 + 3/4*b^2*c^2*d*x^2 + a^2*c^3*x + 1/2*b^2*c^3*x - 3/16*(2*b^
2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 - b^2*d^3)*cos(2*f*x +
2*e)/f^4 - 2*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x + a
*b*c^3*f^3 - 6*a*b*d^3*f*x - 6*a*b*c*d^2*f)*cos(f*x + e)/f^4 - 1/8*(2*b^2*d
^3*f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 6*b^2*c^2*d*f^3*x + 2*b^2*c^3*f^3 - 3*b^
2*d^3*f*x - 3*b^2*c*d^2*f)*sin(2*f*x + 2*e)/f^4 + 6*(a*b*d^3*f^2*x^2 + 2*a*
b*c*d^2*f^2*x + a*b*c^2*d*f^2 - 2*a*b*d^3)*sin(f*x + e)/f^4

**Mupad [B]**
time = 2.64, size = 497, normalized size = 1.99

$$\frac{\frac{3b^2d^3\cos(2e+2fx)}{2}+8a^2c^3f^4x+4b^2c^3f^4x-96abd^3\sin(e+fx)-2b^2c^3f^3\sin(2e+2fx)+2a^2d^3f^4x^4+b^2d^3f^4x^4-16abc^3f^3\cos(e+fx)-3b^2d^3f^2x^2\cos(2e+2fx)-2b^2d^3f^3x^3\sin(2e+2fx)+3b^2cd^2f\sin(2e+2fx)+3b^2d^3fx\sin(2e+2fx)-3b^2c^2df^2\cos(2e+2fx)+12a^2c^2df^4x^2+8a^2cd^2f^4x^3+6b^2c^2df^4x^2+4b^2cd^2f^4x^3+96abcd^2f\cos(e+fx)+96abd^3fx\cos(e+fx)-6b^2cd^2f^2x\cos(2e+2fx)-6b^2c^2df^3x\sin(2e+2fx)+48abc^2df^2\sin(e+fx)-6b^2cd^2f^3x^2\sin(2e+2fx)-16abd^3f^3x^3\cos(e+fx)+48abd^3f^2x^2\sin(e+fx)-48abcd^2f^3x^2\cos(e+fx)-48abc^2df^3x\cos(e+fx)+96abcd^2f^2x\sin(e+fx)}{8f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(e + f*x))^2*(c + d*x)^3,x)

[Out] ((3*b^2*d^3*cos(2*e + 2*f*x))/2 + 8*a^2*c^3*f^4*x + 4*b^2*c^3*f^4*x - 96*a*
b*d^3*sin(e + f*x) - 2*b^2*c^3*f^3*sin(2*e + 2*f*x) + 2*a^2*d^3*f^4*x^4 + b
^2*d^3*f^4*x^4 - 16*a*b*c^3*f^3*cos(e + f*x) - 3*b^2*d^3*f^2*x^2*cos(2*e +
2*f*x) - 2*b^2*d^3*f^3*x^3*sin(2*e + 2*f*x) + 3*b^2*c*d^2*f*sin(2*e + 2*f*x
) + 3*b^2*d^3*f*x*sin(2*e + 2*f*x) - 3*b^2*c^2*d*f^2*cos(2*e + 2*f*x) + 12*
a^2*c^2*d*f^4*x^2 + 8*a^2*c*d^2*f^4*x^3 + 6*b^2*c^2*d*f^4*x^2 + 4*b^2*c*d^2
*f^4*x^3 + 96*a*b*c*d^2*f*cos(e + f*x) + 96*a*b*d^3*f*x*cos(e + f*x) - 6*b^
2*c*d^2*f^2*x*cos(2*e + 2*f*x) - 6*b^2*c^2*d*f^3*x*sin(2*e + 2*f*x) + 48*a*
b*c^2*d*f^2*sin(e + f*x) - 6*b^2*c*d^2*f^3*x^2*sin(2*e + 2*f*x) - 16*a*b*d^
3*f^3*x^3*cos(e + f*x) + 48*a*b*d^3*f^2*x^2*sin(e + f*x) - 48*a*b*c*d^2*f^3
*x^2*cos(e + f*x) - 48*a*b*c^2*d*f^3*x*cos(e + f*x) + 96*a*b*c*d^2*f^2*x*si
n(e + f*x))/(8*f^4)

# 3.158 $\int (c+dx)^2(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=182

$$-\frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}+\frac{b^2(c+dx)^3}{6d}+\frac{4abd^2\cos(e+fx)}{f^3}-\frac{2ab(c+dx)^2\cos(e+fx)}{f}+\frac{4abd(c+dx)\sin(e+f}{f^2}$$

```
[Out] -1/4*b^2*d^2*x/f^2+1/3*a^2*(d*x+c)^3/d+1/6*b^2*(d*x+c)^3/d+4*a*b*d^2*cos(f*
x+e)/f^3-2*a*b*(d*x+c)^2*cos(f*x+e)/f+4*a*b*d*(d*x+c)*sin(f*x+e)/f^2+1/4*b^
2*d^2*cos(f*x+e)*sin(f*x+e)/f^3-1/2*b^2*(d*x+c)^2*cos(f*x+e)*sin(f*x+e)/f+1
/2*b^2*d*(d*x+c)*sin(f*x+e)^2/f^2
```

**Rubi** [A]
time = 0.12, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3398, 3377, 2718, 3392, 32, 2715, 8}

$$\frac{a^2(c+dx)^3}{3d}+\frac{4abd(c+dx)\sin(e+fx)}{f^2}-\frac{2ab(c+dx)^2\cos(e+fx)}{f}+\frac{4abd^2\cos(e+fx)}{f^3}+\frac{b^2d(c+dx)\sin^2(e+fx)}{2f^2}-\frac{b^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{b^2(c+dx)^3}{6d}+\frac{b^2d^2\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{b^2d^2x}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(b^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(3*d) + (b^2*(c + d*x)^3)/(6*d) +
(4*a*b*d^2*Cos[e + f*x])/f^3 - (2*a*b*(c + d*x)^2*Cos[e + f*x])/f + (4*a*b*
d*(c + d*x)*Sin[e + f*x])/f^2 + (b^2*d^2*Cos[e + f*x]*Sin[e + f*x])/(4*f^3)
 - (b^2*(c + d*x)^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b^2*d*(c + d*x)*Sin
[e + f*x]^2)/(2*f^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+b\sin(e+fx))^2\,dx &= \int \left(a^2(c+dx)^2+2ab(c+dx)^2\sin(e+fx)+b^2(c+dx)^2\sin^2(e+fx)\right) \\
&= \frac{a^2(c+dx)^3}{3d}+(2ab)\int (c+dx)^2\sin(e+fx)\,dx+b^2\int (c+dx)^2\sin^2(e \\
&= \frac{a^2(c+dx)^3}{3d}-\frac{2ab(c+dx)^2\cos(e+fx)}{f}-\frac{b^2(c+dx)^2\cos(e+fx)\sin(}{2f} \\
&= \frac{a^2(c+dx)^3}{3d}+\frac{b^2(c+dx)^3}{6d}-\frac{2ab(c+dx)^2\cos(e+fx)}{f}+\frac{4abd(c+dx)}{f^2} \\
&= -\frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}+\frac{b^2(c+dx)^3}{6d}+\frac{4abd^2\cos(e+fx)}{f^3}-\frac{2ab(c+d}{}
\end{aligned}$$

**Mathematica** [A]
time = 0.42, size = 249, normalized size = 1.37

$$\frac{24a^2c^2f^3x+12b^2c^2f^3x+24a^2cdf^3x^2+12b^2cdf^3x^2+8a^2d^2f^3x^3+4b^2d^2f^3x^3-48ab(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)-6b^2df(c+dx)\cos(2(e+fx))+96abcdf\sin(e+fx)+96abd^2fx\sin(e+fx)+3b^2d^2\sin(2(e+fx))-6b^2c^2f^2\sin(2(e+fx))-12b^2cdf^2x\sin(2(e+fx))-6b^2d^2f^2x^2\sin(2(e+fx))}{24f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (24*a^2*c^2*f^3*x + 12*b^2*c^2*f^3*x + 24*a^2*c*d*f^3*x^2 + 12*b^2*c*d*f^3*
x^2 + 8*a^2*d^2*f^3*x^3 + 4*b^2*d^2*f^3*x^3 - 48*a*b*(c^2*f^2 + 2*c*d*f^2*x
 + d^2*(-2 + f^2*x^2))*Cos[e + f*x] - 6*b^2*d*f*(c + d*x)*Cos[2*(e + f*x)]
+ 96*a*b*c*d*f*Sin[e + f*x] + 96*a*b*d^2*f*x*Sin[e + f*x] + 3*b^2*d^2*Sin[2
*(e + f*x)] - 6*b^2*c^2*f^2*Sin[2*(e + f*x)] - 12*b^2*c*d*f^2*x*Sin[2*(e +
f*x)] - 6*b^2*d^2*f^2*x^2*Sin[2*(e + f*x)])/(24*f^3)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(170) = 340$.
time = 0.10, size = 561, normalized size = 3.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c^2*(f*x+e)-2*a^2/f*c*d*e*(f*x+e)+a^2/f*c*d*(f*x+e)^2+a^2/f^2*d^2*
e^2*(f*x+e)-a^2/f^2*d^2*e*(f*x+e)^2+1/3*a^2/f^2*d^2*(f*x+e)^3-2*a*b*c^2*cos
(f*x+e)+4/f*a*b*c*d*e*cos(f*x+e)+4/f*a*b*c*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e)
)-2/f^2*a*b*d^2*e^2*cos(f*x+e)-4/f^2*a*b*d^2*e*(sin(f*x+e)-(f*x+e)*cos(f*x+
e))+2/f^2*a*b*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))
+b^2*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/f*b^2*c*d*e*(-1/2*sin
(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2/f*b^2*c*d*((f*x+e)*(-1/2*sin(f*x+e)*cos
(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+1/f^2*b^2*d^2*e^2*(-
1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/f^2*b^2*d^2*e*((f*x+e)*(-1/2*sin
(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+1/f^2*b^2
*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(
f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(178) = 356$.
time = 0.30, size = 543, normalized size = 2.98

$$\frac{24\,(fx+e)a^2c^2 + 6\,(2\,fx+2\,e-\sin(2\,fx+2\,e))b^2c^2 + \frac{8\,(fx+e)^3a^2d^2}{f^2} + \frac{24\,(fx+e)^2a^2cd}{f} - 48\,abc^2\cos(fx+e) - \frac{24\,(fx+e)^2a^2d^2e}{f^2} - \frac{48\,(fx+e)a^2cde}{f} - \frac{12\,(2\,fx+2\,e-\sin(2\,fx+2\,e))b^2cde}{f} + \frac{96\,abcd\cos(fx+e)e}{f} - \frac{96\,((fx+e)\cos(fx+e)-\sin(fx+e))abcd}{f} + \frac{6\left(2\,(fx+e)^2-2\,(fx+e)\sin(2\,fx+2\,e)-\cos(2\,fx+2\,e)\right)b^2cd}{f} + \frac{24\,(fx+e)a^2d^2e^2}{f^2} + \frac{6\,(2\,fx+2\,e-\sin(2\,fx+2\,e))b^2d^2e^2}{f^2} - \frac{48\,abd^2\cos(fx+e)e^2}{f^2} + \frac{96\,((fx+e)\cos(fx+e)-\sin(fx+e))abd^2e}{f^2} - \frac{6\left(2\,(fx+e)^2-2\,(fx+e)\sin(2\,fx+2\,e)-\cos(2\,fx+2\,e)\right)b^2d^2e}{f^2} - \frac{48\left(\left((fx+e)^2-2\right)\cos(fx+e)-2\,(fx+e)\sin(fx+e)\right)abd^2}{f^2} + \frac{\left(4\,(fx+e)^3-6\,(fx+e)\cos(2\,fx+2\,e)-3\left(2\,(fx+e)^2-1\right)\sin(2\,fx+2\,e)\right)b^2d^2}{f^2}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(24*(f*x + e)*a^2*c^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^2 + 8
*(f*x + e)^3*a^2*d^2/f^2 + 24*(f*x + e)^2*a^2*c*d/f - 48*a*b*c^2*cos(f*x +
e) - 24*(f*x + e)^2*a^2*d^2*e/f^2 - 48*(f*x + e)*a^2*c*d*e/f - 12*(2*f*x +
2*e - sin(2*f*x + 2*e))*b^2*c*d*e/f + 96*a*b*c*d*cos(f*x + e)*e/f - 96*((f*
x + e)*cos(f*x + e) - sin(f*x + e))*a*b*c*d/f + 6*(2*(f*x + e)^2 - 2*(f*x +
 e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*b^2*c*d/f + 24*(f*x + e)*a^2*d^2*e
^2/f^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*d^2*e^2/f^2 - 48*a*b*d^2*co
```

```
s(f*x + e)*e^2/f^2 + 96*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*b*d^2*e/f
^2 - 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*b^
2*d^2*e/f^2 - 48*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e)
)*a*b*d^2/f^2 + (4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x +
 e)^2 - 1)*sin(2*f*x + 2*e))*b^2*d^2/f^2)/f
```

**Fricas [A]**
time = 0.36, size = 230, normalized size = 1.26

$$\frac{2\,(2\,a^{2}+b^{2})d^{2}f^{3}x^{3}+6\,(2\,a^{2}+b^{2})cdf^{3}x^{2}-6\,(b^{2}d^{2}fx+b^{2}cdf)\cos\left(fx+e\right)^{2}+3\,(2\,(2\,a^{2}+b^{2})c^{2}f^{3}+b^{2}d^{2}f)x-24\,(abd^{2}f^{2}x^{2}+2\,abcdf^{2}x+abc^{2}f^{2}-2\,abd^{2})\cos\left(fx+e\right)+3\,(16\,abd^{2}fx+16\,abcdf-(2\,b^{2}d^{2}f^{2}x^{2}+4\,b^{2}cdf^{2}x+2\,b^{2}c^{2}f^{2}-b^{2}d^{2})\cos\left(fx+e\right))\sin\left(fx+e\right)}{12\,f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*a^2 + b^2)*d^2*f^3*x^3 + 6*(2*a^2 + b^2)*c*d*f^3*x^2 - 6*(b^2*d^
2*f*x + b^2*c*d*f)*cos(f*x + e)^2 + 3*(2*(2*a^2 + b^2)*c^2*f^3 + b^2*d^2*f)
*x - 24*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2)*cos(f
*x + e) + 3*(16*a*b*d^2*f*x + 16*a*b*c*d*f - (2*b^2*d^2*f^2*x^2 + 4*b^2*c*d
*f^2*x + 2*b^2*c^2*f^2 - b^2*d^2)*cos(f*x + e))*sin(f*x + e))/f^3
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(177) = 354$.
time = 0.29, size = 456, normalized size = 2.51

$$\begin{cases} a^{2}c^{2}x+a^{2}cdx^{2}+\frac{a^{2}d^{2}x^{3}}{3}-\frac{2abc^{2}\cos\left(e+fx\right)}{f}-\frac{4abcdx\cos\left(e+fx\right)}{f}+\frac{4abcd\sin\left(e+fx\right)}{f^{2}}-\frac{2abd^{2}x^{2}\cos\left(e+fx\right)}{f}+\frac{4abd^{2}x\sin\left(e+fx\right)}{f^{2}}+\frac{4abd^{2}\cos\left(e+fx\right)}{f^{3}}+\frac{b^{2}c^{2}x\sin^{2}\left(e+fx\right)}{2}+\frac{b^{2}c^{2}x\cos^{2}\left(e+fx\right)}{2}-\frac{b^{2}c^{2}\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{b^{2}cdx^{2}\sin^{2}\left(e+fx\right)}{2}+\frac{b^{2}cdx^{2}\cos^{2}\left(e+fx\right)}{2}-\frac{b^{2}cdx\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{b^{2}cd\sin^{2}\left(e+fx\right)}{2f^{2}}+\frac{b^{2}d^{2}x^{3}\sin^{2}\left(e+fx\right)}{6}+\frac{b^{2}d^{2}x^{3}\cos^{2}\left(e+fx\right)}{6}-\frac{b^{2}d^{2}x^{2}\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{b^{2}d^{2}x\sin^{2}\left(e+fx\right)}{4f^{2}}-\frac{b^{2}d^{2}x\cos^{2}\left(e+fx\right)}{4f^{2}}+\frac{b^{2}d^{2}\sin\left(e+fx\right)\cos\left(e+fx\right)}{4f^{3}} & \text{for}\: f \neq 0 \\ \left(a+b\sin\left(e\right)\right)^{2}\left(c^{2}x+cdx^{2}+\frac{d^{2}x^{3}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 - 2*a*b*c**2*cos(
e + f*x)/f - 4*a*b*c*d*x*cos(e + f*x)/f + 4*a*b*c*d*sin(e + f*x)/f**2 - 2*a
*b*d**2*x**2*cos(e + f*x)/f + 4*a*b*d**2*x*sin(e + f*x)/f**2 + 4*a*b*d**2*c
os(e + f*x)/f**3 + b**2*c**2*x*sin(e + f*x)**2/2 + b**2*c**2*x*cos(e + f*x)
**2/2 - b**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + b**2*c*d*x**2*sin(e + f
*x)**2/2 + b**2*c*d*x**2*cos(e + f*x)**2/2 - b**2*c*d*x*sin(e + f*x)*cos(e
+ f*x)/f + b**2*c*d*sin(e + f*x)**2/(2*f**2) + b**2*d**2*x**3*sin(e + f*x)*
*2/6 + b**2*d**2*x**3*cos(e + f*x)**2/6 - b**2*d**2*x**2*sin(e + f*x)*cos(e
 + f*x)/(2*f) + b**2*d**2*x*sin(e + f*x)**2/(4*f**2) - b**2*d**2*x*cos(e +
f*x)**2/(4*f**2) + b**2*d**2*sin(e + f*x)*cos(e + f*x)/(4*f**3), Ne(f, 0)),
 ((a + b*sin(e))**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

**Giac [A]**
time = 4.83, size = 229, normalized size = 1.26

$$\frac{1}{3}\,a^{2}d^{2}x^{3}+\frac{1}{6}\,b^{2}d^{2}x^{3}+a^{2}cdx^{2}+\frac{1}{2}\,b^{2}cdx^{2}+a^{2}c^{2}x+\frac{1}{2}\,b^{2}c^{2}x-\frac{(b^{2}d^{2}fx+b^{2}cdf)\cos\left(2\,fx+2\,e\right)}{4\,f^{3}}-\frac{2\,(abd^{2}f^{2}x^{2}+2\,abcdf^{2}x+abc^{2}f^{2}-2\,abd^{2})\cos\left(fx+e\right)}{f^{3}}-\frac{(2\,b^{2}d^{2}f^{2}x^{2}+4\,b^{2}cdf^{2}x+2\,b^{2}c^{2}f^{2}-b^{2}d^{2})\sin\left(2\,fx+2\,e\right)}{8\,f^{3}}+\frac{4\,(abd^{2}fx+abcdf)\sin\left(fx+e\right)}{f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*d^2*x^3 + 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 + 1/2*b^2*c*d*x^2 + a^2*c^2
*x + 1/2*b^2*c^2*x - 1/4*(b^2*d^2*f*x + b^2*c*d*f)*cos(2*f*x + 2*e)/f^3 - 2
*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2)*cos(f*x + e)
/f^3 - 1/8*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 - b^2*d^2)*
sin(2*f*x + 2*e)/f^3 + 4*(a*b*d^2*f*x + a*b*c*d*f)*sin(f*x + e)/f^3
```

**Mupad [B]**

time = 1.15, size = 281, normalized size = 1.54

$$a^2\,c^2\,x+\frac{b^2\,c^2\,x}{2}+\frac{a^2\,d^2\,x^3}{3}+\frac{b^2\,d^2\,x^3}{6}-\frac{b^2\,c^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{b^2\,d^2\,\sin\left(2\,e+2\,f\,x\right)}{8\,f^3}+a^2\,c\,d\,x^2+\frac{b^2\,c\,d\,x^2}{2}-\frac{2\,a\,b\,c^2\,\cos\left(e+f\,x\right)}{f}+\frac{4\,a\,b\,d^2\,\cos\left(e+f\,x\right)}{f^3}-\frac{b^2\,d^2\,x^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{b^2\,c\,d\,\cos\left(2\,e+2\,f\,x\right)}{4\,f^2}-\frac{b^2\,d^2\,x\,\cos\left(2\,e+2\,f\,x\right)}{4\,f^2}+\frac{4\,a\,b\,c\,d\,\sin\left(e+f\,x\right)}{f^2}+\frac{4\,a\,b\,d^2\,x\,\sin\left(e+f\,x\right)}{f^2}-\frac{2\,a\,b\,d^2\,x^2\,\cos\left(e+f\,x\right)}{f}-\frac{b^2\,c\,d\,x\,\sin\left(2\,e+2\,f\,x\right)}{2\,f}-\frac{4\,a\,b\,c\,d\,x\,\cos\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*x)^2,x)
```

```
[Out] a^2*c^2*x + (b^2*c^2*x)/2 + (a^2*d^2*x^3)/3 + (b^2*d^2*x^3)/6 - (b^2*c^2*si
n(2*e + 2*f*x))/(4*f) + (b^2*d^2*sin(2*e + 2*f*x))/(8*f^3) + a^2*c*d*x^2 +
(b^2*c*d*x^2)/2 - (2*a*b*c^2*cos(e + f*x))/f + (4*a*b*d^2*cos(e + f*x))/f^3
 - (b^2*d^2*x^2*sin(2*e + 2*f*x))/(4*f) - (b^2*c*d*cos(2*e + 2*f*x))/(4*f^2
) - (b^2*d^2*x*cos(2*e + 2*f*x))/(4*f^2) + (4*a*b*c*d*sin(e + f*x))/f^2 + (
4*a*b*d^2*x*sin(e + f*x))/f^2 - (2*a*b*d^2*x^2*cos(e + f*x))/f - (b^2*c*d*x
*sin(2*e + 2*f*x))/(2*f) - (4*a*b*c*d*x*cos(e + f*x))/f
```

# 3.159 $\int (c+dx)(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=116

$$\frac{1}{2}b^2cx+\frac{1}{4}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\cos(e+fx)}{f}+\frac{2abd\sin(e+fx)}{f^2}-\frac{b^2(c+dx)\cos(e+fx)\sin(e+f}{2f}$$

```
[Out] 1/2*b^2*c*x+1/4*b^2*d*x^2+1/2*a^2*(d*x+c)^2/d-2*a*b*(d*x+c)*cos(f*x+e)/f+2*
a*b*d*sin(f*x+e)/f^2-1/2*b^2*(d*x+c)*cos(f*x+e)*sin(f*x+e)/f+1/4*b^2*d*sin(
f*x+e)^2/f^2
```

**Rubi** [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3377, 2717, 3391}

$$\frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\cos(e+fx)}{f}+\frac{2abd\sin(e+fx)}{f^2}-\frac{b^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}b^2cx+\frac{b^2d\sin^2(e+fx)}{4f^2}+\frac{1}{4}b^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (b^2*c*x)/2 + (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a*b*(c + d*x)*Co
s[e + f*x])/f + (2*a*b*d*Sin[e + f*x])/f^2 - (b^2*(c + d*x)*Cos[e + f*x]*Si
n[e + f*x])/(2*f) + (b^2*d*Sin[e + f*x]^2)/(4*f^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)(a+b\sin(e+fx))^2\,dx = \int \left(a^2(c+dx)+2ab(c+dx)\sin(e+fx)+b^2(c+dx)\sin^2(e+fx)\right)\,dx$$
$$= \frac{a^2(c+dx)^2}{2d}+(2ab)\int (c+dx)\sin(e+fx)\,dx+b^2\int (c+dx)\sin^2(e+$$
$$= \frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\cos(e+fx)}{f}-\frac{b^2(c+dx)\cos(e+fx)\sin(e}{2f}$$
$$= \frac{1}{2}b^2cx+\frac{1}{4}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\cos(e+fx)}{f}+\frac{2abd\sin(e}{f^2}$$

**Mathematica [A]**
time = 0.37, size = 96, normalized size = 0.83

$$-\frac{2(2a^2+b^2)(e+fx)(-2cf+d(e-fx))+16abf(c+dx)\cos(e+fx)+b^2d\cos(2(e+fx))-16abd\sin(e+fx)+2b^2f(c+dx)\sin(2(e+fx))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Sin[e + f*x])^2,x]
[Out] -1/8*(2*(2*a^2 + b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*a*b*f*(c + d*x)
*Cos[e + f*x] + b^2*d*Cos[2*(e + f*x)] - 16*a*b*d*Sin[e + f*x] + 2*b^2*f*(c
 + d*x)*Sin[2*(e + f*x)])/f^2
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(106) = 212.
time = 0.06, size = 216, normalized size = 1.86

| method | result |
|---|---|
| risch | $\frac{a^2dx^2}{2}+a^2cx+\frac{b^2dx^2}{4}+\frac{b^2cx}{2}-\frac{2ab(dx+c)\cos(fx+e)}{f}+\frac{2abd\sin(fx+e)}{f^2}-\frac{b^2d\cos(2fx+2e)}{8f^2}-\frac{b^2(dx+c)}{}$ |
| derivativedivides | $a^2c(fx+e)-\frac{a^2de(fx+e)}{f}+\frac{a^2d(fx+e)^2}{2f}-2abc\cos(fx+e)+\frac{2abde\cos(fx+e)}{f}+\frac{2abd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}+b^2c\left(-\frac{\sin(}{}\right.$ |
| default | $a^2c(fx+e)-\frac{a^2de(fx+e)}{f}+\frac{a^2d(fx+e)^2}{2f}-2abc\cos(fx+e)+\frac{2abde\cos(fx+e)}{f}+\frac{2abd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}+b^2c\left(-\frac{\sin(}{}\right.$ |

| | |
|---|---|
| norman | $\frac{\left(\frac{1}{2}a^2d+\frac{1}{4}b^2d\right)x^2+\left(a^2d+\frac{1}{2}b^2d\right)x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{1}{2}a^2d+\frac{1}{4}b^2d\right)x^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{b(-bcf+4da)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f^2}+\frac{b(bcf+4}{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c*(f*x+e)-a^2/f*d*e*(f*x+e)+1/2*a^2/f*d*(f*x+e)^2-2*a*b*c*cos(f*x+
e)+2/f*a*b*d*e*cos(f*x+e)+2/f*a*b*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e))+b^2*c*(
-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/f*b^2*d*e*(-1/2*sin(f*x+e)*cos(
f*x+e)+1/2*f*x+1/2*e)+1/f*b^2*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*
x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2))
```

**Maxima** [A]
time = 0.34, size = 221, normalized size = 1.91

$$\frac{8\,(fx+e)a^2c+2\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))b^2c+\frac{4\,(fx+e)^2a^2d}{f}-16\,abc\cos\left(fx+e\right)-\frac{8\,(fx+e)a^2de}{f}-\frac{2\,(2\,fx+2\,e-\sin(2\,fx+2\,e))b^2de}{f}+\frac{16\,abd\cos(fx+e)e}{f}-\frac{16\,((fx+e)\cos(fx+e)-\sin(fx+e))abd}{f}+\frac{\left(2\,(fx+e)^2-2\,(fx+e)\sin(2\,fx+2\,e)-\cos(2\,fx+2\,e)\right)b^2d}{f}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(f*x + e)*a^2*c + 2*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c + 4*(f*x
+ e)^2*a^2*d/f - 16*a*b*c*cos(f*x + e) - 8*(f*x + e)*a^2*d*e/f - 2*(2*f*x +
 2*e - sin(2*f*x + 2*e))*b^2*d*e/f + 16*a*b*d*cos(f*x + e)*e/f - 16*((f*x +
 e)*cos(f*x + e) - sin(f*x + e))*a*b*d/f + (2*(f*x + e)^2 - 2*(f*x + e)*sin
(2*f*x + 2*e) - cos(2*f*x + 2*e))*b^2*d/f)/f
```

**Fricas** [A]
time = 0.37, size = 113, normalized size = 0.97

$$\frac{(2\,a^2+b^2)df^2x^2+2\,(2\,a^2+b^2)cf^2x-b^2d\cos\left(fx+e\right)^2-8\,(abdfx+abcf)\cos\left(fx+e\right)+2\,(4\,abd-(b^2dfx+b^2cf)\cos\left(fx+e\right))\sin\left(fx+e\right)}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*d*f^2*x^2 + 2*(2*a^2 + b^2)*c*f^2*x - b^2*d*cos(f*x + e)
^2 - 8*(a*b*d*f*x + a*b*c*f)*cos(f*x + e) + 2*(4*a*b*d - (b^2*d*f*x + b^2*c
*f)*cos(f*x + e))*sin(f*x + e))/f^2
```

**Sympy** [A]
time = 0.16, size = 219, normalized size = 1.89

$$\begin{cases} a^2cx+\frac{a^2dx^2}{2}-\frac{2abc\cos\left(e+fx\right)}{f}-\frac{2abdx\cos\left(e+fx\right)}{f}+\frac{2abd\sin\left(e+fx\right)}{f^2}+\frac{b^2cx\sin^2\left(e+fx\right)}{2}+\frac{b^2cx\cos^2\left(e+fx\right)}{2}-\frac{b^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{b^2dx^2\sin^2\left(e+fx\right)}{4}+\frac{b^2dx^2\cos^2\left(e+fx\right)}{4}-\frac{b^2dx\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{b^2d\sin^2\left(e+fx\right)}{4f^2} & \text{for}\ f\neq 0 \\ \left(a+b\sin\left(e\right)\right)^2\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*x**2/2 - 2*a*b*c*cos(e + f*x)/f - 2*a*b*d*x*co
s(e + f*x)/f + 2*a*b*d*sin(e + f*x)/f**2 + b**2*c*x*sin(e + f*x)**2/2 + b**
2*c*x*cos(e + f*x)**2/2 - b**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + b**2*d*x
**2*sin(e + f*x)**2/4 + b**2*d*x**2*cos(e + f*x)**2/4 - b**2*d*x*sin(e + f*
x)*cos(e + f*x)/(2*f) + b**2*d*sin(e + f*x)**2/(4*f**2), Ne(f, 0)), ((a + b
*sin(e))**2*(c*x + d*x**2/2), True))
```

**Giac [A]**
time = 4.86, size = 119, normalized size = 1.03

$$\frac{1}{2}a^2dx^2 + \frac{1}{4}b^2dx^2 + a^2cx + \frac{1}{2}b^2cx - \frac{b^2d\cos(2fx+2e)}{8f^2} + \frac{2abd\sin(fx+e)}{f^2} - \frac{2(abdfx+abcf)\cos(fx+e)}{f^2} - \frac{(b^2dfx+b^2cf)\sin(2fx+2e)}{4f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d*x^2 + 1/4*b^2*d*x^2 + a^2*c*x + 1/2*b^2*c*x - 1/8*b^2*d*cos(2*f*x
 + 2*e)/f^2 + 2*a*b*d*sin(f*x + e)/f^2 - 2*(a*b*d*f*x + a*b*c*f)*cos(f*x +
e)/f^2 - 1/4*(b^2*d*f*x + b^2*c*f)*sin(2*f*x + 2*e)/f^2
```

**Mupad [B]**
time = 0.74, size = 143, normalized size = 1.23

$$\frac{a^2\,d\,x^2}{2} + \frac{b^2\,d\,x^2}{4} + a^2\,c\,x + \frac{b^2\,c\,x}{2} - \frac{b^2\,c\,\sin(2\,e+2\,f\,x)}{4\,f} + \frac{b^2\,d\,\sin(e+f\,x)^2}{4\,f^2} + \frac{4\,a\,b\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{f} - \frac{b^2\,d\,x\,\sin(2\,e+2\,f\,x)}{4\,f} + \frac{2\,a\,b\,d\,\sin(e+f\,x)}{f^2} + \frac{2\,a\,b\,d\,x\,\left(2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*x),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*d*x^2)/4 + a^2*c*x + (b^2*c*x)/2 - (b^2*c*sin(2*e + 2*
f*x))/(4*f) + (b^2*d*sin(e + f*x)^2)/(4*f^2) + (4*a*b*c*sin(e/2 + (f*x)/2)^
2)/f - (b^2*d*x*sin(2*e + 2*f*x))/(4*f) + (2*a*b*d*sin(e + f*x))/f^2 + (2*a
*b*d*x*(2*sin(e/2 + (f*x)/2)^2 - 1))/f
```

# 3.160 $\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx$

**Optimal**. Leaf size=156

$$-\frac{b^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cos\left(e\right.}{}$$

```
[Out] -1/2*b^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/d+a^2*ln(d*x+c)/d+1/2*b^2*ln(d
*x+c)/d+2*a*b*cos(-e+c*f/d)*Si(c*f/d+f*x)/d-1/2*b^2*Si(2*c*f/d+2*f*x)*sin(-
2*e+2*c*f/d)/d-2*a*b*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d
```

**Rubi** [A]
time = 0.21, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3398, 3384, 3380, 3383, 3393}

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}-\frac{b^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{b^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{b^2\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*x),x]
```

```
[Out] -1/2*(b^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d + (a^2*Log
[c + d*x])/d + (b^2*Log[c + d*x])/(2*d) + (2*a*b*CosIntegral[(c*f)/d + f*x]
*Sin[e - (c*f)/d])/d + (2*a*b*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/
d + (b^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx &= \int \left(\frac{a^2}{c+dx}+\frac{2ab\sin(e+fx)}{c+dx}+\frac{b^2\sin^2(e+fx)}{c+dx}\right)dx\\
&= \frac{a^2\log(c+dx)}{d}+(2ab)\int \frac{\sin(e+fx)}{c+dx}\,dx+b^2\int \frac{\sin^2(e+fx)}{c+dx}\,dx\\
&= \frac{a^2\log(c+dx)}{d}+b^2\int \left(\frac{1}{2(c+dx)}-\frac{\cos(2e+2fx)}{2(c+dx)}\right)dx+\left(2ab\cos\left(e-\frac{cf}{d}\right.\right.\\
&= \frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cos\left(e-\right.}{}\\
&= \frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cos\left(e-\right.}{}\\
&= -\frac{b^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Ci}(}{}
\end{aligned}$$

**Mathematica [A]**
time = 0.16, size = 134, normalized size = 0.86

$$\frac{-b^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2f(c+dx)}{d}\right)+2a^2\log(c+dx)+b^2\log(c+dx)+4ab\text{Ci}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+4ab\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+b^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*x),x]
```

```
[Out] (-(b^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*f*(c + d*x))/d]) + 2*a^2*Log[c +
 d*x] + b^2*Log[c + d*x] + 4*a*b*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d]
+ 4*a*b*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + b^2*Sin[2*e - (2*c*f)/d
]*SinIntegral[(2*f*(c + d*x))/d])/(2*d)
```

**Maple [A]**
time = 0.06, size = 221, normalized size = 1.42

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a^{2} f \ln(cf-de+d(fx+e))}{d}+2 f a b\left(\frac{\mathrm{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{f b^{2} \ln(c}{}}{f}$ |
| default | $\frac{\frac{a^{2} f \ln(cf-de+d(fx+e))}{d}+2 f a b\left(\frac{\mathrm{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{f b^{2} \ln(c}{}}{f}$ |
| risch | $-\frac{i a b\, \mathrm{e}^{\frac{i(cf-de)}{d}} \mathrm{expIntegral}\left(1, i f x+i e+\frac{i(cf-de)}{d}\right)}{d}+\frac{a^{2} \ln(dx+c)}{d}+\frac{b^{2} \ln(dx+c)}{2d}+\frac{b^{2} \mathrm{e}^{\frac{2i(cf-de)}{d}} \mathrm{expIntegral}\left(1, 2 i f x\right.}{4d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*f*ln(c*f-d*e+d*(f*x+e))/d+2*f*a*b*(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-
d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)+1/2*f*b^2*ln(c*f-d*e+d*
(f*x+e))/d-1/4*f*b^2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*
Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.40, size = 358, normalized size = 2.29

$$\frac{\frac{4 a^{2} f \log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{4\left(f\left(-i E_{1}\left(\frac{i(fx+e)d+icf-ide}{d}\right)+i E_{1}\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f\left(E_{1}\left(\frac{i(fx+e)d+icf-ide}{d}\right)+E_{1}\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)ab}{d}+\frac{\left(f\left(E_{1}\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+E_{1}\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\cos\left(\frac{2(cf-de)}{d}\right)+f\left(-i E_{1}\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+i E_{1}\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\sin\left(\frac{2(cf-de)}{d}\right)+2 f \log((fx+e)d+cf-de)\right)b^{2}}{d}}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d + 4*(f*(-I*exp_integral_e(1,
(I*(f*x + e)*d + I*c*f - I*d*e)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d +
I*c*f - I*d*e)/d))*cos((c*f - d*e)/d) + f*(exp_integral_e(1, (I*(f*x + e)*d
 + I*c*f - I*d*e)/d) + exp_integral_e(1, -(I*(f*x + e)*d + I*c*f - I*d*e)/d
))*sin((c*f - d*e)/d))*a*b/d + (f*(exp_integral_e(1, 2*(-I*(f*x + e)*d - I*
c*f + I*d*e)/d) + exp_integral_e(1, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d))
*cos(2*(c*f - d*e)/d) + f*(-I*exp_integral_e(1, 2*(-I*(f*x + e)*d - I*c*f +
 I*d*e)/d) + I*exp_integral_e(1, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d))*si
n(2*(c*f - d*e)/d) + 2*f*log((f*x + e)*d + c*f - d*e))*b^2/d)/f
```

**Fricas [A]**
time = 0.37, size = 194, normalized size = 1.24

$$\frac{2 b^{2} \sin\left(-\frac{2(cf-de)}{d}\right) \mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right)+8 a b \cos\left(-\frac{cf-de}{d}\right) \mathrm{Si}\left(\frac{dfx+cf}{d}\right)-\left(b^{2} \mathrm{Ci}\left(\frac{2(dfx+cf)}{d}\right)+b^{2} \mathrm{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\right)\cos\left(-\frac{2(cf-de)}{d}\right)+2\left(2 a^{2}+b^{2}\right)\log(dx+c)+4\left(ab \mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+ab \mathrm{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*sin(-2*(c*f - d*e)/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*a*b*co
s(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) - (b^2*cos_integral(2*(d*f*
x + c*f)/d) + b^2*cos_integral(-2*(d*f*x + c*f)/d))*cos(-2*(c*f - d*e)/d) +
 2*(2*a^2 + b^2)*log(d*x + c) + 4*(a*b*cos_integral((d*f*x + c*f)/d) + a*b*
cos_integral(-(d*f*x + c*f)/d))*sin(-(c*f - d*e)/d))/d
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(d*x+c),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2/(c + d*x), x)
```

**Giac [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.90, size = 7397, normalized size = 47.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(4*a*b*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 - 4*a*b*imag_part(cos_integral(-f*x - c*f/d))*tan(
c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*log(abs(d*x + c))*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 2*b^2*log(abs(d*x + c)
)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - b^2*real_part(cos_i
ntegral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)
^2 - b^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f
/d)^2*tan(1/2*e)^2*tan(e)^2 + 8*a*b*sin_integral((d*f*x + c*f)/d)*tan(c*f/d
)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*b^2*imag_part(cos_integral(2
*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 2*b^2*
imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan
(1/2*e)^2*tan(e) - 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1
/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 8*a*b*real_part(cos_integral(f*x + c*f/d)
)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a*b*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 +
8*a*b*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(
1/2*e)^2*tan(e)^2 + 8*a*b*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^
2*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 2*b^2*imag_part(cos_integral(2*f*x
 + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*b^2*imag
_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e
)^2*tan(e)^2 + 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f
```

/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a*b*imag_part(cos_integral(f*x + c*f/d))*ta
n(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*a*b*imag_part(cos_integral(-f*
x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*log(abs(d*x
+ c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b^2*log(abs(d*x + c))*
tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b^2*real_part(cos_integral(2*f
*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b^2*real_part(c
os_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 +
 8*a*b*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*
e)^2 - 4*b^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*
f/d)^2*tan(1/2*e)^2*tan(e) - 4*b^2*real_part(cos_integral(-2*f*x - 2*c*f/d)
)*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a*b*imag_part(cos_int
egral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 4*a*b*imag_par
t(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 4*a^
2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 2*b^2*log(abs(
d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - b^2*real_part(cos_integr
al(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - b^2*real_part
(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - 8
*a*b*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 +
 16*a*b*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*ta
n(1/2*e)*tan(e)^2 - 16*a*b*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)
^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 32*a*b*sin_integral((d*f*x + c*f)/d
)*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 4*a*b*imag_part(cos_int
egral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a*b*imag_part(co
s_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*log(ab
s(d*x + c))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 2*b^2*log(abs(d*x + c))*ta
n(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - b^2*real_part(cos_integral(2*f*x + 2*c*f
/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - b^2*real_part(cos_integral(-2*f*x
 - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 8*a*b*sin_integral((d*f*x
 + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a*b*imag_part(cos_integra
l(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 4*a*b*imag_part(co
s_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*lo
g(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 2*b^2*log(abs(d*x
+ c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + b^2*real_part(cos_integral(2
*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + b^2*real_part(cos
_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 8*a*b
*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 8*a
*b*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1
/2*e) - 8*a*b*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*
f/d)^2*tan(1/2*e) + 8*a*b*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2
*tan(1/2*c*f/d)*tan(1/2*e)^2 + 8*a*b*real_part(cos_integral(-f*x - c*f/d))*
tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 - 2*b^2*imag_part(cos_integral(2*f
*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b^2*imag_part(c
os_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4
*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)
^2 - 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*

```
f/d)^2*tan(e) + 2*b^2*imag_part(cos_integral(-2...
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*x),x)
```

```
[Out] int((a + b*sin(e + f*x))^2/(c + d*x), x)
```

# 3.161 $\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=183

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}+\frac{b^2f\text{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{2ab\sin(e+fx)}{d(c+dx)}-\frac{b^2\sin^2(e+fx)}{d(c+dx)}$$

```
[Out] -a^2/d/(d*x+c)+2*a*b*f*Ci(c*f/d+f*x)*cos(-e+c*f/d)/d^2+b^2*f*cos(-2*e+2*c*f
/d)*Si(2*c*f/d+2*f*x)/d^2-b^2*f*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/d^2+2*a
*b*f*Si(c*f/d+f*x)*sin(-e+c*f/d)/d^2-2*a*b*sin(f*x+e)/d/(d*x+c)-b^2*sin(f*x
+e)^2/d/(d*x+c)
```

**Rubi** [A]
time = 0.23, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3398, 3378, 3384, 3380, 3383, 3394, 12}

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2abf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\sin(e+fx)}{d(c+dx)}+\frac{b^2f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{b^2f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{b^2\sin^2(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] -(a^2/(d*(c + d*x))) + (2*a*b*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x]
)/d^2 + (b^2*f*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (
2*a*b*Sin[e + f*x])/(d*(c + d*x)) - (b^2*Sin[e + f*x]^2)/(d*(c + d*x)) - (2
*a*b*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Cos[2*e -
(2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx &= \int \left(\frac{a^2}{(c+dx)^2}+\frac{2ab\sin(e+fx)}{(c+dx)^2}+\frac{b^2\sin^2(e+fx)}{(c+dx)^2}\right)dx \\
&= -\frac{a^2}{d(c+dx)}+(2ab)\int \frac{\sin(e+fx)}{(c+dx)^2}\,dx+b^2\int \frac{\sin^2(e+fx)}{(c+dx)^2}\,dx \\
&= -\frac{a^2}{d(c+dx)}-\frac{2ab\sin(e+fx)}{d(c+dx)}-\frac{b^2\sin^2(e+fx)}{d(c+dx)}+\frac{(2abf)\int \frac{\cos(e+fx)}{c+dx}\,dx}{d}+\frac{(2b
\\
&= -\frac{a^2}{d(c+dx)}-\frac{2ab\sin(e+fx)}{d(c+dx)}-\frac{b^2\sin^2(e+fx)}{d(c+dx)}+\frac{(b^2f)\int \frac{\sin(2e+2fx)}{c+dx}\,dx}{d}+\frac{(2
\\
&= -\frac{a^2}{d(c+dx)}+\frac{2abf\cos\left(e-\frac{cf}{d}\right)\operatorname{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{2ab\sin(e+fx)}{d(c+dx)}-\frac{b^2\sin^2(e+
\\
&= -\frac{a^2}{d(c+dx)}+\frac{2abf\cos\left(e-\frac{cf}{d}\right)\operatorname{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}+\frac{b^2f\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2c
\end{aligned}$$

**Mathematica [A]**
time = 0.37, size = 232, normalized size = 1.27

$$\frac{-2a^2d-b^2d+b^2d\cos(2(e+fx))+4abf(c+dx)\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(f\left(\frac{c}{d}+x\right)\right)+2b^2f(c+dx)\text{Ci}\left(\frac{2f(c+dx)}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)-4abd\sin(e+fx)-4abcf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)-4abdfx\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+2b^2cf\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)+2b^2dfx\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)}{2d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-2*a^2*d - b^2*d + b^2*d*Cos[2*(e + f*x)] + 4*a*b*f*(c + d*x)*Cos[e - (c*f
)/d]*CosIntegral[f*(c/d + x)] + 2*b^2*f*(c + d*x)*CosIntegral[(2*f*(c + d*x
))/d]*Sin[2*e - (2*c*f)/d] - 4*a*b*d*Sin[e + f*x] - 4*a*b*c*f*Sin[e - (c*f)
/d]*SinIntegral[f*(c/d + x)] - 4*a*b*d*f*x*Sin[e - (c*f)/d]*SinIntegral[f*(
c/d + x)] + 2*b^2*c*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] +
 2*b^2*d*f*x*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d])/(2*d^2*(c
 + d*x))
```

**Maple [A]**
time = 0.10, size = 301, normalized size = 1.64

| method | result |
|---|---|
| derivativedivides | $-\frac{a^2f^2}{(cf-de+d(fx+e))d}+2f^2ab\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\text{cos}}{d}}{d}\right.$ |
| default | $-\frac{a^2f^2}{(cf-de+d(fx+e))d}+2f^2ab\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(fx+e+\frac{cf-de}{d}\right)\text{cos}}{d}}{d}\right.$ |
| risch | $-\frac{fab\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\text{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{d^2}-\frac{a^2}{d(dx+c)}-\frac{b^2}{2d(dx+c)}-\frac{ib^2f\,\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\text{expIntegral}\left(1,2ifx+\right.}{2d^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*f^2/(c*f-d*e+d*(f*x+e))/d+2*f^2*a*b*(-sin(f*x+e)/(c*f-d*e+d*(f*x+
e))/d+(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*cos((
c*f-d*e)/d)/d)/d)-1/2*f^2*b^2/(c*f-d*e+d*(f*x+e))/d-1/4*f^2*b^2*(-2*cos(2*f
*x+2*e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d
*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.50, size = 395, normalized size = 2.16

$$-\frac{\frac{4\,a^2f^2}{(fx+e)d^2+cdf-d^2e}-\frac{4\left(f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+i\,E_2\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f^2\left(E_2\left(\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)+E_2\left(-\frac{i\,(fx+e)d+i\,cf-i\,de}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)ab}{(fx+e)d^2+cdf-d^2e}-\frac{\left(f^2\left(E_2\left(\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+E_2\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\right)\cos\left(\frac{2\,(cf-de)}{d}\right)-f^2\left(i\,E_2\left(\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)-i\,E_2\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\right)\sin\left(\frac{2\,(cf-de)}{d}\right)-2\,f^2\right)b^2}{(fx+e)d^2+cdf-d^2e}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^2*f^2/((f*x + e)*d^2 + c*d*f - d^2*e) - 4*(f^2*(-I*exp_integral_e
(2, (I*(f*x + e)*d + I*c*f - I*d*e)/d) + I*exp_integral_e(2, -(I*(f*x + e)*
d + I*c*f - I*d*e)/d))*cos((c*f - d*e)/d) + f^2*(exp_integral_e(2, (I*(f*x
+ e)*d + I*c*f - I*d*e)/d) + exp_integral_e(2, -(I*(f*x + e)*d + I*c*f - I*
d*e)/d))*sin((c*f - d*e)/d))*a*b/((f*x + e)*d^2 + c*d*f - d^2*e) - (f^2*(ex
p_integral_e(2, 2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d) + exp_integral_e(2, -
2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d))*cos(2*(c*f - d*e)/d) - f^2*(I*exp_in
tegral_e(2, 2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d) - I*exp_integral_e(2, -2*
(-I*(f*x + e)*d - I*c*f + I*d*e)/d))*sin(2*(c*f - d*e)/d) - 2*f^2)*b^2/((f*
x + e)*d^2 + c*d*f - d^2*e))/f
```

**Fricas** [**A**]
time = 0.37, size = 286, normalized size = 1.56

$$\frac{2\,b^2d\cos\left(fx+e\right)^2-4\,abd\sin\left(fx+e\right)+2\,\left(b^2dfx+b^2cf\right)\cos\left(-\frac{2\,(cf-de)}{d}\right)\operatorname{Si}\left(\frac{2\,(dfx+cf)}{d}\right)-4\,\left(abdfx+abcf\right)\sin\left(-\frac{cf-de}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)-2\,\left(a^2+b^2\right)d+2\,\left(\left(abdfx+abcf\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+\left(abdfx+abcf\right)\operatorname{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\cos\left(-\frac{cf-de}{d}\right)+\left(\left(b^2dfx+b^2cf\right)\operatorname{Ci}\left(\frac{2\,(dfx+cf)}{d}\right)+\left(b^2dfx+b^2cf\right)\operatorname{Ci}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\sin\left(-\frac{2\,(cf-de)}{d}\right)}{2\,\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d*cos(f*x + e)^2 - 4*a*b*d*sin(f*x + e) + 2*(b^2*d*f*x + b^2*c*f
)*cos(-2*(c*f - d*e)/d)*sin_integral(2*(d*f*x + c*f)/d) - 4*(a*b*d*f*x + a*
b*c*f)*sin(-(c*f - d*e)/d)*sin_integral((d*f*x + c*f)/d) - 2*(a^2 + b^2)*d
+ 2*((a*b*d*f*x + a*b*c*f)*cos_integral((d*f*x + c*f)/d) + (a*b*d*f*x + a*b
*c*f)*cos_integral(-(d*f*x + c*f)/d))*cos(-(c*f - d*e)/d) + ((b^2*d*f*x + b
^2*c*f)*cos_integral(2*(d*f*x + c*f)/d) + (b^2*d*f*x + b^2*c*f)*cos_integra
l(-2*(d*f*x + c*f)/d))*sin(-2*(c*f - d*e)/d))/(d^3*x + c*d^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^2}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2/(c + d*x)**2, x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(192) = 384$.
time = 4.29, size = 1135, normalized size = 6.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*x + c)*a*b*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos((c*f - d*e
)/d)*cos_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d
*e)/d) - 4*a*b*c*f^3*cos((c*f - d*e)/d)*cos_integral(-((d*x + c)*(c*f/(d*x
+ c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 4*a*b*d*f^2*cos((c*f - d*e)/d)*
cos_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d
)*e - 2*(d*x + c)*b^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos_integral(
-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*sin(2*(c*
f - d*e)/d) + 2*b^2*c*f^3*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d
*e/(d*x + c)) - c*f + d*e)/d)*sin(2*(c*f - d*e)/d) - 2*b^2*d*f^2*cos_integr
al(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e*sin(
2*(c*f - d*e)/d) + 4*(d*x + c)*a*b*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*
sin((c*f - d*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x +
 c)) - c*f + d*e)/d) - 4*a*b*c*f^3*sin((c*f - d*e)/d)*sin_integral(-((d*x +
 c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 4*a*b*d*f^2*e*sin
((c*f - d*e)/d)*sin_integral(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)
) - c*f + d*e)/d) + 2*(d*x + c)*b^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2
*cos(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(
d*x + c)) - c*f + d*e)/d) - 2*b^2*c*f^3*cos(2*(c*f - d*e)/d)*sin_integral(-
2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 2*b^2*d*
f^2*cos(2*(c*f - d*e)/d)*e*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f -
d*e/(d*x + c)) - c*f + d*e)/d) - b^2*d*f^2*cos(2*(d*x + c)*(c*f/(d*x + c) -
 f - d*e/(d*x + c))/d) - 4*a*b*d*f^2*sin((d*x + c)*(c*f/(d*x + c) - f - d*e
/(d*x + c))/d) + 2*a^2*d*f^2 + b^2*d*f^2)*d^2/(((d*x + c)*d^4*(c*f/(d*x + c
) - f - d*e/(d*x + c)) - c*d^4*f + d^5*e)*f)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x))^2/(c + d*x)^2, x)
```

# 3.162 $\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=245

$$-\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{d^3}-\frac{abf^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}-\frac{ab\sin}{d(c}$$

```
[Out] -1/2*a^2/d/(d*x+c)^2+b^2*f^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/d^3-a*b*f*
cos(f*x+e)/d^2/(d*x+c)-a*b*f^2*cos(-e+c*f/d)*Si(c*f/d+f*x)/d^3+b^2*f^2*Si(2
*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/d^3+a*b*f^2*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^3
-a*b*sin(f*x+e)/d/(d*x+c)^2-b^2*f*cos(f*x+e)*sin(f*x+e)/d^2/(d*x+c)-1/2*b^2
*sin(f*x+e)^2/d/(d*x+c)^2
```

**Rubi** [A]
time = 0.29, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3398, 3378, 3384, 3380, 3383, 3395, 31, 3393}

$$-\frac{a^2}{2d(c+dx)^2}-\frac{abf^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}-\frac{abf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\cos(e+fx)}{d^2(c+dx)}-\frac{ab\sin(e+fx)}{d(c+dx)^2}+\frac{b^2f^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{d^3}-\frac{b^2f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{d^3}-\frac{b^2f\sin(e+fx)\cos(e+fx)}{d^2(c+dx)}-\frac{b^2\sin^2(e+fx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -1/2*a^2/(d*(c + d*x)^2) - (a*b*f*Cos[e + f*x])/(d^2*(c + d*x)) + (b^2*f^2*
Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d^3 - (a*b*f^2*CosInte
gral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d^3 - (a*b*Sin[e + f*x])/(d*(c + d*x)
^2) - (b^2*f*Cos[e + f*x]*Sin[e + f*x])/(d^2*(c + d*x)) - (b^2*Sin[e + f*x]
^2)/(2*d*(c + d*x)^2) - (a*b*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x
])/d^3 - (b^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx &= \int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\sin(e+fx)}{(c+dx)^3}+\frac{b^2\sin^2(e+fx)}{(c+dx)^3}\right)\,dx \\
&= -\frac{a^2}{2d(c+dx)^2}+(2ab)\int \frac{\sin(e+fx)}{(c+dx)^3}\,dx+b^2\int \frac{\sin^2(e+fx)}{(c+dx)^3}\,dx \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\sin(e+fx)}{d(c+dx)^2}-\frac{b^2f\cos(e+fx)\sin(e+fx)}{d^2(c+dx)}-\frac{b^2\sin^2(e+f}{2d(c+dx)^{} } \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\log(c+dx)}{d^3}-\frac{ab\sin(e+fx)}{d(c+dx)^2}-\frac{b^2f\,c}{} \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}-\frac{ab\sin(e+fx)}{d(c+dx)^2}-\frac{b^2f\cos(e+fx)\sin(e+}{d^2(c+dx)} \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}-\frac{abf^2\operatorname{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}-\frac{ab\sin(e+}{d(c+d} \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{d^3}-\frac{abf^2}{}
\end{aligned}
$$

**Mathematica [A]**
time = 0.76, size = 395, normalized size = 1.61

$$
-\frac{2a^2d^2+b^2d^2+4abcdf\cos(e+fx)+4abd^2fx\cos(e+fx)-b^2d^2\cos(2(e+fx))-4b^2f^2(c+dx)^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2f(c+dx)}{d}\right)+4abf^2(c+dx)^2\operatorname{Ci}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+4abd^2\sin(e+fx)+2b^2cdf\sin(2(e+fx))+2b^2d^2fx\sin(2(e+fx))+4abc^2f^2\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)+8abcdf^2x\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)+4abd^2f^2x^2\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)+4b^2c^2f^2\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)+8b^2cdf^2x\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)+4b^2d^2f^2x^2\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)}{4d^3(c+dx)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -1/4*(2*a^2*d^2 + b^2*d^2 + 4*a*b*c*d*f*Cos[e + f*x] + 4*a*b*d^2*f*x*Cos[e
+ f*x] - b^2*d^2*Cos[2*(e + f*x)] - 4*b^2*f^2*(c + d*x)^2*Cos[2*e - (2*c*f)
/d]*CosIntegral[(2*f*(c + d*x))/d] + 4*a*b*f^2*(c + d*x)^2*CosIntegral[f*(c
/d + x)]*Sin[e - (c*f)/d] + 4*a*b*d^2*Sin[e + f*x] + 2*b^2*c*d*f*Sin[2*(e +
 f*x)] + 2*b^2*d^2*f*x*Sin[2*(e + f*x)] + 4*a*b*c^2*f^2*Cos[e - (c*f)/d]*Si
nIntegral[f*(c/d + x)] + 8*a*b*c*d*f^2*x*Cos[e - (c*f)/d]*SinIntegral[f*(c/
d + x)] + 4*a*b*d^2*f^2*x^2*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 4*b
^2*c^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 8*b^2*c*d*
f^2*x*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 4*b^2*d^2*f^2*x
^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d])/(d^3*(c + d*x)^2)
```

**Maple [A]**
time = 0.15, size = 374, normalized size = 1.53

| method | result |
|---|---|
| derivativedivides | $-\frac{a^2 f^3}{2(cf-de+d(fx+e))^2 d}+2f^3 ab\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\text{cosi}}{d}}{d}}{2d}\right.$ |
| default | $-\frac{a^2 f^3}{2(cf-de+d(fx+e))^2 d}+2f^3 ab\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\text{sinIntegral}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\text{cosi}}{d}}{d}}{2d}\right.$ |
| risch | $\frac{if^2 ab\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\text{expIntegral}\left(1,ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^3}-\frac{a^2}{2d(dx+c)^2}-\frac{b^2}{4d(dx+c)^2}-\frac{b^2 f^2\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\text{expIntegral}\left(1,2ifx\right.}{2d^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*a^2*f^3/(c*f-d*e+d*(f*x+e))^2/d+2*f^3*a*b*(-1/2*sin(f*x+e)/(c*f-d
*e+d*(f*x+e))^2/d+1/2*(-cos(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e
)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d)-1/4
*f^3*b^2/(c*f-d*e+d*(f*x+e))^2/d-1/4*f^3*b^2*(-cos(2*f*x+2*e)/(c*f-d*e+d*(f
*x+e))^2/d-(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d+2*(2*Si(2*f*x+2*e+2*(c*
f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e
)/d)/d)/d)/d))
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.51, size = 503, normalized size = 2.05

$$-\frac{\frac{2a^2f^3}{(fx+e)^2d^3+c^2df^2-2cd^2fe+d^3e^2+2(cd^2f-d^3e)(fx+e)}-\frac{4\left(f^3\left(-iE_3\left(\frac{i(fx+e)d+icf-ide}{d}\right)+iE_3\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\cos\left(\frac{cf-de}{d}\right)+f^3\left(E_3\left(\frac{i(fx+e)d+icf-ide}{d}\right)+E_3\left(-\frac{i(fx+e)d+icf-ide}{d}\right)\right)\sin\left(\frac{cf-de}{d}\right)\right)ab}{(fx+e)^2d^3+c^2df^2-2cd^2fe+d^3e^2+2(cd^2f-d^3e)(fx+e)}-\frac{\left(f^3\left(E_3\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+E_3\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\cos\left(\frac{2(cf-de)}{d}\right)-f^3\left(iE_3\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)-iE_3\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\right)\sin\left(\frac{2(cf-de)}{d}\right)-f^3\right)b^2}{(fx+e)^2d^3+c^2df^2-2cd^2fe+d^3e^2+2(cd^2f-d^3e)(fx+e)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*f^3/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*(c
*d^2*f - d^3*e)*(f*x + e)) - 4*(f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d +
I*c*f - I*d*e)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d + I*c*f - I*d*e)/d)
)*cos((c*f - d*e)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d + I*c*f - I*d*
e)/d) + exp_integral_e(3, -(I*(f*x + e)*d + I*c*f - I*d*e)/d))*sin((c*f - d
*e)/d))*a*b/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 + 2*(c*d^2
*f - d^3*e)*(f*x + e)) - (f^3*(exp_integral_e(3, 2*(-I*(f*x + e)*d - I*c*f
+ I*d*e)/d) + exp_integral_e(3, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d))*cos
(2*(c*f - d*e)/d) - f^3*(I*exp_integral_e(3, 2*(-I*(f*x + e)*d - I*c*f + I*
```

d*e)/d) - I*exp_integral_e(3, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d))*sin(2
*(c*f - d*e)/d) - f^3)*b^2/((f*x + e)^2*d^3 + c^2*d*f^2 - 2*c*d^2*f*e + d^3
*e^2 + 2*(c*d^2*f - d^3*e)*(f*x + e)))/f

**Fricas [A]**
time = 0.39, size = 476, normalized size = 1.94

$$\frac{b^2d^2\cos\left(fx+e\right)^2-\left(a^2+b^2\right)d^2-2\left(b^2d^2f^2x^2+2\,b^2cdf^2x+b^2c^2f^2\right)\sin\left(-\frac{2\,(cf-de)}{d}\right)\operatorname{Si}\left(\frac{2\,(dfx+cf)}{d}\right)-2\left(abd^2f^2x^2+2\,abcdf^2x+abc^2f^2\right)\cos\left(-\frac{cf-de}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)-2\left(abd^2fx+abcdf\right)\cos\left(fx+e\right)+\left(\left(b^2d^2f^2x^2+2\,b^2cdf^2x+b^2c^2f^2\right)\operatorname{Ci}\left(\frac{2\,(dfx+cf)}{d}\right)+\left(b^2d^2f^2x^2+2\,b^2cdf^2x+b^2c^2f^2\right)\operatorname{Ci}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\cos\left(-\frac{2\,(cf-de)}{d}\right)-2\left(abd^2+\left(b^2d^2fx+b^2cdf\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)-\left(\left(abd^2f^2x^2+2\,abcdf^2x+abc^2f^2\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+\left(abd^2f^2x^2+2\,abcdf^2x+abc^2f^2\right)\operatorname{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{cf-de}{d}\right)}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*cos(f*x + e)^2 - (a^2 + b^2)*d^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*
c*d*f^2*x + b^2*c^2*f^2)*sin(-2*(c*f - d*e)/d)*sin_integral(2*(d*f*x + c*f)
/d) - 2*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*cos(-(c*f - d*e)/
d)*sin_integral((d*f*x + c*f)/d) - 2*(a*b*d^2*f*x + a*b*c*d*f)*cos(f*x + e)
 + ((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*cos_integral(2*(d*f*x
 + c*f)/d) + (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*cos_integral
(-2*(d*f*x + c*f)/d))*cos(-2*(c*f - d*e)/d) - 2*(a*b*d^2 + (b^2*d^2*f*x + b
^2*c*d*f)*cos(f*x + e))*sin(f*x + e) - ((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x
+ a*b*c^2*f^2)*cos_integral((d*f*x + c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d
*f^2*x + a*b*c^2*f^2)*cos_integral(-(d*f*x + c*f)/d))*sin(-(c*f - d*e)/d))/
(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\sin\left(e + fx\right)\right)^{2}}{\left(c + dx\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2/(c + d*x)**3, x)
```

**Giac [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.92, size = 123654, normalized size = 504.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1
/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a*b*d^2*f^2
*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*
```

f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - b^2*d^2*f^2*x^2*real_part(c
os_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/
2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - b^2*d^2*f^2*x^2*real_part(cos_integral(-
2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*t
an(1/2*e)^2*tan(e)^2 + 2*a*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(
f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 -
2*b^2*d^2*f^2*x^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(
1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 2*b^2*d^2*f^
2*x^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*b^2*d^2*f^2*x^2*sin_in
tegral(2*(d*f*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*
f/d)^2*tan(1/2*e)^2*tan(e) - 2*a*b*d^2*f^2*x^2*real_part(cos_integral(f*x +
c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)
*tan(e)^2 - 2*a*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(f*x
)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 + 2*a*
b*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1/2*f*x)^
2*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 2*a*b*d^2*f^2*x^2*rea
l_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*t
an(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 2*b^2*d^2*f^2*x^2*imag_part(cos_integ
ral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2
*tan(1/2*e)^2*tan(e)^2 - 2*b^2*d^2*f^2*x^2*imag_part(cos_integral(-2*f*x -
2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^
2*tan(e)^2 + 4*b^2*d^2*f^2*x^2*sin_integral(2*(d*f*x + c*f)/d)*tan(f*x)^2*t
an(1/2*f*x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 2*a*b*c*d
*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c
*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*a*b*c*d*f^2*x*imag_part(
cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*
c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*b^2*c*d*f^2*x*real_part(cos_integral(2*f
*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(
1/2*e)^2*tan(e)^2 - 2*b^2*c*d*f^2*x*real_part(cos_integral(-2*f*x - 2*c*f/d
))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan
(e)^2 + 4*a*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*
x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + a*b*d^2*f^2*x^2*
imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2
*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - a*b*d^2*f^2*x^2*imag_part(cos_integral(-f*
x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2
*e)^2 + b^2*d^2*f^2*x^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2
*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b^2*d^2*f^2*x^
2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c
*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*b*d^2*f^2*x^2*sin_integral((d*f
*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/
2*e)^2 - 4*b^2*d^2*f^2*x^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x
)^2*tan(1/2*f*x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*b^2*
d^2*f^2*x^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*
x)^2*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*b^2*c*d*f^2*x*imag

```
_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2
*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 4*b^2*c*d*f^2*x*imag_part(cos_integ
ral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d
)^2*tan(1/2*e)^2*tan(e) - 8*b^2*c*d*f^2*x*sin_integral(2*(d*f*x + c*f)/d)*t
an(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)
- a*b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(1/2*f
*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + a*b*d^2*f^2*x^2*imag_part(co
s_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*
f/d)^2*tan(e)^2 - b^2*d^2*f^2*x^2*real_part(cos_integral(2*f*x + 2*c*f/d))*
tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - b^2*d^2*
f^2*x^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2
*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - 2*a*b*d^2*f^2*x^2*sin_integral((d
*f*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(
e)^2 + 4*a*b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*ta
n(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 4*a*b*d^2*f^
2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(c
*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 8*a*b*d^2*f^2*x^2*sin_integral
((d*f*x + c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(c*f/d)^2*tan(1/2*c*f/d)*tan
(1/2*e)*tan(e)^2 - 4*a*b*c*d*f^2*x*real_part(co...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(e+f\,x))^2}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*x)^3,x)
[Out] int((a + b*sin(e + f*x))^2/(c + d*x)^3, x)
```

# 3.163 $\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=495

$$-\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}+\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}-\frac{3d(c+dx)^2\mathrm{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}+$$

```
[Out] -I*(d*x+c)^3*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)
+I*(d*x+c)^3*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)
-3*d*(d*x+c)^2*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/f^2/(a^2-b
^2)^(1/2)+3*d*(d*x+c)^2*polylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/f
^2/(a^2-b^2)^(1/2)-6*I*d^2*(d*x+c)*polylog(3,I*b*exp(I*(f*x+e))/(a-(a^2-b^2
)^(1/2)))/f^3/(a^2-b^2)^(1/2)+6*I*d^2*(d*x+c)*polylog(3,I*b*exp(I*(f*x+e))/
(a+(a^2-b^2)^(1/2)))/f^3/(a^2-b^2)^(1/2)+6*d^3*polylog(4,I*b*exp(I*(f*x+e))
/(a-(a^2-b^2)^(1/2)))/f^4/(a^2-b^2)^(1/2)-6*d^3*polylog(4,I*b*exp(I*(f*x+e)
)/(a+(a^2-b^2)^(1/2)))/f^4/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.65, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3404, 2296, 2221, 2611, 6744, 2320, 6724}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{6id^2(c+dx)\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^3\sqrt{a^2-b^2}}-\frac{3d(c+dx)^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}+\frac{6d^3\text{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^4\sqrt{a^2-b^2}}-\frac{6d^3\text{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^4\sqrt{a^2-b^2}}-\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}+\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sq
rt[a^2 - b^2]*f) + (I*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a
^2 - b^2])])/(Sqrt[a^2 - b^2]*f) - (3*d*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e
 + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2) + (3*d*(c + d*x)^2*
PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f
^2) - ((6*I)*d^2*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*f^3) + ((6*I)*d^2*(c + d*x)*PolyLog[3, (I*b*E^(I*
(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^3) + (6*d^3*PolyLog[
4, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^4) - (6
*d^3*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b
^2]*f^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx = 2\int \frac{e^{i(e+fx)}(c+dx)^3}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx$$

$$= -\frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)^3}{2a-2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}} + \frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)^3}{2a+2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{(3id)}{}$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{3d(c}{}$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{3d(c}{}$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{3d(c}{}$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{3d(c}{}$$

**Mathematica** [A]
time = 0.17, size = 401, normalized size = 0.81

$$-\frac{i\left((c+dx)^3\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)-(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)+\frac{3d\left(-if^2(c+dx)^2\mathrm{Li}_2\left(-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)+2d\left(f(c+dx)\mathrm{Li}_3\left(-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)+id\mathrm{Li}_4\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)\right)\right)}{f^3}+\frac{3id\left(f^2(c+dx)^2\mathrm{Li}_2\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)+2idf(c+dx)\mathrm{Li}_3\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)-2d^2\mathrm{Li}_4\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)\right)}{f^3}\right)}{\sqrt{a^2-b^2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*((c + d*x)^3*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] -
(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])] + (3*d*((-
I)*f^2*(c + d*x)^2*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2
])] + 2*d*(f*(c + d*x)*PolyLog[3, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 -
```

```
 b^2])] + I*d*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])))/f^
3 + ((3*I)*d*(f^2*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt
[a^2 - b^2])] - 2*d^2*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2]
)]))/f^3))/(Sqrt[a^2 - b^2]*f)
```

**Maple** [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{a+b\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^3/(a+b*sin(f*x+e)),x)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2237 vs. 2(429) = 858.
time = 0.55, size = 2237, normalized size = 4.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(-6*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(f*x + e) + a*si
n(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b)
 + 6*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(f*x + e) + a*sin(f
*x + e) - (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) +
6*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(f*x + e) + a*sin(f*x
 + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*
I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(f*x + e) + a*sin(f*x +
```

```
e) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 3*(I
*b*d^3*f^2*x^2 + 2*I*b*c*d^2*f^2*x + I*b*c^2*d*f^2)*sqrt(-(a^2 - b^2)/b^2)*
dilog((I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x +
e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(-I*b*d^3*f^2*x^2 - 2*I*b*c*d^2*
f^2*x - I*b*c^2*d*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x + e) - a*s
in(f*x + e) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) + 3*(-I*b*d^3*f^2*x^2 - 2*I*b*c*d^2*f^2*x - I*b*c^2*d*f^2)*sqrt(-
(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e
) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(I*b*d^3*f^2*x
^2 + 2*I*b*c*d^2*f^2*x + I*b*c^2*d*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*
cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + (b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^
2 - b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(f*x + e) + 2*I*b*sin(f*x
+ e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*c^3*f^3 - 3*b*c^2*d*f^2*e +
 3*b*c*d^2*f*e^2 - b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(f*x + e) -
 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*c^3*f^3 - 3*
b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*
b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) -
 (b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*sqrt(-(a^2 - b
^2)/b^2)*log(-2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)
/b^2) - 2*I*a) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b
*c^2*d*f^2*e - 3*b*c*d^2*f*e^2 + b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*
a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(
-(a^2 - b^2)/b^2) - b)/b) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*
f^3*x + 3*b*c^2*d*f^2*e - 3*b*c*d^2*f*e^2 + b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^
2)*log(-(I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b*sin(f*x
+ e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 +
 3*b*c^2*d*f^3*x + 3*b*c^2*d*f^2*e - 3*b*c*d^2*f*e^2 + b*d^3*e^3)*sqrt(-(a^
2 - b^2)/b^2)*log(-(-I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) -
I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d^3*f^3*x^3 + 3*b*c*d
^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*c^2*d*f^2*e - 3*b*c*d^2*f*e^2 + b*d^3*e^
3)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos
(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 6*(b*d^3*f*x
 + b*c*d^2*f)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(f*x + e) + a*sin(
f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) -
 6*(b*d^3*f*x + b*c*d^2*f)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(f*x
+ e) + a*sin(f*x + e) - (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^
2)/b^2))/b) + 6*(b*d^3*f*x + b*c*d^2*f)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -
(-I*a*cos(f*x + e) + a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*s
qrt(-(a^2 - b^2)/b^2))/b) - 6*(b*d^3*f*x + b*c*d^2*f)*sqrt(-(a^2 - b^2)/b^2
)*polylog(3, -(-I*a*cos(f*x + e) + a*sin(f*x + e) - (b*cos(f*x + e) + I*b*s
in(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b))/((a^2 - b^2)*f^4)
```

**Sympy** [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**3/(a + b*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{a+b\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*sin(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a + b*sin(e + f*x)), x)
```

# 3.164 $\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=367

$$-\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}+\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}-\frac{2d(c+dx)\text{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}+ \cdots$$

```
[Out] -I*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)
+I*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)
-2*d*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/f^2/(a^2-b^2
)^(1/2)+2*d*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/f^2/(
a^2-b^2)^(1/2)-2*I*d^2*polylog(3,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/f^
3/(a^2-b^2)^(1/2)+2*I*d^2*polylog(3,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))
/f^3/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.53, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$,
Rules used = {3404, 2296, 2221, 2611, 2320, 6724}

$$-\frac{2d(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+\frac{2d(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}-\frac{2id^2\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{2id^2\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^3\sqrt{a^2-b^2}}-\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}+\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sq
rt[a^2 - b^2]*f) + (I*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a
^2 - b^2])])/(Sqrt[a^2 - b^2]*f) - (2*d*(c + d*x)*PolyLog[2, (I*b*E^(I*(e +
 f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2) + (2*d*(c + d*x)*Poly
Log[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2)
- ((2*I)*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*f^3) + ((2*I)*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])])/(Sqrt[a^2 - b^2]*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx = 2\int \frac{e^{i(e+fx)}(c+dx)^2}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx$$

$$= -\frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}} + \frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$= -\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{(2id)}{}$$

$$= -\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{2d(c}{}$$

$$= -\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{2d(c}{}$$

$$= -\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{2d(c}{}$$

**Mathematica** [A]
time = 0.12, size = 296, normalized size = 0.81

$$-\frac{i\left((c+dx)^2\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)-(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)+\frac{2d\left(-if(c+dx)\text{Li}_2\left(-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)+d\text{Li}_3\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)\right)}{f^2}+\frac{2id\left(f(c+dx)\text{Li}_2\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)+id\text{Li}_3\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)\right)}{f^2}\right)}{\sqrt{a^2-b^2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*((c + d*x)^2*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] -
(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])] + (2*d*((-
I)*f*(c + d*x)*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])]
+ d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])]))/f^2 + ((2*I)*
d*(f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])] + I*
d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]))/f^2))/(Sqrt[a^2
 - b^2]*f)
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{a+b\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^2/(a+b*sin(f*x+e)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1599 vs. 2(315) = 630.
time = 0.54, size = 1599, normalized size = 4.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(f*x + e) + a*sin(f
*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) -
2*b*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(f*x + e) + a*sin(f*x +
e) - (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*b*d
^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(f*x + e) + a*sin(f*x + e) +
 (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*b*d^2*s
qrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(f*x + e) + a*sin(f*x + e) - (b*
cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(I*b*d^2*f*
x + I*b*c*d*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x + e) - a*sin(f*x +
 e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) + 2*(-I*b*d^2*f*x - I*b*c*d*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x
+ e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^
2)/b^2) - b)/b + 1) + 2*(-I*b*d^2*f*x - I*b*c*d*f)*sqrt(-(a^2 - b^2)/b^2)*d
ilog((-I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x +
e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(I*b*d^2*f*x + I*b*c*d*f)*sqrt(-
(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e
) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (b*c^2*f^2 - 2*b
*c*d*f*e + b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(f*x + e) + 2*I*b*s
```

```
in(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*c^2*f^2 - 2*b*c*d*f*
e + b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(f*x + e) - 2*I*b*sin(f*x
+ e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*c^2*f^2 - 2*b*c*d*f*e + b*d
^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) +
 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2
)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*s
qrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f
*e - b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(f*x + e) - a*sin(f*x +
 e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) +
(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*e - b*d^2*e^2)*sqrt(-(a^2 - b^2)
/b^2)*log(-(I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b*sin(f
*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
2*b*c*d*f*e - b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(f*x + e) - a
*sin(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)
- b)/b) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*e - b*d^2*e^2)*sqrt(-(
a^2 - b^2)/b^2)*log(-(-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e)
- I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^2 - b^2)*f^3)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*sin(f*x+e)),x)
[Out] Integral((c + d*x)**2/(a + b*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^2/(b*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{a+b\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*sin(e + f*x)),x)
[Out] int((c + d*x)^2/(a + b*sin(e + f*x)), x)
```

# 3.165 $\int \frac{c+dx}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=234

$$-\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}+\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}-\frac{d\mathrm{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}+\frac{d\mathrm{Li}_2\left(\frac{ibe^{i}}{a+\sqrt{}}\right.}{\sqrt{a^2-}}$$

```
[Out] -I*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)+I
*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)-d*p
olylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)+d*poly
log(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.29, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3404, 2296, 2221, 2317, 2438}

$$-\frac{d\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+\frac{d\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}-\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}+\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*f) + (I*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*f) - (d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqr
t[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2) + (d*PolyLog[2, (I*b*E^(I*(e + f*x)))
/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{a+b\sin(e+fx)}\,dx &= 2\int \frac{e^{i(e+fx)}(c+dx)}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx \\
&= -\frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}} + \frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)}{2a+2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}} \\
&= -\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{(id)\int \mathrm{l}\cdots}{\cdots} \\
&= -\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{d\mathrm{Subst}\cdots}{\cdots} \\
&= -\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{d\mathrm{Li}_2\left(\cdots\right)}{\sqrt{a\cdots}}
\end{aligned}$$

**Mathematica [A]**
time = 0.03, size = 182, normalized size = 0.78

$$\frac{-if(c+dx)\left(\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)\right)-d\mathrm{Li}_2\left(-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)+d\mathrm{Li}_2\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*Sin[e + f*x]),x]

[Out] ((-I)*f*(c + d*x)*(Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] - Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]) - d*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] + d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2)

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(204) = 408.
time = 0.08, size = 492, normalized size = 2.10

| method | result |
|---|---|
| risch | $\frac{2ic\arctan\left(\frac{2ib\,e^{i(fx+e)}-2a}{2\sqrt{-a^2+b^2}}\right)}{f\sqrt{-a^2+b^2}}+\frac{d\ln\left(\frac{-ia-b\,e^{i(fx+e)}+\sqrt{-a^2+b^2}}{-ia+\sqrt{-a^2+b^2}}\right)x}{f\sqrt{-a^2+b^2}}+\frac{d\ln\left(\frac{-ia-b\,e^{i(fx+e)}+\sqrt{-a^2+b^2}}{-ia+\sqrt{-a^2+b^2}}\right)e}{f^2\sqrt{-a^2+b^2}}-\frac{d\ln}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 2*I/f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^(1/2))+1/f*d/(-a^2+b^2)^(1/2)*ln((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*x+1/f^2*d/(-a^2+b^2)^(1/2)*ln((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*e-1/f*d/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/f^2*d/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*e+I/f^2*d/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-I/f^2*d/(-a^2+b^2)^(1/2)*dilog((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))-2*I/f^2*d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^(1/2))

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+b*sin(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more de

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(204) = 408$.
time = 0.55, size = 1045, normalized size = 4.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b*d*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x + e) - a*sin(f*x + e)
+ (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) -
I*b*d*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x + e) - a*sin(f*x + e) - (b*
cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*b*d
*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(
f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*b*d*sqr
t(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x
+ e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (b*c*f - b*d*
e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*s
qrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*c*f - b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log
(2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a
) - (b*c*f - b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e) + 2*I*b*si
n(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b*c*f - b*d*e)*sqrt(-(a
^2 - b^2)/b^2)*log(-2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2
- b^2)/b^2) - 2*I*a) - (b*d*f*x + b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*c
os(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a
^2 - b^2)/b^2) - b)/b) + (b*d*f*x + b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a
*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-
(a^2 - b^2)/b^2) - b)/b) - (b*d*f*x + b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-
I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqr
t(-(a^2 - b^2)/b^2) - b)/b) + (b*d*f*x + b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(
-(-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) - I*b*sin(f*x + e))*
sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^2 - b^2)*f^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx}{a + b\sin{(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c + d*x)/(a + b*sin(e + f*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*sin(e + f*x)),x)
```

```
[Out] int((c + d*x)/(a + b*sin(e + f*x)), x)
```

# 3.166 $\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\sin(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Sin[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a + b*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$$

**Mathematica** [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])), x]
```

**Maple** [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a+b\sin(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*sin(f*x+e)),x)
[Out] int(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/((d*x + c)*(b*sin(f*x + e) + a)), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*sin(f*x + e)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \sin{(e + fx)}) (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x)
[Out] Integral(1/((a + b*sin(e + f*x))*(c + d*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*x + c)*(b*sin(f*x + e) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + b\,\sin{(e + f\,x)})\,\,(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*x)),x)
[Out] int(1/((a + b*sin(e + f*x))*(c + d*x)), x)
```

# 3.167 $\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\sin(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*sin(f*x+e)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Sin[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$$

**Mathematica** [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])), x]
```

**Maple** [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx+c\right)^{2}\left(a+b\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*sin(f*x+e)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^2*(b*sin(f*x + e) + a)), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*sin(f*x + e)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sin(e+fx))(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))*(c + d*x)**2), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*sin(f*x + e) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a + b*sin(e + f*x))*(c + d*x)^2), x)
```

# 3.168 $\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=925

$$\frac{i(c+dx)^3}{(a^2-b^2)f}-\frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2}-\frac{ia(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f}-\frac{3d(c+dx)^2\log\left(1+\right.}{(a^2-b}$$

```
[Out] -I*a*(d*x+c)^3*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)
/f-3*d*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^2
+6*I*d^2*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2
)/f^3-3*d*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/
f^2+I*a*(d*x+c)^3*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3
/2)/f+6*I*d^2*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^
2-b^2)/f^3-3*a*d*(d*x+c)^2*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2))
)/(a^2-b^2)^(3/2)/f^2-6*I*a*d^2*(d*x+c)*polylog(3,I*b*exp(I*(f*x+e))/(a-(a^
2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+3*a*d*(d*x+c)^2*polylog(2,I*b*exp(I*(f*x
+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2-6*d^3*polylog(3,I*b*exp(I*(f*
x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^4+I*(d*x+c)^3/(a^2-b^2)/f-6*d^3*poly
log(3,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^4+6*I*a*d^2*(d*x+
c)*polylog(3,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+6*
a*d^3*polylog(4,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^4
-6*a*d^3*polylog(4,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/
f^4+b*(d*x+c)^3*cos(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

**Rubi** [A]
time = 1.02, antiderivative size = 925, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3405, 3404, 2296, 2221, 2611, 6744, 2320, 6724, 4615}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (I*(c + d*x)^3)/((a^2 - b^2)*f) - (3*d*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f
*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (I*a*(c + d*x)^3*Log[1 -
(I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) - (3*d*
(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b
^2)*f^2) + (I*a*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b
^2])])/((a^2 - b^2)^(3/2)*f) + ((6*I)*d^2*(c + d*x)*PolyLog[2, (I*b*E^(I*(e
 + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^3) - (3*a*d*(c + d*x)^2*Po
lyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f
^2) + ((6*I)*d^2*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
```

```
 b^2])])/((a^2 - b^2)*f^3) + (3*a*d*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f
*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^2) - (6*d^3*PolyLog[3, (
I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^4) - ((6*I)*a*d
^2*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*f^3) - (6*d^3*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2
- b^2])])/((a^2 - b^2)*f^4) + ((6*I)*a*d^2*(c + d*x)*PolyLog[3, (I*b*E^(I*(
e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^3) + (6*a*d^3*PolyL
og[4, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^4)
 - (6*a*d^3*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2
- b^2)^(3/2)*f^4) + (b*(c + d*x)^3*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[
e + f*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
```

```
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx = \frac{b(c+dx)^3\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{a\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx}{a^2-b^2} - \frac{(3bd)\int \frac{(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}\,d}{(a^2-b^2)\,f}$$

$$= \frac{i(c+dx)^3}{(a^2-b^2)\,f} + \frac{b(c+dx)^3\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{(2a)\int \frac{e^{i(e+fx)}(c+dx)^3}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx}{a^2-b^2}$$

$$= \frac{i(c+dx)^3}{(a^2-b^2)\,f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{3d(c+dx)^2\log\left(1-\frac{}{a+}\right.}{(a^2-b^2)\,f^2}$$

$$= \frac{i(c+dx)^3}{(a^2-b^2)\,f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{i}{a-}\right.}{(a^2-b^2)^{3/2}\,f}$$

$$= \frac{i(c+dx)^3}{(a^2-b^2)\,f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{i}{a-}\right.}{(a^2-b^2)^{3/2}\,f}$$

$$= \frac{i(c+dx)^3}{(a^2-b^2)\,f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{i}{a-}\right.}{(a^2-b^2)^{3/2}\,f}$$

$$= \frac{i(c+dx)^3}{(a^2-b^2)\,f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{i}{a-}\right.}{(a^2-b^2)^{3/2}\,f}$$

$$= \frac{i(c+dx)^3}{(a^2-b^2)\,f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{i}{a-}\right.}{(a^2-b^2)^{3/2}\,f}$$

**Mathematica [A]**
time = 2.14, size = 742, normalized size = 0.80

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (I*f^3*(c + d*x)^3 - 3*d*f^2*(c + d*x)^2*Log[1 + (I*b*E^(I*(e + f*x)))/(-a
+ Sqrt[a^2 - b^2])] - 3*d*f^2*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a
+ Sqrt[a^2 - b^2])] + (6*I)*d^2*(f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)
```

```
))/(a - Sqrt[a^2 - b^2])] + I*d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[
a^2 - b^2])]) + (6*I)*d^2*(f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a
+ Sqrt[a^2 - b^2])] + I*d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
b^2])]) - (I*a*(f^3*(c + d*x)^3*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^
2 - b^2])] - f^3*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
b^2])] - (3*I)*d*(f^2*(c + d*x)^2*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a +
 Sqrt[a^2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[3, ((-I)*b*E^(I*(e + f*x))
)/(-a + Sqrt[a^2 - b^2])] - 2*d^2*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a - Sqr
t[a^2 - b^2])]) + (3*I)*d*(f^2*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f*x)))
/(a + Sqrt[a^2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x
)))/(a + Sqrt[a^2 - b^2])] - 2*d^2*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sq
rt[a^2 - b^2])])))/Sqrt[a^2 - b^2] + (b*f^3*(c + d*x)^3*Cos[e + f*x])/(a +
b*Sin[e + f*x]))/((a^2 - b^2)*f^4)
```

**Maple** [**F**]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{(a+b\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^3/(a+b*sin(f*x+e))^2,x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5206 vs. 2(823) = 1646.
time = 0.73, size = 5206, normalized size = 5.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(6*(I*a*b^2*d^3*sin(f*x + e) + I*a^2*b*d^3)*sqrt(-(a^2 - b^2)/b^2)*pol
ylog(4, -(I*a*cos(f*x + e) + a*sin(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x
 + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(-I*a*b^2*d^3*sin(f*x + e) - I*a^2*b*
d^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(f*x + e) + a*sin(f*x + e)
- (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(-I*a*
b^2*d^3*sin(f*x + e) - I*a^2*b*d^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*
a*cos(f*x + e) + a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(
-(a^2 - b^2)/b^2))/b) + 6*(I*a*b^2*d^3*sin(f*x + e) + I*a^2*b*d^3)*sqrt(-(a
^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(f*x + e) + a*sin(f*x + e) - (b*cos(f*x
 + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*((a^2*b - b^3)*d^3
*f^3*x^3 + 3*(a^2*b - b^3)*c*d^2*f^3*x^2 + 3*(a^2*b - b^3)*c^2*d*f^3*x + (a
^2*b - b^3)*c^3*f^3)*cos(f*x + e) + 3*(-2*I*(a^3 - a*b^2)*d^3*f*x - 2*I*(a^
3 - a*b^2)*c*d^2*f + 2*(-I*(a^2*b - b^3)*d^3*f*x - I*(a^2*b - b^3)*c*d^2*f)
*sin(f*x + e) + (-I*a^2*b*d^3*f^2*x^2 - 2*I*a^2*b*c*d^2*f^2*x - I*a^2*b*c^2
*d*f^2 + (-I*a*b^2*d^3*f^2*x^2 - 2*I*a*b^2*c*d^2*f^2*x - I*a*b^2*c^2*d*f^2)
*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a*cos(f*x + e) - a*sin(f*x
+ e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
1) + 3*(-2*I*(a^3 - a*b^2)*d^3*f*x - 2*I*(a^3 - a*b^2)*c*d^2*f + 2*(-I*(a^2
*b - b^3)*d^3*f*x - I*(a^2*b - b^3)*c*d^2*f)*sin(f*x + e) + (I*a^2*b*d^3*f^
2*x^2 + 2*I*a^2*b*c*d^2*f^2*x + I*a^2*b*c^2*d*f^2 + (I*a*b^2*d^3*f^2*x^2 +
2*I*a*b^2*c*d^2*f^2*x + I*a*b^2*c^2*d*f^2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/
b^2))*dilog((I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b*sin(
f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(2*I*(a^3 - a*b^2)*d^3*f*x
 + 2*I*(a^3 - a*b^2)*c*d^2*f + 2*(I*(a^2*b - b^3)*d^3*f*x + I*(a^2*b - b^3)
*c*d^2*f)*sin(f*x + e) + (I*a^2*b*d^3*f^2*x^2 + 2*I*a^2*b*c*d^2*f^2*x + I*a
^2*b*c^2*d*f^2 + (I*a*b^2*d^3*f^2*x^2 + 2*I*a*b^2*c*d^2*f^2*x + I*a*b^2*c^2
*d*f^2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(f*x + e) - a*
sin(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) -
 b)/b + 1) + 3*(2*I*(a^3 - a*b^2)*d^3*f*x + 2*I*(a^3 - a*b^2)*c*d^2*f + 2*(
I*(a^2*b - b^3)*d^3*f*x + I*(a^2*b - b^3)*c*d^2*f)*sin(f*x + e) + (-I*a^2*b
*d^3*f^2*x^2 - 2*I*a^2*b*c*d^2*f^2*x - I*a^2*b*c^2*d*f^2 + (-I*a*b^2*d^3*f^
2*x^2 - 2*I*a*b^2*c*d^2*f^2*x - I*a*b^2*c^2*d*f^2)*sin(f*x + e))*sqrt(-(a^2
 - b^2)/b^2))*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) -
 I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (3*(a^3 - a*b^2)*c^
2*d*f^2 - 6*(a^3 - a*b^2)*c*d^2*f*e + 3*(a^3 - a*b^2)*d^3*e^2 + 3*((a^2*b -
 b^3)*c^2*d*f^2 - 2*(a^2*b - b^3)*c*d^2*f*e + (a^2*b - b^3)*d^3*e^2)*sin(f*
x + e) - (a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*e + 3*a^2*b*c*d^2*f*e^2 - a^2*b
*d^3*e^3 + (a*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*e + 3*a*b^2*c*d^2*f*e^2 - a*b
^2*d^3*e^3)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(f*x + e) + 2*
I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (3*(a^3 - a*b^2)*c
^2*d*f^2 - 6*(a^3 - a*b^2)*c*d^2*f*e + 3*(a^3 - a*b^2)*d^3*e^2 + 3*((a^2*b
- b^3)*c^2*d*f^2 - 2*(a^2*b - b^3)*c*d^2*f*e + (a^2*b - b^3)*d^3*e^2)*sin(f
*x + e) - (a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*e + 3*a^2*b*c*d^2*f*e^2 - a^2*
b*d^3*e^3 + (a*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*e + 3*a*b^2*c*d^2*f*e^2 - a*
b^2*d^3*e^3)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(f*x + e) - 2
```

```
*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (3*(a^3 - a*b^2)*
c^2*d*f^2 - 6*(a^3 - a*b^2)*c*d^2*f*e + 3*(a^3 - a*b^2)*d^3*e^2 + 3*((a^2*b
 - b^3)*c^2*d*f^2 - 2*(a^2*b - b^3)*c*d^2*f*e + (a^2*b - b^3)*d^3*e^2)*sin(
f*x + e) + (a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*e + 3*a^2*b*c*d^2*f*e^2 - a^2
*b*d^3*e^3 + (a*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*e + 3*a*b^2*c*d^2*f*e^2 - a
*b^2*d^3*e^3)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(f*x + e) +
 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (3*(a^3 - a*b^2
)*c^2*d*f^2 - 6*(a^3 - a*b^2)*c*d^2*f*e + 3*(a^3 - a*b^2)*d^3*e^2 + 3*((a^2
*b - b^3)*c^2*d*f^2 - 2*(a^2*b - b^3)*c*d^2*f*e + (a^2*b - b^3)*d^3*e^2)*si
n(f*x + e) + (a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*e + 3*a^2*b*c*d^2*f*e^2 - a
^2*b*d^3*e^3 + (a*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*e + 3*a*b^2*c*d^2*f*e^2 -
 a*b^2*d^3*e^3)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(f*x + e)
 - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (3*(a^3 - a*b
^2)*d^3*f^2*x^2 + 6*(a^3 - a*b^2)*c*d^2*f^2*x + 6*(a^3 - a*b^2)*c*d^2*f*e -
 3*(a^3 - a*b^2)*d^3*e^2 + 3*((a^2*b - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c
*d^2*f^2*x + 2*(a^2*b - b^3)*c*d^2*f*e - (a^2*b - b^3)*d^3*e^2)*sin(f*x + e
) + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b*c^2*d*f^3*x + 3*a^
2*b*c^2*d*f^2*e - 3*a^2*b*c*d^2*f*e^2 + a^2*b*d^3*e^3 + (a*b^2*d^3*f^3*x^3
+ 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + 3*a*b^2*c^2*d*f^2*e - 3*a*b
^2*c*d^2*f*e^2 + a*b^2*d^3*e^3)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-
(I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sq
rt(-(a^2 - b^2)/b^2) - b)/b) + (3*(a^3 - a*b^2)...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*sin(e + f*x))^2,x)
```

```
[Out] \text{Hanged}
```

# 3.169 $\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=671

$$\frac{i(c+dx)^2}{(a^2-b^2)\,f}-\frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2}-\frac{ia(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f}-\frac{2d(c+dx)\log\left(1-\right.}{(a^2-b^2)}$$

```
[Out] I*(d*x+c)^2/(a^2-b^2)/f-2*d*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1
/2)))/(a^2-b^2)/f^2-I*a*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2
)))/(a^2-b^2)^(3/2)/f-2*d*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2
)))/(a^2-b^2)/f^2+I*a*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2))
)/(a^2-b^2)^(3/2)/f+2*I*d^2*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)
))/(a^2-b^2)/f^3-2*a*d*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1
/2)))/(a^2-b^2)^(3/2)/f^2+2*I*d^2*polylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)
^(1/2)))/(a^2-b^2)/f^3+2*a*d*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b
^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2-2*I*a*d^2*polylog(3,I*b*exp(I*(f*x+e))/(a-(
a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+2*I*a*d^2*polylog(3,I*b*exp(I*(f*x+e))
/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+b*(d*x+c)^2*cos(f*x+e)/(a^2-b^2)/
f/(a+b*sin(f*x+e))
```

**Rubi** [A]

time = 0.76, antiderivative size = 671, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3405, 3404, 2296, 2221, 2611, 2320, 6724, 4615, 2317, 2438}

$$-\frac{2ad(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}}+\frac{2ad(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2(a^2-b^2)^{3/2}}+\frac{2id^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3(a^2-b^2)}+\frac{2id^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^3(a^2-b^2)}-\frac{2iad^2\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3(a^2-b^2)^{3/2}}+\frac{2iad^2\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^3(a^2-b^2)^{3/2}}-\frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)}-\frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2(a^2-b^2)}-\frac{ia(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{3/2}}+\frac{ia(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f(a^2-b^2)^{3/2}}+\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}+\frac{i(c+dx)^2}{f(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (I*(c + d*x)^2)/((a^2 - b^2)*f) - (2*d*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x
)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (I*a*(c + d*x)^2*Log[1 - (I
*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) - (2*d*(c
 + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*
f^2) + (I*a*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])
])/((a^2 - b^2)^(3/2)*f) + ((2*I)*d^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a -
 Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, (I*b*E^
(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^2) + ((2*I)*d^2
*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^3)
 + (2*a*d*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]
)/((a^2 - b^2)^(3/2)*f^2) - ((2*I)*a*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(
a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^3) + ((2*I)*a*d^2*PolyLog[3, (I
*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^3) + (b*(c
 + d*x)^2*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
```

```
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx = \frac{b(c+dx)^2\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{a\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx}{a^2-b^2} - \frac{(2bd)\int \frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}\,dx}{(a^2-b^2)\,f}$$

$$= \frac{i(c+dx)^2}{(a^2-b^2)\,f} + \frac{b(c+dx)^2\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{(2a)\int \frac{e^{i(e+fx)}(c+dx)^2}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx}{a^2-b^2}$$

$$= \frac{i(c+dx)^2}{(a^2-b^2)\,f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{2d(c+dx)\log\left(1-\frac{ibe}{a+\sqrt{}}\right.}{(a^2-b^2)\,f^2}$$

$$= \frac{i(c+dx)^2}{(a^2-b^2)\,f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{ia(c+dx)^2\log\left(1-\frac{ibe}{a-\sqrt{}}\right.}{(a^2-b^2)^{3/2}\,f}$$

$$= \frac{i(c+dx)^2}{(a^2-b^2)\,f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{ia(c+dx)^2\log\left(1-\frac{ibe}{a-\sqrt{}}\right.}{(a^2-b^2)^{3/2}\,f}$$

$$= \frac{i(c+dx)^2}{(a^2-b^2)\,f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{ia(c+dx)^2\log\left(1-\frac{ibe}{a-\sqrt{}}\right.}{(a^2-b^2)^{3/2}\,f}$$

$$= \frac{i(c+dx)^2}{(a^2-b^2)\,f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} - \frac{ia(c+dx)^2\log\left(1-\frac{ibe}{a-\sqrt{}}\right.}{(a^2-b^2)^{3/2}\,f}$$

**Mathematica [A]**
time = 1.11, size = 530, normalized size = 0.79

$$\frac{if^2(c+dx)^2 - 2df(c+dx)\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right) - 2df(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right) + 2id^2\text{Li}_2\left(-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right) + 2id^2\text{Li}_2\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right) - \frac{ia\left(f^2(c+dx)^2\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right) - f^2(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right) - 2idf(c+dx)\text{Li}_2\left(-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right) + 2idf(c+dx)\text{Li}_2\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right) + 2d^2\text{Li}_3\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right) - 2d^2\text{Li}_3\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}}{(a^2-b^2)\,f^3} + \frac{bf^2(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (I*f^2*(c + d*x)^2 - 2*d*f*(c + d*x)*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sq
rt[a^2 - b^2])] - 2*d*f*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a
^2 - b^2])] + (2*I)*d^2*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2
- b^2])] + (2*I)*d^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])
] - (I*a*(f^2*(c + d*x)^2*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^
2])] - f^2*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]
 - (2*I)*d*f*(c + d*x)*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 -
```

```
 b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])] + 2*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])]
- 2*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]))/Sqrt[a^2
- b^2] + (b*f^2*(c + d*x)^2*Cos[e + f*x])/(a + b*Sin[e + f*x]))/((a^2 - b^2
)*f^3)
```

**Maple** [**F**]
time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{(a+b\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3197 vs. 2(593) = 1186.
time = 0.60, size = 3197, normalized size = 4.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*b^2*d^2*sin(f*x + e) + a^2*b*d^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(
3, -(I*a*cos(f*x + e) + a*sin(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e)
)*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(a*b^2*d^2*sin(f*x + e) + a^2*b*d^2)*sqrt(
-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(f*x + e) + a*sin(f*x + e) - (b*cos(f
*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a*b^2*d^2*sin(f
*x + e) + a^2*b*d^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(f*x + e)
+ a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^
```

```
2))/b) - 2*(a*b^2*d^2*sin(f*x + e) + a^2*b*d^2)*sqrt(-(a^2 - b^2)/b^2)*poly
log(3, -(-I*a*cos(f*x + e) + a*sin(f*x + e) - (b*cos(f*x + e) + I*b*sin(f*x
 + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b
 - b^3)*c*d*f^2*x + (a^2*b - b^3)*c^2*f^2)*cos(f*x + e) - 2*(-I*(a^2*b - b^
3)*d^2*sin(f*x + e) - I*(a^3 - a*b^2)*d^2 + (-I*a^2*b*d^2*f*x - I*a^2*b*c*d
*f + (-I*a*b^2*d^2*f*x - I*a*b^2*c*d*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2
))*dilog((I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x
 + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*(a^2*b - b^3)*d^2*sin(f*x
 + e) - I*(a^3 - a*b^2)*d^2 + (I*a^2*b*d^2*f*x + I*a^2*b*c*d*f + (I*a*b^2*d
^2*f*x + I*a*b^2*c*d*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a*co
s(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^
2 - b^2)/b^2) - b)/b + 1) - 2*(I*(a^2*b - b^3)*d^2*sin(f*x + e) + I*(a^3 -
a*b^2)*d^2 + (I*a^2*b*d^2*f*x + I*a^2*b*c*d*f + (I*a*b^2*d^2*f*x + I*a*b^2*
c*d*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(f*x + e) - a*s
in(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) - 2*(I*(a^2*b - b^3)*d^2*sin(f*x + e) + I*(a^3 - a*b^2)*d^2 + (-I
*a^2*b*d^2*f*x - I*a^2*b*c*d*f + (-I*a*b^2*d^2*f*x - I*a*b^2*c*d*f)*sin(f*x
 + e))*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) -
(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (2
*(a^3 - a*b^2)*c*d*f - 2*(a^3 - a*b^2)*d^2*e + 2*((a^2*b - b^3)*c*d*f - (a^
2*b - b^3)*d^2*e)*sin(f*x + e) - (a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*e + a^2*b*d
^2*e^2 + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*e + a*b^2*d^2*e^2)*sin(f*x + e))*sq
rt(-(a^2 - b^2)/b^2))*log(2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(
-(a^2 - b^2)/b^2) + 2*I*a) - (2*(a^3 - a*b^2)*c*d*f - 2*(a^3 - a*b^2)*d^2*e
 + 2*((a^2*b - b^3)*c*d*f - (a^2*b - b^3)*d^2*e)*sin(f*x + e) - (a^2*b*c^2*
f^2 - 2*a^2*b*c*d*f*e + a^2*b*d^2*e^2 + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*e +
a*b^2*d^2*e^2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(f*x + e) -
 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (2*(a^3 - a*b^2
)*c*d*f - 2*(a^3 - a*b^2)*d^2*e + 2*((a^2*b - b^3)*c*d*f - (a^2*b - b^3)*d^
2*e)*sin(f*x + e) + (a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*e + a^2*b*d^2*e^2 + (a*b
^2*c^2*f^2 - 2*a*b^2*c*d*f*e + a*b^2*d^2*e^2)*sin(f*x + e))*sqrt(-(a^2 - b^
2)/b^2))*log(-2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)
/b^2) + 2*I*a) - (2*(a^3 - a*b^2)*c*d*f - 2*(a^3 - a*b^2)*d^2*e + 2*((a^2*b
 - b^3)*c*d*f - (a^2*b - b^3)*d^2*e)*sin(f*x + e) + (a^2*b*c^2*f^2 - 2*a^2*
b*c*d*f*e + a^2*b*d^2*e^2 + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*e + a*b^2*d^2*e^
2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(f*x + e) - 2*I*b*sin(
f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (2*(a^3 - a*b^2)*d^2*f*x +
 2*(a^3 - a*b^2)*d^2*e + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*si
n(f*x + e) + (a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x + 2*a^2*b*c*d*f*e - a^2
*b*d^2*e^2 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x + 2*a*b^2*c*d*f*e - a*b
^2*d^2*e^2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-(I*a*cos(f*x + e) -
a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)
 - b)/b) - (2*(a^3 - a*b^2)*d^2*f*x + 2*(a^3 - a*b^2)*d^2*e + 2*((a^2*b - b
^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*sin(f*x + e) - (a^2*b*d^2*f^2*x^2 + 2*a^
2*b*c*d*f^2*x + 2*a^2*b*c*d*f*e - a^2*b*d^2*e^2 + (a*b^2*d^2*f^2*x^2 + 2*a*
```

```
b^2*c*d*f^2*x + 2*a*b^2*c*d*f*e - a*b^2*d^2*e^2)*sin(f*x + e))*sqrt(-(a^2 -
 b^2)/b^2))*log(-(I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b
*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (2*(a^3 - a*b^2)*d^2*f*x +
2*(a^3 - a*b^2)*d^2*e + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*sin
(f*x + e) + (a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x + 2*a^2*b*c*d*f*e - a^2*
b*d^2*e^2 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x + 2*a*b^2*c*d*f*e - a*b^
2*d^2*e^2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-(-I*a*cos(f*x + e) -
a*sin(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)
 - b)/b) - (2*(a^3 - a*b^2)*d^2*f*x + 2*(a^3 - a*b^2)*d^2*e + 2*((a^2*b - b
^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*sin(f*x + e) - (a^2*b*d^2*f^2*x^2 + 2*a^
2*b*c*d*f^2*x + 2*a^2*b*c*d*f*e - a^2*b*d^2*e^2 + (a*b^2*d^2*f^2*x^2 + 2*a*
b^2*c*d*f^2*x + 2*a*b^2*c*d*f*e - a*b^2*d^2*e^2)*sin(f*x + e))*sqrt(-(a^2 -
 b^2)/b^2))*log(-(-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) - I*
b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^4*b - 2*a^2*b^3 + b^5)*
f^3*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f^3)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*sin(e + f*x))^2,x)
```

```
[Out] \text{Hanged}
```

# 3.170 $\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=305

$$-\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f}+\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f}-\frac{d\log(a+b\sin(e+fx))}{(a^2-b^2)f^2}-\frac{ad\mathrm{Li}_2}{(a}$$

```
[Out] -d*ln(a+b*sin(f*x+e))/(a^2-b^2)/f^2-I*a*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a-
(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f+I*a*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a+
(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f-a*d*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^
2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+a*d*polylog(2,I*b*exp(I*(f*x+e))/(a+(a^2
-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+b*(d*x+c)*cos(f*x+e)/(a^2-b^2)/f/(a+b*sin
(f*x+e))
```

**Rubi** [A]

time = 0.34, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3405, 3404, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{ad\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}}+\frac{ad\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2(a^2-b^2)^{3/2}}-\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{3/2}}+\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f(a^2-b^2)^{3/2}}+\frac{b(c+dx)\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}-\frac{d\log(a+b\sin(e+fx))}{f^2(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((-I)*a*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a
^2 - b^2)^(3/2)*f) + (I*a*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt
[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) - (d*Log[a + b*Sin[e + f*x]])/((a^2 -
b^2)*f^2) - (a*d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(
(a^2 - b^2)^(3/2)*f^2) + (a*d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])])/((a^2 - b^2)^(3/2)*f^2) + (b*(c + d*x)*Cos[e + f*x])/((a^2 - b^
2)*f*(a + b*Sin[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx = \frac{b(c+dx)\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{a\int \frac{c+dx}{a+b\sin(e+fx)}\,dx}{a^2-b^2} - \frac{(bd)\int \frac{\cos(e+fx)}{a+b\sin(e+fx)}\,dx}{(a^2-b^2)\,f}$$

$$= \frac{b(c+dx)\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{(2a)\int \frac{e^{i(e+fx)}(c+dx)}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx}{a^2-b^2} - \frac{d\text{Subst}(\int \cdots}{\cdots}$$

$$= -\frac{d\log(a+b\sin(e+fx))}{(a^2-b^2)\,f^2} + \frac{b(c+dx)\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{(2iab)\int \frac{e^{i\cdots}}{2a-2\sqrt{a\cdots}}}{(a^2\cdots}$$

$$= -\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} + \frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} - \cdots$$

$$= -\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} + \frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} - \cdots$$

$$= -\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} + \frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} - \cdots$$

**Mathematica** [**A**]
time = 0.69, size = 236, normalized size = 0.77

$$\frac{-d\log(a+b\sin(e+fx)) + \frac{a\left(-if(c+dx)\left(\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)\right)-d\text{Li}_2\left(-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)+d\text{Li}_2\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}} + \frac{bf(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}}{(a^2-b^2)\,f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-(d*Log[a + b*Sin[e + f*x]]) + (a*((-I)*f*(c + d*x)*(Log[1 + (I*b*E^(I*(e
+ f*x)))/(-a + Sqrt[a^2 - b^2])] - Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[
a^2 - b^2])]) - d*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2]
)] + d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]))/Sqrt[a^2 -
 b^2] + (b*f*(c + d*x)*Cos[e + f*x])/(a + b*Sin[e + f*x]))/((a^2 - b^2)*f^2
)
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(275) = 550.
time = 1.44, size = 641, normalized size = 2.10

| method | result |
|---|---|
| risch | $\frac{2(dx+c)\left(ib+a\,\mathrm{e}^{i(fx+e)}\right)}{(a^2-b^2)f\left(b\,\mathrm{e}^{2i(fx+e)}-b+2ia\,\mathrm{e}^{i(fx+e)}\right)}+\frac{d\ln\left(i\mathrm{e}^{2i(fx+e)}b-ib-2a\,\mathrm{e}^{i(fx+e)}\right)}{f^2(-a^2+b^2)}-\frac{2d\ln\left(\mathrm{e}^{i(fx+e)}\right)}{f^2(-a^2+b^2)}+\frac{da\ln\left(\frac{ia+b\,\mathrm{e}^{i(fx+e)}+\sqrt{-a^2 \ldots}}{ia+\sqrt{-a^2+b^2}}\ldots\right.}{f(-a^2+b^2)^{\frac{3}{2}}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d*x+c)*(I*b+a*exp(I*(f*x+e)))/(a^2-b^2)/f/(b*exp(2*I*(f*x+e))-b+2*I*a*ex
p(I*(f*x+e)))+1/f^2/(-a^2+b^2)*d*ln(I*exp(2*I*(f*x+e))*b-I*b-2*a*exp(I*(f*x
+e)))-2/f^2/(-a^2+b^2)*d*ln(exp(I*(f*x+e)))+1/f/(-a^2+b^2)^(3/2)*d*a*ln((I*
a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/f^2/(-a^2+
b^2)^(3/2)*d*a*ln((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(
1/2)))*e-2*I/f/(-a^2+b^2)^(3/2)*a*c*arctan(1/2*(2*I*b*exp(I*(f*x+e))-2*a)/(
-a^2+b^2)^(1/2))-1/f/(-a^2+b^2)^(3/2)*d*a*ln((-I*a-b*exp(I*(f*x+e))+(-a^2+b
^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*x-1/f^2/(-a^2+b^2)^(3/2)*d*a*ln((-I*a-b
*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*e+I/f^2/(-a^2+b^
2)^(3/2)*d*a*dilog((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2
)^(1/2)))-I/f^2/(-a^2+b^2)^(3/2)*d*a*dilog((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)
^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+2*I/f^2/(-a^2+b^2)^(3/2)*a*d*e*arctan(1/2*(
2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^(1/2))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(274) = 548$.
time = 0.57, size = 1586, normalized size = 5.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((I*a*b^2*d*sin(f*x + e) + I*a^2*b*d)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a
*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*d*sin(f*x + e) - I*a^2*b*d)*sqrt(-
(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e)
 + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*d*sin(f
*x + e) - I*a^2*b*d)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*si
n(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b
)/b + 1) + (I*a*b^2*d*sin(f*x + e) + I*a^2*b*d)*sqrt(-(a^2 - b^2)/b^2)*dilo
g((-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) - I*b*sin(f*x + e))
*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f
*x + a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(f*x + e)
 - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b
^2) - b)/b) + (a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*sin(f*x
+ e))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(f*x + e) - a*sin(f*x + e) - (b*c
os(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a^2*b*d*f
*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/
b^2)*log(-(-I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) - I*b*sin(f
*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a^2*b*d*f*x + a^2*b*d*e + (a*b^2
*d*f*x + a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(f*x
 + e) - a*sin(f*x + e) - (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b
^2)/b^2) - b)/b) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*cos(f*x + e)
 - ((a^2*b - b^3)*d*sin(f*x + e) + (a^3 - a*b^2)*d - (a^2*b*c*f - a^2*b*d*e
 + (a*b^2*c*f - a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*co
s(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - ((a
^2*b - b^3)*d*sin(f*x + e) + (a^3 - a*b^2)*d - (a^2*b*c*f - a^2*b*d*e + (a*
b^2*c*f - a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(f*x
+ e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - ((a^2*b -
 b^3)*d*sin(f*x + e) + (a^3 - a*b^2)*d + (a^2*b*c*f - a^2*b*d*e + (a*b^2*c*
f - a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(f*x + e)
+ 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - ((a^2*b - b^3)
*d*sin(f*x + e) + (a^3 - a*b^2)*d + (a^2*b*c*f - a^2*b*d*e + (a*b^2*c*f - a
*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(f*x + e) - 2*I
*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b - 2*a^2*b^3
+ b^5)*f^2*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f^2)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*sin(e + f*x))^2,x)
```

```
[Out] \text{Hanged}
```

# 3.171 $\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*sin(f*x+e))^2,x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Sin[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)*(a + b*Sin[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$$

**Mathematica** [A]
time = 23.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])^2), x]
```

**Maple** [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+b\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+b*sin(f*x+e))^2,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*cos(2*f*x + 2*e)*cos(f*x + e) + 2*a*b*cos(f*x + e) - ((a^2*b^2 - b^4
)*d*f*x + (a^2*b^2 - b^4)*c*f + ((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*
f)*cos(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d*f*x + (a^4 - a^2*b^2)*c*f)*cos
(f*x + e)^2 + 4*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*c*f)*cos(f*x + e)*
sin(2*f*x + 2*e) + ((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f)*sin(2*f*x
+ 2*e)^2 + 4*((a^4 - a^2*b^2)*d*f*x + (a^4 - a^2*b^2)*c*f)*sin(f*x + e)^2 -
 2*((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f + 2*((a^3*b - a*b^3)*d*f*x
+ (a^3*b - a*b^3)*c*f)*sin(f*x + e))*cos(2*f*x + 2*e) + 4*((a^3*b - a*b^3)*
d*f*x + (a^3*b - a*b^3)*c*f)*sin(f*x + e))*integrate(-2*(a*b*d*cos(f*x + e)
 + 2*(a^2*d*f*x + a^2*c*f)*cos(f*x + e)^2 + 2*(a^2*d*f*x + a^2*c*f)*sin(f*x
 + e)^2 + (a*b*d*cos(f*x + e) - (a*b*d*f*x + a*b*c*f)*sin(f*x + e))*cos(2*f
*x + 2*e) + (a*b*d*sin(f*x + e) + b^2*d + (a*b*d*f*x + a*b*c*f)*cos(f*x + e
))*sin(2*f*x + 2*e) + (a*b*d*f*x + a*b*c*f)*sin(f*x + e))/((a^2*b^2 - b^4)*
d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + ((a^2*b^2 -
 b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f)*cos(2*
f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^2)*c*d*f*x + (
a^4 - a^2*b^2)*c^2*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^
3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*cos(f*x + e)*sin(2*f*x + 2*e)
 + ((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)
*c^2*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^
2)*c*d*f*x + (a^4 - a^2*b^2)*c^2*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d^2
*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + 2*((a^3*b - a*
b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f*x
 + e))*cos(2*f*x + 2*e) + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*
c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f*x + e)), x) + 2*(a*b*sin(f*x + e) +
b^2)*sin(2*f*x + 2*e))/((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f + ((a^2
*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f)*cos(2*f*x + 2*e)^2 + 4*((a^4 - a^2
*b^2)*d*f*x + (a^4 - a^2*b^2)*c*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d*f*
x + (a^3*b - a*b^3)*c*f)*cos(f*x + e)*sin(2*f*x + 2*e) + ((a^2*b^2 - b^4)*d
*f*x + (a^2*b^2 - b^4)*c*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d*f*x +
 (a^4 - a^2*b^2)*c*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d*f*x + (a^2*b^2
- b^4)*c*f + 2*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*c*f)*sin(f*x + e))*
cos(2*f*x + 2*e) + 4*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*c*f)*sin(f*x
+ e))
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(1/((a^2 + b^2)*d*x - (b^2*d*x + b^2*c)*cos(f*x + e)^2 + (a^2 + b^2
)*c + 2*(a*b*d*x + a*b*c)*sin(f*x + e)), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/((d*x + c)*(b*sin(f*x + e) + a)^2), x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*x)),x)
[Out] int(1/((a + b*sin(e + f*x))^2*(c + d*x)), x)
```

# 3.172 $\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\sin(e+fx))^2},x\right)$$

[Out] Unintegrable(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x)

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2),x]

[Out] Defer[Int][1/((c + d*x)^2*(a + b*Sin[e + f*x])^2), x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$$

**Mathematica** [A]
time = 69.62, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2),x]

[Out] Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2), x]

**Maple** [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+b\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*cos(2*f*x + 2*e)*cos(f*x + e) + 2*a*b*cos(f*x + e) - ((a^2*b^2 - b^4
)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + ((a^2*b^2
 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f)*cos(
2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^2)*c*d*f*x +
 (a^4 - a^2*b^2)*c^2*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(
a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*cos(f*x + e)*sin(2*f*x + 2*
e) + ((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^
4)*c^2*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*
b^2)*c*d*f*x + (a^4 - a^2*b^2)*c^2*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d
^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + 2*((a^3*b -
a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f
*x + e))*cos(2*f*x + 2*e) + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3
)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f*x + e))*integrate(-2*(2*a*b*d*cos(
f*x + e) + 2*(a^2*d*f*x + a^2*c*f)*cos(f*x + e)^2 + 2*(a^2*d*f*x + a^2*c*f)
*sin(f*x + e)^2 + (2*a*b*d*cos(f*x + e) - (a*b*d*f*x + a*b*c*f)*sin(f*x + e
))*cos(2*f*x + 2*e) + (2*a*b*d*sin(f*x + e) + 2*b^2*d + (a*b*d*f*x + a*b*c*
f)*cos(f*x + e))*sin(2*f*x + 2*e) + (a*b*d*f*x + a*b*c*f)*sin(f*x + e))/((a
^2*b^2 - b^4)*d^3*f*x^3 + 3*(a^2*b^2 - b^4)*c*d^2*f*x^2 + 3*(a^2*b^2 - b^4)
*c^2*d*f*x + (a^2*b^2 - b^4)*c^3*f + ((a^2*b^2 - b^4)*d^3*f*x^3 + 3*(a^2*b^
2 - b^4)*c*d^2*f*x^2 + 3*(a^2*b^2 - b^4)*c^2*d*f*x + (a^2*b^2 - b^4)*c^3*f)
*cos(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^3*f*x^3 + 3*(a^4 - a^2*b^2)*c*d^
2*f*x^2 + 3*(a^4 - a^2*b^2)*c^2*d*f*x + (a^4 - a^2*b^2)*c^3*f)*cos(f*x + e)
^2 + 4*((a^3*b - a*b^3)*d^3*f*x^3 + 3*(a^3*b - a*b^3)*c*d^2*f*x^2 + 3*(a^3*
b - a*b^3)*c^2*d*f*x + (a^3*b - a*b^3)*c^3*f)*cos(f*x + e)*sin(2*f*x + 2*e)
 + ((a^2*b^2 - b^4)*d^3*f*x^3 + 3*(a^2*b^2 - b^4)*c*d^2*f*x^2 + 3*(a^2*b^2
- b^4)*c^2*d*f*x + (a^2*b^2 - b^4)*c^3*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^
2*b^2)*d^3*f*x^3 + 3*(a^4 - a^2*b^2)*c*d^2*f*x^2 + 3*(a^4 - a^2*b^2)*c^2*d*
f*x + (a^4 - a^2*b^2)*c^3*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d^3*f*x^3
+ 3*(a^2*b^2 - b^4)*c*d^2*f*x^2 + 3*(a^2*b^2 - b^4)*c^2*d*f*x + (a^2*b^2 -
b^4)*c^3*f + 2*((a^3*b - a*b^3)*d^3*f*x^3 + 3*(a^3*b - a*b^3)*c*d^2*f*x^2 +
 3*(a^3*b - a*b^3)*c^2*d*f*x + (a^3*b - a*b^3)*c^3*f)*sin(f*x + e))*cos(2*f
*x + 2*e) + 4*((a^3*b - a*b^3)*d^3*f*x^3 + 3*(a^3*b - a*b^3)*c*d^2*f*x^2 +
3*(a^3*b - a*b^3)*c^2*d*f*x + (a^3*b - a*b^3)*c^3*f)*sin(f*x + e)), x) + 2*
(a*b*sin(f*x + e) + b^2)*sin(2*f*x + 2*e))/((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(
```

```
a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + ((a^2*b^2 - b^4)*d^2*f*x^2
 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f)*cos(2*f*x + 2*e)^2 +
4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^2)*c*d*f*x + (a^4 - a^2*b^2)*
c^2*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*
d*f*x + (a^3*b - a*b^3)*c^2*f)*cos(f*x + e)*sin(2*f*x + 2*e) + ((a^2*b^2 -
b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f)*sin(2*f
*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^2)*c*d*f*x + (a
^4 - a^2*b^2)*c^2*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2
*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + 2*((a^3*b - a*b^3)*d^2*f*x^2
+ 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f*x + e))*cos(2*f*
x + 2*e) + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*
b - a*b^3)*c^2*f)*sin(f*x + e))
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2 + b^2)*d^2*x^2 + 2*(a^2 + b^2)*c*d*x + (a^2 + b^2)*c^2 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*
b*c*d*x + a*b*c^2)*sin(f*x + e)), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*sin(f*x + e) + a)^2), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\sin\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*x)^2),x)
[Out] int(1/((a + b*sin(e + f*x))^2*(c + d*x)^2), x)
```

# 3.173 $\int (c+dx)^m(a+b\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=23

$$\text{Int}((c+dx)^m(a+b\sin(e+fx))^n,x)$$

```
[Out] Unintegrable((d*x+c)^m*(a+b*sin(f*x+e))^n,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + b*Sin[e + f*x])^n,x]
[Out] Defer[Int][(c + d*x)^m*(a + b*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx = \int (c+dx)^m(a+b\sin(e+fx))^n\,dx$$

**Mathematica** [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^n,x]
[Out] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x]
```

**Maple** [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,(a+b\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sin(f*x+e))^n,x)
[Out] int((d*x+c)^m*(a+b*sin(f*x+e))^n,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(b*sin(f*x + e) + a)^n, x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(b*sin(f*x + e) + a)^n, x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sin(f*x+e))**n,x)
[Out] Timed out
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(b*sin(f*x + e) + a)^n, x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + b \sin(e + f x))^n (c + d x)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^n*(c + d*x)^m,x)
[Out] int((a + b*sin(e + f*x))^n*(c + d*x)^m, x)
```

# 3.174 $\int (c+dx)^m (a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=607

$$\frac{a^3(c+dx)^{1+m}}{d(1+m)}+\frac{3ab^2(c+dx)^{1+m}}{2d(1+m)}-\frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{3b^3e^{i\left(e-\frac{cf}{d}\right)}(}{}$$

```
[Out] a^3*(d*x+c)^(1+m)/d/(1+m)+3/2*a*b^2*(d*x+c)^(1+m)/d/(1+m)-3/2*a^2*b*exp(I*(
e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-3/8*b^
3*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^
m)-3/2*a^2*b*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)/exp(I*(e-c*f/d))/f/((I*f*(d
*x+c)/d)^m)-3/8*b^3*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)/exp(I*(e-c*f/d))/f/(
(I*f*(d*x+c)/d)^m)+3*I*2^(-3-m)*a*b^2*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+
m,-2*I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-3*I*2^(-3-m)*a*b^2*(d*x+c)^m*GAM
MA(1+m,2*I*f*(d*x+c)/d)/exp(2*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+1/8*3^(-1-
m)*b^3*exp(3*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-3*I*f*(d*x+c)/d)/f/((-I*f*(d
*x+c)/d)^m)+1/8*3^(-1-m)*b^3*(d*x+c)^m*GAMMA(1+m,3*I*f*(d*x+c)/d)/exp(3*I*(
e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

**Rubi** [A]
time = 0.54, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3398, 3389, 2212, 3393, 3388}

$$-\frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{3a^2be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{2f}+\frac{3iab^2 2^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{f}-\frac{3iab^2 2^{-m-3}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2if(c+dx)}{d}\right)}{f}-\frac{3b^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{8f}+\frac{b^3 3^{-m-1}e^{3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3if(c+dx)}{d}\right)}{8f}-\frac{3b^3e^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{8f}+\frac{b^3 3^{-m-1}e^{-3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3if(c+dx)}{d}\right)}{8f}+\frac{a^3(c+dx)^{m+1}}{d(m+1)}+\frac{3ab^2(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1 +
 m)) - (3*a^2*b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*f*(c + d
*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (3*b^3*E^(I*(e - (c*f)/d))*(c + d
*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) -
(3*a^2*b*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d)
)*f*((I*f*(c + d*x))/d)^m) - (3*b^3*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x)
)/d])/(8*E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + ((3*I)*2^(-3 - m)*a
*b^2*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/
d])/(f*(((-I)*f*(c + d*x))/d)^m) - ((3*I)*2^(-3 - m)*a*b^2*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*f*(c + d*x))/d])/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x)
)/d)^m) + (3^(-1 - m)*b^3*E^((3*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m,
((-3*I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) + (3^(-1 - m)*b^3*(
c + d*x)^m*Gamma[1 + m, ((3*I)*f*(c + d*x))/d])/(8*E^((3*I)*(e - (c*f)/d))*
f*((I*f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^m (a+b\sin(e+fx))^3\,dx = \int \left(a^3(c+dx)^m + 3a^2b(c+dx)^m\sin(e+fx) + 3ab^2(c+dx)^m\sin^2(e+\right.$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \left(3a^2b\right)\int (c+dx)^m \sin(e+fx)\,dx + \left(3ab^2\right)\int (c+d$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}\left(3ia^2b\right)\int e^{-i(e+fx)}(c+dx)^m\,dx - \frac{1}{2}\left(3ia^2b\right)\int e^{i(e+}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} - \frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right.}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} - \frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right.}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} - \frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right.}{2f}$$

**Mathematica** [**A**]
time = 11.85, size = 552, normalized size = 0.91

$$\frac{i(c+dx)^m\left(-24ia^3f(c+dx)-36iab^2f(c+dx)+36ia^2bde^{i\left(e-\frac{cf}{d}\right)}(1+m)\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)+9ib^3de^{i\left(e-\frac{cf}{d}\right)}(1+m)\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)+36ia^2bde^{-i\left(e-\frac{cf}{d}\right)}(1+m)\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)+9ib^3de^{-i\left(e-\frac{cf}{d}\right)}(1+m)\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)+9\,2^{-m}ab^2de^{2i\left(e-\frac{cf}{d}\right)}(1+m)\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)-9\,2^{-m}ab^2de^{-2i\left(e-\frac{cf}{d}\right)}(1+m)\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)-i3^{-m}b^3de^{3i\left(e-\frac{cf}{d}\right)}(1+m)\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3if(c+dx)}{d}\right)-i3^{-m}b^3de^{-3i\left(e-\frac{cf}{d}\right)}(1+m)\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3if(c+dx)}{d}\right)\right)}{24df(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((I/24)*(c + d*x)^m*((-24*I)*a^3*f*(c + d*x) - (36*I)*a*b^2*f*(c + d*x) + (
(36*I)*a^2*b*d*E^(I*(e - (c*f)/d))*(1 + m)*Gamma[1 + m, ((-I)*f*(c + d*x))/
d])/(((-I)*f*(c + d*x))/d)^m + ((9*I)*b^3*d*E^(I*(e - (c*f)/d))*(1 + m)*Gam
ma[1 + m, ((-I)*f*(c + d*x))/d])/(((-I)*f*(c + d*x))/d)^m + ((36*I)*a^2*b*d
*(1 + m)*Gamma[1 + m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*((I*f*(c + d
*x))/d)^m) + ((9*I)*b^3*d*(1 + m)*Gamma[1 + m, (I*f*(c + d*x))/d])/(E^(I*(e
 - (c*f)/d))*((I*f*(c + d*x))/d)^m) + (9*a*b^2*d*E^((2*I)*(e - (c*f)/d))*(1
 + m)*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m*(((-I)*f*(c + d*x))/d)^m)
- (9*a*b^2*d*(1 + m)*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(2^m*E^((2*I)*(e
- (c*f)/d))*((I*f*(c + d*x))/d)^m) - (I*b^3*d*E^((3*I)*(e - (c*f)/d))*(1 +
m)*Gamma[1 + m, ((-3*I)*f*(c + d*x))/d])/(3^m*(((-I)*f*(c + d*x))/d)^m) - (
I*b^3*d*(1 + m)*Gamma[1 + m, ((3*I)*f*(c + d*x))/d])/(3^m*E^((3*I)*(e - (c*
f)/d))*((I*f*(c + d*x))/d)^m)))/(d*f*(1 + m))
```

**Maple** [**F**]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\left(a+b\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+b*sin(f*x+e))^3,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(6*a*b^2*e^(m*log(d*x + c) + log(d*
x + c)) - 6*(a*b^2*d*m + a*b^2*d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x
) - (b^3*d*m + b^3*d)*integrate((d*x + c)^m*sin(3*f*x + 3*e), x) + 3*((4*a^
2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*integrate((d*x + c)^m*sin(f*x + e), x))
/(d*m + d)
```

**Fricas** [A]
time = 0.12, size = 442, normalized size = 0.73

$$-\frac{9\left((4a^2b+b^3)dm+(4a^2b+b^3)d\right)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)-icf+ide}{d}\right)}\Gamma\left(m+1,\frac{idfx+icf}{d}\right)+9\left(-iab^2dm-iab^2d\right)e^{\left(-\frac{dm\log\left(-\frac{2if}{d}\right)+2icf-2ide}{d}\right)}\Gamma\left(m+1,-\frac{2(idfx+icf)}{d}\right)-(b^3dm+b^3d)e^{\left(-\frac{dm\log\left(-\frac{3if}{d}\right)+3icf-3ide}{d}\right)}\Gamma\left(m+1,-\frac{3(idfx+icf)}{d}\right)+9\left((4a^2b+b^3)dm+(4a^2b+b^3)d\right)e^{\left(-\frac{dm\log\left(-\frac{if}{d}\right)+icf-ide}{d}\right)}\Gamma\left(m+1,\frac{-idfx-icf}{d}\right)+9\left(iab^2dm+iab^2d\right)e^{\left(-\frac{dm\log\left(\frac{2if}{d}\right)-2icf+2ide}{d}\right)}\Gamma\left(m+1,-\frac{2(-idfx-icf)}{d}\right)-(b^3dm+b^3d)e^{\left(-\frac{dm\log\left(\frac{3if}{d}\right)-3icf+3ide}{d}\right)}\Gamma\left(m+1,-\frac{3(-idfx-icf)}{d}\right)-12\left((2a^3+3ab^2)dfx+(2a^3+3ab^2)cf\right)(dx+c)^m}{24\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/24*(9*((4*a^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*e^(-(d*m*log(I*f/d) - I*
c*f + I*d*e)/d)*gamma(m + 1, (I*d*f*x + I*c*f)/d) + 9*(-I*a*b^2*d*m - I*a*b
^2*d)*e^(-(d*m*log(-2*I*f/d) + 2*I*c*f - 2*I*d*e)/d)*gamma(m + 1, -2*(I*d*f
*x + I*c*f)/d) - (b^3*d*m + b^3*d)*e^(-(d*m*log(-3*I*f/d) + 3*I*c*f - 3*I*d
*e)/d)*gamma(m + 1, -3*(I*d*f*x + I*c*f)/d) + 9*((4*a^2*b + b^3)*d*m + (4*a
^2*b + b^3)*d)*e^(-(d*m*log(-I*f/d) + I*c*f - I*d*e)/d)*gamma(m + 1, (-I*d*
f*x - I*c*f)/d) + 9*(I*a*b^2*d*m + I*a*b^2*d)*e^(-(d*m*log(2*I*f/d) - 2*I*c
*f + 2*I*d*e)/d)*gamma(m + 1, -2*(-I*d*f*x - I*c*f)/d) - (b^3*d*m + b^3*d)*
e^(-(d*m*log(3*I*f/d) - 3*I*c*f + 3*I*d*e)/d)*gamma(m + 1, -3*(-I*d*f*x - I
*c*f)/d) - 12*((2*a^3 + 3*a*b^2)*d*f*x + (2*a^3 + 3*a*b^2)*c*f)*(d*x + c)^m
)/(d*f*m + d*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(e + fx))^3 (c + dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*(c + d*x)**m, x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*x + c)^m, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin\left(e+f\,x\right))^3\,{\left(c+d\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*x)^m,x)
```

```
[Out] int((a + b*sin(e + f*x))^3*(c + d*x)^m, x)
```

# 3.175 $\int (c+dx)^m (a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=318

$$\frac{a^2(c+dx)^{1+m}}{d(1+m)}+\frac{b^2(c+dx)^{1+m}}{2d(1+m)}-\frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f}-\frac{abe^{-i\left(e-\frac{cf}{d}\right)}(c+$$

```
[Out] a^2*(d*x+c)^(1+m)/d/(1+m)+1/2*b^2*(d*x+c)^(1+m)/d/(1+m)-a*b*exp(I*(e-c*f/d)
)*(d*x+c)^m*GAMMA(1+m,-I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-a*b*(d*x+c)^m*
GAMMA(1+m,I*f*(d*x+c)/d)/exp(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+I*2^(-3-m)*
b^2*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-2*I*f*(d*x+c)/d)/f/((-I*f*(d*x+
c)/d)^m)-I*2^(-3-m)*b^2*(d*x+c)^m*GAMMA(1+m,2*I*f*(d*x+c)/d)/exp(2*I*(e-c*f
/d))/f/((I*f*(d*x+c)/d)^m)
```

**Rubi** [A]
time = 0.26, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3398, 3389, 2212, 3393, 3388}

$$-\frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{f}-\frac{abe^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{f}+\frac{ib^2 2^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{f}-\frac{ib^2 2^{-m-3}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2if(c+dx)}{d}\right)}{f}+\frac{a^2(c+dx)^{m+1}}{d(m+1)}+\frac{b^2(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m))
 - (a*b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])
/(f*(((-I)*f*(c + d*x))/d)^m) - (a*b*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x
))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (I*2^(-3 - m)*b^2*E^
((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*
(((-I)*f*(c + d*x))/d)^m) - (I*2^(-3 - m)*b^2*(c + d*x)^m*Gamma[1 + m, ((2*
I)*f*(c + d*x))/d])/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^m(a+b\sin(e+fx))^2\,dx &= \int \left(a^2(c+dx)^m + 2ab(c+dx)^m\sin(e+fx) + b^2(c+dx)^m\sin^2(e+fx\right. \\
&= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (2ab)\int (c+dx)^m\sin(e+fx)\,dx + b^2\int (c+dx)^m \sin \\
&= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (iab)\int e^{-i(e+fx)}(c+dx)^m\,dx - (iab)\int e^{i(e+fx)}(c+ \\
&= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} - \frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}}{f} \\
&= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} - \frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}}{f} \\
&= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} - \frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}}{f}
\end{aligned}
$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 707 vs. 2(318) = 636.
time = 9.48, size = 707, normalized size = 2.22

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (2^(-3 - m)*(c + d*x)^m*(2^(3 + m)*a^2*c*f*((f^2*(c + d*x)^2)/d^2)^m + 2^(2
 + m)*b^2*c*f*((f^2*(c + d*x)^2)/d^2)^m + 2^(3 + m)*a^2*d*f*x*((f^2*(c + d*
x)^2)/d^2)^m + 2^(2 + m)*b^2*d*f*x*((f^2*(c + d*x)^2)/d^2)^m + I*b^2*d*((I*
f*(c + d*x))/d)^m*Cos[2*e - (2*c*f)/d]*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d]
 + I*b^2*d*m*((I*f*(c + d*x))/d)^m*Cos[2*e - (2*c*f)/d]*Gamma[1 + m, ((-2*I
)*f*(c + d*x))/d] - I*b^2*d*(((-I)*f*(c + d*x))/d)^m*Cos[2*e - (2*c*f)/d]*G
amma[1 + m, ((2*I)*f*(c + d*x))/d] - I*b^2*d*m*(((-I)*f*(c + d*x))/d)^m*Cos
[2*e - (2*c*f)/d]*Gamma[1 + m, ((2*I)*f*(c + d*x))/d] - b^2*d*((I*f*(c + d*
x))/d)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d]*Sin[2*e - (2*c*f)/d] - b^2*d*
m*((I*f*(c + d*x))/d)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d]*Sin[2*e - (2*c
*f)/d] - b^2*d*(((-I)*f*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d]
*Sin[2*e - (2*c*f)/d] - b^2*d*m*(((-I)*f*(c + d*x))/d)^m*Gamma[1 + m, ((2*I
)*f*(c + d*x))/d]*Sin[2*e - (2*c*f)/d] - 2^(3 + m)*a*b*d*(1 + m)*(((-I)*f*(
c + d*x))/d)^m*Gamma[1 + m, (I*f*(c + d*x))/d]*(Cos[e - (c*f)/d] - I*Sin[e
- (c*f)/d]) - 2^(3 + m)*a*b*d*(1 + m)*((I*f*(c + d*x))/d)^m*Gamma[1 + m, ((
-I)*f*(c + d*x))/d]*(Cos[e - (c*f)/d] + I*Sin[e - (c*f)/d])))/(d*f*(1 + m)*
((f^2*(c + d*x)^2)/d^2)^m)
```

**Maple [F]**
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m} \left(a+b\sin\left(fx+e\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+b*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^2/(d*(m + 1)) + 1/2*(b^2*e^(m*log(d*x + c) + log(d*x +
c)) - (b^2*d*m + b^2*d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) + 4*(a*b
*d*m + a*b*d)*integrate((d*x + c)^m*sin(f*x + e), x))/(d*m + d)
```

**Fricas [A]**
time = 0.10, size = 282, normalized size = 0.89

$$-\frac{8\,(abdm+abd)e^{\left(-\frac{dm\log\left(\frac{i\,f}{d}\right)-i\,cf+i\,de}{d}\right)}\Gamma\left(m+1,\frac{i\,dfx+i\,cf}{d}\right)-(i\,b^2dm+i\,b^2d)e^{\left(-\frac{dm\log\left(-\frac{2i\,f}{d}\right)+2i\,cf-2i\,de}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,dfx+i\,cf)}{d}\right)+8\,(abdm+abd)e^{\left(-\frac{dm\log\left(-\frac{i\,f}{d}\right)+i\,cf-i\,de}{d}\right)}\Gamma\left(m+1,\frac{-i\,dfx-i\,cf}{d}\right)-(-i\,b^2dm-i\,b^2d)e^{\left(-\frac{dm\log\left(\frac{2i\,f}{d}\right)-2i\,cf+2i\,de}{d}\right)}\Gamma\left(m+1,-\frac{2\,(-i\,dfx-i\,cf)}{d}\right)-4\,((2\,a^2+b^2)dfx+(2\,a^2+b^2)cf)(dx+c)^m}{8\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*(a*b*d*m + a*b*d)*e^(-(d*m*log(I*f/d) - I*c*f + I*d*e)/d)*gamma(m +
 1, (I*d*f*x + I*c*f)/d) - (I*b^2*d*m + I*b^2*d)*e^(-(d*m*log(-2*I*f/d) + 2
*I*c*f - 2*I*d*e)/d)*gamma(m + 1, -2*(I*d*f*x + I*c*f)/d) + 8*(a*b*d*m + a*
b*d)*e^(-(d*m*log(-I*f/d) + I*c*f - I*d*e)/d)*gamma(m + 1, (-I*d*f*x - I*c*
f)/d) - (-I*b^2*d*m - I*b^2*d)*e^(-(d*m*log(2*I*f/d) - 2*I*c*f + 2*I*d*e)/d
)*gamma(m + 1, -2*(-I*d*f*x - I*c*f)/d) - 4*((2*a^2 + b^2)*d*f*x + (2*a^2 +
 b^2)*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(e + fx))^2 (c + dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*(c + d*x)**m, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*x + c)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\sin(e + f\,x))^2\,(c + d\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*x)^m,x)
```

```
[Out] int((a + b*sin(e + f*x))^2*(c + d*x)^m, x)
```

# 3.176 $\int (c+dx)^m(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=148

$$\frac{a(c+dx)^{1+m}}{d(1+m)}-\frac{be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}}{2f}$$

```
[Out] a*(d*x+c)^(1+m)/d/(1+m)-1/2*b*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*f*(d*
x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-1/2*b*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)/exp
(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

**Rubi** [A]

time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3389, 2212}

$$-\frac{be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{2f}+\frac{a(c+dx)^{m+1}}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) - (b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamm
a[1 + m, ((-I)*f*(c + d*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (b*(c + d*
x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d))*f*((I*f*(c + d
*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^m (a+b\sin(e+fx))\,dx &= \int \left(a(c+dx)^m + b(c+dx)^m \sin(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^{1+m}}{d(1+m)} + b\int (c+dx)^m \sin(e+fx)\,dx \\
&= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}(ib)\int e^{-i(e+fx)}(c+dx)^m\,dx - \frac{1}{2}(ib)\int e^{i(e+fx)}(c+d \\
&= \frac{a(c+dx)^{1+m}}{d(1+m)} - \frac{be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{if(c+dx)}{d}\right.}{2f}
\end{aligned}
$$

**Mathematica [A]**
time = 2.23, size = 166, normalized size = 1.12

$$\frac{(c+dx)^m \left(2acf+2adfx-bd(1+m)\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)\left(\cos\left(e-\frac{cf}{d}\right)-i\sin\left(e-\frac{cf}{d}\right)\right)-bd(1+m)\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)\left(\cos\left(e-\frac{cf}{d}\right)+i\sin\left(e-\frac{cf}{d}\right)\right)\right)}{2df(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x]),x]
```

```
[Out] ((c + d*x)^m*(2*a*c*f + 2*a*d*f*x - (b*d*(1 + m)*Gamma[1 + m, (I*f*(c + d*x
))/d]*(Cos[e - (c*f)/d] - I*Sin[e - (c*f)/d]))/((I*f*(c + d*x))/d)^m - (b*d
*(1 + m)*Gamma[1 + m, ((-I)*f*(c + d*x))/d]*(Cos[e - (c*f)/d] + I*Sin[e - (
c*f)/d]))/(((-I)*f*(c + d*x))/d)^m))/(2*d*f*(1 + m))
```

**Maple [F]**
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx+c)^m (a+b\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+b*sin(f*x+e)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] b*integrate((d*x + c)^m*sin(f*x + e), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

**Fricas [A]**
time = 0.10, size = 138, normalized size = 0.93

$$-\frac{(bdm+bd)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)-icf+ide}{d}\right)}\Gamma\left(m+1,\frac{idfx+icf}{d}\right)+(bdm+bd)e^{\left(-\frac{dm\log\left(-\frac{if}{d}\right)+icf-ide}{d}\right)}\Gamma\left(m+1,\frac{-idfx-icf}{d}\right)-2\,(adfx+acf)(dx+c)^m}{2\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] -1/2*((b*d*m + b*d)*e^(-(d*m*log(I*f/d) - I*c*f + I*d*e)/d)*gamma(m + 1, (I
*d*f*x + I*c*f)/d) + (b*d*m + b*d)*e^(-(d*m*log(-I*f/d) + I*c*f - I*d*e)/d)
*gamma(m + 1, (-I*d*f*x - I*c*f)/d) - 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(d*f
*m + d*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin{(e+fx)})\,(c+dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sin(f*x+e)),x)
[Out] Integral((a + b*sin(e + f*x))*(c + d*x)**m, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)*(d*x + c)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\sin\left(e+f\,x\right))\,{\left(c+d\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*x)^m,x)
[Out] int((a + b*sin(e + f*x))*(c + d*x)^m, x)
```

# 3.177 $\int \frac{(c+dx)^m}{a+b\sin(e+fx)} \, dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{a+b\sin(e+fx)}, x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+b*sin(f*x+e)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Sin[e + f*x]),x]
[Out] Defer[Int][(c + d*x)^m/(a + b*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)} \, dx = \int \frac{(c+dx)^m}{a+b\sin(e+fx)} \, dx$$

**Mathematica** [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Sin[e + f*x]),x]
[Out] Integrate[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
```

**Maple** [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+b\sin(fx+e)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*sin(f*x+e)),x)
[Out] int((d*x+c)^m/(a+b*sin(f*x+e)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(b*sin(f*x + e) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((d*x + c)^m/(b*sin(f*x + e) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+b\sin{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*sin(f*x+e)),x)
[Out] Integral((c + d*x)**m/(a + b*sin(e + f*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^m/(b*sin(f*x + e) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(c+d\,x\right)}^m}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + b*sin(e + f*x)),x)
[Out] int((c + d*x)^m/(a + b*sin(e + f*x)), x)
```

# 3.178 $\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=23

$$\operatorname{Int}\left(\frac{(c+dx)^m}{(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+b*sin(f*x+e))^2,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Sin[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$$

**Mathematica** [A]
time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Sin[e + f*x])^2, x]
```

**Maple** [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a+b\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+b*sin(f*x+e))^2,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(b*sin(f*x + e) + a)^2, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*x + c)^m/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)
, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+b\sin{(e+fx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**m/(a + b*sin(e + f*x))**2, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(b*sin(f*x + e) + a)^2, x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{(a+b\,\sin\left(e+f\,x\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + b*sin(e + f*x))^2,x)
[Out] int((c + d*x)^m/(a + b*sin(e + f*x))^2, x)
```

# 3.179 $\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=164

$$\frac{i(e+fx)^3}{ad}+\frac{(e+fx)^4}{4af}+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{12if^2(e+fx)\text{Li}_2\left(ie^{i(c}{ad^3}$$

```
[Out] I*(f*x+e)^3/a/d+1/4*(f*x+e)^4/a/f+(f*x+e)^3*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-6
*f*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a/d^2+12*I*f^2*(f*x+e)*polylog(2,I*exp(
I*(d*x+c)))/a/d^3-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4
```

**Rubi** [A]
time = 0.22, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4611, 32, 3399, 4269, 3798, 2221, 2611, 2320, 6724}

$$-\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}+\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}+\frac{i(e+fx)^3}{ad}+\frac{(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (I*(e + f*x)^3)/(a*d) + (e + f*x)^4/(4*a*f) + ((e + f*x)^3*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) - (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + (
(12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - (12*f^3*PolyL
og[3, I*E^(I*(c + d*x))])/(a*d^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^3\,dx}{a} - \int \frac{(e+fx)^3}{a+a\sin(c+dx)}\,dx \\
&= \frac{(e+fx)^4}{4af} - \frac{\int (e+fx)^3\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)\,dx}{2a} \\
&= \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(3f)\int (e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\, d}{ad} \\
&= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(6f)\int \frac{e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}(e+fx)^2}{1-ie^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}}{ad} \\
&= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2\log\left(1-\right.}{ad^2} \\
&= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2\log\left(1-\right.}{ad^2} \\
&= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2\log\left(1-\right.}{ad^2} \\
&= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2\log\left(1-\right.}{ad^2}
\end{aligned}
$$

**Mathematica [A]**
time = 1.19, size = 240, normalized size = 1.46

$$
\frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)+\frac{8f\left(-3d^2(e+fx)^2\log(1-i\cos(c+dx)+\sin(c+dx))+6idf(e+fx)\text{Li}_2(i\cos(c+dx)-\sin(c+dx))-6f^2\text{Li}_3(i\cos(c+dx)-\sin(c+dx))+\frac{id^3x(3e^2+3efx+f^2x^2)(\cos(c)+i\sin(c))}{\cos(c)+i(1+\sin(c))}\right)}{d^4}-\frac{8(e+fx)^3\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + (8*f*(-3*d^2*(e + f*x)^2*L
og[1 - I*Cos[c + d*x] + Sin[c + d*x]] + (6*I)*d*f*(e + f*x)*PolyLog[2, I*Co
s[c + d*x] - Sin[c + d*x]] - 6*f^2*PolyLog[3, I*Cos[c + d*x] - Sin[c + d*x]
] + (I*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*(Cos[c] + I*Sin[c]))/(Cos[c] + I*(
1 + Sin[c]))))/d^4 - (8*(e + f*x)^3*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(4*a)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(145) = 290$.
time = 0.17, size = 537, normalized size = 3.27

| method | result |
| --- | --- |
| risch | $\frac{f^3x^4}{4a}+\frac{e^4}{4af}-\frac{6if^3c^2x}{a\,d^3}-\frac{6f^3\ln\left(1-ie^{i(dx+c)}\right)x^2}{a\,d^2}+\frac{6f^3\ln\left(1-ie^{i(dx+c)}\right)c^2}{a\,d^4}+\frac{6f\ln\left(e^{i(dx+c)}\right)e^2}{a\,d^2}+\frac{6f^3c^2\ln\left(e^{i(dx+c)}\right)}{a\,d^4}-6$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*f^3*x^4+1/4/a/f*e^4-6/a/d^2*f^3*ln(1-I*exp(I*(d*x+c)))*x^2+6/a/d^4*f^
3*ln(1-I*exp(I*(d*x+c)))*c^2+6/a/d^2*f*ln(exp(I*(d*x+c)))*e^2+6/a/d^4*f^3*c
^2*ln(exp(I*(d*x+c)))-6/a/d^4*f^3*c^2*ln(exp(I*(d*x+c))+I)-6/a/d^2*f*ln(exp
(I*(d*x+c))+I)*e^2+2*I/a/d*f^3*x^3-4*I/a/d^4*f^3*c^3+12*I/a/d^2*f^2*e*c*x+1
/a*f^2*e*x^3+3/2/a*f*e^2*x^2+1/a*e^3*x-12/a/d^3*f^2*e*c*ln(exp(I*(d*x+c)))+
12*I/a/d^3*f^3*polylog(2,I*exp(I*(d*x+c)))*x-6*I/a/d^3*f^3*c^2*x+6*I/a/d*f^
2*e*x^2+6*I/a/d^3*f^2*e*c^2+12*I/a/d^3*f^2*e*polylog(2,I*exp(I*(d*x+c)))+2*
(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(I*(d*x+c))+I)-12*f^3*polylog(3
,I*exp(I*(d*x+c)))/a/d^4+12/a/d^3*f^2*e*c*ln(exp(I*(d*x+c))+I)-12/a/d^2*f^2
*e*ln(1-I*exp(I*(d*x+c)))*x-12/a/d^3*f^2*e*ln(1-I*exp(I*(d*x+c)))*c
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(144) = 288$.
time = 0.63, size = 1316, normalized size = 8.02

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(12*c^2*f^2*(1/(a*d^2 + a*d^2*sin(d*x + c)/(cos(d*x + c) + 1)) + arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^2))*e - 12*c*f*(1/(a*d + a*d*sin(d*x
 + c)/(cos(d*x + c) + 1)) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d))*
e^2 - 6*((d*x + c)^2*cos(d*x + c)^2 + (d*x + c)^2*sin(d*x + c)^2 + 2*(d*x +
 c)^2*sin(d*x + c) + (d*x + c)^2 + 4*(d*x + c)*cos(d*x + c) - 2*(cos(d*x +
c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)*log(cos(d*x + c)^2 + sin(d*x +
c)^2 + 2*sin(d*x + c) + 1))*c*f^2*e/(a*d^2*cos(d*x + c)^2 + a*d^2*sin(d*x +
 c)^2 + 2*a*d^2*sin(d*x + c) + a*d^2) + 4*(arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a + 1/(a + a*sin(d*x + c)/(cos(d*x + c) + 1)))*e^3 + 3*((d*x + c)^2
*cos(d*x + c)^2 + (d*x + c)^2*sin(d*x + c)^2 + 2*(d*x + c)^2*sin(d*x + c) +
 (d*x + c)^2 + 4*(d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^2 + sin(d*x + c)^
2 + 2*sin(d*x + c) + 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c
) + 1))*f*e^2/(a*d*cos(d*x + c)^2 + a*d*sin(d*x + c)^2 + 2*a*d*sin(d*x + c)
 + a*d) + 2*((d*x + c)^4*f^3 + 6*(d*x + c)^2*c^2*f^3 - 4*(d*x + c)*c^3*f^3
+ 8*I*c^3*f^3 - 4*(c*f^3 - d*f^2*e)*(d*x + c)^3 - 24*(c^2*f^3*cos(d*x + c)
+ I*c^2*f^3*sin(d*x + c) + I*c^2*f^3)*arctan2(sin(d*x + c) + 1, cos(d*x + c
)) + 24*(I*(d*x + c)^2*f^3 + 2*(-I*c*f^3 + I*d*f^2*e)*(d*x + c) + ((d*x + c
```

```
)^2*f^3 - 2*(c*f^3 - d*f^2*e)*(d*x + c))*cos(d*x + c) + (I*(d*x + c)^2*f^3
+ 2*(-I*c*f^3 + I*d*f^2*e)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c), s
in(d*x + c) + 1) - (I*(d*x + c)^4*f^3 - 4*(I*c^3 + 6*c^2)*(d*x + c)*f^3 - 4
*((I*c + 2)*f^3 - I*d*f^2*e)*(d*x + c)^3 - 6*((-I*c^2 - 4*c)*f^3 + 4*d*f^2*
e)*(d*x + c)^2)*cos(d*x + c) + 48*(I*(d*x + c)*f^3 - I*c*f^3 + I*d*f^2*e +
((d*x + c)*f^3 - c*f^3 + d*f^2*e)*cos(d*x + c) + (I*(d*x + c)*f^3 - I*c*f^3
 + I*d*f^2*e)*sin(d*x + c))*dilog(I*e^(I*d*x + I*c)) - 12*((d*x + c)^2*f^3
+ c^2*f^3 - 2*(c*f^3 - d*f^2*e)*(d*x + c) - (I*(d*x + c)^2*f^3 + I*c^2*f^3
+ 2*(-I*c*f^3 + I*d*f^2*e)*(d*x + c))*cos(d*x + c) + ((d*x + c)^2*f^3 + c^2
*f^3 - 2*(c*f^3 - d*f^2*e)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + si
n(d*x + c)^2 + 2*sin(d*x + c) + 1) + 48*(I*f^3*cos(d*x + c) - f^3*sin(d*x +
 c) - f^3)*polylog(3, I*e^(I*d*x + I*c)) + ((d*x + c)^4*f^3 - 4*(c^3 - 6*I*
c^2)*(d*x + c)*f^3 - 4*((c - 2*I)*f^3 - d*f^2*e)*(d*x + c)^3 + 6*((c^2 - 4*
I*c)*f^3 + 4*I*d*f^2*e)*(d*x + c)^2)*sin(d*x + c))/(-4*I*a*d^3*cos(d*x + c)
 + 4*a*d^3*sin(d*x + c) + 4*a*d^3))/d
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(144) = 288$.
time = 0.39, size = 1051, normalized size = 6.41

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^4*f^3*x^4 + 4*d^3*f^3*x^3 + (d^4*f^3*x^4 + 4*d^3*f^3*x^3 + 4*(d^4*x
+ d^3)*e^3 + 6*(d^4*f*x^2 + 2*d^3*f*x)*e^2 + 4*(d^4*f^2*x^3 + 3*d^3*f^2*x^2
)*e)*cos(d*x + c) - 24*(-I*d*f^3*x - I*d*f^2*e + (-I*d*f^3*x - I*d*f^2*e)*c
os(d*x + c) + (-I*d*f^3*x - I*d*f^2*e)*sin(d*x + c))*dilog(I*cos(d*x + c) -
 sin(d*x + c)) - 24*(I*d*f^3*x + I*d*f^2*e + (I*d*f^3*x + I*d*f^2*e)*cos(d*
x + c) + (I*d*f^3*x + I*d*f^2*e)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(
d*x + c)) + 4*(d^4*x + d^3)*e^3 + 6*(d^4*f*x^2 + 2*d^3*f*x)*e^2 + 4*(d^4*f^
2*x^3 + 3*d^3*f^2*x^2)*e - 12*(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + (c^2*f^3
 - 2*c*d*f^2*e + d^2*f*e^2)*cos(d*x + c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e
^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 12*(d^2*f^3*x^2
- c^2*f^3 + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*x + c
) + 2*(d^2*f^2*x + c*d*f^2)*e + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d
*f^2)*e)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 12*(d^2*f^3
*x^2 - c^2*f^3 + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*
x + c) + 2*(d^2*f^2*x + c*d*f^2)*e + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x
+ c*d*f^2)*e)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) - 12*(c
^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(
d*x + c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*sin(d*x + c))*log(-cos(d*x +
 c) + I*sin(d*x + c) + I) - 24*(f^3*cos(d*x + c) + f^3*sin(d*x + c) + f^3)*
polylog(3, I*cos(d*x + c) - sin(d*x + c)) - 24*(f^3*cos(d*x + c) + f^3*sin(
```

```
d*x + c) + f^3)*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) + (d^4*f^3*x^4 -
 4*d^3*f^3*x^3 + 4*(d^4*x - d^3)*e^3 + 6*(d^4*f*x^2 - 2*d^3*f*x)*e^2 + 4*(d
^4*f^2*x^3 - 3*d^3*f^2*x^2)*e)*sin(d*x + c))/(a*d^4*cos(d*x + c) + a*d^4*si
n(d*x + c) + a*d^4)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3 \sin (c+dx)}{\sin (c+dx)+1}\, dx + \int \frac{f^3x^3 \sin (c+dx)}{\sin (c+dx)+1}\, dx + \int \frac{3ef^2x^2 \sin (c+dx)}{\sin (c+dx)+1}\, dx + \int \frac{3e^2fx \sin (c+dx)}{\sin (c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*sin(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*sin
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sin(c + d*x)/(sin
(c + d*x) + 1), x) + Integral(3*e**2*f*x*sin(c + d*x)/(sin(c + d*x) + 1), x
))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (c+d\,x)\,{\left(e+f\,x\right)}^3}{a+a\,\sin (c+d\,x)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((sin(c + d*x)*(e + f*x)^3)/(a + a*sin(c + d*x)), x)
```

# 3.180 $\int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=129

$$\frac{i(e+fx)^2}{ad}+\frac{(e+fx)^3}{3af}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{4if^2\text{Li}_2\left(ie^{i(c+dx)}\right)}{ad^3}$$

```
[Out] I*(f*x+e)^2/a/d+1/3*(f*x+e)^3/a/f+(f*x+e)^2*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-4
*f*(f*x+e)*ln(1-I*exp(I*(d*x+c)))/a/d^2+4*I*f^2*polylog(2,I*exp(I*(d*x+c)))
/a/d^3
```

**Rubi** [A]

time = 0.16, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4611, 32, 3399, 4269, 3798, 2221, 2317, 2438}

$$\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}+\frac{i(e+fx)^2}{ad}+\frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (I*(e + f*x)^2)/(a*d) + (e + f*x)^3/(3*a*f) + ((e + f*x)^2*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) - (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + ((4
*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^2 \sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^2\,dx}{a} - \int \frac{(e+fx)^2}{a+a\sin(c+dx)}\,dx \\
&= \frac{(e+fx)^3}{3af} - \frac{\int (e+fx)^2 \csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)\,dx}{2a} \\
&= \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(2f)\int (e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{ad} \\
&= \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(4f)\int \frac{e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}(e+fx)}{1-ie^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}}{ad} \\
&= \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log\left(1-i\right.}{ad^2} \\
&= \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log\left(1-i\right.}{ad^2} \\
&= \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log\left(1-i\right.}{ad^2}
\end{aligned}
$$

**Mathematica [A]**
time = 0.99, size = 181, normalized size = 1.40

$$
\frac{x(3e^2+3efx+f^2x^2)+\frac{6if\left(2id(e+fx)\log(1-i\cos(c+dx)+\sin(c+dx))+2f\text{Li}_2(i\cos(c+dx)-\sin(c+dx))+\frac{d^2x(2e+fx)(\cos(c)+i\sin(c))}{\cos(c)+i(1+\sin(c))}\right)}{d^3}-\frac{6(e+fx)^2\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2) + ((6*I)*f*((2*I)*d*(e + f*x)*Log[1 - I*Cos[
c + d*x] + Sin[c + d*x]] + 2*f*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]] +
(d^2*x*(2*e + f*x)*(Cos[c] + I*Sin[c]))/(Cos[c] + I*(1 + Sin[c]))))/d^3 - (
6*(e + f*x)^2*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])))/(3*a)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(113) = 226$.
time = 0.11, size = 293, normalized size = 2.27

| method | result |
|---|---|
| risch | $\frac{f^2x^3}{3a}+\frac{fe\,x^2}{a}+\frac{e^2x}{a}+\frac{e^3}{3af}+\frac{2x^2f^2+4efx+2e^2}{da\left(e^{i(dx+c)}+i\right)}-\frac{4f\ln\left(e^{i(dx+c)}+i\right)e}{a\,d^2}+\frac{4f\ln\left(e^{i(dx+c)}\right)e}{a\,d^2}+\frac{2if^2x^2}{ad}+\frac{4if^2cx}{a\,d^2}+\frac{2if^2c^2}{a\,d^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/3/a*f^2*x^3+1/a*f*e*x^2+1/a*e^2*x+1/3/a/f*e^3+2*(f^2*x^2+2*e*f*x+e^2)/d/a
/(exp(I*(d*x+c))+I)-4/a/d^2*f*ln(exp(I*(d*x+c))+I)*e+4/a/d^2*f*ln(exp(I*(d*
x+c)))*e+2*I/a/d*f^2*x^2+4*I/a/d^2*f^2*c*x+2*I/a/d^3*f^2*c^2-4/a/d^2*f^2*ln
(1-I*exp(I*(d*x+c)))*x-4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c+4*I*f^2*polylog
(2,I*exp(I*(d*x+c)))/a/d^3+4/a/d^3*f^2*c*ln(exp(I*(d*x+c))+I)-4/a/d^3*f^2*c
*ln(exp(I*(d*x+c)))

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(112) = 224.
time = 0.60, size = 409, normalized size = 3.17

$$\frac{d^3f^2x^3 + 3d^3fx^2e + 3d^3xe^2 - 6id^2e^2 - 12(df\cos(dx+c)e + idfe\sin(dx+c) + idfe)\arctan(\sin(dx+c)+1, \cos(dx+c)) + 12(df^2x\cos(dx+c) + idf^2x\sin(dx+c) + idf^2x)\arctan(\cos(dx+c), \sin(dx+c)+1) - (id^3f^2x^3 - 3(-id^3fe + 2d^2f^2)x^2 - 3(-id^3e^2 + 4d^2fe)x)\cos(dx+c) + 12(f^2\cos(dx+c) + if^2\sin(dx+c) + if^2)\mathrm{Li}_2(ie^{(idx+ic)}) - 6(df^2x + dfe - (idf^2x + idfe)\cos(dx+c) + (df^2x + dfe)\sin(dx+c))\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2\sin(dx+c) + 1) + (d^3f^2x^3 + 3(d^3fe + 2id^2f^2)x^2 + 3(d^3e^2 + 4id^2fe)x)\sin(dx+c)}{-3iad^3\cos(dx+c) + 3ad^3\sin(dx+c) + 3ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] (d^3*f^2*x^3 + 3*d^3*f*x^2*e + 3*d^3*x*e^2 - 6*I*d^2*e^2 - 12*(d*f*cos(d*x
+ c)*e + I*d*f*e*sin(d*x + c) + I*d*f*e)*arctan2(sin(d*x + c) + 1, cos(d*x
+ c)) + 12*(d*f^2*x*cos(d*x + c) + I*d*f^2*x*sin(d*x + c) + I*d*f^2*x)*arct
an2(cos(d*x + c), sin(d*x + c) + 1) - (I*d^3*f^2*x^3 - 3*(-I*d^3*f*e + 2*d^
2*f^2)*x^2 - 3*(-I*d^3*e^2 + 4*d^2*f*e)*x)*cos(d*x + c) + 12*(f^2*cos(d*x +
 c) + I*f^2*sin(d*x + c) + I*f^2)*dilog(I*e^(I*d*x + I*c)) - 6*(d*f^2*x + d
*f*e - (I*d*f^2*x + I*d*f*e)*cos(d*x + c) + (d*f^2*x + d*f*e)*sin(d*x + c))
*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + (d^3*f^2*x^3 +
 3*(d^3*f*e + 2*I*d^2*f^2)*x^2 + 3*(d^3*e^2 + 4*I*d^2*f*e)*x)*sin(d*x + c))
/(-3*I*a*d^3*cos(d*x + c) + 3*a*d^3*sin(d*x + c) + 3*a*d^3)

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 587 vs. 2(112) = 224.
time = 0.39, size = 587, normalized size = 4.55

$$\frac{d^3f^2x^3 + 3d^2f^2x^2 + (d^3f^2x^3 + 3d^2f^2x^2 + 3(d^3x + d^2)e^2 + 3(d^3fx^2 + 2d^2fx)e)\cos(dx+c) - 6(-if^2\cos(dx+c) - if^2\sin(dx+c) - if^2)\mathrm{Li}_2(i\cos(dx+c) - \sin(dx+c)) - 6(if^2\cos(dx+c) + if^2\sin(dx+c) + if^2)\mathrm{Li}_2(-i\cos(dx+c) - \sin(dx+c)) + 3(d^3x + d^2)e^2 + 3(d^3fx^2 + 2d^2fx)e + 6(cf^2 - dfe + (cf^2 - dfe)\cos(dx+c) + (cf^2 - dfe)\sin(dx+c))\log(\cos(dx+c) + i\sin(dx+c) + i) - 6(df^2x + cf^2 + (df^2x + cf^2)\cos(dx+c) + (df^2x + cf^2)\sin(dx+c))\log(i\cos(dx+c) + \sin(dx+c) + 1) - 6(df^2x + cf^2 + (df^2x + cf^2)\cos(dx+c) + (df^2x + cf^2)\sin(dx+c))\log(-i\cos(dx+c) + \sin(dx+c) + 1) + 6(cf^2 - dfe + (cf^2 - dfe)\cos(dx+c) + (cf^2 - dfe)\sin(dx+c))\log(-\cos(dx+c) + i\sin(dx+c) + i) + (d^3f^2x^3 - 3d^2f^2x^2 + 3(d^3x - d^2)e^2 + 3(d^3fx^2 - 2d^2fx)e)\sin(dx+c)}{3(ad^3\cos(dx+c) + ad^3\sin(dx+c) + ad^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/3*(d^3*f^2*x^3 + 3*d^2*f^2*x^2 + (d^3*f^2*x^3 + 3*d^2*f^2*x^2 + 3*(d^3*x
+ d^2)*e^2 + 3*(d^3*f*x^2 + 2*d^2*f*x)*e)*cos(d*x + c) - 6*(-I*f^2*cos(d*x
+ c) - I*f^2*sin(d*x + c) - I*f^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 6
*(I*f^2*cos(d*x + c) + I*f^2*sin(d*x + c) + I*f^2)*dilog(-I*cos(d*x + c) -

```
sin(d*x + c)) + 3*(d^3*x + d^2)*e^2 + 3*(d^3*f*x^2 + 2*d^2*f*x)*e + 6*(c*f^
2 - d*f*e + (c*f^2 - d*f*e)*cos(d*x + c) + (c*f^2 - d*f*e)*sin(d*x + c))*lo
g(cos(d*x + c) + I*sin(d*x + c) + I) - 6*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^
2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(I*cos(d*x + c) + sin(
d*x + c) + 1) - 6*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^
2*x + c*f^2)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) + 6*(c*f
^2 - d*f*e + (c*f^2 - d*f*e)*cos(d*x + c) + (c*f^2 - d*f*e)*sin(d*x + c))*l
og(-cos(d*x + c) + I*sin(d*x + c) + I) + (d^3*f^2*x^3 - 3*d^2*f^2*x^2 + 3*(
d^3*x - d^2)*e^2 + 3*(d^3*f*x^2 - 2*d^2*f*x)*e)*sin(d*x + c))/(a*d^3*cos(d*
x + c) + a*d^3*sin(d*x + c) + a*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2 \sin(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \sin(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \sin(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sin(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*sin
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sin(c + d*x)/(sin(c + d
*x) + 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)\,{(e+f\,x)}^2}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((sin(c + d*x)*(e + f*x)^2)/(a + a*sin(c + d*x)), x)
```

# 3.181 $\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=76

$$\frac{ex}{a}+\frac{fx^2}{2a}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}$$

[Out] e*x/a+1/2*f*x^2/a+(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2

**Rubi** [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4611, 3399, 4269, 3556}

$$-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

[In] Int[((e + f*x)*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]

[Out] (e*x)/a + (f*x^2)/(2*a) + ((e + f*x)*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/(a*d^2)

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
```

```
b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\,dx}{a} - \int \frac{e+fx}{a+a\sin(c+dx)}\,dx \\
&= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{\int (e+fx)\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)\,dx}{2a} \\
&= \frac{ex}{a} + \frac{fx^2}{2a} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{f\int \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{ad} \\
&= \frac{ex}{a} + \frac{fx^2}{2a} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}
\end{aligned}
$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(76) = 152.
time = 0.33, size = 199, normalized size = 2.62

$$
\frac{2dfx\cos\left(c+\frac{dx}{2}\right)+\cos\left(\frac{dx}{2}\right)\left(d^2x(2e+fx)-4f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-4de\sin\left(\frac{dx}{2}\right)-2dfx\sin\left(\frac{dx}{2}\right)+2d^2ex\sin\left(c+\frac{dx}{2}\right)+d^2fx^2\sin\left(c+\frac{dx}{2}\right)-4f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(c+\frac{dx}{2}\right)}{2ad^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*d*f*x*Cos[c + (d*x)/2] + Cos[(d*x)/2]*(d^2*x*(2*e + f*x) - 4*f*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]) - 4*d*e*Sin[(d*x)/2] - 2*d*f*x*Sin[(d*x)/2
] + 2*d^2*e*x*Sin[c + (d*x)/2] + d^2*f*x^2*Sin[c + (d*x)/2] - 4*f*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + (d*x)/2])/(2*a*d^2*(Cos[c/2] + Sin[
c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple** [C] Result contains complex when optimal does not.
time = 0.13, size = 88, normalized size = 1.16

| method | result |
|---|---|
| risch | $\frac{f\,x^{2}}{2a}+\frac{ex}{a}+\frac{2ifx}{ad}+\frac{2ifc}{a\,d^{2}}+\frac{2fx+2e}{da\left(\mathrm{e}^{i(dx+c)}+i\right)}-\frac{2f\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a\,d^{2}}$ |
| norman | $\frac{-\frac{2e\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{2e\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{(de+f)x}{da}+\frac{(de-f)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{(de-f)x\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{(de+f)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{f\,x^{2}}{2a}+\frac{f\,x^{2}\tan(\cdots}{2\cdots}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/a+e*x/a+2*I*f/a/d*x+2*I*f/a/d^2*c+2*(f*x+e)/d/a/(exp(I*(d*x+c))+I
)-2*f/a/d^2*ln(exp(I*(d*x+c))+I)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(64) = 128$.
time = 0.50, size = 274, normalized size = 3.61

$$-\frac{4\,cf\left(\frac{1}{ad+\frac{ad\sin(dx+c)}{\cos(dx+c)+1}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right)-4\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}\right)e-\frac{\left((dx+c)^2\cos(dx+c)^2+(dx+c)^2\sin(dx+c)^2+2\,(dx+c)^2\sin(dx+c)+(dx+c)^2+4\,(dx+c)\cos(dx+c)-2\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)\right)f}{ad\cos(dx+c)^2+ad\sin(dx+c)^2+2\,ad\sin(dx+c)+ad}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(4*c*f*(1/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1)) + arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/(a*d)) - 4*(arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a + 1/(a + a*sin(d*x + c)/(cos(d*x + c) + 1)))*e - ((d*x + c)^2*cos(d*x
+ c)^2 + (d*x + c)^2*sin(d*x + c)^2 + 2*(d*x + c)^2*sin(d*x + c) + (d*x + c
)^2 + 4*(d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin
(d*x + c) + 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1))*f
/(a*d*cos(d*x + c)^2 + a*d*sin(d*x + c)^2 + 2*a*d*sin(d*x + c) + a*d))/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(64) = 128$.
time = 0.35, size = 152, normalized size = 2.00

$$\frac{d^2fx^2+2\,dfx+(d^2fx^2+2\,dfx+2\,(d^2x+d)e)\cos(dx+c)+2\,(d^2x+d)e-2\,(f\cos(dx+c)+f\sin(dx+c)+f)\log(\sin(dx+c)+1)+(d^2fx^2-2\,dfx+2\,(d^2x-d)e)\sin(dx+c)}{2\,(ad^2\cos(dx+c)+ad^2\sin(dx+c)+ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*f*x^2 + 2*d*f*x + (d^2*f*x^2 + 2*d*f*x + 2*(d^2*x + d)*e)*cos(d*x
+ c) + 2*(d^2*x + d)*e - 2*(f*cos(d*x + c) + f*sin(d*x + c) + f)*log(sin(d*
x + c) + 1) + (d^2*f*x^2 - 2*d*f*x + 2*(d^2*x - d)*e)*sin(d*x + c))/(a*d^2*
cos(d*x + c) + a*d^2*sin(d*x + c) + a*d^2)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(58) = 116$.
time = 0.72, size = 456, normalized size = 6.00

$$\begin{cases}\frac{2d^2ex\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{2d^2ex}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{d^2fx^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{d^2fx^2}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{4de}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}-\frac{2dfx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{2dfx}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}-\frac{4f\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}-\frac{4f\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{2f\log\left(\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{2f\log\left(\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} & \text{for } d\neq 0\\ \frac{\left(ex+\frac{fx^2}{2}\right)\sin(c)}{a\sin(c)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*d**2*e*x*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**
2) + 2*d**2*e*x/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + d**2*f*x**2*tan(c/
```

```
2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + d**2*f*x**2/(2*a*d**2*t
an(c/2 + d*x/2) + 2*a*d**2) + 4*d*e/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2)
- 2*d*f*x*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 2*d*f*x
/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*f*log(tan(c/2 + d*x/2) + 1)*tan
(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*f*log(tan(c/2 + d*
x/2) + 1)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 2*f*log(tan(c/2 + d*x/2)
**2 + 1)*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 2*f*log(
tan(c/2 + d*x/2)**2 + 1)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2), Ne(d, 0)),
 ((e*x + f*x**2/2)*sin(c)/(a*sin(c) + a), True))
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(64) = 128$.
time = 3.26, size = 772, normalized size = 10.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*f*x^2*tan(1/2*d*x)*tan(1/2*c) - d^2*f*x^2*tan(1/2*d*x) - d^2*f*x^2
*tan(1/2*c) + 2*d^2*x*e*tan(1/2*d*x)*tan(1/2*c) - d^2*f*x^2 - 2*d^2*x*e*tan
(1/2*d*x) - 2*d^2*x*e*tan(1/2*c) + 2*d*f*x*tan(1/2*d*x)*tan(1/2*c) - 2*d^2*
x*e + 2*d*f*x*tan(1/2*d*x) + 2*d*f*x*tan(1/2*c) + 2*d*e*tan(1/2*d*x)*tan(1/
2*c) - 2*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c)
 - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan
(1/2*d*x)*tan(1/2*c) - 2*d*f*x + 2*d*e*tan(1/2*d*x) + 2*f*log(2*(tan(1/2*d*
x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x) + 2*d*e*tan(1/2*c)
 + 2*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2
*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2
*c) - 2*d*e + 2*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan
(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 +
1)))/(a*d^2*tan(1/2*d*x)*tan(1/2*c) - a*d^2*tan(1/2*d*x) - a*d^2*tan(1/2*c)
 - a*d^2)
```

**Mupad** [**B**]

time = 1.16, size = 80, normalized size = 1.05

$$\frac{f\,x^2}{2\,a}-\frac{2\,f\,\ln\left({\mathrm{e}}^{c\,1\mathrm{i}}\,{\mathrm{e}}^{d\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{a\,d^2}+\frac{2\,\left(e+f\,x\right)}{a\,d\,\left({\mathrm{e}}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\mathrm{i}\right)}+\frac{x\,\left(d\,e+f\,2\mathrm{i}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] (f*x^2)/(2*a) - (2*f*log(exp(c*1i)*exp(d*x*1i) + 1i))/(a*d^2) + (2*(e + f*x
))/(a*d*(exp(c*1i + d*x*1i) + 1i)) + (x*(f*2i + d*e))/(a*d)
```

# 3.182 $\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=28

$$\frac{x}{a}+\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

```
[Out] x/a+cos(d*x+c)/d/(a+a*sin(d*x+c))
```

**Rubi** [A]

time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2814, 2727}

$$\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(d*(a + a*Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\sin(c+dx)}\,dx\\ &= \frac{x}{a}+\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(28) = 56.

time = 0.08, size = 72, normalized size = 2.57

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left((c+dx)\cos\left(\frac{1}{2}(c+dx)\right)+(-2+c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*((c + d*x)*Cos[(c + d*x)/2] + (-2 +
c + d*x)*Sin[(c + d*x)/2]))/(a*d*(1 + Sin[c + d*x]))
```

**Maple** [A]
time = 0.06, size = 37, normalized size = 1.32

| method | result | size |
|---|---|---|
| risch | $\frac{x}{a}+\frac{2}{da\left(e^{i(dx+c)}+i\right)}$ | 29 |
| derivativedivides | $\frac{\frac{4}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ | 37 |
| default | $\frac{\frac{4}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ | 37 |
| norman | $\frac{\frac{x}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2}{ad}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$ | 109 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*(1/2/(tan(1/2*d*x+1/2*c)+1)+1/2*arctan(tan(1/2*d*x+1/2*c)))
```

**Maxima** [A]
time = 0.49, size = 50, normalized size = 1.79

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)/(cos(d
*x + c) + 1)))/d
```

**Fricas** [A]
time = 0.34, size = 54, normalized size = 1.93

$$\frac{dx+(dx+1)\cos(dx+c)+(dx-1)\sin(dx+c)+1}{ad\cos(dx+c)+ad\sin(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x + (d*x + 1)*cos(d*x + c) + (d*x - 1)*sin(d*x + c) + 1)/(a*d*cos(d*x +
c) + a*d*sin(d*x + c) + a*d)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(20) = 40$.
time = 0.62, size = 80, normalized size = 2.86

$$\begin{cases} \frac{dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{dx}{ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2}{ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\sin(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((d*x*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2) + a*d) + d*x/(a*d*tan
(c/2 + d*x/2) + a*d) + 2/(a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x*sin(c)
/(a*sin(c) + a), True))
```

**Giac** [A]
time = 3.43, size = 32, normalized size = 1.14

$$\frac{\frac{dx+c}{a} + \frac{2}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a + 2/(a*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

**Mupad** [B]
time = 0.74, size = 27, normalized size = 0.96

$$\frac{x}{a} + \frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x)),x)
```

```
[Out] x/a + 2/(a*d*(tan(c/2 + (d*x)/2) + 1))
```

# 3.183 $\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=29

$$\text{Int}\left(\frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 5.84, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(fx+e)\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*f*cos(d*x + c) + 2*(a*d*f^3*x + a*d*f^2*e + (a*d*f^3*x + a*d*f^2*e)*cos(
d*x + c)^2 + (a*d*f^3*x + a*d*f^2*e)*sin(d*x + c)^2 + 2*(a*d*f^3*x + a*d*f^
2*e)*sin(d*x + c))*integrate(cos(d*x + c)/(a*d*f^2*x^2 + 2*a*d*f*x*e + (a*d
*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*cos(d*x + c)^2 + a*d*e^2 + (a*d*f^2*x^2 +
 2*a*d*f*x*e + a*d*e^2)*sin(d*x + c)^2 + 2*(a*d*f^2*x^2 + 2*a*d*f*x*e + a*d
*e^2)*sin(d*x + c)), x) + (d*f*x + (d*f*x + d*e)*cos(d*x + c)^2 + (d*f*x +
d*e)*sin(d*x + c)^2 + d*e + 2*(d*f*x + d*e)*sin(d*x + c))*log(f*x + e))/(a*
d*f^2*x + a*d*f*e + (a*d*f^2*x + a*d*f*e)*cos(d*x + c)^2 + (a*d*f^2*x + a*d
*f*e)*sin(d*x + c)^2 + 2*(a*d*f^2*x + a*d*f*e)*sin(d*x + c))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin\left(c+d\,x\right)}{\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/((e + f*x)*(a + a*sin(c + d*x))),x)
[Out] int(sin(c + d*x)/((e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.184 $\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=29

$$\text{Int}\left(\frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 5.79, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(fx+e)^2\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(d*f*x + (d*f*x + d*e)*cos(d*x + c)^2 + (d*f*x + d*e)*sin(d*x + c)^2 - 2*f
*cos(d*x + c) + d*e - 4*(a*d*f^4*x^2 + 2*a*d*f^3*x*e + a*d*f^2*e^2 + (a*d*f
^4*x^2 + 2*a*d*f^3*x*e + a*d*f^2*e^2)*cos(d*x + c)^2 + (a*d*f^4*x^2 + 2*a*d
*f^3*x*e + a*d*f^2*e^2)*sin(d*x + c)^2 + 2*(a*d*f^4*x^2 + 2*a*d*f^3*x*e + a
*d*f^2*e^2)*sin(d*x + c))*integrate(cos(d*x + c)/(a*d*f^3*x^3 + 3*a*d*f^2*x
^2*e + 3*a*d*f*x*e^2 + (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d
*e^3)*cos(d*x + c)^2 + a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x
*e^2 + a*d*e^3)*sin(d*x + c)^2 + 2*(a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f
*x*e^2 + a*d*e^3)*sin(d*x + c)), x) + 2*(d*f*x + d*e)*sin(d*x + c))/(a*d*f^
3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^2 + (a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^
2)*cos(d*x + c)^2 + (a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^2)*sin(d*x + c)^
2 + 2*(a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^2)*sin(d*x + c))
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2 + 2*a*f*x
*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin (c+dx)}{e^2 \sin (c+dx)+e^2+2efx \sin (c+dx)+2efx+f^2x^2 \sin (c+dx)+f^2x^2}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2*
e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin\left(c+d\,x\right)}{{\left(e+f\,x\right)}^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/((e + f*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] int(sin(c + d*x)/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.185 $\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+a\sin(c+dx)} \, dx$

**Optimal**. Leaf size=247

$$-\frac{i(e+fx)^3}{ad}-\frac{(e+fx)^4}{4af}+\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}-\frac{(e+fx)^3\cos(c+dx)}{ad}-\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+6$$

```
[Out] -I*(f*x+e)^3/a/d-1/4*(f*x+e)^4/a/f+6*f^2*(f*x+e)*cos(d*x+c)/a/d^3-(f*x+e)^3
*cos(d*x+c)/a/d-(f*x+e)^3*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d+6*f*(f*x+e)^2*ln(1-
I*exp(I*(d*x+c)))/a/d^2-12*I*f^2*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/a/d^3+
12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4-6*f^3*sin(d*x+c)/a/d^4+3*f*(f*x+e)
^2*sin(d*x+c)/a/d^2
```

**Rubi** [A]
time = 0.32, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {4611, 3377, 2717, 32, 3399, 4269, 3798, 2221, 2611, 2320, 6724}

$$\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}-\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{6f^3\sin(c+dx)}{ad^4}+\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}+\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{3f(e+fx)^2\sin(c+dx)}{ad^2}-\frac{(e+fx)^3\cos(c+dx)}{ad}-\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{i(e+fx)^3}{ad}-\frac{(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (e + f*x)^4/(4*a*f) + (6*f^2*(e + f*x)*Cos[c + d
*x])/(a*d^3) - ((e + f*x)^3*Cos[c + d*x])/(a*d) - ((e + f*x)^3*Cot[c/2 + Pi
/4 + (d*x)/2])/(a*d) + (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2)
 - ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) + (12*f^3*P
olyLog[3, I*E^(I*(c + d*x))])/(a*d^4) - (6*f^3*Sin[c + d*x])/(a*d^4) + (3*f
*(e + f*x)^2*Sin[c + d*x])/(a*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
```

```
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sin(c+dx)\,dx}{a} - \int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{\int (e+fx)^3\,dx}{a} + \frac{(3f)\int (e+fx)^2\cos(c+dx)\,dx}{ad}$$

$$= -\frac{(e+fx)^4}{4af} - \frac{(e+fx)^3\cos(c+dx)}{ad} + \frac{3f(e+fx)^2\sin(c+dx)}{ad^2} + \frac{\int (e+fx}{}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{(e+fx)}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+d}{ad}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+d}{ad}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+d}{ad}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+d}{ad}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+d}{ad}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1378 vs. 2(247) = 494.
time = 2.70, size = 1378, normalized size = 5.58

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(-6*d^2*e^2*f*Cos[(d*x)/2] + 12*f^3*Cos[(d*x)/2] + 4*d^4*e^3*x*Cos[(d*
x)/2] + (12*I)*d^3*e^2*f*x*Cos[(d*x)/2] - 12*d^2*e*f^2*x*Cos[(d*x)/2] + 6*d
^4*e^2*f*x^2*Cos[(d*x)/2] + (12*I)*d^3*e*f^2*x^2*Cos[(d*x)/2] - 6*d^2*f^3*x
^2*Cos[(d*x)/2] + 4*d^4*e*f^2*x^3*Cos[(d*x)/2] + (4*I)*d^3*f^3*x^3*Cos[(d*x
)/2] + d^4*f^3*x^4*Cos[(d*x)/2] + 2*d^3*e^3*Cos[c + (d*x)/2] - 12*d*e*f^2*C
os[c + (d*x)/2] + 18*d^3*e^2*f*x*Cos[c + (d*x)/2] - 12*d*f^3*x*Cos[c + (d*x
)/2] + 18*d^3*e*f^2*x^2*Cos[c + (d*x)/2] + 6*d^3*f^3*x^3*Cos[c + (d*x)/2] +
 2*d^3*e^3*Cos[c + (3*d*x)/2] - 12*d*e*f^2*Cos[c + (3*d*x)/2] + 6*d^3*e^2*f
*x*Cos[c + (3*d*x)/2] - 12*d*f^3*x*Cos[c + (3*d*x)/2] + 6*d^3*e*f^2*x^2*Cos
[c + (3*d*x)/2] + 2*d^3*f^3*x^3*Cos[c + (3*d*x)/2] + 6*d^2*e^2*f*Cos[2*c +
(3*d*x)/2] - 12*f^3*Cos[2*c + (3*d*x)/2] + 12*d^2*e*f^2*x*Cos[2*c + (3*d*x)
/2] + 6*d^2*f^3*x^2*Cos[2*c + (3*d*x)/2] - 24*d^2*e^2*f*Cos[(d*x)/2]*Log[1
- I*Cos[c + d*x] + Sin[c + d*x]] - 48*d^2*e*f^2*x*Cos[(d*x)/2]*Log[1 - I*Co
s[c + d*x] + Sin[c + d*x]] - 24*d^2*f^3*x^2*Cos[(d*x)/2]*Log[1 - I*Cos[c +
d*x] + Sin[c + d*x]] - 10*d^3*e^3*Sin[(d*x)/2] + 12*d*e*f^2*Sin[(d*x)/2] -
18*d^3*e^2*f*x*Sin[(d*x)/2] + 12*d*f^3*x*Sin[(d*x)/2] - 18*d^3*e*f^2*x^2*Si
n[(d*x)/2] - 6*d^3*f^3*x^3*Sin[(d*x)/2] - 6*d^2*e^2*f*Sin[c + (d*x)/2] + 12
*f^3*Sin[c + (d*x)/2] + 4*d^4*e^3*x*Sin[c + (d*x)/2] + (12*I)*d^3*e^2*f*x*S
in[c + (d*x)/2] - 12*d^2*e*f^2*x*Sin[c + (d*x)/2] + 6*d^4*e^2*f*x^2*Sin[c +
 (d*x)/2] + (12*I)*d^3*e*f^2*x^2*Sin[c + (d*x)/2] - 6*d^2*f^3*x^2*Sin[c + (
d*x)/2] + 4*d^4*e*f^2*x^3*Sin[c + (d*x)/2] + (4*I)*d^3*f^3*x^3*Sin[c + (d*x
)/2] + d^4*f^3*x^4*Sin[c + (d*x)/2] - 24*d^2*e^2*f*Log[1 - I*Cos[c + d*x] +
 Sin[c + d*x]]*Sin[c + (d*x)/2] - 48*d^2*e*f^2*x*Log[1 - I*Cos[c + d*x] + S
in[c + d*x]]*Sin[c + (d*x)/2] - 24*d^2*f^3*x^2*Log[1 - I*Cos[c + d*x] + Sin
[c + d*x]]*Sin[c + (d*x)/2] - 48*f^3*PolyLog[3, I*Cos[c + d*x] - Sin[c + d*
x]]*(Cos[(d*x)/2] + Sin[c + (d*x)/2]) + (48*I)*d*f^2*(e + f*x)*PolyLog[2, I
*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]) - 6*d^2*e^2*f*Sin[c + (3*d*x)/2] + 12*f^3*Sin[c + (3*d*x)/2]
 - 12*d^2*e*f^2*x*Sin[c + (3*d*x)/2] - 6*d^2*f^3*x^2*Sin[c + (3*d*x)/2] + 2
*d^3*e^3*Sin[2*c + (3*d*x)/2] - 12*d*e*f^2*Sin[2*c + (3*d*x)/2] + 6*d^3*e^2
*f*x*Sin[2*c + (3*d*x)/2] - 12*d*f^3*x*Sin[2*c + (3*d*x)/2] + 6*d^3*e*f^2*x
^2*Sin[2*c + (3*d*x)/2] + 2*d^3*f^3*x^3*Sin[2*c + (3*d*x)/2])/(a*d^4*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(228) = 456$.
time = 0.27, size = 759, normalized size = 3.07

| method | result |
|---|---|
| risch | $-\frac{\left(d^{3} x^{3} f^{3}+3 d^{3} e\, f^{2} x^{2}-3 i d^{2} f^{3} x^{2}+3 d^{3} e^{2} f x-6 i d^{2} e\, f^{2} x+d^{3} e^{3}-3 i d^{2} e^{2} f-6 d\, f^{3} x-6 d e\, f^{2}+6 i f^{3}\right) {\mathrm e}^{-i \left(d x+c\right)}}{2 a\, d^{4}}-\frac{f^{3} x^{4}}{4 a}-\frac{e^{4}}{4 a f}+\frac{6 i f^{3}}{a d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d/a*f^3*x^3+4*I/d^4/a*f^3*c^3-1/4/a*f^3*x^4-1/4/a/f*e^4+6/a/d^2*f^3*ln
(1-I*exp(I*(d*x+c)))*x^2-6/a/d^4*f^3*ln(1-I*exp(I*(d*x+c)))*c^2-6/a/d^2*f*l
n(exp(I*(d*x+c)))*e^2-6/a/d^4*f^3*c^2*ln(exp(I*(d*x+c)))+6/a/d^4*f^3*c^2*ln
(exp(I*(d*x+c))+I)+6/a/d^2*f*ln(exp(I*(d*x+c))+I)*e^2-1/a*f^2*e*x^3-3/2/a*f
*e^2*x^2-1/a*e^3*x-12*I/d^3/a*e*f^2*polylog(2,I*exp(I*(d*x+c)))+12/a/d^3*f^
2*e*c*ln(exp(I*(d*x+c)))-2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(I*(
d*x+c))+I)-12*I/d^3/a*f^3*polylog(2,I*exp(I*(d*x+c)))*x+6*I/d^3/a*f^3*c^2*x
-6*I/d/a*e*f^2*x^2-6*I/d^3/a*f^2*e*c^2-12*I/d^2/a*f^2*e*c*x+12*f^3*polylog(
3,I*exp(I*(d*x+c)))/a/d^4-12/a/d^3*f^2*e*c*ln(exp(I*(d*x+c))+I)+12/a/d^2*f^
2*e*ln(1-I*exp(I*(d*x+c)))*x+12/a/d^3*f^2*e*ln(1-I*exp(I*(d*x+c)))*c-1/2*(d
^3*x^3*f^3+3*I*d^2*f^3*x^2+3*d^3*e*f^2*x^2+6*I*d^2*e*f^2*x+3*d^3*e^2*f*x+3*
I*d^2*e^2*f+d^3*e^3-6*d*f^3*x-6*I*f^3-6*d*e*f^2)/a/d^4*exp(I*(d*x+c))-1/2*(
d^3*x^3*f^3-3*I*d^2*f^3*x^2+3*d^3*e*f^2*x^2-6*I*d^2*e*f^2*x+3*d^3*e^2*f*x-3
*I*d^2*e^2*f+d^3*e^3-6*d*f^3*x+6*I*f^3-6*d*e*f^2)/a/d^4*exp(-I*(d*x+c))
```

**<u>Maxima</u>** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4616 vs. $2(230) = 460$.
time = 0.84, size = 4616, normalized size = 18.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(12*c^2*f^2*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 2)/(a*d^2 + a*d^2*sin(d*x + c)/(cos(d*x + c) + 1) + a*d^2*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*d^2*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^2))*e - 12*c*f*((sin(d*
x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2)/(a*d +
 a*d*sin(d*x + c)/(cos(d*x + c) + 1) + a*d*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/(a*d))*e^2 - 6*(((d*x + c)^2 - 1)*cos(d*x + c)^4 + ((d*x + c)
^2 - 1)*sin(d*x + c)^4 + ((d*x + c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*
d*x + 2*c)^3 + 7*(d*x + c)*cos(d*x + c)^3 + (d*x + (d*x + c)*sin(d*x + c) +
 c - cos(d*x + c))*sin(2*d*x + 2*c)^3 + (2*(d*x + c)^2 - 3)*sin(d*x + c)^3
+ (((d*x + c)^2 - 1)*cos(d*x + c)^2 + ((d*x + c)^2 - 3)*sin(d*x + c)^2 + (d
*x + c)^2 + 6*(d*x + c)*cos(d*x + c) + 2*((d*x + c)^2 - (d*x + c)*cos(d*x +
 c) - 2)*sin(d*x + c) - 1)*cos(2*d*x + 2*c)^2 + ((d*x + c)^2 - 1)*cos(d*x +
 c)^2 + (((d*x + c)^2 - 3)*cos(d*x + c)^2 + ((d*x + c)^2 - 1)*sin(d*x + c)^
2 + (d*x + c)^2 + ((d*x + c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2
*c) + 8*(d*x + c)*cos(d*x + c) + 2*((d*x + c)^2 + (d*x + c)*cos(d*x + c) -
1)*sin(d*x + c) - 1)*sin(2*d*x + 2*c)^2 + (2*((d*x + c)^2 - 1)*cos(d*x + c)
^2 + (d*x + c)^2 + 7*(d*x + c)*cos(d*x + c) - 3)*sin(d*x + c)^2 + ((d*x + c
)*cos(d*x + c)^3 - (2*(d*x + c)^2 - 3)*sin(d*x + c)^3 - (4*(d*x + c)^2 - (d
```

```
*x + c)*cos(d*x + c) - 6)*sin(d*x + c)^2 + 2*cos(d*x + c)^2 - ((2*(d*x + c)
^2 - 3)*cos(d*x + c)^2 + 2*(d*x + c)^2 + 12*(d*x + c)*cos(d*x + c) - 4)*sin
(d*x + c) + 1)*cos(2*d*x + 2*c) + (d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^
4 + sin(d*x + c)^4 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
*cos(2*d*x + 2*c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1
)*sin(2*d*x + 2*c)^2 + 2*cos(d*x + c)^2*sin(d*x + c) + (2*cos(d*x + c)^2 +
1)*sin(d*x + c)^2 + 2*sin(d*x + c)^3 - 2*(sin(d*x + c)^3 + (cos(d*x + c)^2
+ 1)*sin(d*x + c) + 2*sin(d*x + c)^2)*cos(2*d*x + 2*c) + cos(d*x + c)^2 + 2
*(cos(d*x + c)^3 + cos(d*x + c)*sin(d*x + c)^2 + 2*cos(d*x + c)*sin(d*x + c
) + cos(d*x + c))*sin(2*d*x + 2*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2
*sin(d*x + c) + 1) + ((2*(d*x + c)^2 - 3)*cos(d*x + c)^3 + (d*x + c)*sin(d*
x + c)^3 + (d*x + (d*x + c)*sin(d*x + c) + c - cos(d*x + c))*cos(2*d*x + 2*
c)^2 + 14*(d*x + c)*cos(d*x + c)^2 + (2*d*x + (2*(d*x + c)^2 - 3)*cos(d*x +
 c) + 2*c)*sin(d*x + c)^2 + d*x + 2*((d*x + c)*cos(d*x + c)^2 - (d*x + c)*s
in(d*x + c)^2 - (d*x + c - 2*cos(d*x + c))*sin(d*x + c) + cos(d*x + c))*cos
(2*d*x + 2*c) + 2*((d*x + c)^2 - 1)*cos(d*x + c) + ((d*x + c)*cos(d*x + c)^
2 + 2*d*x + 4*((d*x + c)^2 - 1)*cos(d*x + c) + 2*c)*sin(d*x + c) + c)*sin(2
*d*x + 2*c) + ((2*(d*x + c)^2 - 3)*cos(d*x + c)^2 + 2*(d*x + c)*cos(d*x + c
) - 1)*sin(d*x + c))*c*f^2*e/(a*d^2*cos(d*x + c)^4 + a*d^2*sin(d*x + c)^4 +
 2*a*d^2*cos(d*x + c)^2*sin(d*x + c) + 2*a*d^2*sin(d*x + c)^3 + a*d^2*cos(d
*x + c)^2 + (a*d^2*cos(d*x + c)^2 + a*d^2*sin(d*x + c)^2 + 2*a*d^2*sin(d*x
+ c) + a*d^2)*cos(2*d*x + 2*c)^2 + (a*d^2*cos(d*x + c)^2 + a*d^2*sin(d*x +
c)^2 + 2*a*d^2*sin(d*x + c) + a*d^2)*sin(2*d*x + 2*c)^2 + (2*a*d^2*cos(d*x
+ c)^2 + a*d^2)*sin(d*x + c)^2 - 2*(a*d^2*sin(d*x + c)^3 + 2*a*d^2*sin(d*x
+ c)^2 + (a*d^2*cos(d*x + c)^2 + a*d^2)*sin(d*x + c))*cos(2*d*x + 2*c) + 2*
(a*d^2*cos(d*x + c)^3 + a*d^2*cos(d*x + c)*sin(d*x + c)^2 + 2*a*d^2*cos(d*x
 + c)*sin(d*x + c) + a*d^2*cos(d*x + c))*sin(2*d*x + 2*c)) + 4*((sin(d*x +
c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2)/(a + a*sin
(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(sin(d*x + c)/(cos(d*x + c) + 1)
)/a)*e^3 + 3*(((d*x + c)^2 - 1)*cos(d*x + c)^4 + ((d*x + c)^2 - 1)*sin(d*x
+ c)^4 + ((d*x + c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c)^3 + 7
*(d*x + c)*cos(d*x + c)^3 + (d*x + (d*x + c)*sin(d*x + c) + c - cos(d*x + c
))*sin(2*d*x + 2*c)^3 + (2*(d*x + c)^2 - 3)*sin(d*x + c)^3 + (((d*x + c)^2
- 1)*cos(d*x + c)^2 + ((d*x + c)^2 - 3)*sin(d*x + c)^2 + (d*x + c)^2 + 6*(d
*x + c)*cos(d*x + c) + 2*((d*x + c)^2 - (d*x + c)*cos(d*x + c) - 2)*sin(d*x
 + c) - 1)*cos(2*d*x + 2*c)^2 + ((d*x + c)^2 - 1)*cos(d*x + c)^2 + (((d*x +
 c)^2 - 3)*cos(d*x + c)^2 + ((d*x + c)^2 - 1)*sin(d*x + c)^2 + (d*x + c)^2
+ ((d*x + c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c) + 8*(d*x + c
)*cos(d*x + c) + 2*((d*x + c)^2 + (d*x + c)*cos(d*x + c) - 1)*sin(d*x + c)
- 1)*sin(2*d*x + 2*c)^2 + (2*((d*x + c)^2 - 1)*cos(d*x + c)^2 + (d*x + c)^2
 + 7*(d*x + c)*cos(d*x + c) - 3)*sin(d*x + c)^2 + ((d*x + c)*cos(d*x + c)^3
 - (2*(d*x + c)^2 - 3)*sin(d*x + c)^3 - (4*(d*x + c)^2 - (d*x + c)*cos(d*x
+ c) - 6)*sin(d*x + c)^2 + 2*cos(d*x + c)^2 - ((2*(d*x + c)^2 - 3)*cos(d*x
+ c)^2 + 2*(d*x + c)^2 + 12*(d*x + c)*cos(d*x + c) - 4)*sin(d*x + c) + 1)*c
```

```
os(2*d*x + 2*c) + (d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^4 + sin(d*x + c)
^4 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d...
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1319 vs. $2(230) = 460$.
time = 0.42, size = 1319, normalized size = 5.34

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^4*f^3*x^4 + 4*d^3*f^3*x^3 - 12*d^2*f^3*x^2 + 24*f^3 + 4*(d^3*f^3*x^
3 + 3*d^2*f^3*x^2 - 6*d*f^3*x + d^3*e^3 - 6*f^3 + 3*(d^3*f*x + d^2*f)*e^2 +
 3*(d^3*f^2*x^2 + 2*d^2*f^2*x - 2*d*f^2)*e)*cos(d*x + c)^2 + (d^4*f^3*x^4 +
 8*d^3*f^3*x^3 - 24*d*f^3*x + 4*(d^4*x + 2*d^3)*e^3 + 6*(d^4*f*x^2 + 4*d^3*
f*x)*e^2 + 4*(d^4*f^2*x^3 + 6*d^3*f^2*x^2 - 6*d*f^2)*e)*cos(d*x + c) + 24*(
I*d*f^3*x + I*d*f^2*e + (I*d*f^3*x + I*d*f^2*e)*cos(d*x + c) + (I*d*f^3*x +
 I*d*f^2*e)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) + 24*(-I*d*f
^3*x - I*d*f^2*e + (-I*d*f^3*x - I*d*f^2*e)*cos(d*x + c) + (-I*d*f^3*x - I*
d*f^2*e)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) + 4*(d^4*x + d
^3)*e^3 + 6*(d^4*f*x^2 + 2*d^3*f*x - 2*d^2*f)*e^2 + 4*(d^4*f^2*x^3 + 3*d^3*
f^2*x^2 - 6*d^2*f^2*x)*e - 12*(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + (c^2*f^3
 - 2*c*d*f^2*e + d^2*f*e^2)*cos(d*x + c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e
^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 12*(d^2*f^3*x^2
 - c^2*f^3 + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*x + c
) + 2*(d^2*f^2*x + c*d*f^2)*e + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d
*f^2)*e)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 12*(d^2*f^3
*x^2 - c^2*f^3 + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*
x + c) + 2*(d^2*f^2*x + c*d*f^2)*e + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x
 + c*d*f^2)*e)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) - 12*(c
^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(
d*x + c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*sin(d*x + c))*log(-cos(d*x +
 c) + I*sin(d*x + c) + I) - 24*(f^3*cos(d*x + c) + f^3*sin(d*x + c) + f^3)*
polylog(3, I*cos(d*x + c) - sin(d*x + c)) - 24*(f^3*cos(d*x + c) + f^3*sin(
d*x + c) + f^3)*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) + (d^4*f^3*x^4 -
 4*d^3*f^3*x^3 - 12*d^2*f^3*x^2 + 24*f^3 + 4*(d^3*f^3*x^3 - 3*d^2*f^3*x^2 -
 6*d*f^3*x + d^3*e^3 + 6*f^3 + 3*(d^3*f*x - d^2*f)*e^2 + 3*(d^3*f^2*x^2 - 2
*d^2*f^2*x - 2*d*f^2)*e)*cos(d*x + c) + 4*(d^4*x - d^3)*e^3 + 6*(d^4*f*x^2
 - 2*d^3*f*x - 2*d^2*f)*e^2 + 4*(d^4*f^2*x^3 - 3*d^3*f^2*x^2 - 6*d^2*f^2*x)*
e)*sin(d*x + c))/(a*d^4*cos(d*x + c) + a*d^4*sin(d*x + c) + a*d^4)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3 \sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3 \sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2 \sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx \sin^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*
sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sin(c + d*x
)**2/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sin(c + d*x)**2/(sin(c +
d*x) + 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^2\,{\left(e+f\,x\right)}^3}{a+a\,\sin\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((sin(c + d*x)^2*(e + f*x)^3)/(a + a*sin(c + d*x)), x)
```

# 3.186 $\int \frac{(e+fx)^2 \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=188

$$-\frac{i(e+fx)^2}{ad}-\frac{(e+fx)^3}{3af}+\frac{2f^2\cos(c+dx)}{ad^3}-\frac{(e+fx)^2\cos(c+dx)}{ad}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{4f(e+fx}{}$$

```
[Out] -I*(f*x+e)^2/a/d-1/3*(f*x+e)^3/a/f+2*f^2*cos(d*x+c)/a/d^3-(f*x+e)^2*cos(d*x
+c)/a/d-(f*x+e)^2*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d+4*f*(f*x+e)*ln(1-I*exp(I*(d
*x+c)))/a/d^2-4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3+2*f*(f*x+e)*sin(d*x
+c)/a/d^2
```

**Rubi** [A]

time = 0.24, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4611, 3377, 2718, 32, 3399, 4269, 3798, 2221, 2317, 2438}

$$-\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}+\frac{2f^2\cos(c+dx)}{ad^3}+\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{2f(e+fx)\sin(c+dx)}{ad^2}-\frac{(e+fx)^2\cos(c+dx)}{ad}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{i(e+fx)^2}{ad}-\frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^2)/(a*d) - (e + f*x)^3/(3*a*f) + (2*f^2*Cos[c + d*x])/(a*d^
3) - ((e + f*x)^2*Cos[c + d*x])/(a*d) - ((e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)
/2])/(a*d) + (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) - ((4*I)*f^
2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) + (2*f*(e + f*x)*Sin[c + d*x])/(a*
d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^2\sin(c+dx)\,dx}{a} - \int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx \\
&= -\frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{\int (e+fx)^2\,dx}{a} + \frac{(2f)\int (e+fx)\cos(c+dx)\,dx}{ad} \\
&= -\frac{(e+fx)^3}{3af} - \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{2f(e+fx)\sin(c+dx)}{ad^2} + \frac{\int (e+fx)}{} \\
&= -\frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}\right.}{ad} \\
&= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e}{} \\
&= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e}{} \\
&= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e}{} \\
&= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e}{}
\end{aligned}
$$

**Mathematica [A]**
time = 1.73, size = 263, normalized size = 1.40

$$
-\frac{x(3e^2+3efx+f^2x^2)+\frac{3\cos(dx)\left((-2f^2+d^2(e+fx)^2)\cos(c)-2df(e+fx)\sin(c)\right)}{d^3}+\frac{6if\left(2id(e+fx)\log(1-i\cos(c+dx)+\sin(c+dx))+2f\,\mathrm{Li}_2(i\cos(c+dx)-\sin(c+dx))+\frac{d^2x(2e+fx)(\cos(c)+i\sin(c))}{\cos(c)+i(1+\sin(c))}\right)}{d^3}-\frac{3\left(2df(e+fx)\cos(c)+(-2f^2+d^2(e+fx)^2)\sin(c)\right)\sin(dx)}{d^3}-\frac{6(e+fx)^2\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(x*(3*e^2 + 3*e*f*x + f^2*x^2) + (3*Cos[d*x]*((-2*f^2 + d^2*(e + f*x)^
2)*Cos[c] - 2*d*f*(e + f*x)*Sin[c]))/d^3 + ((6*I)*f*((2*I)*d*(e + f*x)*Log[
1 - I*Cos[c + d*x] + Sin[c + d*x]] + 2*f*PolyLog[2, I*Cos[c + d*x] - Sin[c
+ d*x]] + (d^2*x*(2*e + f*x)*(Cos[c] + I*Sin[c]))/(Cos[c] + I*(1 + Sin[c]))
))/d^3 - (3*(2*d*f*(e + f*x)*Cos[c] + (-2*f^2 + d^2*(e + f*x)^2)*Sin[c])*Si
n[d*x])/d^3 - (6*(e + f*x)^2*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])))/a
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(172) = 344.
time = 0.37, size = 419, normalized size = 2.23

| method | result |
|---|---|
| risch | $-\frac{f^2x^3}{3a}-\frac{fe\,x^2}{a}-\frac{e^2x}{a}-\frac{e^3}{3af}-\frac{(d^2x^2f^2+2d^2efx+2id\,f^2x+d^2e^2+2idef-2f^2)\,\mathrm{e}^{i(dx+c)}}{2a\,d^3}-\frac{(d^2x^2f^2+2d^2efx-2id\,f^2x+d^2e^2-}{2a\,d^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a*f^2*x^3-1/a*f*e*x^2-1/a*e^2*x-1/3/a/f*e^3-1/2*(d^2*x^2*f^2+2*I*d*f^2
*x+2*d^2*e*f*x+2*I*d*e*f+d^2*e^2-2*f^2)/a/d^3*exp(I*(d*x+c))-1/2*(d^2*x^2*f
^2-2*I*d*f^2*x+2*d^2*e*f*x-2*I*d*e*f+d^2*e^2-2*f^2)/a/d^3*exp(-I*(d*x+c))-2
*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(I*(d*x+c))+I)-4/a/d^2*f*ln(exp(I*(d*x+c)))*
e+4/a/d^2*f*ln(exp(I*(d*x+c))+I)*e-4*I/d^2/a*f^2*c*x-2*I/d/a*f^2*x^2-4*I*f^
2*polylog(2,I*exp(I*(d*x+c)))/a/d^3+4/a/d^2*f^2*ln(1-I*exp(I*(d*x+c)))*x+4/
a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c-2*I/d^3/a*f^2*c^2+4/a/d^3*f^2*c*ln(exp(I
*(d*x+c)))-4/a/d^3*f^2*c*ln(exp(I*(d*x+c))+I)
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(173) = 346.
time = 0.71, size = 613, normalized size = 3.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*d^3*f^2*x^3 + 3*(2*d^3*f*e - I*d^2*f^2)*x^2 - 15*I*d^2*e^2 - 6*d*f*e +
6*I*f^2 + 6*(d^3*e^2 - I*d^2*f*e - d*f^2)*x - 24*(d*f*cos(d*x + c)*e + I*d*
f*e*sin(d*x + c) + I*d*f*e)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) + 24*(d
*f^2*x*cos(d*x + c) + I*d*f^2*x*sin(d*x + c) + I*d*f^2*x)*arctan2(cos(d*x +
 c), sin(d*x + c) + 1) + 3*(-I*d^2*f^2*x^2 - I*d^2*e^2 + 2*d*f*e + 2*I*f^2
+ 2*(-I*d^2*f*e + d*f^2)*x)*cos(2*d*x + 2*c) - (2*I*d^3*f^2*x^3 - 3*(-2*I*d
^3*f*e + 5*d^2*f^2)*x^2 - 3*d^2*e^2 - 6*I*d*f*e + 6*f^2 - 6*(-I*d^3*e^2 + 5
*d^2*f*e + I*d*f^2)*x)*cos(d*x + c) + 24*(f^2*cos(d*x + c) + I*f^2*sin(d*x
+ c) + I*f^2)*dilog(I*e^(I*d*x + I*c)) - 12*(d*f^2*x + d*f*e - (I*d*f^2*x +
 I*d*f*e)*cos(d*x + c) + (d*f^2*x + d*f*e)*sin(d*x + c))*log(cos(d*x + c)^2
 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(d^2*f^2*x^2 + d^2*e^2 + 2*I*d*
f*e - 2*f^2 + 2*(d^2*f*e + I*d*f^2)*x)*sin(2*d*x + 2*c) + (2*d^3*f^2*x^3 +
3*(2*d^3*f*e + 5*I*d^2*f^2)*x^2 + 3*I*d^2*e^2 - 6*d*f*e - 6*I*f^2 + 6*(d^3*
e^2 + 5*I*d^2*f*e - d*f^2)*x)*sin(d*x + c))/(-6*I*a*d^3*cos(d*x + c) + 6*a*
d^3*sin(d*x + c) + 6*a*d^3)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(173) = 346.
time = 0.41, size = 721, normalized size = 3.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(d^3*f^2*x^3 + 3*d^2*f^2*x^2 - 6*d*f^2*x + 3*(d^2*f^2*x^2 + 2*d*f^2*x
+ d^2*e^2 - 2*f^2 + 2*(d^2*f*x + d*f)*e)*cos(d*x + c)^2 + (d^3*f^2*x^3 + 6*
d^2*f^2*x^2 - 6*f^2 + 3*(d^3*x + 2*d^2)*e^2 + 3*(d^3*f*x^2 + 4*d^2*f*x)*e)*
cos(d*x + c) + 6*(I*f^2*cos(d*x + c) + I*f^2*sin(d*x + c) + I*f^2)*dilog(I*
cos(d*x + c) - sin(d*x + c)) + 6*(-I*f^2*cos(d*x + c) - I*f^2*sin(d*x + c)
- I*f^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + 3*(d^3*x + d^2)*e^2 + 3*(d
^3*f*x^2 + 2*d^2*f*x - 2*d*f)*e + 6*(c*f^2 - d*f*e + (c*f^2 - d*f*e)*cos(d*
x + c) + (c*f^2 - d*f*e)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) +
I) - 6*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2
)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 6*(d*f^2*x + c*f^2
 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(-I*
cos(d*x + c) + sin(d*x + c) + 1) + 6*(c*f^2 - d*f*e + (c*f^2 - d*f*e)*cos(d
*x + c) + (c*f^2 - d*f*e)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c)
+ I) + (d^3*f^2*x^3 - 3*d^2*f^2*x^2 - 6*d*f^2*x + 3*(d^2*f^2*x^2 - 2*d*f^2*
x + d^2*e^2 - 2*f^2 + 2*(d^2*f*x - d*f)*e)*cos(d*x + c) + 3*(d^3*x - d^2)*e
^2 + 3*(d^3*f*x^2 - 2*d^2*f*x - 2*d*f)*e)*sin(d*x + c))/(a*d^3*cos(d*x + c)
 + a*d^3*sin(d*x + c) + a*d^3)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2 \sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \sin^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sin(c + d*x)**2/(
sin(c + d*x) + 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(c+d\,x\right)}^2\,{\left(e+f\,x\right)}^2}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
[Out] int((sin(c + d*x)^2*(e + f*x)^2)/(a + a*sin(c + d*x)), x)
```

# 3.187 $\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=111

$$-\frac{ex}{a}-\frac{fx^2}{2a}-\frac{(e+fx)\cos(c+dx)}{ad}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{f\sin(c+dx)}{ad^2}$$

```
[Out] -e*x/a-1/2*f*x^2/a-(f*x+e)*cos(d*x+c)/a/d-(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)
/a/d+2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2+f*sin(d*x+c)/a/d^2
```

**Rubi** [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4611, 3377, 2717, 3399, 4269, 3556}

$$\frac{f\sin(c+dx)}{ad^2}+\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}-\frac{(e+fx)\cos(c+dx)}{ad}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{ex}{a}-\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((e*x)/a) - (f*x^2)/(2*a) - ((e + f*x)*Cos[c + d*x])/(a*d) - ((e + f*x)*Co
t[c/2 + Pi/4 + (d*x)/2])/(a*d) + (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/(a*d^
2) + (f*Sin[c + d*x])/(a*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\sin(c+dx)\,dx}{a}-\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx\\ &= -\frac{(e+fx)\cos(c+dx)}{ad}-\frac{\int (e+fx)\,dx}{a}+\frac{f\int \cos(c+dx)\,dx}{ad}+\int \frac{e+f}{a+a\sin(c}\\ &= -\frac{ex}{a}-\frac{fx^2}{2a}-\frac{(e+fx)\cos(c+dx)}{ad}+\frac{f\sin(c+dx)}{ad^2}+\frac{\int (e+fx)\csc^2\left(\frac{1}{2}(c+\right.}{2a}\\ &= -\frac{ex}{a}-\frac{fx^2}{2a}-\frac{(e+fx)\cos(c+dx)}{ad}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{f\sin(c+}{ad^2}\\ &= -\frac{ex}{a}-\frac{fx^2}{2a}-\frac{(e+fx)\cos(c+dx)}{ad}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{2f\log\left(\sin\right.}{}\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(111) = 222.
time = 0.46, size = 236, normalized size = 2.13

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\left(-4de+2cde+2cf-c^2f+2d^2ex-2dfx+d^2fx^2+2d(e+fx)\cos(c+dx)-4f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2f\sin(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(2cde+2cf-c^2f+2d^2ex+2dfx+d^2fx^2+2d(e+fx)\cos(c+dx)-4f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2f\sin(c+dx)\right)\right)}{2ad^2(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sin[(c + d*x)/2]*(-4*d*e + 2*c
*d*e + 2*c*f - c^2*f + 2*d^2*e*x - 2*d*f*x + d^2*f*x^2 + 2*d*(e + f*x)*Cos[
c + d*x] - 4*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*f*Sin[c + d*x])
 + Cos[(c + d*x)/2]*(2*c*d*e + 2*c*f - c^2*f + 2*d^2*e*x + 2*d*f*x + d^2*f*
x^2 + 2*d*(e + f*x)*Cos[c + d*x] - 4*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] - 2*f*Sin[c + d*x])))/(a*d^2*(1 + Sin[c + d*x]))
```

**Maple [A]**
time = 0.22, size = 192, normalized size = 1.73

| method | result |
|---|---|
| risch | $-\frac{f\,x^{2}}{2a}-\frac{ex}{a}-\frac{(dxf+de+if)\mathrm{e}^{i(dx+c)}}{2a\,d^{2}}-\frac{(dxf+de-if)\mathrm{e}^{-i(dx+c)}}{2a\,d^{2}}-\frac{2ifx}{ad}-\frac{2ifc}{a\,d^{2}}-\frac{2(fx+e)}{da\left(\mathrm{e}^{i(dx+c)}+i\right)}+\frac{2f\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a\,d^{2}}$ |
| default | $-\frac{\frac{4e}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\frac{2fx}{d}-\frac{2fx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{4f\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d^{2}}+\frac{2f\ln\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^{2}}+\frac{4e}{d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{4e\arctan\left(\tan\left(\frac{dx}{2}+\right.\right.}{d}}{2a}$ |
| norman | $\frac{\frac{-2de-2f}{a\,d^{2}}+\frac{(2de-2f)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,d^{2}}-\frac{f\,x^{2}}{2a}-\frac{ex\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{ex\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{f\,x^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{f\,x^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{f\,x^{2}\left(\tan^{3}\left(\frac{dx}{2}\right.\right.}{a}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(4*e/d/(tan(1/2*d*x+1/2*c)+1)+(2*f/d*x-2*f/d*x*tan(1/2*d*x+1/2*c))/(
tan(1/2*d*x+1/2*c)+1)-4*f/d^2*ln(tan(1/2*d*x+1/2*c)+1)+2*f/d^2*ln(1+tan(1/2
*d*x+1/2*c)^2)+4*e/d/(1+tan(1/2*d*x+1/2*c)^2)+4*e/d*arctan(tan(1/2*d*x+1/2*
c))+2*f/d^2*(-sin(d*x+c)+(d*x+c)*cos(d*x+c)+1/2*(d*x+c)^2-c*cos(d*x+c)-(d*x
+c)*c))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 1763 vs. 2(100) = 200.
time = 0.55, size = 1763, normalized size = 15.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*c*f*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2)/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1) + a*d*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(
sin(d*x + c)/(cos(d*x + c) + 1))/(a*d)) - 4*((sin(d*x + c)/(cos(d*x + c) +
1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2)/(a + a*sin(d*x + c)/(cos(d*x
+ c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)*e - (((d*x + c
)^2 - 1)*cos(d*x + c)^4 + ((d*x + c)^2 - 1)*sin(d*x + c)^4 + ((d*x + c)*cos
(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c)^3 + 7*(d*x + c)*cos(d*x + c)
^3 + (d*x + (d*x + c)*sin(d*x + c) + c - cos(d*x + c))*sin(2*d*x + 2*c)^3 +
 (2*(d*x + c)^2 - 3)*sin(d*x + c)^3 + (((d*x + c)^2 - 1)*cos(d*x + c)^2 + (
(d*x + c)^2 - 3)*sin(d*x + c)^2 + (d*x + c)^2 + 6*(d*x + c)*cos(d*x + c) +
2*((d*x + c)^2 - (d*x + c)*cos(d*x + c) - 2)*sin(d*x + c) - 1)*cos(2*d*x +
2*c)^2 + ((d*x + c)^2 - 1)*cos(d*x + c)^2 + (((d*x + c)^2 - 3)*cos(d*x + c)
```

```
^2 + ((d*x + c)^2 - 1)*sin(d*x + c)^2 + (d*x + c)^2 + ((d*x + c)*cos(d*x +
c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c) + 8*(d*x + c)*cos(d*x + c) + 2*((d*
x + c)^2 + (d*x + c)*cos(d*x + c) - 1)*sin(d*x + c) - 1)*sin(2*d*x + 2*c)^2
 + (2*((d*x + c)^2 - 1)*cos(d*x + c)^2 + (d*x + c)^2 + 7*(d*x + c)*cos(d*x
+ c) - 3)*sin(d*x + c)^2 + ((d*x + c)*cos(d*x + c)^3 - (2*(d*x + c)^2 - 3)*
sin(d*x + c)^3 - (4*(d*x + c)^2 - (d*x + c)*cos(d*x + c) - 6)*sin(d*x + c)^
2 + 2*cos(d*x + c)^2 - ((2*(d*x + c)^2 - 3)*cos(d*x + c)^2 + 2*(d*x + c)^2
+ 12*(d*x + c)*cos(d*x + c) - 4)*sin(d*x + c) + 1)*cos(2*d*x + 2*c) + (d*x
+ c)*cos(d*x + c) - 2*(cos(d*x + c)^4 + sin(d*x + c)^4 + (cos(d*x + c)^2 +
sin(d*x + c)^2 + 2*sin(d*x + c) + 1)*cos(2*d*x + 2*c)^2 + (cos(d*x + c)^2 +
 sin(d*x + c)^2 + 2*sin(d*x + c) + 1)*sin(2*d*x + 2*c)^2 + 2*cos(d*x + c)^2
*sin(d*x + c) + (2*cos(d*x + c)^2 + 1)*sin(d*x + c)^2 + 2*sin(d*x + c)^3 -
2*(sin(d*x + c)^3 + (cos(d*x + c)^2 + 1)*sin(d*x + c) + 2*sin(d*x + c)^2)*c
os(2*d*x + 2*c) + cos(d*x + c)^2 + 2*(cos(d*x + c)^3 + cos(d*x + c)*sin(d*x
 + c)^2 + 2*cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*sin(2*d*x + 2*c))*log
(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + ((2*(d*x + c)^2 -
3)*cos(d*x + c)^3 + (d*x + c)*sin(d*x + c)^3 + (d*x + (d*x + c)*sin(d*x + c
) + c - cos(d*x + c))*cos(2*d*x + 2*c)^2 + 14*(d*x + c)*cos(d*x + c)^2 + (2
*d*x + (2*(d*x + c)^2 - 3)*cos(d*x + c) + 2*c)*sin(d*x + c)^2 + d*x + 2*((d
*x + c)*cos(d*x + c)^2 - (d*x + c)*sin(d*x + c)^2 - (d*x + c - 2*cos(d*x +
c))*sin(d*x + c) + cos(d*x + c))*cos(2*d*x + 2*c) + 2*((d*x + c)^2 - 1)*cos
(d*x + c) + ((d*x + c)*cos(d*x + c)^2 + 2*d*x + 4*((d*x + c)^2 - 1)*cos(d*x
 + c) + 2*c)*sin(d*x + c) + c)*sin(2*d*x + 2*c) + ((2*(d*x + c)^2 - 3)*cos(
d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 1)*sin(d*x + c))*f/(a*d*cos(d*x + c
)^4 + a*d*sin(d*x + c)^4 + 2*a*d*cos(d*x + c)^2*sin(d*x + c) + 2*a*d*sin(d*
x + c)^3 + a*d*cos(d*x + c)^2 + (a*d*cos(d*x + c)^2 + a*d*sin(d*x + c)^2 +
2*a*d*sin(d*x + c) + a*d)*cos(2*d*x + 2*c)^2 + (a*d*cos(d*x + c)^2 + a*d*si
n(d*x + c)^2 + 2*a*d*sin(d*x + c) + a*d)*sin(2*d*x + 2*c)^2 + (2*a*d*cos(d*
x + c)^2 + a*d)*sin(d*x + c)^2 - 2*(a*d*sin(d*x + c)^3 + 2*a*d*sin(d*x + c)
^2 + (a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))*cos(2*d*x + 2*c) + 2*(a*d*cos
(d*x + c)^3 + a*d*cos(d*x + c)*sin(d*x + c)^2 + 2*a*d*cos(d*x + c)*sin(d*x
+ c) + a*d*cos(d*x + c))*sin(2*d*x + 2*c)))/d
```

**Fricas** [A]
time = 0.35, size = 200, normalized size = 1.80

$$-\frac{d^2 f x^2 + 2\,dfx + 2\,(dfx + de + f)\cos\left(dx + c\right)^2 + (d^2 f x^2 + 4\,dfx + 2\,(d^2 x + 2\,d)e)\cos\left(dx + c\right) + 2\,(d^2 x + d)e - 2\,(f\cos\left(dx + c\right) + f\sin\left(dx + c\right) + f)\log\left(\sin\left(dx + c\right) + 1\right) + (d^2 f x^2 - 2\,dfx + 2\,(dfx + de - f)\cos\left(dx + c\right) + 2\,(d^2 x - d)e - 2\,f)\sin\left(dx + c\right) - 2\,f}{2\,(ad^2\cos\left(dx + c\right) + ad^2\sin\left(dx + c\right) + ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*f*x^2 + 2*d*f*x + 2*(d*f*x + d*e + f)*cos(d*x + c)^2 + (d^2*f*x^2
 + 4*d*f*x + 2*(d^2*x + 2*d)*e)*cos(d*x + c) + 2*(d^2*x + d)*e - 2*(f*cos(d
*x + c) + f*sin(d*x + c) + f)*log(sin(d*x + c) + 1) + (d^2*f*x^2 - 2*d*f*x
+ 2*(d*f*x + d*e - f)*cos(d*x + c) + 2*(d^2*x - d)*e - 2*f)*sin(d*x + c) -
2*f)/(a*d^2*cos(d*x + c) + a*d^2*sin(d*x + c) + a*d^2)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. $2(87) = 174$.
time = 1.41, size = 1867, normalized size = 16.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-2*d**2*e*x*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 +
2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*d*
*2*e*x*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2
 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*d**2*e*x*tan(c/2 +
 d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*
d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*d**2*e*x/(2*a*d**2*tan(c/2 + d*x/2)**
3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) -
d**2*f*x**2*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*ta
n(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - d**2*f*x**2*tan
(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)*
*2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - d**2*f*x**2*tan(c/2 + d*x/2)/(
2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(
c/2 + d*x/2) + 2*a*d**2) - d**2*f*x**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*
d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*d*e*ta
n(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)
**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*d*e*tan(c/2 + d*x/2)/(2*a*d
**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 +
 d*x/2) + 2*a*d**2) - 8*d*e/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/
2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*d*f*x*tan(c/2 + d
*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a
*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*d*f*x/(2*a*d**2*tan(c/2 + d*x/2)**3
+ 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*
f*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)*
*3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) +
 4*f*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/
2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2
) + 4*f*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/
2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2
) + 4*f*log(tan(c/2 + d*x/2) + 1)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*
tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*f*log(tan(c
/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a
*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*f*log
(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3
 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2
*f*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)
**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2)
```

```
- 2*f*log(tan(c/2 + d*x/2)**2 + 1)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2
*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*f*tan(c/2
+ d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 +
2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*f*tan(c/2 + d*x/2)/(2*a*d**2*tan(
c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2)
+ 2*a*d**2), Ne(d, 0)), ((e*x + f*x**2/2)*sin(c)**2/(a*sin(c) + a), True))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 20204 vs. $2(100) = 200$.
time = 5.82, size = 20204, normalized size = 182.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - d^2*f*x^2*tan(1/2*d*
x)^3*tan(1/2*c)^4*tan(c)^2 - d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2
 - 2*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 + d^2*f*x^2*tan(1/2*d*x)^
3*tan(1/2*c)^5 + 2*d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - d^2*f*x
^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 2*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*
c)^4*tan(c)^2 + d^2*f*x^2*tan(1/2*d*x)*tan(1/2*c)^5*tan(c)^2 + 2*d^2*x*e*ta
n(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 + 4*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^5*ta
n(c)^2 - d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - d^2*f*x^2*tan(1/2*d*x)^2*t
an(1/2*c)^5 + 2*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*c)^5 - 16*d^2*x*e*tan(1/2*d*
x)^3*tan(1/2*c)^4*tan(c) + 4*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c) - 2*d^2*f*x^2*tan(1/2
*d*x)^3*tan(1/2*c)^2*tan(c)^2 - 2*d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan
(c)^2 + 12*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - d^2*f*x^2*tan(1/2
*d*x)*tan(1/2*c)^4*tan(c)^2 + 2*d^2*x*e*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^
2 - 8*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c)
- 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 - d^2*f*x^2*tan(1/2*c)^5*tan(c)^2 - 2*d^2*
x*e*tan(1/2*d*x)*tan(1/2*c)^5*tan(c)^2 - 2*d*e*tan(1/2*d*x)^3*tan(1/2*c)^5*
tan(c)^2 - 2*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 + 2*d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*
c)^3 - d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^4 - 2*d^2*x*e*tan(1/2*d*x)^3*tan
```

```
(1/2*c)^4 + d^2*f*x^2*tan(1/2*d*x)*tan(1/2*c)^5 - 2*d^2*x*e*tan(1/2*d*x)^2*
tan(1/2*c)^5 + 4*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^5 + 16*d^2*x*e*tan(1/2*d*x
)^3*tan(1/2*c)^3*tan(c) + 16*d^2*x*e*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c) - 4
*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*t
an(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^3*tan(1/2*c)^4*tan(c) - 4*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/
(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c) + d^2*f*x^2*tan(1/2*
d*x)^3*tan(1/2*c)*tan(c)^2 - 2*d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)
^2 - 12*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*c)^2*tan(c)^2 + 2*d^2*f*x^2*tan(1/2*
d*x)*tan(1/2*c)^3*tan(c)^2 - 12*d^2*x*e*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(c)^
2 + 4*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 8*d*e*log(2*(tan(1/2*d*x
)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c
)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2
*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*
x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(
c)^2 - d^2*f*x^2*tan(1/2*c)^4*tan(c)^2 + 2*d^2*x*e*tan(1/2*d*x)*tan(1/2*c)^
4*tan(c)^2 - 12*d*f*x*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 8*d*e*log(2*(t
an(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2
*c)^4*tan(c)^2 + 2*d*e*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 + 2*f*log(2*(ta
n(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*
tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*
c)^4*tan(c)^2 + 2*d^2*x*e*tan(1/2*c)^5*tan(c)^2 + 2*d*e*tan(1/2*d*x)^2*tan(
1/2*c)^5*tan(c)^2 + 2*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 - 2*d^2*f*x^2*tan(1/2*d*x)^3
*tan(1/2*c)^2 - 2*d^2*f*x^2*tan(1/2*d*x)^2*tan(...
```

**Mupad [B]**

time = 1.77, size = 164, normalized size = 1.48

$$-e^{c\,1i+d\,x\,1i}\,\left(\frac{d\,e+f\,1i}{2\,a\,d^2}+\frac{f\,x}{2\,a\,d}\right)+e^{-c\,1i-d\,x\,1i}\,\left(\frac{-d\,e+f\,1i}{2\,a\,d^2}-\frac{f\,x}{2\,a\,d}\right)-\frac{f\,x^2}{2\,a}+\frac{2\,f\,\ln\left(e^{c\,1i}\,e^{d\,x\,1i}+1i\right)}{a\,d^2}-\frac{x\,\left(d\,e+f\,2i\right)}{a\,d}-\frac{\left(e+f\,x\right)\,2i}{a\,d\,\left(-1+e^{c\,1i+d\,x\,1i}\,1i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(e + f*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] exp(- c*1i - d*x*1i)*((f*1i - d*e)/(2*a*d^2) - (f*x)/(2*a*d)) - exp(c*1i +
d*x*1i)*((f*1i + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) - (f*x^2)/(2*a) + (2*f*log
(exp(c*1i)*exp(d*x*1i) + 1i))/(a*d^2) - (x*(f*2i + d*e))/(a*d) - ((e + f*x)
*2i)/(a*d*(exp(c*1i + d*x*1i)*1i - 1))
```

# 3.188 $\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=45

$$-\frac{x}{a}-\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)}{ad(1+\sin(c+dx))}$$

```
[Out] -x/a-cos(d*x+c)/a/d-cos(d*x+c)/a/d/(1+sin(d*x+c))
```

**Rubi** [A]
time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2825, 12, 2814, 2727}

$$-\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)}{ad(\sin(c+dx)+1)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x]),x]
[Out] -(x/a) - Cos[c + d*x]/(a*d) - Cos[c + d*x]/(a*d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\cos(c+dx)}{ad} - \frac{\int \frac{a\sin(c+dx)}{a+a\sin(c+dx)}\,dx}{a}$$
$$= -\frac{\cos(c+dx)}{ad} - \int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{x}{a} - \frac{\cos(c+dx)}{ad} + \int \frac{1}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{x}{a} - \frac{\cos(c+dx)}{ad} - \frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

**Mathematica** [A]
time = 0.10, size = 85, normalized size = 1.89

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)(c+dx+\cos(c+dx))+(-2+c+dx+\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2]*(c + d*x + Cos[c
 + d*x]) + (-2 + c + d*x + Cos[c + d*x])*Sin[(c + d*x)/2]))/(a*d*(1 + Sin[c
 + d*x])))
```

**Maple** [A]
time = 0.07, size = 54, normalized size = 1.20

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ |
| default | $\frac{-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ |
| risch | $-\frac{x}{a}-\frac{e^{i(dx+c)}}{2ad}-\frac{e^{-i(dx+c)}}{2ad}-\frac{2}{da\left(e^{i(dx+c)}+i\right)}$ |
| norman | $\frac{-\frac{2}{ad}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{x}{a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-2}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a*(-1/4/(tan(1/2*d*x+1/2*c)+1)-1/4/(1+tan(1/2*d*x+1/2*c)^2)-1/4*arctan(
tan(1/2*d*x+1/2*c)))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(45) = 90$.

time = 0.50, size = 129, normalized size = 2.87

$$-\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+2}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 2)/(a + a*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a)/d
```

**Fricas [A]**

time = 0.35, size = 69, normalized size = 1.53

$$-\frac{dx+(dx+2)\cos(dx+c)+\cos(dx+c)^2+(dx+\cos(dx+c)-1)\sin(dx+c)+1}{ad\cos(dx+c)+ad\sin(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x + (d*x + 2)*cos(d*x + c) + cos(d*x + c)^2 + (d*x + cos(d*x + c) - 1)*
sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(32) = 64$.

time = 1.25, size = 422, normalized size = 9.38

$$\begin{cases} -\frac{dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{dx}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{4}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\sin^2(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2
+ d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) - d*x*tan(c/2 + d*x/2)**2/(a*d*ta
n(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) -
 d*x*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 +
a*d*tan(c/2 + d*x/2) + a*d) - d*x/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 +
d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c/
2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) - 2*t
an(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 + a*d*ta
```

```
n(c/2 + d*x/2) + a*d) - 4/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**
2 + a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x*sin(c)**2/(a*sin(c) + a), Tr
ue))
```

**Giac** [A]
time = 5.19, size = 77, normalized size = 1.71

$$-\frac{\frac{dx+c}{a}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a + 2*(tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 2)/((tan
(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 1)*a)
)/d
```

**Mupad** [B]
time = 1.18, size = 69, normalized size = 1.53

$$-\frac{x}{a}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

```
[Out] - x/a - (2*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^2 + 4)/(a*d*(tan(c/2 +
 (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

# 3.189 $\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\mathrm{Int}\left(\frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 5.82, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(dx+c)}{(fx+e)\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
[Out] int(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(((I*e*exp_integral_e(1, (I*d*f*x + I*d*e)/f) - I*e*exp_integral_e(1,
-(I*d*f*x + I*d*e)/f))*d*cos((c*f - d*e)/f) + (e*exp_integral_e(1, (I*d*f*x
 + I*d*e)/f) + e*exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*d*sin((c*f - d*e)
/f) + (d*f*(I*exp_integral_e(1, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(1,
-(I*d*f*x + I*d*e)/f))*cos((c*f - d*e)/f) + d*f*(exp_integral_e(1, (I*d*f*x
 + I*d*e)/f) + exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*sin((c*f - d*e)/f))
*x)*cos(d*x + c)^2 + (I*e*exp_integral_e(1, (I*d*f*x + I*d*e)/f) - I*e*exp_
integral_e(1, -(I*d*f*x + I*d*e)/f))*d*cos((c*f - d*e)/f) + ((I*e*exp_integ
ral_e(1, (I*d*f*x + I*d*e)/f) - I*e*exp_integral_e(1, -(I*d*f*x + I*d*e)/f)
)*d*cos((c*f - d*e)/f) + (e*exp_integral_e(1, (I*d*f*x + I*d*e)/f) + e*exp_
integral_e(1, -(I*d*f*x + I*d*e)/f))*d*sin((c*f - d*e)/f) + (d*f*(I*exp_int
egral_e(1, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(1, -(I*d*f*x + I*d*e)/f)
)*cos((c*f - d*e)/f) + d*f*(exp_integral_e(1, (I*d*f*x + I*d*e)/f) + exp_in
tegral_e(1, -(I*d*f*x + I*d*e)/f))*sin((c*f - d*e)/f))*x)*sin(d*x + c)^2 +
(e*exp_integral_e(1, (I*d*f*x + I*d*e)/f) + e*exp_integral_e(1, -(I*d*f*x +
 I*d*e)/f))*d*sin((c*f - d*e)/f) + (d*f*(I*exp_integral_e(1, (I*d*f*x + I*d
*e)/f) - I*exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*cos((c*f - d*e)/f) + d*
f*(exp_integral_e(1, (I*d*f*x + I*d*e)/f) + exp_integral_e(1, -(I*d*f*x + I
*d*e)/f))*sin((c*f - d*e)/f))*x + 4*f*cos(d*x + c) + 4*(a*d*f^3*x + a*d*f^2
*e + (a*d*f^3*x + a*d*f^2*e)*cos(d*x + c)^2 + (a*d*f^3*x + a*d*f^2*e)*sin(d
*x + c)^2 + 2*(a*d*f^3*x + a*d*f^2*e)*sin(d*x + c))*integrate(cos(d*x + c)/
(a*d*f^2*x^2 + 2*a*d*f*x*e + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*cos(d*x
+ c)^2 + a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*sin(d*x + c)^2 + 2
*(a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*sin(d*x + c)), x) + 2*(d*f*x + (d*f*
x + d*e)*cos(d*x + c)^2 + (d*f*x + d*e)*sin(d*x + c)^2 + d*e + 2*(d*f*x + d
*e)*sin(d*x + c))*log(f*x + e) - 2*((-I*e*exp_integral_e(1, (I*d*f*x + I*d*
e)/f) + I*e*exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*d*cos((c*f - d*e)/f) -
 (e*exp_integral_e(1, (I*d*f*x + I*d*e)/f) + e*exp_integral_e(1, -(I*d*f*x
+ I*d*e)/f))*d*sin((c*f - d*e)/f) + (d*f*(-I*exp_integral_e(1, (I*d*f*x + I
*d*e)/f) + I*exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*cos((c*f - d*e)/f) -
d*f*(exp_integral_e(1, (I*d*f*x + I*d*e)/f) + exp_integral_e(1, -(I*d*f*x +
 I*d*e)/f))*sin((c*f - d*e)/f))*x)*sin(d*x + c))/(a*d*f^2*x + a*d*f*e + (a*
```

```
d*f^2*x + a*d*f*e)*cos(d*x + c)^2 + (a*d*f^2*x + a*d*f*e)*sin(d*x + c)^2 +
2*(a*d*f^2*x + a*d*f*e)*sin(d*x + c))
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)),
x)
```

**Sympy** [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
[Out] Integral(sin(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^2/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(c+d\,x)^2}{(e+f\,x)\,(a+a\,\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/((e + f*x)*(a + a*sin(c + d*x))),x)
[Out] int(sin(c + d*x)^2/((e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.190 $\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 6.50, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(dx+c)}{(fx+e)^2\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(((I*e*exp_integral_e(2, (I*d*f*x + I*d*e)/f) - I*e*exp_integral_e(2,
-(I*d*f*x + I*d*e)/f))*d*cos((c*f - d*e)/f) + (e*exp_integral_e(2, (I*d*f*x
 + I*d*e)/f) + e*exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*d*sin((c*f - d*e)
/f) + (d*f*(I*exp_integral_e(2, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(2,
-(I*d*f*x + I*d*e)/f))*cos((c*f - d*e)/f) + d*f*(exp_integral_e(2, (I*d*f*x
 + I*d*e)/f) + exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*sin((c*f - d*e)/f)
- 2*d*f)*x - 2*d*e)*cos(d*x + c)^2 + (I*e*exp_integral_e(2, (I*d*f*x + I*d*
e)/f) - I*e*exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*d*cos((c*f - d*e)/f) +
 ((I*e*exp_integral_e(2, (I*d*f*x + I*d*e)/f) - I*e*exp_integral_e(2, -(I*d
*f*x + I*d*e)/f))*d*cos((c*f - d*e)/f) + (e*exp_integral_e(2, (I*d*f*x + I*
d*e)/f) + e*exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*d*sin((c*f - d*e)/f) +
 (d*f*(I*exp_integral_e(2, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(2, -(I*d
*f*x + I*d*e)/f))*cos((c*f - d*e)/f) + d*f*(exp_integral_e(2, (I*d*f*x + I*
d*e)/f) + exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*sin((c*f - d*e)/f) - 2*d
*f)*x - 2*d*e)*sin(d*x + c)^2 + (e*exp_integral_e(2, (I*d*f*x + I*d*e)/f) +
 e*exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*d*sin((c*f - d*e)/f) + (d*f*(I*
exp_integral_e(2, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(2, -(I*d*f*x + I*
d*e)/f))*cos((c*f - d*e)/f) + d*f*(exp_integral_e(2, (I*d*f*x + I*d*e)/f) +
 exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*sin((c*f - d*e)/f) - 2*d*f)*x + 4
*f*cos(d*x + c) - 2*d*e + 8*(a*d*f^4*x^2 + 2*a*d*f^3*x*e + a*d*f^2*e^2 + (a
*d*f^4*x^2 + 2*a*d*f^3*x*e + a*d*f^2*e^2)*cos(d*x + c)^2 + (a*d*f^4*x^2 + 2
*a*d*f^3*x*e + a*d*f^2*e^2)*sin(d*x + c)^2 + 2*(a*d*f^4*x^2 + 2*a*d*f^3*x*e
 + a*d*f^2*e^2)*sin(d*x + c))*integrate(cos(d*x + c)/(a*d*f^3*x^3 + 3*a*d*f
^2*x^2*e + 3*a*d*f*x*e^2 + (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 +
 a*d*e^3)*cos(d*x + c)^2 + a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d
*f*x*e^2 + a*d*e^3)*sin(d*x + c)^2 + 2*(a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a
*d*f*x*e^2 + a*d*e^3)*sin(d*x + c)), x) - 2*((-I*e*exp_integral_e(2, (I*d*f
*x + I*d*e)/f) + I*e*exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*d*cos((c*f -
d*e)/f) - (e*exp_integral_e(2, (I*d*f*x + I*d*e)/f) + e*exp_integral_e(2, -
(I*d*f*x + I*d*e)/f))*d*sin((c*f - d*e)/f) + (d*f*(-I*exp_integral_e(2, (I*
d*f*x + I*d*e)/f) + I*exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*cos((c*f - d
*e)/f) - d*f*(exp_integral_e(2, (I*d*f*x + I*d*e)/f) + exp_integral_e(2, -(
```

```
I*d*f*x + I*d*e)/f))*sin((c*f - d*e)/f) + 2*d*f)*x + 2*d*e)*sin(d*x + c))/(
a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^2 + (a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*
d*f*e^2)*cos(d*x + c)^2 + (a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^2)*sin(d*x
 + c)^2 + 2*(a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^2)*sin(d*x + c))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2
+ 2*a*f*x*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin\left(c+d\,x\right)^2}{{\left(e+f\,x\right)}^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/((e + f*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] int(sin(c + d*x)^2/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.191 $\int \frac{(e+fx)^3\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=382

$$-\frac{3ef^2x}{4ad^2}-\frac{3f^3x^2}{8ad^2}+\frac{i(e+fx)^3}{ad}+\frac{3(e+fx)^4}{8af}-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}+\frac{(e+fx)^3\cos(c+dx)}{ad}+\frac{(e+fx)^3\cot}{a}$$

```
[Out] -3/4*e*f^2*x/a/d^2-3/8*f^3*x^2/a/d^2+12*I*f^2*(f*x+e)*polylog(2,I*exp(I*(d*
x+c)))/a/d^3+3/8*(f*x+e)^4/a/f-6*f^2*(f*x+e)*cos(d*x+c)/a/d^3+(f*x+e)^3*cos
(d*x+c)/a/d+(f*x+e)^3*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-6*f*(f*x+e)^2*ln(1-I*ex
p(I*(d*x+c)))/a/d^2+I*(f*x+e)^3/a/d-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^
4+6*f^3*sin(d*x+c)/a/d^4-3*f*(f*x+e)^2*sin(d*x+c)/a/d^2+3/4*f^2*(f*x+e)*cos
(d*x+c)*sin(d*x+c)/a/d^3-1/2*(f*x+e)^3*cos(d*x+c)*sin(d*x+c)/a/d-3/8*f^3*si
n(d*x+c)^2/a/d^4+3/4*f*(f*x+e)^2*sin(d*x+c)^2/a/d^2
```

**Rubi** [A]
time = 0.41, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {4611, 3392, 32, 3391, 3377, 2717, 3399, 4269, 3798, 2221, 2611, 2320, 6724}

$$-\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}+\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{3f^3\sin^2(c+dx)}{8ad^4}+\frac{6f^3\sin(c+dx)}{ad^4}-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}+\frac{3f^2(e+fx)\sin(c+dx)\cos(c+dx)}{4ad^3}-\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4ad^2}-\frac{3f(e+fx)^2\sin(c+dx)}{ad^2}+\frac{(e+fx)^3\cos(c+dx)}{ad}+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{3ef^2x}{4ad^2}-\frac{3f^3x^2}{8ad^2}+\frac{i(e+fx)^3}{ad}+\frac{3(e+fx)^4}{8af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*e*f^2*x)/(4*a*d^2) - (3*f^3*x^2)/(8*a*d^2) + (I*(e + f*x)^3)/(a*d) + (3
*(e + f*x)^4)/(8*a*f) - (6*f^2*(e + f*x)*Cos[c + d*x])/(a*d^3) + ((e + f*x)
^3*Cos[c + d*x])/(a*d) + ((e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (6
*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + ((12*I)*f^2*(e + f*x)*
PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - (12*f^3*PolyLog[3, I*E^(I*(c + d*x
))])/(a*d^4) + (6*f^3*Sin[c + d*x])/(a*d^4) - (3*f*(e + f*x)^2*Sin[c + d*x]
)/(a*d^2) + (3*f^2*(e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^3) - ((e + f
*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - (3*f^3*Sin[c + d*x]^2)/(8*a*d^4)
 + (3*f*(e + f*x)^2*Sin[c + d*x]^2)/(4*a*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sin^2(c+dx)\,dx}{a} - \int \frac{(e+fx)^3\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)^3\cos(c+dx)\sin(c+dx)}{2ad} + \frac{3f(e+fx)^2\sin^2(c+dx)}{4ad^2} + \frac{\int (e+fx)^3}{2a}$$

$$= \frac{(e+fx)^4}{8af} + \frac{(e+fx)^3\cos(c+dx)}{ad} + \frac{3f^2(e+fx)\cos(c+dx)\sin(c+dx)}{4ad^3} -$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{3(e+fx)^4}{8af} + \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{3f(e+fx)^2\sin(c}{ad^2}$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c}{ad}$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} +$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} +$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} +$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} +$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} +$$

**<u>Mathematica</u> [A]**
time = 3.37, size = 391, normalized size = 1.02

$$\frac{24e^3x+36e^2fx^2+24ef^2x^3+6f^3x^4-\frac{96f^2(e+fx)\cos(c+dx)}{d^3}+\frac{16(e+fx)^3\cos(c+dx)}{d}+\frac{3f^3\cos(2(c+dx))}{d^4}-\frac{6f(e+fx)^2\cos(2(c+dx))}{d^2}-\frac{96f(e+fx)^2\log(1-i\cos(c+dx)+\sin(c+dx))}{d^2}+\frac{192if^2(e+fx)\text{Li}_2(i\cos(c+dx)-\sin(c+dx))}{d^3}-\frac{192f^3\text{Li}_3(i\cos(c+dx)-\sin(c+dx))}{d^4}+\frac{32ifx(3e^2+3efx+f^2x^2)(\cos(c)+i\sin(c))}{d(\cos(c)+i(1+\sin(c)))}-\frac{32(e+fx)^3\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{96f^3\sin(c+dx)}{d^4}-\frac{48f(e+fx)^2\sin(c+dx)}{d^2}+\frac{6f^2(e+fx)\sin(2(c+dx))}{d^3}-\frac{4(e+fx)^3\sin(2(c+dx))}{d}}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (24*e^3*x + 36*e^2*f*x^2 + 24*e*f^2*x^3 + 6*f^3*x^4 - (96*f^2*(e + f*x)*Cos
[c + d*x])/d^3 + (16*(e + f*x)^3*Cos[c + d*x])/d + (3*f^3*Cos[2*(c + d*x)])
/d^4 - (6*f*(e + f*x)^2*Cos[2*(c + d*x)])/d^2 - (96*f*(e + f*x)^2*Log[1 - I
*Cos[c + d*x] + Sin[c + d*x]])/d^2 + ((192*I)*f^2*(e + f*x)*PolyLog[2, I*Co
s[c + d*x] - Sin[c + d*x]])/d^3 - (192*f^3*PolyLog[3, I*Cos[c + d*x] - Sin[
c + d*x]])/d^4 + ((32*I)*f*x*(3*e^2 + 3*e*f*x + f^2*x^2)*(Cos[c] + I*Sin[c]
```

```
))/(d*(Cos[c] + I*(1 + Sin[c]))) - (32*(e + f*x)^3*Sin[(d*x)/2])/(d*(Cos[c/
2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (96*f^3*Sin[c + d*x
])/d^4 - (48*f*(e + f*x)^2*Sin[c + d*x])/d^2 + (6*f^2*(e + f*x)*Sin[2*(c +
d*x)])/d^3 - (4*(e + f*x)^3*Sin[2*(c + d*x)])/d)/(16*a)
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(351) = 702$.
time = 0.21, size = 881, normalized size = 2.31

| method | result |
|---|---|
| risch | $-\frac{3f(2d^2x^2f^2+4d^2efx+2d^2e^2-f^2)\cos(2dx+2c)}{16a\,d^4}+\frac{(d^3x^3f^3+3d^3e\,f^2x^2-3id^2f^3x^2+3d^3e^2fx-6id^2e\,f^2x+d^3e^3-3id^2e^2f-6d\,f^3}{2a\,d^4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*f*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-f^2)/a/d^4*cos(2*d*x+2*c)+3/8/
a*f^3*x^4+3/8/a/f*e^4-6/a/d^2*f^3*ln(1-I*exp(I*(d*x+c)))*x^2+6/a/d^4*f^3*ln
(1-I*exp(I*(d*x+c)))*c^2+6/a/d^2*f*ln(exp(I*(d*x+c)))*e^2+6/a/d^4*f^3*c^2*l
n(exp(I*(d*x+c)))-6/a/d^4*f^3*c^2*ln(exp(I*(d*x+c))+I)-6/a/d^2*f*ln(exp(I*(
d*x+c))+I)*e^2+2*I/a/d*f^3*x^3-4*I/a/d^4*f^3*c^3+12*I/a/d^2*f^2*e*c*x+3/2/a
*f^2*e*x^3+9/4/a*f*e^2*x^2+3/2/a*e^3*x-12/a/d^3*f^2*e*c*ln(exp(I*(d*x+c)))+
12*I/a/d^3*f^3*polylog(2,I*exp(I*(d*x+c)))*x-6*I/a/d^3*f^3*c^2*x+6*I/a/d*f^
2*e*x^2+6*I/a/d^3*f^2*e*c^2+12*I/a/d^3*f^2*e*polylog(2,I*exp(I*(d*x+c)))+2*
(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(I*(d*x+c))+I)-1/8/d^3*(2*d^2*f
^3*x^3+6*d^2*e*f^2*x^2+6*d^2*e^2*f*x+2*d^2*e^3-3*f^3*x-3*e*f^2)/a*sin(2*d*x
+2*c)-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+12/a/d^3*f^2*e*c*ln(exp(I*(d
*x+c))+I)-12/a/d^2*f^2*e*ln(1-I*exp(I*(d*x+c)))*x-12/a/d^3*f^2*e*ln(1-I*exp
(I*(d*x+c)))*c+1/2*(d^3*x^3*f^3+3*I*d^2*f^3*x^2+3*d^3*e*f^2*x^2+6*I*d^2*e*f
^2*x+3*d^3*e^2*f*x+3*I*d^2*e^2*f+d^3*e^3-6*d*f^3*x-6*I*f^3-6*d*e*f^2)/a/d^4
*exp(I*(d*x+c))+1/2*(d^3*x^3*f^3-3*I*d^2*f^3*x^2+3*d^3*e*f^2*x^2-6*I*d^2*e*
f^2*x+3*d^3*e^2*f*x-3*I*d^2*e^2*f+d^3*e^3-6*d*f^3*x+6*I*f^3-6*d*e*f^2)/a/d^
4*exp(-I*(d*x+c))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(357) = 714$.
time = 0.44, size = 1569, normalized size = 4.11

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*d^4*f^3*x^4 + 16*d^3*f^3*x^3 - 42*d^2*f^3*x^2 + 2*(4*d^3*f^3*x^3 -
6*d^2*f^3*x^2 - 6*d*f^3*x + 4*d^3*e^3 + 3*f^3 + 6*(2*d^3*f*x - d^2*f)*e^2 +
 6*(2*d^3*f^2*x^2 - 2*d^2*f^2*x - d*f^2)*e)*cos(d*x + c)^3 + 93*f^3 + 2*(8*
d^3*f^3*x^3 + 18*d^2*f^3*x^2 - 48*d*f^3*x + 8*d^3*e^3 - 45*f^3 + 6*(4*d^3*f
*x + 3*d^2*f)*e^2 + 12*(2*d^3*f^2*x^2 + 3*d^2*f^2*x - 4*d*f^2)*e)*cos(d*x +
 c)^2 + 3*(2*d^4*f^3*x^4 + 8*d^3*f^3*x^3 + 2*d^2*f^3*x^2 - 28*d*f^3*x - f^3
 + 8*(d^4*x + d^3)*e^3 + 2*(6*d^4*f*x^2 + 12*d^3*f*x + d^2*f)*e^2 + 4*(2*d^
4*f^2*x^3 + 6*d^3*f^2*x^2 + d^2*f^2*x - 7*d*f^2)*e)*cos(d*x + c) - 96*(-I*d
*f^3*x - I*d*f^2*e + (-I*d*f^3*x - I*d*f^2*e)*cos(d*x + c) + (-I*d*f^3*x -
I*d*f^2*e)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - 96*(I*d*f^3
*x + I*d*f^2*e + (I*d*f^3*x + I*d*f^2*e)*cos(d*x + c) + (I*d*f^3*x + I*d*f^
2*e)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) + 8*(3*d^4*x + 2*d
^3)*e^3 + 6*(6*d^4*f*x^2 + 8*d^3*f*x - 7*d^2*f)*e^2 + 12*(2*d^4*f^2*x^3 + 4
*d^3*f^2*x^2 - 7*d^2*f^2*x)*e - 48*(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + (c^
2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(d*x + c) + (c^2*f^3 - 2*c*d*f^2*e + d^
2*f*e^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 48*(d^2*f^3
*x^2 - c^2*f^3 + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*
x + c) + 2*(d^2*f^2*x + c*d*f^2)*e + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x
+ c*d*f^2)*e)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 48*(d^
2*f^3*x^2 - c^2*f^3 + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*c
os(d*x + c) + 2*(d^2*f^2*x + c*d*f^2)*e + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f
^2*x + c*d*f^2)*e)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) -
48*(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)
*cos(d*x + c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*sin(d*x + c))*log(-cos(
d*x + c) + I*sin(d*x + c) + I) - 96*(f^3*cos(d*x + c) + f^3*sin(d*x + c) +
f^3)*polylog(3, I*cos(d*x + c) - sin(d*x + c)) - 96*(f^3*cos(d*x + c) + f^3
*sin(d*x + c) + f^3)*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) + (6*d^4*f^
3*x^4 - 16*d^3*f^3*x^3 - 42*d^2*f^3*x^2 + 93*f^3 - 2*(4*d^3*f^3*x^3 + 6*d^2
*f^3*x^2 - 6*d*f^3*x + 4*d^3*e^3 - 3*f^3 + 6*(2*d^3*f*x + d^2*f)*e^2 + 6*(2
*d^3*f^2*x^2 + 2*d^2*f^2*x - d*f^2)*e)*cos(d*x + c)^2 + 4*(2*d^3*f^3*x^3 -
12*d^2*f^3*x^2 - 21*d*f^3*x + 2*d^3*e^3 + 24*f^3 + 6*(d^3*f*x - 2*d^2*f)*e^
2 + 3*(2*d^3*f^2*x^2 - 8*d^2*f^2*x - 7*d*f^2)*e)*cos(d*x + c) + 8*(3*d^4*x
- 2*d^3)*e^3 + 6*(6*d^4*f*x^2 - 8*d^3*f*x - 7*d^2*f)*e^2 + 12*(2*d^4*f^2*x^
3 - 4*d^3*f^2*x^2 - 7*d^2*f^2*x)*e)*sin(d*x + c))/(a*d^4*cos(d*x + c) + a*d
^4*sin(d*x + c) + a*d^4)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3 \sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3 x^3 \sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2 \sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx \sin^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*
sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sin(c + d*x
)**3/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sin(c + d*x)**3/(sin(c +
d*x) + 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)^3\,(e+f\,x)^3}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((sin(c + d*x)^3*(e + f*x)^3)/(a + a*sin(c + d*x)), x)
```

# 3.192 $\int \frac{(e+fx)^2 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=278

$$-\frac{f^2x}{4ad^2}+\frac{i(e+fx)^2}{ad}+\frac{(e+fx)^3}{2af}-\frac{2f^2\cos(c+dx)}{ad^3}+\frac{(e+fx)^2\cos(c+dx)}{ad}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{4f(e$$

```
[Out] -1/4*f^2*x/a/d^2+I*(f*x+e)^2/a/d+1/2*(f*x+e)^3/a/f-2*f^2*cos(d*x+c)/a/d^3+(
f*x+e)^2*cos(d*x+c)/a/d+(f*x+e)^2*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-4*f*(f*x+e)
*ln(1-I*exp(I*(d*x+c)))/a/d^2+4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3-2*f
*(f*x+e)*sin(d*x+c)/a/d^2+1/4*f^2*cos(d*x+c)*sin(d*x+c)/a/d^3-1/2*(f*x+e)^2
*cos(d*x+c)*sin(d*x+c)/a/d+1/2*f*(f*x+e)*sin(d*x+c)^2/a/d^2
```

**Rubi** [A]
time = 0.32, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$,
Rules used = {4611, 3392, 32, 2715, 8, 3377, 2718, 3399, 4269, 3798, 2221, 2317, 2438}

$$\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{2f^2\cos(c+dx)}{ad^3}+\frac{f^2\sin(c+dx)\cos(c+dx)}{4ad^3}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{f(e+fx)\sin^2(c+dx)}{2ad^2}-\frac{2f(e+fx)\sin(c+dx)}{ad^2}+\frac{(e+fx)^2\cos(c+dx)}{ad}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2ad}-\frac{f^2x}{4ad^2}+\frac{i(e+fx)^2}{ad}+\frac{(e+fx)^3}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(f^2*x)/(a*d^2) + (I*(e + f*x)^2)/(a*d) + (e + f*x)^3/(2*a*f) - (2*f^2
*Cos[c + d*x])/(a*d^3) + ((e + f*x)^2*Cos[c + d*x])/(a*d) + ((e + f*x)^2*Co
t[c/2 + Pi/4 + (d*x)/2])/(a*d) - (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])
/(a*d^2) + ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - (2*f*(e + f*
x)*Sin[c + d*x])/(a*d^2) + (f^2*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^3) - ((e
+ f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) + (f*(e + f*x)*Sin[c + d*x]^2)/
(2*a*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^2\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^2\sin^2(c+dx)\,dx}{a} - \int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx \\
&= -\frac{(e+fx)^2\cos(c+dx)\sin(c+dx)}{2ad} + \frac{f(e+fx)\sin^2(c+dx)}{2ad^2} + \frac{\int (e+fx)^2}{2a} \\
&= \frac{(e+fx)^3}{6af} + \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{f^2\cos(c+dx)\sin(c+dx)}{4ad^3} - \frac{(e+fx)}{} \\
&= -\frac{f^2x}{4ad^2} + \frac{(e+fx)^3}{2af} + \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{2f(e+fx)\sin(c+dx)}{ad^2} + f \\
&= -\frac{f^2x}{4ad^2} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{(e+fx)^2}{} \\
&= -\frac{f^2x}{4ad^2} + \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx}{ad} \\
&= -\frac{f^2x}{4ad^2} + \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx}{ad} \\
&= -\frac{f^2x}{4ad^2} + \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx}{ad} \\
&= -\frac{f^2x}{4ad^2} + \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx}{ad}
\end{aligned}
$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 931 vs. $2(278) = 556$.
time = 1.52, size = 931, normalized size = 3.35

---

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (8*d^2*e^2*Cos[c + (d*x)/2] - 16*f^2*Cos[c + (d*x)/2] + 48*d^2*e*f*x*Cos[c
+ (d*x)/2] + 24*d^2*f^2*x^2*Cos[c + (d*x)/2] + 6*d^2*e^2*Cos[c + (3*d*x)/2]
 - 15*f^2*Cos[c + (3*d*x)/2] + 12*d^2*e*f*x*Cos[c + (3*d*x)/2] + 6*d^2*f^2*
x^2*Cos[c + (3*d*x)/2] + 14*d*e*f*Cos[2*c + (3*d*x)/2] + 14*d*f^2*x*Cos[2*c
 + (3*d*x)/2] - 2*d*e*f*Cos[2*c + (5*d*x)/2] - 2*d*f^2*x*Cos[2*c + (5*d*x)/
2] + 2*d^2*e^2*Cos[3*c + (5*d*x)/2] - f^2*Cos[3*c + (5*d*x)/2] + 4*d^2*e*f*
x*Cos[3*c + (5*d*x)/2] + 2*d^2*f^2*x^2*Cos[3*c + (5*d*x)/2] + 8*d*Cos[(d*x)
/2]*(3*d^2*e^2*x + f^2*x*(-2 + (2*I)*d*x + d^2*x^2) + e*f*(-2 + (4*I)*d*x +
```

```
 3*d^2*x^2) - 8*f*(e + f*x)*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]) - 40*d^
2*e^2*Sin[(d*x)/2] + 16*f^2*Sin[(d*x)/2] - 48*d^2*e*f*x*Sin[(d*x)/2] - 24*d
^2*f^2*x^2*Sin[(d*x)/2] - 16*d*e*f*Sin[c + (d*x)/2] + 24*d^3*e^2*x*Sin[c +
(d*x)/2] + (32*I)*d^2*e*f*x*Sin[c + (d*x)/2] - 16*d*f^2*x*Sin[c + (d*x)/2]
+ 24*d^3*e*f*x^2*Sin[c + (d*x)/2] + (16*I)*d^2*f^2*x^2*Sin[c + (d*x)/2] + 8
*d^3*f^2*x^3*Sin[c + (d*x)/2] - 64*d*e*f*Log[1 - I*Cos[c + d*x] + Sin[c + d
*x]]*Sin[c + (d*x)/2] - 64*d*f^2*x*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]*S
in[c + (d*x)/2] + (64*I)*f^2*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]]*(Cos
[(d*x)/2] + Sin[c + (d*x)/2]) - 14*d*e*f*Sin[c + (3*d*x)/2] - 14*d*f^2*x*Si
n[c + (3*d*x)/2] + 6*d^2*e^2*Sin[2*c + (3*d*x)/2] - 15*f^2*Sin[2*c + (3*d*x
)/2] + 12*d^2*e*f*x*Sin[2*c + (3*d*x)/2] + 6*d^2*f^2*x^2*Sin[2*c + (3*d*x)/
2] - 2*d^2*e^2*Sin[2*c + (5*d*x)/2] + f^2*Sin[2*c + (5*d*x)/2] - 4*d^2*e*f*
x*Sin[2*c + (5*d*x)/2] - 2*d^2*f^2*x^2*Sin[2*c + (5*d*x)/2] - 2*d*e*f*Sin[3
*c + (5*d*x)/2] - 2*d*f^2*x*Sin[3*c + (5*d*x)/2])/(16*a*d^3*(Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple [A]**
time = 0.52, size = 492, normalized size = 1.77

| method | result |
|---|---|
| risch | $\frac{f^{2} x^{3}}{2 a}+\frac{3 f e \,x^{2}}{2 a}+\frac{3 e^{2} x}{2 a}+\frac{e^{3}}{2 a f}+\frac{\left(d^{2} x^{2} f^{2}+2 d^{2} e f x +2 i d \,f^{2} x +d^{2} e^{2}+2 i d e f -2 f^{2}\right) {\mathrm e}^{i \left(d x +c \right)}}{2 a \,d^{3}}+\frac{\left(d^{2} x^{2} f^{2}+2 d^{2} e f x -2 i d \,f^{2} x +d^{2} e^{2}-\right.}{2 a \,d^{3}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*f^2*x^3+3/2/a*f*e*x^2+3/2/a*e^2*x+1/2/a/f*e^3+1/2*(d^2*x^2*f^2+2*I*d*
f^2*x+2*d^2*e*f*x+2*I*d*e*f+d^2*e^2-2*f^2)/a/d^3*exp(I*(d*x+c))+1/2*(d^2*x^
2*f^2-2*I*d*f^2*x+2*d^2*e*f*x-2*I*d*e*f+d^2*e^2-2*f^2)/a/d^3*exp(-I*(d*x+c)
)+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(I*(d*x+c))+I)-4/a/d^2*f*ln(exp(I*(d*x+c)
)+I)*e+4/a/d^2*f*ln(exp(I*(d*x+c)))*e+4*I/a/d^2*f^2*c*x+2*I/a/d^3*f^2*c^2+4
*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3-4/a/d^2*f^2*ln(1-I*exp(I*(d*x+c)))
*x-4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c+2*I/a/d*f^2*x^2+4/a/d^3*f^2*c*ln(ex
p(I*(d*x+c))+I)-4/a/d^3*f^2*c*ln(exp(I*(d*x+c)))-1/4/d^2*f*(f*x+e)/a*cos(2*
d*x+2*c)-1/8*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-f^2)/a/d^3*sin(2*d*x+2*c)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 851 vs. 2(257) = 514.
time = 0.41, size = 851, normalized size = 3.06

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^3*f^2*x^3 + 4*d^2*f^2*x^2 - 7*d*f^2*x + (2*d^2*f^2*x^2 - 2*d*f^2*x
 + 2*d^2*e^2 - f^2 + 2*(2*d^2*f*x - d*f)*e)*cos(d*x + c)^3 + 2*(2*d^2*f^2*x
^2 + 3*d*f^2*x + 2*d^2*e^2 - 4*f^2 + (4*d^2*f*x + 3*d*f)*e)*cos(d*x + c)^2
+ (2*d^3*f^2*x^3 + 6*d^2*f^2*x^2 + d*f^2*x - 7*f^2 + 6*(d^3*x + d^2)*e^2 +
(6*d^3*f*x^2 + 12*d^2*f*x + d*f)*e)*cos(d*x + c) - 8*(-I*f^2*cos(d*x + c) -
 I*f^2*sin(d*x + c) - I*f^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 8*(I*f^
2*cos(d*x + c) + I*f^2*sin(d*x + c) + I*f^2)*dilog(-I*cos(d*x + c) - sin(d*
x + c)) + 2*(3*d^3*x + 2*d^2)*e^2 + (6*d^3*f*x^2 + 8*d^2*f*x - 7*d*f)*e + 8
*(c*f^2 - d*f*e + (c*f^2 - d*f*e)*cos(d*x + c) + (c*f^2 - d*f*e)*sin(d*x +
c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 8*(d*f^2*x + c*f^2 + (d*f^2*x
+ c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(I*cos(d*x + c)
+ sin(d*x + c) + 1) - 8*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c) +
 (d*f^2*x + c*f^2)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) +
8*(c*f^2 - d*f*e + (c*f^2 - d*f*e)*cos(d*x + c) + (c*f^2 - d*f*e)*sin(d*x +
 c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (2*d^3*f^2*x^3 - 4*d^2*f^2*x
^2 - 7*d*f^2*x - (2*d^2*f^2*x^2 + 2*d*f^2*x + 2*d^2*e^2 - f^2 + 2*(2*d^2*f*
x + d*f)*e)*cos(d*x + c)^2 + (2*d^2*f^2*x^2 - 8*d*f^2*x + 2*d^2*e^2 - 7*f^2
 + 4*(d^2*f*x - 2*d*f)*e)*cos(d*x + c) + 2*(3*d^3*x - 2*d^2)*e^2 + (6*d^3*f
*x^2 - 8*d^2*f*x - 7*d*f)*e)*sin(d*x + c))/(a*d^3*cos(d*x + c) + a*d^3*sin(
d*x + c) + a*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2 \sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \sin^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sin(c + d*x)**3/(
sin(c + d*x) + 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^3\,{\left(e+f\,x\right)}^2}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((sin(c + d*x)^3*(e + f*x)^2)/(a + a*sin(c + d*x)), x)
```

# 3.193 $\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=158

$$\frac{3ex}{2a}+\frac{3fx^2}{4a}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{f\sin(c+dx)}{ad^2}-$$

```
[Out] 3/2*e*x/a+3/4*f*x^2/a+(f*x+e)*cos(d*x+c)/a/d+(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d
*x)/a/d-2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2-f*sin(d*x+c)/a/d^2-1/2*(f*x
+e)*cos(d*x+c)*sin(d*x+c)/a/d+1/4*f*sin(d*x+c)^2/a/d^2
```

**Rubi** [A]
time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4611, 3391, 3377, 2717, 3399, 4269, 3556}

$$\frac{f\sin^2(c+dx)}{4ad^2}-\frac{f\sin(c+dx)}{ad^2}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2ad}+\frac{3ex}{2a}+\frac{3fx^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*e*x)/(2*a) + (3*f*x^2)/(4*a) + ((e + f*x)*Cos[c + d*x])/(a*d) + ((e + f*
x)*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/
(a*d^2) - (f*Sin[c + d*x])/(a*d^2) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/
(2*a*d) + (f*Sin[c + d*x]^2)/(4*a*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\sin^2(c+dx)\,dx}{a}-\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx\\
&= -\frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2ad}+\frac{f\sin^2(c+dx)}{4ad^2}+\frac{\int (e+fx)\,dx}{2a}-\frac{\int (e+\cdots}{}\\
&= \frac{ex}{2a}+\frac{fx^2}{4a}+\frac{(e+fx)\cos(c+dx)}{ad}-\frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2ad}+\frac{f\sin^2\cdots}{4\cdots}\\
&= \frac{3ex}{2a}+\frac{3fx^2}{4a}+\frac{(e+fx)\cos(c+dx)}{ad}-\frac{f\sin(c+dx)}{ad^2}-\frac{(e+fx)\cos(c+dx)\sin\cdots}{2ad}\\
&= \frac{3ex}{2a}+\frac{3fx^2}{4a}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{f\sin(c+\cdots}{ad^2}\\
&= \frac{3ex}{2a}+\frac{3fx^2}{4a}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{2f\log\left(\sin\cdots\right.}{}\end{aligned}$$

**Mathematica [A]**
time = 0.83, size = 298, normalized size = 1.89

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\left(8d(e+fx)\cos(c+dx)-f\cos(2(c+dx))+2\left(-8de+6cde+4cf-3c^2f+6d^2ex-4dfx+3d^2fx^2-8f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-4f\sin(c+dx)-d(e+fx)\sin(2(c+dx))\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(8d(e+fx)\cos(c+dx)-f\cos(2(c+dx))+2\left(6cde+4cf-3c^2f+6d^2ex+4dfx+3d^2fx^2-8f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-4f\sin(c+dx)-d(e+fx)\sin(2(c+dx))\right)\right)\right)}{8ad^2(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sin[(c + d*x)/2]*(8*d*(e + f*x)*Cos
[c + d*x] - f*Cos[2*(c + d*x)] + 2*(-8*d*e + 6*c*d*e + 4*c*f - 3*c^2*f + 6*
d^2*e*x - 4*d*f*x + 3*d^2*f*x^2 - 8*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] - 4*f*Sin[c + d*x] - d*(e + f*x)*Sin[2*(c + d*x)])) + Cos[(c + d*x)/2]*
(8*d*(e + f*x)*Cos[c + d*x] - f*Cos[2*(c + d*x)] + 2*(6*c*d*e + 4*c*f - 3*c
^2*f + 6*d^2*e*x + 4*d*f*x + 3*d^2*f*x^2 - 8*f*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - 4*f*Sin[c + d*x] - d*(e + f*x)*Sin[2*(c + d*x)]))))/(8*a*d^2
*(1 + Sin[c + d*x]))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(138) = 276$.
time = 0.27, size = 452, normalized size = 2.86

| method | result |
|---|---|
| risch | $\frac{3f\,x^2}{4a}+\frac{3ex}{2a}+\frac{(dxf+de+if)\mathrm{e}^{i(dx+c)}}{2a\,d^2}+\frac{(dxf+de-if)\mathrm{e}^{-i(dx+c)}}{2a\,d^2}+\frac{2ifx}{ad}+\frac{2ifc}{a\,d^2}+\frac{2fx+2e}{da\left(\mathrm{e}^{i(dx+c)}+i\right)}-\frac{2f\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a\,d^2}$ |
| default | $-\frac{-\frac{8e}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{12e\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{-2f\,x^2-2f\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2f\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2f\,x^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4fx}{d}+\frac{4fx\tan(}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{}$ |
| norman | $\frac{\frac{de+2f}{a\,d^2}-\frac{2e\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{5f\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,d^2}+\frac{(-3de+2f)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,d^2}+\frac{(-6de+5f)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,d^2}+\frac{(-5de+3f)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,d^2}+(-de+}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a*(-8*e/d/(tan(1/2*d*x+1/2*c)+1)-12*e/d*arctan(tan(1/2*d*x+1/2*c))+(-2
*f*x^2-2*f*x^2*tan(1/2*d*x+1/2*c)-2*f*x^2*tan(1/2*d*x+1/2*c)^2-2*f*x^2*tan(
1/2*d*x+1/2*c)^3-4*f/d*x+4*f/d*x*tan(1/2*d*x+1/2*c)-4*f/d*x*tan(1/2*d*x+1/2
*c)^2+4*f/d*x*tan(1/2*d*x+1/2*c)^3)/(tan(1/2*d*x+1/2*c)+1)/(1+tan(1/2*d*x+1
/2*c)^2)+8*f/d^2*ln(tan(1/2*d*x+1/2*c)+1)-4*f/d^2*ln(1+tan(1/2*d*x+1/2*c)^2
)-4*e/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-8*e/d/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+4*e/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2
*d*x+1/2*c)-8*e/d/(1+tan(1/2*d*x+1/2*c)^2)^2+4*f/d^2*((d*x+c)*(1/2*cos(d*x+
c)*sin(d*x+c)+1/2*d*x+1/2*c)-3/4*(d*x+c)^2-1/4*sin(d*x+c)^2-c*(1/2*cos(d*x+
c)*sin(d*x+c)+1/2*d*x+1/2*c)+sin(d*x+c)-(d*x+c)*cos(d*x+c)+c*cos(d*x+c)+(d*
x+c)*c))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [A]
time = 0.37, size = 256, normalized size = 1.62

$$\frac{6\,d^2fx^2+2\,(2\,dfx+2\,de-f)\cos(dx+c)^3+8\,dfx+2\,(4\,dfx+4\,de+3\,f)\cos(dx+c)^2+(6\,d^2fx^2+12\,dfx+12\,(d^2x+d)e+f)\cos(dx+c)+4\,(3\,d^2x+2\,d)e-8\,(f\cos(dx+c)+f\sin(dx+c)+f)\log(\sin(dx+c)+1)+(6\,d^2fx^2-8\,dfx-2\,(2\,dfx+2\,de+f)\cos(dx+c)^2+4\,(dfx+de-2\,f)\cos(dx+c)+4\,(3\,d^2x-2\,d)e-7\,f)\sin(dx+c)-7\,f}{8\,(ad^2\cos(dx+c)+ad^2\sin(dx+c)+ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(6*d^2*f*x^2 + 2*(2*d*f*x + 2*d*e - f)*cos(d*x + c)^3 + 8*d*f*x + 2*(4*
d*f*x + 4*d*e + 3*f)*cos(d*x + c)^2 + (6*d^2*f*x^2 + 12*d*f*x + 12*(d^2*x +
 d)*e + f)*cos(d*x + c) + 4*(3*d^2*x + 2*d)*e - 8*(f*cos(d*x + c) + f*sin(d
*x + c) + f)*log(sin(d*x + c) + 1) + (6*d^2*f*x^2 - 8*d*f*x - 2*(2*d*f*x +
2*d*e + f)*cos(d*x + c)^2 + 4*(d*f*x + d*e - 2*f)*cos(d*x + c) + 4*(3*d^2*x
 - 2*d)*e - 7*f)*sin(d*x + c) - 7*f)/(a*d^2*cos(d*x + c) + a*d^2*sin(d*x +
c) + a*d^2)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 4653 vs. $2(134) = 268$.
time = 2.92, size = 4653, normalized size = 29.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((6*d**2*e*x*tan(c/2 + d*x/2)**5/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4
*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c
/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 6*d**2*e*x*tan(c/2
 + d*x/2)**4/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 +
 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan
(c/2 + d*x/2) + 4*a*d**2) + 12*d**2*e*x*tan(c/2 + d*x/2)**3/(4*a*d**2*tan(c
/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**
3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) +
12*d**2*e*x*tan(c/2 + d*x/2)**2/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*ta
n(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2
)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 6*d**2*e*x*tan(c/2 + d*x/2)/
(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan
(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2)
 + 4*a*d**2) + 6*d**2*e*x/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2
```

```
+ d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 +
 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 3*d**2*f*x**2*tan(c/2 + d*x/2)**5/
(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan
(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2)
 + 4*a*d**2) + 3*d**2*f*x**2*tan(c/2 + d*x/2)**4/(4*a*d**2*tan(c/2 + d*x/2)
**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**
2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 6*d**2*f*x*
*2*tan(c/2 + d*x/2)**3/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d
*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*
a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 6*d**2*f*x**2*tan(c/2 + d*x/2)**2/(4*
a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/
2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) +
4*a*d**2) + 3*d**2*f*x**2*tan(c/2 + d*x/2)/(4*a*d**2*tan(c/2 + d*x/2)**5 +
4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(
c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 3*d**2*f*x**2/(4*
a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/
2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) +
4*a*d**2) + 12*d*e*tan(c/2 + d*x/2)**4/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*
d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2
 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 12*d*e*tan(c/2 + d*x/
2)**3/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d*
*2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 +
d*x/2) + 4*a*d**2) + 20*d*e*tan(c/2 + d*x/2)**2/(4*a*d**2*tan(c/2 + d*x/2)*
*5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2
*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 4*d*e*tan(c/
2 + d*x/2)/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8
*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c
/2 + d*x/2) + 4*a*d**2) + 16*d*e/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*t
an(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/
2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) - 8*d*f*x*tan(c/2 + d*x/2)**5
/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*ta
n(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2
) + 4*a*d**2) + 4*d*f*x*tan(c/2 + d*x/2)**4/(4*a*d**2*tan(c/2 + d*x/2)**5 +
 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan
(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) - 4*d*f*x*tan(c/2
+ d*x/2)**3/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 +
8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(
c/2 + d*x/2) + 4*a*d**2) + 4*d*f*x*tan(c/2 + d*x/2)**2/(4*a*d**2*tan(c/2 +
d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8
*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) - 4*d*f
*x*tan(c/2 + d*x/2)/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/
2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d
**2*tan(c/2 + d*x/2) + 4*a*d**2) + 8*d*f*x/(4*a*d**2*tan(c/2 + d*x/2)**5 +
4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(
c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) - 8*f*log(tan(c/2 +
```

```
d*x/2) + 1)*tan(c/2 + d*x/2)**5/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*t
an(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/
2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) - 8*f*log(tan(c/2 + d*x/2) +
1)*tan(c/2 + d*x/2)**4/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d
*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*
a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) - 16*f*log(tan(c/2 + d*x/2) + 1)*tan(c/
2 + d*x/2)**3/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4
+ 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c...
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 44105 vs. $2(142) = 284$.
time = 12.41, size = 44105, normalized size = 279.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(6*d^2*f*x^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 - 6*d^2*f*x^2*tan
(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 - 6*d^2*f*x^2*tan(1/2*d*x)^4*tan(3/2*
c)^2*tan(1/2*c)^7 + 12*d^2*x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 + 1
8*d^2*f*x^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 - 6*d^2*f*x^2*tan(1/2*
d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 - 12*d^2*x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*t
an(1/2*c)^6 + 6*d^2*f*x^2*tan(1/2*d*x)^5*tan(1/2*c)^7 + 12*d^2*f*x^2*tan(1/
2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^7 - 12*d^2*x*e*tan(1/2*d*x)^4*tan(3/2*c)^2
*tan(1/2*c)^7 + 16*d*f*x*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 - 18*d^2*
f*x^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 - 18*d^2*f*x^2*tan(1/2*d*x)^
4*tan(3/2*c)^2*tan(1/2*c)^5 - 72*d^2*x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/
2*c)^5 - 6*d^2*f*x^2*tan(1/2*d*x)^5*tan(1/2*c)^6 + 72*d^2*x*e*tan(1/2*d*x)^
5*tan(3/2*c)*tan(1/2*c)^6 - 12*d^2*f*x^2*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/
2*c)^6 - 12*d^2*x*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 + 8*d*f*x*tan(
1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 + 36*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1
/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(
1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 -
6*d^2*f*x^2*tan(1/2*d*x)^4*tan(1/2*c)^7 - 12*d*e*log(2*(tan(1/2*d*x)^4*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^7 -
12*d^2*f*x^2*tan(1/2*d*x)^2*tan(3/2*c)^2*tan(1/2*c)^7 + 24*d^2*x*e*tan(1/2
*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^7 + 8*d*f*x*tan(1/2*d*x)^4*tan(3/2*c)^2*tan
(1/2*c)^7 + 16*d*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 - 8*f*log(2*(ta
n(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*
```

```
tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*
c)^2*tan(1/2*c)^7 + 18*d^2*f*x^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^3 -
 18*d^2*f*x^2*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^4 + 72*d^2*x*e*tan(1/2
*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 + 18*d^2*f*x^2*tan(1/2*d*x)^5*tan(1/2*c)^
5 - 72*d^2*x*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^5 + 36*d^2*f*x^2*tan(1/
2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^5 + 72*d^2*x*e*tan(1/2*d*x)^4*tan(3/2*c)^2
*tan(1/2*c)^5 + 24*d*f*x*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 - 36*d*e*
log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/
2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5
*tan(3/2*c)^2*tan(1/2*c)^5 - 6*d^2*f*x^2*tan(1/2*d*x)^4*tan(1/2*c)^6 - 72*d
^2*x*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^6 + 12*d*e*log(2*(tan(1/2*d*x)^
4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^
2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*t
an(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
 + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*
c)^6 - 12*d^2*f*x^2*tan(1/2*d*x)^2*tan(3/2*c)^2*tan(1/2*c)^6 - 24*d^2*x*e*t
an(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^6 - 64*d*f*x*tan(1/2*d*x)^4*tan(3/2*c
)^2*tan(1/2*c)^6 - 36*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 - 4*d*e*tan(1/2*d*x)^5
*tan(3/2*c)^2*tan(1/2*c)^6 + 8*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 + 12*d^2*f*x^2*
tan(1/2*d*x)^3*tan(1/2*c)^7 + 16*d*f*x*tan(1/2*d*x)^5*tan(1/2*c)^7 + 12*d*e
*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
4*tan(3/2*c)*tan(1/2*c)^7 + 4*d*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^7 +
6*d^2*f*x^2*tan(1/2*d*x)*tan(3/2*c)^2*tan(1/2*c)^7 - 24*d^2*x*e*tan(1/2*d*x
)^2*tan(3/2*c)^2*tan(1/2*c)^7 + 8*d*f*x*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2
*c)^7 + 8*d*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1...
```

**Mupad [B]**
time = 1.94, size = 246, normalized size = 1.56

$$\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(\frac{d\,e+f\,1\mathrm{i}}{2\,a\,d^2}+\frac{f\,x}{2\,a\,d}\right)-\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\frac{-d\,e+f\,1\mathrm{i}}{2\,a\,d^2}-\frac{f\,x}{2\,a\,d}\right)+\mathrm{e}^{-c\,2\mathrm{i}-d\,x\,2\mathrm{i}}\,\left(\frac{\left(-2\,d\,e+f\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,a\,d^2}-\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right)+\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,\left(\frac{\left(2\,d\,e+f\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,a\,d^2}+\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right)+\frac{3\,f\,x^2}{4\,a}-\frac{2\,f\,\ln\left(\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{a\,d^2}+\frac{2\,\left(e+f\,x\right)}{a\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\mathrm{i}\right)}+\frac{x\,\left(3\,d\,e+f\,4\mathrm{i}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(e + f*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] exp(c*1i + d*x*1i)*((f*1i + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) - exp(- c*1i -
d*x*1i)*((f*1i - d*e)/(2*a*d^2) - (f*x)/(2*a*d)) + exp(- c*2i - d*x*2i)*(((
f*1i - 2*d*e)*1i)/(16*a*d^2) - (f*x*1i)/(8*a*d)) + exp(c*2i + d*x*2i)*(((f*
1i + 2*d*e)*1i)/(16*a*d^2) + (f*x*1i)/(8*a*d)) + (3*f*x^2)/(4*a) - (2*f*log
(exp(c*1i)*exp(d*x*1i) + 1i))/(a*d^2) + (2*(e + f*x))/(a*d*(exp(c*1i + d*x*
1i) + 1i)) + (x*(f*4i + 3*d*e))/(2*a*d)
```

# 3.194 $\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=75

$$\frac{3x}{2a}+\frac{2\cos(c+dx)}{ad}-\frac{3\cos(c+dx)\sin(c+dx)}{2ad}+\frac{\cos(c+dx)\sin^2(c+dx)}{d(a+a\sin(c+dx))}$$

```
[Out] 3/2*x/a+2*cos(d*x+c)/a/d-3/2*cos(d*x+c)*sin(d*x+c)/a/d+cos(d*x+c)*sin(d*x+c
)^2/d/(a+a*sin(d*x+c))
```

**Rubi** [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2846, 2813}

$$\frac{2\cos(c+dx)}{ad}+\frac{\sin^2(c+dx)\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(2*a) + (2*Cos[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) + (Cos[c + d*x]*Sin[c + d*x]^2)/(d*(a + a*Sin[c + d*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\cos(c+dx)\sin^2(c+dx)}{d(a+a\sin(c+dx))} - \frac{\int \sin(c+dx)(2a-3a\sin(c+dx))\,dx}{a^2}$$
$$= \frac{3x}{2a} + \frac{2\cos(c+dx)}{ad} - \frac{3\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\cos(c+dx)\sin^2(c+dx)}{d(a+a\sin(c+dx))}$$

**Mathematica [A]**
time = 0.14, size = 117, normalized size = 1.56

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)(-8+6c+6dx+4\cos(c+dx)-\sin(2(c+dx)))+\cos\left(\frac{1}{2}(c+dx)\right)(6c+6dx+4\cos(c+dx)-\sin(2(c+dx)))\right)}{4ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sin[(c + d*x)/2]*(-8 + 6*c + 6*d*x
+ 4*Cos[c + d*x] - Sin[2*(c + d*x)]) + Cos[(c + d*x)/2]*(6*c + 6*d*x + 4*Co
s[c + d*x] - Sin[2*(c + d*x)])))/(4*a*d*(1 + Sin[c + d*x]))
```

**Maple [A]**
time = 0.09, size = 91, normalized size = 1.21

| method | result |
|---|---|
| risch | $\frac{3x}{2a}+\frac{e^{i(dx+c)}}{2ad}+\frac{e^{-i(dx+c)}}{2ad}+\frac{2}{da\left(e^{i(dx+c)}+i\right)}-\frac{\sin(2dx+2c)}{4da}$ |
| derivativedivides | $\frac{\frac{16}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8}+\frac{2\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+1\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ |
| default | $\frac{\frac{16}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8}+\frac{2\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+1\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ |
| norman | $\frac{-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{3x}{2a}+\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{9x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{9x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{9x(\tan^4}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a*(1/8/(tan(1/2*d*x+1/2*c)+1)+1/8*(1/2*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*
x+1/2*c)^2-1/2*tan(1/2*d*x+1/2*c)+1)/(1+tan(1/2*d*x+1/2*c)^2)^2+3/16*arctan
(tan(1/2*d*x+1/2*c)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(71) = 142$.

time = 0.54, size = 212, normalized size = 2.83

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+4}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 4)/(a + a*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 3*arctan(s
in(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

**Fricas** [A]

time = 0.35, size = 92, normalized size = 1.23

$$\frac{\cos(dx+c)^3+3\,dx+3\,(dx+1)\cos(dx+c)+2\,\cos(dx+c)^2+\left(3\,dx-\cos(dx+c)^2+\cos(dx+c)-2\right)\sin(dx+c)+2}{2\,(ad\cos(dx+c)+ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(d*x + c)^3 + 3*d*x + 3*(d*x + 1)*cos(d*x + c) + 2*cos(d*x + c)^2 +
 (3*d*x - cos(d*x + c)^2 + cos(d*x + c) - 2)*sin(d*x + c) + 2)/(a*d*cos(d*x
 + c) + a*d*sin(d*x + c) + a*d)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(65) = 130$.

time = 2.48, size = 1127, normalized size = 15.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**5/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan
(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 +
2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 3*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2
+ d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d
*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 6*d*x*tan(c/2 + d*
x/2)**3/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(
```

```
c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*
d) + 6*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 +
 d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*
tan(c/2 + d*x/2) + 2*a*d) + 3*d*x*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)*
*5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2
+ d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 3*d*x/(2*a*d*tan(c/2 + d*x/
2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c
/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 6*tan(c/2 + d*x/2)**4/(2
*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/
2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 6*tan
(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4
*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/
2) + 2*a*d) + 10*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan
(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 +
2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 2*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2
)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/
2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 8/(2*a*d*tan(c/2 + d*x/2)
**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2
 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d), Ne(d, 0)), (x*sin(c)**3/(a*
sin(c) + a), True))
```

**Giac [A]**
time = 5.17, size = 91, normalized size = 1.21

$$\frac{\frac{3\,(dx+c)}{a}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a}+\frac{4}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a + 2*(tan(1/2*d*x + 1/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 -
 tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a) + 4/(a*(tan(1
/2*d*x + 1/2*c) + 1)))/d
```

**Mupad [B]**
time = 3.29, size = 92, normalized size = 1.23

$$\frac{3\,x}{2\,a}+\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*x)/(2*a) + (tan(c/2 + (d*x)/2) + 5*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d
*x)/2)^3 + 3*tan(c/2 + (d*x)/2)^4 + 4)/(a*d*(tan(c/2 + (d*x)/2) + 1)*(tan(c
/2 + (d*x)/2)^2 + 1)^2)
```

# 3.195 $\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 4.19, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(dx+c)}{(fx+e)\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*si
n(d*x + c)), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(c+dx)^3}{(e+fx)\,(a+a\sin(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/((e + f*x)*(a + a*sin(c + d*x))),x)
[Out] int(sin(c + d*x)^3/((e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.196 $\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 3.84, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(dx+c)}{(fx+e)^2\left(a+a\sin(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a*f^2*x^2 + 2*a*f*x*e + a*e^2
+ (a*f^2*x^2 + 2*a*f*x*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin (c+d x)^3}{(e+f x)^2 (a+a \sin (c+d x))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/((e + f*x)^2*(a + a*sin(c + d*x))),x)
[Out] int(sin(c + d*x)^3/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.197 $\int \frac{(e+fx)^3 \csc(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=352

$$\frac{i(e+fx)^3}{ad}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{3if}{}$$

```
[Out] I*(f*x+e)^3/a/d-2*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)^3*cot(1/2*c
+1/4*Pi+1/2*d*x)/a/d-6*f*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a/d^2+3*I*f*(f*x+
e)^2*polylog(2,-exp(I*(d*x+c)))/a/d^2+12*I*f^2*(f*x+e)*polylog(2,I*exp(I*(d
*x+c)))/a/d^3-3*I*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a/d^2-6*f^2*(f*x+e)
*polylog(3,-exp(I*(d*x+c)))/a/d^3-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+
6*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a/d^3-6*I*f^3*polylog(4,-exp(I*(d*x
+c)))/a/d^4+6*I*f^3*polylog(4,exp(I*(d*x+c)))/a/d^4
```

**Rubi** [A]

time = 0.33, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4631, 4268, 2611, 6744, 2320, 6724, 3399, 4269, 3798, 2221}

$$-\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}-\frac{6if^3\text{PolyLog}\left(4,-e^{i(c+dx)}\right)}{ad^4}+\frac{6if^3\text{PolyLog}\left(4,e^{i(c+dx)}\right)}{ad^4}+\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}+\frac{3if(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{i(e+fx)^3}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (I*(e + f*x)^3)/(a*d) - (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((
e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (6*f*(e + f*x)^2*Log[1 - I*E^
(I*(c + d*x))])/(a*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))]
)/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) -
((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (6*f^2*(e + f*x
)*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) - (12*f^3*PolyLog[3, I*E^(I*(c + d*
x))])/(a*d^4) + (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) - ((6
*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*PolyLog[4, E^(I*
(c + d*x))])/(a*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
```

```
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\csc(c+dx)\,dx}{a} - \int \frac{(e+fx)^3}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{\int (e+fx)^3\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)\,dx}{2a} - \frac{(3f)\ldots}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{3if(e+fx)^2\ldots}{a\ldots}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \ldots$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6\ldots}{}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6\ldots}{}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6\ldots}{}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6\ldots}{}$$

**Mathematica** [A]

time = 4.03, size = 565, normalized size = 1.61

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*d^3*e^3*ArcTanh[E^(I*(c + d*x))] + 3*d^3*e^2*f*x*Log[1 - E^(I*(c + d*x)
)] + 3*d^3*e*f^2*x^2*Log[1 - E^(I*(c + d*x))] + d^3*f^3*x^3*Log[1 - E^(I*(c
 + d*x))] - 3*d^3*e^2*f*x*Log[1 + E^(I*(c + d*x))] - 3*d^3*e*f^2*x^2*Log[1
+ E^(I*(c + d*x))] - d^3*f^3*x^3*Log[1 + E^(I*(c + d*x))] + (3*I)*d^2*f*(e
+ f*x)^2*PolyLog[2, -E^(I*(c + d*x))] - (3*I)*d^2*f*(e + f*x)^2*PolyLog[2,
E^(I*(c + d*x))] - 6*d*e*f^2*PolyLog[3, -E^(I*(c + d*x))] - 6*d*f^3*x*PolyL
og[3, -E^(I*(c + d*x))] + 6*d*e*f^2*PolyLog[3, E^(I*(c + d*x))] + 6*d*f^3*x
*PolyLog[3, E^(I*(c + d*x))] - (6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))] + (6*
I)*f^3*PolyLog[4, E^(I*(c + d*x))] + 2*f*(-3*d^2*(e + f*x)^2*Log[1 - I*Cos[
c + d*x] + Sin[c + d*x]] + (6*I)*d*f*(e + f*x)*PolyLog[2, I*Cos[c + d*x] -
Sin[c + d*x]] - 6*f^2*PolyLog[3, I*Cos[c + d*x] - Sin[c + d*x]] + (I*d^3*x*
(3*e^2 + 3*e*f*x + f^2*x^2)*(Cos[c] + I*Sin[c]))/(Cos[c] + I*(1 + Sin[c])))
 - (2*d^3*(e + f*x)^3*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])))/(a*d^4)
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1150 vs. $2(317) = 634$.
time = 0.26, size = 1151, normalized size = 3.27

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1151 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 6/d^3/a*e*f^2*polylog(3,exp(I*(d*x+c)))-6/d^3/a*e*f^2*polylog(3,-exp(I*(d*x
+c)))-1/d^4/a*f^3*c^3*ln(exp(I*(d*x+c))-1)+6/d^3/a*f^3*polylog(3,exp(I*(d*x
+c)))*x-6/d^3/a*f^3*polylog(3,-exp(I*(d*x+c)))*x+1/d/a*e^3*ln(exp(I*(d*x+c)
)-1)-1/d/a*e^3*ln(exp(I*(d*x+c))+1)-6/a/d^2*f^3*ln(1-I*exp(I*(d*x+c)))*x^2+
6/a/d^4*f^3*ln(1-I*exp(I*(d*x+c)))*c^2+6/a/d^2*f*ln(exp(I*(d*x+c)))*e^2+6/a
/d^4*f^3*c^2*ln(exp(I*(d*x+c)))-6/a/d^4*f^3*c^2*ln(exp(I*(d*x+c))+I)+6*I/d^
2/a*e*f^2*polylog(2,-exp(I*(d*x+c)))*x-6*I/d^2/a*e*f^2*polylog(2,exp(I*(d*x
+c)))*x+6*I*f^3*polylog(4,exp(I*(d*x+c)))/a/d^4-6/a/d^2*f*ln(exp(I*(d*x+c))
+I)*e^2+2*I/a/d*f^3*x^3-4*I/a/d^4*f^3*c^3+12*I/a/d^2*f^2*e*c*x-12/a/d^3*f^2
*e*c*ln(exp(I*(d*x+c)))+12*I/a/d^3*f^3*polylog(2,I*exp(I*(d*x+c)))*x-6*I/a/
d^3*f^3*c^2*x+6*I/a/d*f^2*e*x^2+6*I/a/d^3*f^2*e*c^2+12*I/a/d^3*f^2*e*polylo
g(2,I*exp(I*(d*x+c)))+3/d^2/a*ln(1-exp(I*(d*x+c)))*c*e^2*f-3/d/a*e*f^2*ln(e
xp(I*(d*x+c))+1)*x^2+3/d/a*ln(1-exp(I*(d*x+c)))*e^2*f*x-3/d/a*ln(exp(I*(d*x
+c))+1)*e^2*f*x+3/d^3/a*e*f^2*c^2*ln(exp(I*(d*x+c))-1)-3/d^2/a*e^2*f*c*ln(e
xp(I*(d*x+c))-1)-3*I/d^2/a*f^3*polylog(2,exp(I*(d*x+c)))*x^2+3*I/d^2/a*f^3*
polylog(2,-exp(I*(d*x+c)))*x^2-3*I/d^2/a*e^2*f*polylog(2,exp(I*(d*x+c)))+3*
I/d^2/a*e^2*f*polylog(2,-exp(I*(d*x+c)))+2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e
```

```
^3)/d/a/(exp(I*(d*x+c))+I)+1/d/a*f^3*ln(1-exp(I*(d*x+c)))*x^3+1/d^4/a*f^3*l
n(1-exp(I*(d*x+c)))*c^3-1/d/a*f^3*ln(exp(I*(d*x+c))+1)*x^3-3/d^3/a*e*f^2*c^
2*ln(1-exp(I*(d*x+c)))+3/d/a*e*f^2*ln(1-exp(I*(d*x+c)))*x^2-12*f^3*polylog(
3,I*exp(I*(d*x+c)))/a/d^4+12/a/d^3*f^2*e*c*ln(exp(I*(d*x+c))+I)-6*I*f^3*pol
ylog(4,-exp(I*(d*x+c)))/a/d^4-12/a/d^2*f^2*e*ln(1-I*exp(I*(d*x+c)))*x-12/a/
d^3*f^2*e*ln(1-I*exp(I*(d*x+c)))*c
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2842 vs. $2(314) = 628$.
time = 0.82, size = 2842, normalized size = 8.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(3*c*f*(2/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1)) + log(sin(d*x + c)/(
cos(d*x + c) + 1))/(a*d))*e^2 - (log(sin(d*x + c)/(cos(d*x + c) + 1))/a + 2
/(a + a*sin(d*x + c)/(cos(d*x + c) + 1)))*e^3 + (-4*I*c^3*f^3 + 12*I*c^2*d*
f^2*e - 12*(-I*c^2*f^3 + 2*I*c*d*f^2*e - I*d^2*f*e^2 - (c^2*f^3 - 2*c*d*f^2
*e + d^2*f*e^2)*cos(d*x + c) + (-I*c^2*f^3 + 2*I*c*d*f^2*e - I*d^2*f*e^2)*s
in(d*x + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 12*(I*(d*x + c)^2*f^
3 + 2*(-I*c*f^3 + I*d*f^2*e)*(d*x + c) + ((d*x + c)^2*f^3 - 2*(c*f^3 - d*f^
2*e)*(d*x + c))*cos(d*x + c) + (I*(d*x + c)^2*f^3 + 2*(-I*c*f^3 + I*d*f^2*e
)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*(-I*
(d*x + c)^3*f^3 + I*c^3*f^3 - 3*I*c^2*d*f^2*e + 3*(I*c*f^3 - I*d*f^2*e)*(d*
x + c)^2 + 3*(-I*c^2*f^3 + 2*I*c*d*f^2*e - I*d^2*f*e^2)*(d*x + c) - ((d*x +
 c)^3*f^3 - c^3*f^3 + 3*c^2*d*f^2*e - 3*(c*f^3 - d*f^2*e)*(d*x + c)^2 + 3*(
c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*(d*x + c))*cos(d*x + c) + (-I*(d*x + c)^
3*f^3 + I*c^3*f^3 - 3*I*c^2*d*f^2*e + 3*(I*c*f^3 - I*d*f^2*e)*(d*x + c)^2 +
 3*(-I*c^2*f^3 + 2*I*c*d*f^2*e - I*d^2*f*e^2)*(d*x + c))*sin(d*x + c))*arct
an2(sin(d*x + c), cos(d*x + c) + 1) - 2*(-I*c^3*f^3 + 3*I*c^2*d*f^2*e - (c^
3*f^3 - 3*c^2*d*f^2*e)*cos(d*x + c) + (-I*c^3*f^3 + 3*I*c^2*d*f^2*e)*sin(d*
x + c))*arctan2(sin(d*x + c), cos(d*x + c) - 1) - 2*(-I*(d*x + c)^3*f^3 + 3
*(I*c*f^3 - I*d*f^2*e)*(d*x + c)^2 + 3*(-I*c^2*f^3 + 2*I*c*d*f^2*e - I*d^2*
f*e^2)*(d*x + c) - ((d*x + c)^3*f^3 - 3*(c*f^3 - d*f^2*e)*(d*x + c)^2 + 3*(
c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*(d*x + c))*cos(d*x + c) + (-I*(d*x + c)^
3*f^3 + 3*(I*c*f^3 - I*d*f^2*e)*(d*x + c)^2 + 3*(-I*c^2*f^3 + 2*I*c*d*f^2*e
 - I*d^2*f*e^2)*(d*x + c))*sin(d*x + c))*arctan2(sin(d*x + c), -cos(d*x + c
) + 1) - 4*((d*x + c)^3*f^3 - 3*(c*f^3 - d*f^2*e)*(d*x + c)^2 + 3*(c^2*f^3
- 2*c*d*f^2*e + d^2*f*e^2)*(d*x + c))*cos(d*x + c) - 24*(I*(d*x + c)*f^3 -
I*c*f^3 + I*d*f^2*e + ((d*x + c)*f^3 - c*f^3 + d*f^2*e)*cos(d*x + c) + (I*(
d*x + c)*f^3 - I*c*f^3 + I*d*f^2*e)*sin(d*x + c))*dilog(I*e^(I*d*x + I*c))
- 6*(I*(d*x + c)^2*f^3 + I*c^2*f^3 - 2*I*c*d*f^2*e + I*d^2*f*e^2 + 2*(-I*c*
f^3 + I*d*f^2*e)*(d*x + c) + ((d*x + c)^2*f^3 + c^2*f^3 - 2*c*d*f^2*e + d^2
```

```
*f*e^2 - 2*(c*f^3 - d*f^2*e)*(d*x + c))*cos(d*x + c) + (I*(d*x + c)^2*f^3 +
 I*c^2*f^3 - 2*I*c*d*f^2*e + I*d^2*f*e^2 + 2*(-I*c*f^3 + I*d*f^2*e)*(d*x +
c))*sin(d*x + c))*dilog(-e^(I*d*x + I*c)) - 6*(-I*(d*x + c)^2*f^3 - I*c^2*f
^3 + 2*I*c*d*f^2*e - I*d^2*f*e^2 + 2*(I*c*f^3 - I*d*f^2*e)*(d*x + c) - ((d*
x + c)^2*f^3 + c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 - 2*(c*f^3 - d*f^2*e)*(d*x
 + c))*cos(d*x + c) + (-I*(d*x + c)^2*f^3 - I*c^2*f^3 + 2*I*c*d*f^2*e - I*d
^2*f*e^2 + 2*(I*c*f^3 - I*d*f^2*e)*(d*x + c))*sin(d*x + c))*dilog(e^(I*d*x
+ I*c)) + ((d*x + c)^3*f^3 - c^3*f^3 + 3*c^2*d*f^2*e - 3*(c*f^3 - d*f^2*e)*
(d*x + c)^2 + 3*(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*(d*x + c) + (-I*(d*x +
c)^3*f^3 + I*c^3*f^3 - 3*I*c^2*d*f^2*e - 3*(-I*c*f^3 + I*d*f^2*e)*(d*x + c)
^2 - 3*(I*c^2*f^3 - 2*I*c*d*f^2*e + I*d^2*f*e^2)*(d*x + c))*cos(d*x + c) +
((d*x + c)^3*f^3 - c^3*f^3 + 3*c^2*d*f^2*e - 3*(c*f^3 - d*f^2*e)*(d*x + c)^
2 + 3*(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*(d*x + c))*sin(d*x + c))*log(cos(
d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1) - ((d*x + c)^3*f^3 - c^3*
f^3 + 3*c^2*d*f^2*e - 3*(c*f^3 - d*f^2*e)*(d*x + c)^2 + 3*(c^2*f^3 - 2*c*d*
f^2*e + d^2*f*e^2)*(d*x + c) - (I*(d*x + c)^3*f^3 - I*c^3*f^3 + 3*I*c^2*d*f
^2*e - 3*(I*c*f^3 - I*d*f^2*e)*(d*x + c)^2 - 3*(-I*c^2*f^3 + 2*I*c*d*f^2*e
- I*d^2*f*e^2)*(d*x + c))*cos(d*x + c) + ((d*x + c)^3*f^3 - c^3*f^3 + 3*c^2
*d*f^2*e - 3*(c*f^3 - d*f^2*e)*(d*x + c)^2 + 3*(c^2*f^3 - 2*c*d*f^2*e + d^2
*f*e^2)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*co
s(d*x + c) + 1) + 6*((d*x + c)^2*f^3 + c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 -
2*(c*f^3 - d*f^2*e)*(d*x + c) - (I*(d*x + c)^2*f^3 + I*c^2*f^3 - 2*I*c*d*f^
2*e + I*d^2*f*e^2 + 2*(-I*c*f^3 + I*d*f^2*e)*(d*x + c))*cos(d*x + c) + ((d*
x + c)^2*f^3 + c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 - 2*(c*f^3 - d*f^2*e)*(d*x
 + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) +
 1) + 12*(f^3*cos(d*x + c) + I*f^3*sin(d*x + c) + I*f^3)*polylog(4, -e^(I*d
*x + I*c)) - 12*(f^3*cos(d*x + c) + I*f^3*sin(d*x + c) + I*f^3)*polylog(4,
e^(I*d*x + I*c)) - 24*(I*f^3*cos(d*x + c) - f^3*sin(d*x + c) - f^3)*polylog
(3, I*e^(I*d*x + I*c)) + 12*((d*x + c)*f^3 - c*f^3 + d*f^2*e - (I*(d*x + c)
*f^3 - I*c*f^3 + I*d*f^2*e)*cos(d*x + c) + ((d*x + c)*f^3 - c*f^3 + d*f^2*e
)*sin(d*x + c))*polylog(3, -e^(I*d*x + I*c)) - 12*((d*x + c)*f^3 - c*f^3 +
d*f^2*e + (-I*(d*x + c)*f^3 + I*c*f^3 - I*d*f^2*e)*cos(d*x + c) + ((d*x + c
)*f^3 - c*f^3 + d*f^2*e)*sin(d*x + c))*polylog(3, e^(I*d*x + I*c)) - 4*(I*(
d*x + c)^3*f^3 + 3*(-I*c*f^3 + I*d*f^2*e)*(d*x + c)^2 + 3*(I*c^2*f^3 - 2*I*
c*d*f^2*e + I*d^2*f*e^2)*(d*x + c))*sin(d*x + c))/(-2*I*a*d^3*cos(d*x + c)
+ 2*a*d^3*sin(d*x + c) + 2*a*d^3))/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2929 vs. $2(314) = 628$.
time = 0.51, size = 2929, normalized size = 8.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^3*f^3*x^3 + 6*d^3*f^2*x^2*e + 6*d^3*f*x*e^2 + 2*d^3*e^3 + 2*(d^3*f
^3*x^3 + 3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*cos(d*x + c) - 3*(I*d^2
*f^3*x^2 + 2*I*d^2*f^2*x*e + I*d^2*f*e^2 + (I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e
 + I*d^2*f*e^2)*cos(d*x + c) + (I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e + I*d^2*f*e
^2)*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x + c)) - 3*(-I*d^2*f^3*x^2
- 2*I*d^2*f^2*x*e - I*d^2*f*e^2 + (-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2
*f*e^2)*cos(d*x + c) + (-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2)*sin
(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x + c)) - 12*(-I*d*f^3*x - I*d*f^2*
e + (-I*d*f^3*x - I*d*f^2*e)*cos(d*x + c) + (-I*d*f^3*x - I*d*f^2*e)*sin(d*
x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - 12*(I*d*f^3*x + I*d*f^2*e +
(I*d*f^3*x + I*d*f^2*e)*cos(d*x + c) + (I*d*f^3*x + I*d*f^2*e)*sin(d*x + c)
)*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 3*(I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*
e + I*d^2*f*e^2 + (I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e + I*d^2*f*e^2)*cos(d*x +
 c) + (I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e + I*d^2*f*e^2)*sin(d*x + c))*dilog(-
cos(d*x + c) + I*sin(d*x + c)) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^
2*f*e^2 + (-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2)*cos(d*x + c) + (
-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2)*sin(d*x + c))*dilog(-cos(d*
x + c) - I*sin(d*x + c)) - (d^3*f^3*x^3 + 3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 +
 d^3*e^3 + (d^3*f^3*x^3 + 3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*cos(d*
x + c) + (d^3*f^3*x^3 + 3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*sin(d*x
+ c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) - 6*(c^2*f^3 - 2*c*d*f^2*e + d
^2*f*e^2 + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(d*x + c) + (c^2*f^3 - 2*
c*d*f^2*e + d^2*f*e^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I)
 - (d^3*f^3*x^3 + 3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3 + (d^3*f^3*x^3
+ 3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*cos(d*x + c) + (d^3*f^3*x^3 +
3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*sin(d*x + c))*log(cos(d*x + c) -
 I*sin(d*x + c) + 1) - 6*(d^2*f^3*x^2 - c^2*f^3 + (d^2*f^3*x^2 - c^2*f^3 +
2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*x + c) + 2*(d^2*f^2*x + c*d*f^2)*e + (d^2*
f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*sin(d*x + c))*log(I*cos(d*x
+ c) + sin(d*x + c) + 1) - 6*(d^2*f^3*x^2 - c^2*f^3 + (d^2*f^3*x^2 - c^2*f^
3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*x + c) + 2*(d^2*f^2*x + c*d*f^2)*e + (
d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*sin(d*x + c))*log(-I*cos
(d*x + c) + sin(d*x + c) + 1) - (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 -
d^3*e^3 + (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3)*cos(d*x + c)
+ (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) - (c^3*f^3 - 3*c^2*d*f^2*e + 3*c
*d^2*f*e^2 - d^3*e^3 + (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3)*
cos(d*x + c) + (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3)*sin(d*x
+ c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + (d^3*f^3*x^3 + c^
3*f^3 + (d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2
 - c^2*d*f^2)*e)*cos(d*x + c) + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2
- c^2*d*f^2)*e + (d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^
3*f^2*x^2 - c^2*d*f^2)*e)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c)
+ 1) - 6*(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + (c^2*f^3 - 2*c*d*f^2*e + d^2*
f*e^2)*cos(d*x + c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*sin(d*x + c))*log
```

```
(-cos(d*x + c) + I*sin(d*x + c) + I) + (d^3*f^3*x^3 + c^3*f^3 + (d^3*f^3*x^
3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*co
s(d*x + c) + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e + (d
^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f
^2)*e)*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + 1) - 6*(-I*f^3*co
s(d*x + c) - I*f^3*sin(d*x + c) - I*f^3)*polylog(4, cos(d*x + c) + I*sin(d*
x + c)) - 6*(I*f^3*cos(d*x + c) + I*f^3*sin(d*x + c) + I*f^3)*polylog(4, co
s(d*x + c) - I*sin(d*x + c)) - 6*(-I*f^3*cos(d*x + c) - I*f^3*sin(d*x + c)
- I*f^3)*polylog(4, -cos(d*x + c) + I*sin(d*x + c)) - 6*(I*f^3*cos(d*x + c)
 + I*f^3*sin(d*x + c) + I*f^3)*polylog(4, -cos(d*x + c) - I*sin(d*x + c)) +
 6*(d*f^3*x + d*f^2*e + (d*f^3*x + d*f^2*e)*cos(d*x + c) + (d*f^3*x + d*f^2
*e)*sin(d*x + c))*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 6*(d*f^3*x +
d*f^2*e + (d*f^3*x + d*f^2*e)*cos(d*x + c) + (d*f^3*x + d*f^2*e)*sin(d*x +
c))*polylog(3, cos(d*x + c) - I*sin(d*x + c)) - 12*(f^3*cos(d*x + c) + f^3*
sin(d*x + c) + f^3)*polylog(3, I*cos(d*x + c) - sin(d*x + c)) - 12*(f^3*cos
(d*x + c) + f^3*sin(d*x + c) + f^3)*polylog(3, -I*cos(d*x + c) - sin(d*x +
c)) - 6*(d*f^3*x + d*f^2*e + (d*f^3*x + d*f^2*e)*cos(d*x + c) + (d*f^3*x +
d*f^2*e)*sin(d*x + c))*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) - 6*(d*f^
3*x + d*f^2*e + (d*f^3*x + d*f^2*e)*cos(d*x + c) + (d*f^3*x + d*f^2*e)*sin(
d*x + c))*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(d^3*f^3*x^3 + 3*d
^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*sin(d*x + c))/(a*d^4*cos(d*x + c) +
 a*d^4*sin(d*x + c) + a*d^4)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx \csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*csc(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*csc
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*csc(c + d*x)/(sin
(c + d*x) + 1), x) + Integral(3*e**2*f*x*csc(c + d*x)/(sin(c + d*x) + 1), x
))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.198 $\int \frac{(e+fx)^2 \csc(c+dx)}{a+a\sin(c+dx)}\, dx$

**Optimal**. Leaf size=249

$$\frac{i(e+fx)^2}{ad}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{2if(e$$

```
[Out] I*(f*x+e)^2/a/d-2*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)^2*cot(1/2*c
+1/4*Pi+1/2*d*x)/a/d-4*f*(f*x+e)*ln(1-I*exp(I*(d*x+c)))/a/d^2+2*I*f*(f*x+e)
*polylog(2,-exp(I*(d*x+c)))/a/d^2+4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3
-2*I*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2-2*f^2*polylog(3,-exp(I*(d*x+
c)))/a/d^3+2*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3
```

**Rubi** [A]
time = 0.23, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {4631, 4268, 2611, 2320, 6724, 3399, 4269, 3798, 2221, 2317, 2438}

$$\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{2f^2\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{2f^2\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}+\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{i(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (I*(e + f*x)^2)/(a*d) - (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((
e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (4*f*(e + f*x)*Log[1 - I*E^(I
*(c + d*x))])/(a*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a
*d^2) + ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - ((2*I)*f*(e + f
*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (2*f^2*PolyLog[3, -E^(I*(c + d*x
))])/(a*d^3) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\csc(c+dx)\,dx}{a} - \int \frac{(e+fx)^2}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{\int (e+fx)^2\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)\,dx}{2a} - \frac{(2f)\int}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{2if(e+fx)\mathrm{Li}_2}{ad^2}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + 2if$$

$$= \frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - 4f$$

$$= \frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - 4f$$

$$= \frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - 4f$$

**Mathematica [A]**
time = 1.74, size = 363, normalized size = 1.46

$$\frac{-2d^2e^2\tanh^{-1}\left(e^{i(c+dx)}\right)+2d^2efx\log\left(1-e^{i(c+dx)}\right)+d^2f^2x^2\log\left(1-e^{i(c+dx)}\right)-2d^2efx\log\left(1+e^{i(c+dx)}\right)-d^2f^2x^2\log\left(1+e^{i(c+dx)}\right)+2idf(e+fx)\mathrm{Li}_2\left(-e^{i(c+dx)}\right)-2idf(e+fx)\mathrm{Li}_2\left(e^{i(c+dx)}\right)-2f^2\mathrm{Li}_3\left(-e^{i(c+dx)}\right)+2f^2\mathrm{Li}_3\left(e^{i(c+dx)}\right)+2if\left(2id(e+fx)\log(1-i\cos(c+dx)+\sin(c+dx))+2f\mathrm{Li}_2(i\cos(c+dx)-\sin(c+dx))+\frac{d^2x(2e+fx)(\cos(c)+i\sin(c))}{\cos(c)+i(1+\sin(c))}\right)-\frac{2d^2(e+fx)^2\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*d^2*e^2*ArcTanh[E^(I*(c + d*x))] + 2*d^2*e*f*x*Log[1 - E^(I*(c + d*x))]
 + d^2*f^2*x^2*Log[1 - E^(I*(c + d*x))] - 2*d^2*e*f*x*Log[1 + E^(I*(c + d*x
))] - d^2*f^2*x^2*Log[1 + E^(I*(c + d*x))] + (2*I)*d*f*(e + f*x)*PolyLog[2,
 -E^(I*(c + d*x))] - (2*I)*d*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))] - 2*f^
2*PolyLog[3, -E^(I*(c + d*x))] + 2*f^2*PolyLog[3, E^(I*(c + d*x))] + (2*I)*
f*((2*I)*d*(e + f*x)*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] + 2*f*PolyLog[2
, I*Cos[c + d*x] - Sin[c + d*x]] + (d^2*x*(2*e + f*x)*(Cos[c] + I*Sin[c]))/
(Cos[c] + I*(1 + Sin[c]))) - (2*d^2*(e + f*x)^2*Sin[(d*x)/2])/((Cos[c/2] +
Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(a*d^3)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(223) = 446$.
time = 0.15, size = 643, normalized size = 2.58

| method | result |
|---|---|
| risch | $\frac{4 i f^2 c x}{a\, d^2}+\frac{2 x^2 f^2+4 e f x+2 e^2}{d a \left(\mathrm{e}^{i (d x+c)}+i\right)}-\frac{4 f^2 \ln\left(1-i \mathrm{e}^{i (d x+c)}\right) x}{a\, d^2}-\frac{4 f^2 \ln\left(1-i \mathrm{e}^{i (d x+c)}\right) c}{a\, d^3}+\frac{4 i f^2 \,\mathrm{polylog}\left(2, i \mathrm{e}^{i (d x+c)}\right)}{a\, d^3}+\frac{4 f^2 c \ln\left(\mathrm{e}^{i (d x+c}\right.}{a\, d^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*I/a/d^2*f^2*c*x+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(I*(d*x+c))+I)+4*I*f^2*po
lylog(2,I*exp(I*(d*x+c)))/a/d^3+1/d/a*e^2*ln(exp(I*(d*x+c))-1)-1/d/a*e^2*ln
(exp(I*(d*x+c))+1)+4/a/d^2*f*ln(exp(I*(d*x+c)))*e-4/a/d^2*f^2*ln(1-I*exp(I*
(d*x+c)))*x-4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c+4/a/d^3*f^2*c*ln(exp(I*(d*
x+c))+I)-4/a/d^3*f^2*c*ln(exp(I*(d*x+c)))+2*I/a/d*f^2*x^2+2*I/a/d^3*f^2*c^2
-4/a/d^2*f*ln(exp(I*(d*x+c))+I)*e+2/d/a*ln(1-exp(I*(d*x+c)))*e*f*x-2/d/a*ln
(exp(I*(d*x+c))+1)*e*f*x+2/d^2/a*ln(1-exp(I*(d*x+c)))*c*e*f-2/d^2/a*e*f*c*l
n(exp(I*(d*x+c))-1)-2*I/d^2/a*e*f*polylog(2,exp(I*(d*x+c)))+2*I/d^2/a*e*f*p
olylog(2,-exp(I*(d*x+c)))-2*I/d^2/a*f^2*polylog(2,exp(I*(d*x+c)))*x+2*I/d^2
/a*f^2*polylog(2,-exp(I*(d*x+c)))*x-1/d^3/a*f^2*c^2*ln(1-exp(I*(d*x+c)))+1/
d/a*f^2*ln(1-exp(I*(d*x+c)))*x^2-1/d/a*f^2*ln(exp(I*(d*x+c))+1)*x^2+1/d^3/a
*f^2*c^2*ln(exp(I*(d*x+c))-1)-2*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3+2*f^2*
polylog(3,exp(I*(d*x+c)))/a/d^3
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1443 vs. $2(220) = 440$.
time = 0.49, size = 1443, normalized size = 5.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*c*f*(2/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1)) + log(sin(d*x + c)/(
cos(d*x + c) + 1))/(a*d))*e - (log(sin(d*x + c)/(cos(d*x + c) + 1))/a + 2/(
```

```
a + a*sin(d*x + c)/(cos(d*x + c) + 1)))*e^2 + (4*I*c^2*f^2 - 8*(I*c*f^2 - I
*d*f*e + (c*f^2 - d*f*e)*cos(d*x + c) + (I*c*f^2 - I*d*f*e)*sin(d*x + c))*a
rctan2(sin(d*x + c) + 1, cos(d*x + c)) - 8*((d*x + c)*f^2*cos(d*x + c) + I*
(d*x + c)*f^2*sin(d*x + c) + I*(d*x + c)*f^2)*arctan2(cos(d*x + c), sin(d*x
 + c) + 1) - 2*(-I*(d*x + c)^2*f^2 - I*c^2*f^2 + 2*(I*c*f^2 - I*d*f*e)*(d*x
 + c) - ((d*x + c)^2*f^2 + c^2*f^2 - 2*(c*f^2 - d*f*e)*(d*x + c))*cos(d*x +
 c) + (-I*(d*x + c)^2*f^2 - I*c^2*f^2 + 2*(I*c*f^2 - I*d*f*e)*(d*x + c))*si
n(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1) - 2*(c^2*f^2*cos(d*x +
c) + I*c^2*f^2*sin(d*x + c) + I*c^2*f^2)*arctan2(sin(d*x + c), cos(d*x + c)
 - 1) - 2*(-I*(d*x + c)^2*f^2 + 2*(I*c*f^2 - I*d*f*e)*(d*x + c) - ((d*x + c
)^2*f^2 - 2*(c*f^2 - d*f*e)*(d*x + c))*cos(d*x + c) + (-I*(d*x + c)^2*f^2 +
 2*(I*c*f^2 - I*d*f*e)*(d*x + c))*sin(d*x + c))*arctan2(sin(d*x + c), -cos(
d*x + c) + 1) - 4*((d*x + c)^2*f^2 - 2*(c*f^2 - d*f*e)*(d*x + c))*cos(d*x +
 c) - 8*(f^2*cos(d*x + c) + I*f^2*sin(d*x + c) + I*f^2)*dilog(I*e^(I*d*x +
I*c)) - 4*(I*(d*x + c)*f^2 - I*c*f^2 + I*d*f*e + ((d*x + c)*f^2 - c*f^2 + d
*f*e)*cos(d*x + c) + (I*(d*x + c)*f^2 - I*c*f^2 + I*d*f*e)*sin(d*x + c))*di
log(-e^(I*d*x + I*c)) - 4*(-I*(d*x + c)*f^2 + I*c*f^2 - I*d*f*e - ((d*x + c
)*f^2 - c*f^2 + d*f*e)*cos(d*x + c) + (-I*(d*x + c)*f^2 + I*c*f^2 - I*d*f*e
)*sin(d*x + c))*dilog(e^(I*d*x + I*c)) + ((d*x + c)^2*f^2 + c^2*f^2 - 2*(c*
f^2 - d*f*e)*(d*x + c) + (-I*(d*x + c)^2*f^2 - I*c^2*f^2 - 2*(-I*c*f^2 + I*
d*f*e)*(d*x + c))*cos(d*x + c) + ((d*x + c)^2*f^2 + c^2*f^2 - 2*(c*f^2 - d*
f*e)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d
*x + c) + 1) - ((d*x + c)^2*f^2 + c^2*f^2 - 2*(c*f^2 - d*f*e)*(d*x + c) - (
I*(d*x + c)^2*f^2 + I*c^2*f^2 - 2*(I*c*f^2 - I*d*f*e)*(d*x + c))*cos(d*x +
c) + ((d*x + c)^2*f^2 + c^2*f^2 - 2*(c*f^2 - d*f*e)*(d*x + c))*sin(d*x + c)
)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1) + 4*((d*x + c)*
f^2 - c*f^2 + d*f*e - (I*(d*x + c)*f^2 - I*c*f^2 + I*d*f*e)*cos(d*x + c) +
((d*x + c)*f^2 - c*f^2 + d*f*e)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x
+ c)^2 + 2*sin(d*x + c) + 1) - 4*(I*f^2*cos(d*x + c) - f^2*sin(d*x + c) - f
^2)*polylog(3, -e^(I*d*x + I*c)) - 4*(-I*f^2*cos(d*x + c) + f^2*sin(d*x + c
) + f^2)*polylog(3, e^(I*d*x + I*c)) - 4*(I*(d*x + c)^2*f^2 + 2*(-I*c*f^2 +
 I*d*f*e)*(d*x + c))*sin(d*x + c))/(-2*I*a*d^2*cos(d*x + c) + 2*a*d^2*sin(d
*x + c) + 2*a*d^2))/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1666 vs. $2(220) = 440$.
time = 0.44, size = 1666, normalized size = 6.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*f^2*x^2 + 4*d^2*f*x*e + 2*d^2*e^2 + 2*(d^2*f^2*x^2 + 2*d^2*f*x*e
 + d^2*e^2)*cos(d*x + c) - 2*(I*d*f^2*x + I*d*f*e + (I*d*f^2*x + I*d*f*e)*c
```

```
os(d*x + c) + (I*d*f^2*x + I*d*f*e)*sin(d*x + c))*dilog(cos(d*x + c) + I*si
n(d*x + c)) - 2*(-I*d*f^2*x - I*d*f*e + (-I*d*f^2*x - I*d*f*e)*cos(d*x + c)
 + (-I*d*f^2*x - I*d*f*e)*sin(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x + c)
) - 4*(-I*f^2*cos(d*x + c) - I*f^2*sin(d*x + c) - I*f^2)*dilog(I*cos(d*x +
c) - sin(d*x + c)) - 4*(I*f^2*cos(d*x + c) + I*f^2*sin(d*x + c) + I*f^2)*di
log(-I*cos(d*x + c) - sin(d*x + c)) - 2*(I*d*f^2*x + I*d*f*e + (I*d*f^2*x +
 I*d*f*e)*cos(d*x + c) + (I*d*f^2*x + I*d*f*e)*sin(d*x + c))*dilog(-cos(d*x
 + c) + I*sin(d*x + c)) - 2*(-I*d*f^2*x - I*d*f*e + (-I*d*f^2*x - I*d*f*e)*
cos(d*x + c) + (-I*d*f^2*x - I*d*f*e)*sin(d*x + c))*dilog(-cos(d*x + c) - I
*sin(d*x + c)) - (d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2 + (d^2*f^2*x^2 + 2*d^
2*f*x*e + d^2*e^2)*cos(d*x + c) + (d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2)*sin
(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) + 4*(c*f^2 - d*f*e + (c*f
^2 - d*f*e)*cos(d*x + c) + (c*f^2 - d*f*e)*sin(d*x + c))*log(cos(d*x + c) +
 I*sin(d*x + c) + I) - (d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2 + (d^2*f^2*x^2
+ 2*d^2*f*x*e + d^2*e^2)*cos(d*x + c) + (d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^
2)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x + c) + 1) - 4*(d*f^2*x + c*f^
2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(I*
cos(d*x + c) + sin(d*x + c) + 1) - 4*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*c
os(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x
 + c) + 1) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2 + (c^2*f^2 - 2*c*d*f*e + d^2*e^
2)*cos(d*x + c) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*sin(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2 + (
c^2*f^2 - 2*c*d*f*e + d^2*e^2)*cos(d*x + c) + (c^2*f^2 - 2*c*d*f*e + d^2*e^
2)*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + (d^2*f
^2*x^2 - c^2*f^2 + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x
+ c) + 2*(d^2*f*x + c*d*f)*e + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)
*e)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + 4*(c*f^2 - d*f*
e + (c*f^2 - d*f*e)*cos(d*x + c) + (c*f^2 - d*f*e)*sin(d*x + c))*log(-cos(d
*x + c) + I*sin(d*x + c) + I) + (d^2*f^2*x^2 - c^2*f^2 + (d^2*f^2*x^2 - c^2
*f^2 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x + c) + 2*(d^2*f*x + c*d*f)*e + (d^2*f
^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*sin(d*x + c))*log(-cos(d*x + c) -
 I*sin(d*x + c) + 1) + 2*(f^2*cos(d*x + c) + f^2*sin(d*x + c) + f^2)*polylo
g(3, cos(d*x + c) + I*sin(d*x + c)) + 2*(f^2*cos(d*x + c) + f^2*sin(d*x + c
) + f^2)*polylog(3, cos(d*x + c) - I*sin(d*x + c)) - 2*(f^2*cos(d*x + c) +
f^2*sin(d*x + c) + f^2)*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) - 2*(f^2
*cos(d*x + c) + f^2*sin(d*x + c) + f^2)*polylog(3, -cos(d*x + c) - I*sin(d*
x + c)) - 2*(d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2)*sin(d*x + c))/(a*d^3*cos(
d*x + c) + a*d^3*sin(d*x + c) + a*d^3)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*csc(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*csc
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*csc(c + d*x)/(sin(c + d
*x) + 1), x))/a
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.199 $\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=134

$$-\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{if\text{Li}_2\left(-e^{i(c+dx)}\right)}{ad^2}-if$$

```
[Out] -2*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)/a/
d-2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2+I*f*polylog(2,-exp(I*(d*x+c)))/a/
d^2-I*f*polylog(2,exp(I*(d*x+c)))/a/d^2
```

**Rubi** [A]

time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4631, 4268, 2317, 2438, 3399, 4269, 3556}

$$\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) - (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/(a*d^2) + (I*f*Poly
Log[2, -E^(I*(c + d*x))])/(a*d^2) - (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^
2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\csc(c+dx)\,dx}{a} - \int \frac{e+fx}{a+a\sin(c+dx)}\,dx \\
&= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{\int (e+fx)\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)\,dx}{2a} - \frac{f\int \log\Big(\cdots}{\cdots} \\
&= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(if)\text{Subst}\Big(\int \frac{\log(1\cdots}{x}\cdots}{a\cdots} \\
&= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}\cdots\right.\right.}{ad^2}
\end{aligned}
$$

**<u>Mathematica</u>** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 300 vs. $2(134) = 268$.
time = 0.62, size = 300, normalized size = 2.24

$$
\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-2d(e+fx)\sin\left(\frac{1}{2}(c+dx)\right)+f(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+de\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-cf\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+f\left((c+dx)\left(\log\left(1-e^{i(c+dx)}\right)-\log\left(1+e^{i(c+dx)}\right)\right)+i\left(\text{Li}_2\left(-e^{i(c+dx)}\right)-\text{Li}_2\left(e^{i(c+dx)}\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{ad^2(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-2*d*(e + f*x)*Sin[(c + d*x)/2] + f
*(c + d*x)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 2*f*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + d*e*Log[Tan[(c
 + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - c*f*Log[Tan[(c + d*x)/2
]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + f*((c + d*x)*(Log[1 - E^(I*(c +
d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - Poly
Log[2, E^(I*(c + d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(a*d^2*(1
 + Sin[c + d*x]))
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(114) = 228.
time = 0.17, size = 245, normalized size = 1.83

| method | result |
|---|---|
| risch | $\frac{2fx+2e}{da\left(e^{i(dx+c)}+i\right)}+\frac{e\ln\left(e^{i(dx+c)}-1\right)}{da}-\frac{e\ln\left(e^{i(dx+c)}+1\right)}{da}-\frac{fc\ln\left(e^{i(dx+c)}-1\right)}{d^2a}-\frac{if\,\mathrm{polylog}\left(2,e^{i(dx+c)}\right)}{a\,d^2}+\frac{if\,\mathrm{polylog}\left(2,-e^{i(d}\right.}{a\,d^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(f*x+e)/d/a/(exp(I*(d*x+c))+I)+1/d/a*e*ln(exp(I*(d*x+c))-1)-1/d/a*e*ln(ex
p(I*(d*x+c))+1)-1/d^2/a*f*c*ln(exp(I*(d*x+c))-1)-I*f*polylog(2,exp(I*(d*x+c
)))/a/d^2+I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2+2/d^2/a*f*ln(exp(I*(d*x+c)))
+1/d/a*ln(1-exp(I*(d*x+c)))*f*x+1/d^2/a*ln(1-exp(I*(d*x+c)))*c*f-1/d/a*ln(e
xp(I*(d*x+c))+1)*f*x-2*f/a/d^2*ln(exp(I*(d*x+c))+I)
```

**Maxima** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(112) = 224.
time = 0.40, size = 527, normalized size = 3.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (4*d*f*x*cos(d*x + c) + 4*I*d*f*x*sin(d*x + c) - 4*(f*cos(d*x + c) + I*f*si
n(d*x + c) + I*f)*arctan2(cos(c) + sin(d*x), cos(d*x) + sin(c)) + 2*(-I*d*f
*x - (d*f*x + d*e)*cos(d*x + c) - I*d*e + (-I*d*f*x - I*d*e)*sin(d*x + c))*
arctan2(sin(d*x + c), cos(d*x + c) + 1) + 2*(d*cos(d*x + c)*e + I*d*e*sin(d
*x + c) + I*d*e)*arctan2(sin(d*x + c), cos(d*x + c) - 1) - 2*(d*f*x*cos(d*x
 + c) + I*d*f*x*sin(d*x + c) + I*d*f*x)*arctan2(sin(d*x + c), -cos(d*x + c)
 + 1) + 2*(f*cos(d*x + c) + I*f*sin(d*x + c) + I*f)*dilog(-e^(I*d*x + I*c))
```

```
 - 2*(f*cos(d*x + c) + I*f*sin(d*x + c) + I*f)*dilog(e^(I*d*x + I*c)) - 4*I
*d*e - (d*f*x + (-I*d*f*x - I*d*e)*cos(d*x + c) + d*e + (d*f*x + d*e)*sin(d
*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1) + (d*f*x
 - (I*d*f*x + I*d*e)*cos(d*x + c) + d*e + (d*f*x + d*e)*sin(d*x + c))*log(c
os(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1) + 2*(I*f*cos(d*x + c)
- f*sin(d*x + c) - f)*log(cos(d*x)^2 + cos(c)^2 + 2*cos(c)*sin(d*x) + sin(d
*x)^2 + 2*cos(d*x)*sin(c) + sin(c)^2))/(-2*I*a*d^2*cos(d*x + c) + 2*a*d^2*s
in(d*x + c) + 2*a*d^2)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(112) = 224$.
time = 0.40, size = 626, normalized size = 4.67

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*f*x + 2*(d*f*x + d*e)*cos(d*x + c) + (-I*f*cos(d*x + c) - I*f*sin(
d*x + c) - I*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) + (I*f*cos(d*x + c) +
I*f*sin(d*x + c) + I*f)*dilog(cos(d*x + c) - I*sin(d*x + c)) + (-I*f*cos(d*
x + c) - I*f*sin(d*x + c) - I*f)*dilog(-cos(d*x + c) + I*sin(d*x + c)) + (I
*f*cos(d*x + c) + I*f*sin(d*x + c) + I*f)*dilog(-cos(d*x + c) - I*sin(d*x +
 c)) + 2*d*e - (d*f*x + (d*f*x + d*e)*cos(d*x + c) + d*e + (d*f*x + d*e)*si
n(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) - (d*f*x + (d*f*x + d*e)
*cos(d*x + c) + d*e + (d*f*x + d*e)*sin(d*x + c))*log(cos(d*x + c) - I*sin(
d*x + c) + 1) - (c*f + (c*f - d*e)*cos(d*x + c) - d*e + (c*f - d*e)*sin(d*x
 + c))*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) - (c*f + (c*f - d*
e)*cos(d*x + c) - d*e + (c*f - d*e)*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1
/2*I*sin(d*x + c) + 1/2) + (d*f*x + c*f + (d*f*x + c*f)*cos(d*x + c) + (d*f
*x + c*f)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + (d*f*x +
c*f + (d*f*x + c*f)*cos(d*x + c) + (d*f*x + c*f)*sin(d*x + c))*log(-cos(d*x
 + c) - I*sin(d*x + c) + 1) - 2*(f*cos(d*x + c) + f*sin(d*x + c) + f)*log(s
in(d*x + c) + 1) - 2*(d*f*x + d*e)*sin(d*x + c))/(a*d^2*cos(d*x + c) + a*d^
2*sin(d*x + c) + a*d^2)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e\csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*csc(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f*x*csc(c + d*x)
/(sin(c + d*x) + 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad** [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.200 $\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=38

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+cos(d*x+c)/d/(a+a*sin(d*x+c))
```

**Rubi** [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2826, 3855, 2727}

$$\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Cos[c + d*x]/(d*(a + a*Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \csc(c+dx)\,dx}{a}-\int \frac{1}{a+a\sin(c+dx)}\,dx\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}\end{aligned}$$

**Mathematica [A]**
time = 0.05, size = 48, normalized size = 1.26

$$-\frac{\sec(c+dx)\left(-1+\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)\sqrt{\cos^2(c+dx)}+\sin(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Sec[c + d*x]*(-1 + ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[c + d*x]^2] +
Sin[c + d*x]))/(a*d))
```

**Maple [A]**
time = 0.08, size = 34, normalized size = 0.89

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ | 34 |
| default | $\frac{\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ | 34 |
| norman | $-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$ | 49 |
| risch | $\frac{2}{da\left(e^{i(dx+c)}+i\right)}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{da}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{da}$ | 63 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(2/(tan(1/2*d*x+1/2*c)+1)+ln(tan(1/2*d*x+1/2*c)))
```

**Maxima [A]**
time = 0.29, size = 51, normalized size = 1.34

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{2}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1))/a + 2/(a + a*sin(d*x + c)/(cos(d*x +
c) + 1)))/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(38) = 76$.
time = 0.34, size = 97, normalized size = 2.55

$$-\frac{(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+\sin(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\cos(dx+c)+2\sin(dx+c)-2}{2\,(ad\cos(dx+c)+ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c) + sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(
d*x + c) + sin(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*cos(d*x + c)
+ 2*sin(d*x + c) - 2)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

**Giac** [A]
time = 6.06, size = 38, normalized size = 1.00

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{2}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c)))/a + 2/(a*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

**Mupad** [B]
time = 1.21, size = 39, normalized size = 1.03

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + 2/(a*d*(tan(c/2 + (d*x)/2) + 1))
```

# 3.201 $\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=29

$$\text{Int}\left(\frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 7.07, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(dx+c\right)}{\left(fx+e\right)\left(a+a\sin\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*(a*d*f^2*x + a*d*f*e + (a*d*f^2*x + a*d*f*e)*cos(d*x + c)^2 + (a*d*f^2*x
 + a*d*f*e)*sin(d*x + c)^2 + 2*(a*d*f^2*x + a*d*f*e)*sin(d*x + c))*integrat
e(cos(d*x + c)/(a*d*f^2*x^2 + 2*a*d*f*x*e + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*
d*e^2)*cos(d*x + c)^2 + a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*sin
(d*x + c)^2 + 2*(a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*sin(d*x + c)), x) + (
a*d*f*x + (a*d*f*x + a*d*e)*cos(d*x + c)^2 + a*d*e + (a*d*f*x + a*d*e)*sin(
d*x + c)^2 + 2*(a*d*f*x + a*d*e)*sin(d*x + c))*integrate(sin(d*x + c)/(a*f*
x + (a*f*x + a*e)*cos(d*x + c)^2 + (a*f*x + a*e)*sin(d*x + c)^2 + 2*(a*f*x
+ a*e)*cos(d*x + c) + a*e), x) + (a*d*f*x + (a*d*f*x + a*d*e)*cos(d*x + c)^
2 + a*d*e + (a*d*f*x + a*d*e)*sin(d*x + c)^2 + 2*(a*d*f*x + a*d*e)*sin(d*x
+ c))*integrate(sin(d*x + c)/(a*f*x + (a*f*x + a*e)*cos(d*x + c)^2 + (a*f*x
 + a*e)*sin(d*x + c)^2 - 2*(a*f*x + a*e)*cos(d*x + c) + a*e), x) + 2*cos(d*
x + c))/(a*d*f*x + (a*d*f*x + a*d*e)*cos(d*x + c)^2 + a*d*e + (a*d*f*x + a*
d*e)*sin(d*x + c)^2 + 2*(a*d*f*x + a*d*e)*sin(d*x + c))
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
[Out] Integral(csc(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(csc(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sin\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(e + f*x)*(a + a*sin(c + d*x))),x)
[Out] int(1/(sin(c + d*x)*(e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.202 $\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=29

$$\mathrm{Int}\left(\frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 8.15, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(fx+e)^2\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (4*(a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^2 + (a*d*f^3*x^2 + 2*a*d*f^2*x*e
+ a*d*f*e^2)*cos(d*x + c)^2 + (a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^2)*sin
(d*x + c)^2 + 2*(a*d*f^3*x^2 + 2*a*d*f^2*x*e + a*d*f*e^2)*sin(d*x + c))*int
egrate(cos(d*x + c)/(a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + (a*d*f
^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*e^3)*cos(d*x + c)^2 + a*d*e^
3 + (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*e^3)*sin(d*x + c)^
2 + 2*(a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*e^3)*sin(d*x + c
)), x) + (a*d*f^2*x^2 + 2*a*d*f*x*e + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)
*cos(d*x + c)^2 + a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*sin(d*x +
 c)^2 + 2*(a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*sin(d*x + c))*integrate(sin
(d*x + c)/(a*f^2*x^2 + 2*a*f*x*e + (a*f^2*x^2 + 2*a*f*x*e + a*e^2)*cos(d*x
+ c)^2 + (a*f^2*x^2 + 2*a*f*x*e + a*e^2)*sin(d*x + c)^2 + 2*(a*f^2*x^2 + 2*
a*f*x*e + a*e^2)*cos(d*x + c) + a*e^2), x) + (a*d*f^2*x^2 + 2*a*d*f*x*e + (
a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*cos(d*x + c)^2 + a*d*e^2 + (a*d*f^2*x^
2 + 2*a*d*f*x*e + a*d*e^2)*sin(d*x + c)^2 + 2*(a*d*f^2*x^2 + 2*a*d*f*x*e +
a*d*e^2)*sin(d*x + c))*integrate(sin(d*x + c)/(a*f^2*x^2 + 2*a*f*x*e + (a*f
^2*x^2 + 2*a*f*x*e + a*e^2)*cos(d*x + c)^2 + (a*f^2*x^2 + 2*a*f*x*e + a*e^2
)*sin(d*x + c)^2 - 2*(a*f^2*x^2 + 2*a*f*x*e + a*e^2)*cos(d*x + c) + a*e^2),
 x) + 2*cos(d*x + c))/(a*d*f^2*x^2 + 2*a*d*f*x*e + (a*d*f^2*x^2 + 2*a*d*f*x
*e + a*d*e^2)*cos(d*x + c)^2 + a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e
^2)*sin(d*x + c)^2 + 2*(a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*sin(d*x + c))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2 + 2*a*f*x
*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2*
e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sin(c+d\,x)\,{(e+f\,x)}^2\,(a+a\,\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(e + f*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] int(1/(sin(c + d*x)*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.203 $\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=463

$$-\frac{2i(e+fx)^3}{ad}+\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{(e+fx)^3\cot(c+dx)}{ad}+\frac{6f(e+f}{}$$

```
[Out] -12*I*f^2*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/a/d^3+2*(f*x+e)^3*arctanh(exp
(I*(d*x+c)))/a/d-(f*x+e)^3*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-(f*x+e)^3*cot(d*x+
c)/a/d+6*f*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a/d^2+3*f*(f*x+e)^2*ln(1-exp(2*
I*(d*x+c)))/a/d^2+6*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/d^4-6*I*f^3*polylog(
4,exp(I*(d*x+c)))/a/d^4-2*I*(f*x+e)^3/a/d-3*I*f*(f*x+e)^2*polylog(2,-exp(I*
(d*x+c)))/a/d^2+6*f^2*(f*x+e)*polylog(3,-exp(I*(d*x+c)))/a/d^3+12*f^3*polyl
og(3,I*exp(I*(d*x+c)))/a/d^4-6*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a/d^3+
3/2*f^3*polylog(3,exp(2*I*(d*x+c)))/a/d^4+3*I*f*(f*x+e)^2*polylog(2,exp(I*(
d*x+c)))/a/d^2-3*I*f^2*(f*x+e)*polylog(2,exp(2*I*(d*x+c)))/a/d^3
```

**Rubi** [A]
time = 0.50, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4631, 4269, 3798, 2221, 2611, 2320, 6724, 4268, 6744, 3399}

$$\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}+\frac{3f^3\text{PolyLog}\left(3,e^{2i(c+dx)}\right)}{2ad^4}+\frac{6if^3\text{PolyLog}\left(4,-e^{i(c+dx)}\right)}{ad^4}-\frac{6if^3\text{PolyLog}\left(4,e^{i(c+dx)}\right)}{ad^4}-\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}-\frac{6f^2(e+fx)\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}+\frac{3if(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{3f(e+fx)^2\log\left(1-e^{2i(c+dx)}\right)}{ad^2}-\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{(e+fx)^3\cot(c+dx)}{ad}+\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{2i(e+fx)^3}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-2*I)*(e + f*x)^3)/(a*d) + (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d)
 - ((e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - ((e + f*x)^3*Cot[c + d*x
])/(a*d) + (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + (3*f*(e +
 f*x)^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((3*I)*f*(e + f*x)^2*PolyLo
g[2, -E^(I*(c + d*x))])/(a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(
c + d*x))])/(a*d^3) + ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*
d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^3) + (6*f
^2*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) + (12*f^3*PolyLog[3, I*E
^(I*(c + d*x))])/(a*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a
*d^3) + (3*f^3*PolyLog[3, E^((2*I)*(c + d*x))])/(2*a*d^4) + ((6*I)*f^3*Poly
Log[4, -E^(I*(c + d*x))])/(a*d^4) - ((6*I)*f^3*PolyLog[4, E^(I*(c + d*x))])
/(a*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3 \csc^2(c+dx)\,dx}{a} - \int \frac{(e+fx)^3 \csc(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{(e+fx)^3 \cot(c+dx)}{ad} - \frac{\int (e+fx)^3 \csc(c+dx)\,dx}{a} + \frac{(3f)\int (e+fx)^2 \cot(c}{ad}$$
$$= -\frac{i(e+fx)^3}{ad} + \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3 \cot(c+dx)}{ad} + \frac{\int (e+}{}$$
$$= -\frac{i(e+fx)^3}{ad} + \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} -$$
$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} -$$
$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} -$$
$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} -$$
$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} -$$
$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} -$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1208 vs. $2(463) = 926$.
time = 20.54, size = 1208, normalized size = 2.61

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((e^3*Log[Tan[(c + d*x)/2]])/(a*d)) - (3*e^2*f*((c + d*x)*(Log[1 - E^(I*(c
 + d*x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLo
g[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^2) - (f^3*Csc[
c]*(2*d^2*x^2*(2*d*E^((2*I)*c)*x + (3*I)*(-1 + E^((2*I)*c))*Log[1 - E^((2*I
)*(c + d*x))]) + 6*d*(-1 + E^((2*I)*c))*x*PolyLog[2, E^((2*I)*(c + d*x))] +
 (3*I)*(-1 + E^((2*I)*c))*PolyLog[3, E^((2*I)*(c + d*x))]))/(4*a*d^4*E^(I*c
)) + (6*e*f^2*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - I*d*x*PolyL
og[2, -Cos[c + d*x] - I*Sin[c + d*x]] + I*d*x*PolyLog[2, Cos[c + d*x] + I*S
```

```
in[c + d*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - PolyLog[3, Cos[
c + d*x] + I*Sin[c + d*x]]))/(a*d^3) - (f^3*(-2*d^3*x^3*ArcTanh[Cos[c + d*x
] + I*Sin[c + d*x]] + (3*I)*d^2*x^2*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*
x]] - (3*I)*d^2*x^2*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*x]] - 6*d*x*PolyL
og[3, -Cos[c + d*x] - I*Sin[c + d*x]] + 6*d*x*PolyLog[3, Cos[c + d*x] + I*S
in[c + d*x]] - (6*I)*PolyLog[4, -Cos[c + d*x] - I*Sin[c + d*x]] + (6*I)*Pol
yLog[4, Cos[c + d*x] + I*Sin[c + d*x]]))/(a*d^4) + (3*e^2*f*Csc[c]*(-(d*x*C
os[c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*d^2*(Cos[c]^2 +
 Sin[c]^2)) + (2*f*(3*d^2*(e + f*x)^2*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]
] - (6*I)*d*f*(e + f*x)*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]] + 6*f^2*P
olyLog[3, I*Cos[c + d*x] - Sin[c + d*x]] + (d^3*x*(3*e^2 + 3*e*f*x + f^2*x^
2)*((-I)*Cos[c] + Sin[c]))/(Cos[c] + I*(1 + Sin[c]))))/(a*d^4) + (Csc[c/2]*
Csc[c/2 + (d*x)/2]*(e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2
*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2]))/(2*a*d) + (Sec[c/2]*Sec[c/2 + (d*x)/
2]*(e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] +
f^3*x^3*Sin[(d*x)/2]))/(2*a*d) + (2*(e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)
/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2]))/(a*d*(Cos[c/2] + Si
n[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])) - (3*e*f^2*Csc[c]*Sec[c]
*(d^2*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi + 2*ArcTan[Tan[c]]) - Pi*Log[
1 + E^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*Log[1 - E^((2*I)*(d*x + ArcT
an[Tan[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan[c]]*Log[Sin[d*x + ArcTan[Ta
n[c]]]] + I*PolyLog[2, E^((2*I)*(d*x + ArcTan[Tan[c]]))])*Tan[c])/Sqrt[1 +
Tan[c]^2]))/(a*d^3*Sqrt[Sec[c]^2*(Cos[c]^2 + Sin[c]^2)])
```

<u>**Maple**</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1704 vs. $2(419) = 838$.
time = 0.29, size = 1705, normalized size = 3.68

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1705 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -6/d^3/a*e*f^2*polylog(3,exp(I*(d*x+c)))+6/d^3/a*e*f^2*polylog(3,-exp(I*(d*
x+c)))+1/d^4/a*f^3*c^3*ln(exp(I*(d*x+c))-1)-6/d^3/a*f^3*polylog(3,exp(I*(d*
x+c)))*x+6/d^3/a*f^3*polylog(3,-exp(I*(d*x+c)))*x+6/a/d^2*e*f^2*ln(1-exp(I*
(d*x+c)))*x+6/a/d^3*e*f^2*ln(1-exp(I*(d*x+c)))*c+6/a/d^2*e*f^2*ln(exp(I*(d*
x+c))+1)*x+3*I/a/d^2*e^2*f*polylog(2,exp(I*(d*x+c)))-3*I/a/d^2*e^2*f*polylo
g(2,-exp(I*(d*x+c)))-12*I/a/d*e*f^2*x^2-6*I/a/d^3*e*f^2*polylog(2,exp(I*(d*
x+c)))+3*I/a/d^2*f^3*polylog(2,exp(I*(d*x+c)))*x^2-1/d/a*e^3*ln(exp(I*(d*x+
c))-1)+1/d/a*e^3*ln(exp(I*(d*x+c))+1)+6*f^3*polylog(3,-exp(I*(d*x+c)))/a/d^
4+6*f^3*polylog(3,exp(I*(d*x+c)))/a/d^4+6/a/d^2*f^3*ln(1-I*exp(I*(d*x+c)))*
x^2-6/a/d^4*f^3*ln(1-I*exp(I*(d*x+c)))*c^2-6*I/a/d^3*f^3*polylog(2,-exp(I*(
d*x+c)))*x-6*I/a/d^3*f^3*polylog(2,exp(I*(d*x+c)))*x-3*I/a/d^2*f^3*polylog(
```

```
2,-exp(I*(d*x+c)))*x^2+12*I/a/d^3*f^3*c^2*x-12*I/a/d^3*e*f^2*c^2-6*I/a/d^3*
e*f^2*polylog(2,-exp(I*(d*x+c)))-12/a/d^2*f*ln(exp(I*(d*x+c)))*e^2-12/a/d^4
*f^3*c^2*ln(exp(I*(d*x+c)))+6/a/d^4*f^3*c^2*ln(exp(I*(d*x+c))+I)+6*I*f^3*po
lylog(4,-exp(I*(d*x+c)))/a/d^4+6/a/d^2*f*ln(exp(I*(d*x+c))+I)*e^2-12*I/d^3/
a*e*f^2*polylog(2,I*exp(I*(d*x+c)))+24/a/d^3*f^2*e*c*ln(exp(I*(d*x+c)))-3/d
^2/a*ln(1-exp(I*(d*x+c)))*c*e^2*f+3/d/a*e*f^2*ln(exp(I*(d*x+c))+1)*x^2-3/d/
a*ln(1-exp(I*(d*x+c)))*e^2*f*x+3/d/a*ln(exp(I*(d*x+c))+1)*e^2*f*x-3/d^3/a*e
*f^2*c^2*ln(exp(I*(d*x+c))-1)+3/d^2/a*e^2*f*c*ln(exp(I*(d*x+c))-1)-1/d/a*f^
3*ln(1-exp(I*(d*x+c)))*x^3-1/d^4/a*f^3*ln(1-exp(I*(d*x+c)))*c^3+1/d/a*f^3*l
n(exp(I*(d*x+c))+1)*x^3+3/d^3/a*e*f^2*c^2*ln(1-exp(I*(d*x+c)))-3/d/a*e*f^2*
ln(1-exp(I*(d*x+c)))*x^2-12*I/d^3/a*f^3*polylog(2,I*exp(I*(d*x+c)))*x-2*(-2
*f^3*x^3+I*exp(I*(d*x+c))*f^3*x^3-6*e*f^2*x^2+3*I*exp(I*(d*x+c))*e*f^2*x^2-
6*e^2*f*x+3*I*exp(I*(d*x+c))*e^2*f*x-2*e^3+I*exp(I*(d*x+c))*e^3+f^3*x^3*exp
(2*I*(d*x+c))+3*e*f^2*x^2*exp(2*I*(d*x+c))+3*e^2*f*x*exp(2*I*(d*x+c))+e^3*e
xp(2*I*(d*x+c)))/(exp(2*I*(d*x+c))-1)/(exp(I*(d*x+c))+I)/d/a+12*f^3*polylog
(3,I*exp(I*(d*x+c)))/a/d^4-12/a/d^3*f^2*e*c*ln(exp(I*(d*x+c))+I)+6*I/a/d^2*
e*f^2*polylog(2,exp(I*(d*x+c)))*x-6*I/a/d^2*e*f^2*polylog(2,-exp(I*(d*x+c))
)*x-24*I/a/d^2*e*f^2*c*x-6/a/d^3*e*f^2*c*ln(exp(I*(d*x+c))-1)-6*I*f^3*polyl
og(4,exp(I*(d*x+c)))/a/d^4+12/a/d^2*f^2*e*ln(1-I*exp(I*(d*x+c)))*x+12/a/d^3
*f^2*e*ln(1-I*exp(I*(d*x+c)))*c+3/a/d^2*e^2*f*ln(exp(I*(d*x+c))-1)+3/a/d^2*
e^2*f*ln(exp(I*(d*x+c))+1)+3/a/d^4*f^3*c^2*ln(exp(I*(d*x+c))-1)+3/a/d^2*f^3
*ln(1-exp(I*(d*x+c)))*x^2-3/a/d^4*f^3*ln(1-exp(I*(d*x+c)))*c^2+3/a/d^2*f^3*
ln(exp(I*(d*x+c))+1)*x^2-4*I/a/d*f^3*x^3+8*I/a/d^4*f^3*c^3
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4792 vs. $2(417) = 834$.
time = 0.53, size = 4792, normalized size = 10.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^3*f^3*x^3 + 6*d^3*f^2*x^2*e + 6*d^3*f*x*e^2 + 2*d^3*e^3 - 4*(d^3*
f^3*x^3 + 3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*cos(d*x + c)^2 - 2*(d^
```

```
3*f^3*x^3 + 3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*cos(d*x + c) - 3*(-I
*d^2*f^3*x^2 + 2*I*d*f^3*x - I*d^2*f*e^2 + (I*d^2*f^3*x^2 - 2*I*d*f^3*x + I
*d^2*f*e^2 + 2*I*(d^2*f^2*x - d*f^2)*e)*cos(d*x + c)^2 - 2*I*(d^2*f^2*x - d
*f^2)*e + (-I*d^2*f^3*x^2 + 2*I*d*f^3*x - I*d^2*f*e^2 + (-I*d^2*f^3*x^2 + 2
*I*d*f^3*x - I*d^2*f*e^2 - 2*I*(d^2*f^2*x - d*f^2)*e)*cos(d*x + c) - 2*I*(d
^2*f^2*x - d*f^2)*e)*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x + c)) - 3
*(I*d^2*f^3*x^2 - 2*I*d*f^3*x + I*d^2*f*e^2 + (-I*d^2*f^3*x^2 + 2*I*d*f^3*x
 - I*d^2*f*e^2 - 2*I*(d^2*f^2*x - d*f^2)*e)*cos(d*x + c)^2 + 2*I*(d^2*f^2*x
 - d*f^2)*e + (I*d^2*f^3*x^2 - 2*I*d*f^3*x + I*d^2*f*e^2 + (I*d^2*f^3*x^2 -
 2*I*d*f^3*x + I*d^2*f*e^2 + 2*I*(d^2*f^2*x - d*f^2)*e)*cos(d*x + c) + 2*I*
(d^2*f^2*x - d*f^2)*e)*sin(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x + c)) -
 12*(I*d*f^3*x + I*d*f^2*e + (-I*d*f^3*x - I*d*f^2*e)*cos(d*x + c)^2 + (I*d
*f^3*x + I*d*f^2*e + (I*d*f^3*x + I*d*f^2*e)*cos(d*x + c))*sin(d*x + c))*di
log(I*cos(d*x + c) - sin(d*x + c)) - 12*(-I*d*f^3*x - I*d*f^2*e + (I*d*f^3*
x + I*d*f^2*e)*cos(d*x + c)^2 + (-I*d*f^3*x - I*d*f^2*e + (-I*d*f^3*x - I*d
*f^2*e)*cos(d*x + c))*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) -
 3*(-I*d^2*f^3*x^2 - 2*I*d*f^3*x - I*d^2*f*e^2 + (I*d^2*f^3*x^2 + 2*I*d*f^3
*x + I*d^2*f*e^2 + 2*I*(d^2*f^2*x + d*f^2)*e)*cos(d*x + c)^2 - 2*I*(d^2*f^2
*x + d*f^2)*e + (-I*d^2*f^3*x^2 - 2*I*d*f^3*x - I*d^2*f*e^2 + (-I*d^2*f^3*x
^2 - 2*I*d*f^3*x - I*d^2*f*e^2 - 2*I*(d^2*f^2*x + d*f^2)*e)*cos(d*x + c) -
2*I*(d^2*f^2*x + d*f^2)*e)*sin(d*x + c))*dilog(-cos(d*x + c) + I*sin(d*x +
c)) - 3*(I*d^2*f^3*x^2 + 2*I*d*f^3*x + I*d^2*f*e^2 + (-I*d^2*f^3*x^2 - 2*I*
d*f^3*x - I*d^2*f*e^2 - 2*I*(d^2*f^2*x + d*f^2)*e)*cos(d*x + c)^2 + 2*I*(d^
2*f^2*x + d*f^2)*e + (I*d^2*f^3*x^2 + 2*I*d*f^3*x + I*d^2*f*e^2 + (I*d^2*f^
3*x^2 + 2*I*d*f^3*x + I*d^2*f*e^2 + 2*I*(d^2*f^2*x + d*f^2)*e)*cos(d*x + c)
 + 2*I*(d^2*f^2*x + d*f^2)*e)*sin(d*x + c))*dilog(-cos(d*x + c) - I*sin(d*x
 + c)) + (d^3*f^3*x^3 + 3*d^2*f^3*x^2 + d^3*e^3 - (d^3*f^3*x^3 + 3*d^2*f^3*
x^2 + d^3*e^3 + 3*(d^3*f*x + d^2*f)*e^2 + 3*(d^3*f^2*x^2 + 2*d^2*f^2*x)*e)*
cos(d*x + c)^2 + 3*(d^3*f*x + d^2*f)*e^2 + 3*(d^3*f^2*x^2 + 2*d^2*f^2*x)*e
+ (d^3*f^3*x^3 + 3*d^2*f^3*x^2 + d^3*e^3 + (d^3*f^3*x^3 + 3*d^2*f^3*x^2 + d
^3*e^3 + 3*(d^3*f*x + d^2*f)*e^2 + 3*(d^3*f^2*x^2 + 2*d^2*f^2*x)*e)*cos(d*x
 + c) + 3*(d^3*f*x + d^2*f)*e^2 + 3*(d^3*f^2*x^2 + 2*d^2*f^2*x)*e)*sin(d*x
+ c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) + 6*(c^2*f^3 - 2*c*d*f^2*e + d
^2*f*e^2 - (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(d*x + c)^2 + (c^2*f^3 -
2*c*d*f^2*e + d^2*f*e^2 + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(d*x + c))
*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) + (d^3*f^3*x^3 + 3*d^
2*f^3*x^2 + d^3*e^3 - (d^3*f^3*x^3 + 3*d^2*f^3*x^2 + d^3*e^3 + 3*(d^3*f*x +
 d^2*f)*e^2 + 3*(d^3*f^2*x^2 + 2*d^2*f^2*x)*e)*cos(d*x + c)^2 + 3*(d^3*f*x
+ d^2*f)*e^2 + 3*(d^3*f^2*x^2 + 2*d^2*f^2*x)*e + (d^3*f^3*x^3 + 3*d^2*f^3*x
^2 + d^3*e^3 + (d^3*f^3*x^3 + 3*d^2*f^3*x^2 + d^3*e^3 + 3*(d^3*f*x + d^2*f)
*e^2 + 3*(d^3*f^2*x^2 + 2*d^2*f^2*x)*e)*cos(d*x + c) + 3*(d^3*f*x + d^2*f)*
e^2 + 3*(d^3*f^2*x^2 + 2*d^2*f^2*x)*e)*sin(d*x + c))*log(cos(d*x + c) - I*s
in(d*x + c) + 1) + 6*(d^2*f^3*x^2 - c^2*f^3 - (d^2*f^3*x^2 - c^2*f^3 + 2*(d
^2*f^2*x + c*d*f^2)*e)*cos(d*x + c)^2 + 2*(d^2*f^2*x + c*d*f^2)*e + (d^2*f^
3*x^2 - c^2*f^3 + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d
```

```
*x + c) + 2*(d^2*f^2*x + c*d*f^2)*e)*sin(d*x + c))*log(I*cos(d*x + c) + sin
(d*x + c) + 1) + 6*(d^2*f^3*x^2 - c^2*f^3 - (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2
*f^2*x + c*d*f^2)*e)*cos(d*x + c)^2 + 2*(d^2*f^2*x + c*d*f^2)*e + (d^2*f^3*
x^2 - c^2*f^3 + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*x
 + c) + 2*(d^2*f^2*x + c*d*f^2)*e)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(
d*x + c) + 1) + (3*(c + 1)*d^2*f*e^2 - 3*(c^2 + 2*c)*d*f^2*e + (c^3 + 3*c^2
)*f^3 - d^3*e^3 - (3*(c + 1)*d^2*f*e^2 - 3*(c^2 + 2*c)*d*f^2*e + (c^3 + 3*c
^2)*f^3 - d^3*e^3)*cos(d*x + c)^2 + (3*(c + 1)*d^2*f*e^2 - 3*(c^2 + 2*c)*d*
f^2*e + (c^3 + 3*c^2)*f^3 - d^3*e^3 + (3*(c + 1)*d^2*f*e^2 - 3*(c^2 + 2*c)*
d*f^2*e + (c^3 + 3*c^2)*f^3 - d^3*e^3)*cos(d*x + c))*sin(d*x + c))*log(-1/2
*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + (3*(c + 1)*d^2*f*e^2 - 3*(c^2 +
 2*c)*d*f^2*e + (c^3 + 3*c^2)*f^3 - d^3*e^3 - (3*(c + 1)*d^2*f*e^2 - 3*(c^2
 + 2*c)*d*f^2*e + (c^3 + 3*c^2)*f^3 - d^3*e^3)*cos(d*x + c)^2 + (3*(c + 1)*
d^2*f*e^2 - 3*(c^2 + 2*c)*d*f^2*e + (c^3 + 3*c^2)*f^3 - d^3*e^3 + (3*(c + 1
)*d^2*f*e^2 - 3*(c^2 + 2*c)*d*f^2*e + (c^3 + 3*c^2)*f^3 - d^3*e^3)*cos(d*x
+ c))*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - (d^
3*f^3*x^3 - 3*d^2*f^3*x^2 + (c^3 + 3*c^2)*f^3 - (d^3*f^3*x^3 - 3*d^2*f^3*x^
2 + (c^3 + 3*c^2)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - 2*d^2*
f^2*x - (c^2 + 2*c)*d*f^2)*e)*cos(d*x + c)^2 + 3*(d^3*f*x + c*d^2*f)*e^2 +
3*(d^3*f^2*x^2 - 2*d^2*f^2*x - (c^2 + 2*c)*d*f^...
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3 \csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*
csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*csc(c + d*x
)**2/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*csc(c + d*x)**2/(sin(c +
d*x) + 1), x))/a
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.204 $\int \frac{(e+fx)^2 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=327

$$-\frac{2i(e+fx)^2}{ad}+\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{(e+fx)^2\cot(c+dx)}{ad}+\frac{4f(e+fx)}{}$$

```
[Out] -2*I*(f*x+e)^2/a/d+2*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d-(f*x+e)^2*cot(1/
2*c+1/4*Pi+1/2*d*x)/a/d-(f*x+e)^2*cot(d*x+c)/a/d+4*f*(f*x+e)*ln(1-I*exp(I*(
d*x+c)))/a/d^2+2*f*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/a/d^2-2*I*f*(f*x+e)*polyl
og(2,-exp(I*(d*x+c)))/a/d^2-4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3+2*I*f
*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2-I*f^2*polylog(2,exp(2*I*(d*x+c)))/
a/d^3+2*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3-2*f^2*polylog(3,exp(I*(d*x+c))
)/a/d^3
```

**Rubi** [A]
time = 0.34, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {4631, 4269, 3798, 2221, 2317, 2438, 4268, 2611, 2320, 6724, 3399}

$$-\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{if^2\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}+\frac{2f^2\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}-\frac{2f^2\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}-\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}+\frac{2if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{2f(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{ad^2}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{(e+fx)^2\cot(c+dx)}{ad}+\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{2i(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-2*I)*(e + f*x)^2)/(a*d) + (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d)
 - ((e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - ((e + f*x)^2*Cot[c + d*x
])/(a*d) + (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + (2*f*(e + f
*x)*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, -
E^(I*(c + d*x))])/(a*d^2) - ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^
3) + ((2*I)*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (I*f^2*PolyL
og[2, E^((2*I)*(c + d*x))])/(a*d^3) + (2*f^2*PolyLog[3, -E^(I*(c + d*x))])/
(a*d^3) - (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^2\csc^2(c+dx)\,dx}{a}-\int \frac{(e+fx)^2\csc(c+dx)}{a+a\sin(c+dx)}\,dx\\
&=-\frac{(e+fx)^2\cot(c+dx)}{ad}-\frac{\int (e+fx)^2\csc(c+dx)\,dx}{a}+\frac{(2f)\int (e+fx)\cot(c
\\
&=-\frac{i(e+fx)^2}{ad}+\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^2\cot(c+dx)}{ad}+\frac{\int (e+
\\
&=-\frac{i(e+fx)^2}{ad}+\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-
\\
&=-\frac{2i(e+fx)^2}{ad}+\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-
\\
&=-\frac{2i(e+fx)^2}{ad}+\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-
\\
&=-\frac{2i(e+fx)^2}{ad}+\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-
\\
&=-\frac{2i(e+fx)^2}{ad}+\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-
\end{aligned}$$

**Mathematica [A]**

time = 9.85, size = 559, normalized size = 1.71

$$\frac{-4id^2efx-2id^2f^2x^2+4d^2e^2\tanh^{-1}(\cos(c+dx)+i\sin(c+dx))+8d^2efx\tanh^{-1}(\cos(c+dx)+i\sin(c+dx))+4d^2f^2x^2\tanh^{-1}(\cos(c+dx)+i\sin(c+dx))-4d^2efx\cot(c)-2d^2f^2x^2\cot(c)+8df(e+fx)\log(1-i\cos(c+dx)+\sin(c+dx))+4def\log(1-\cos(2(c+dx))-i\sin(2(c+dx)))+4df^2x\log(1-\cos(2(c+dx))-i\sin(2(c+dx)))-8if^2\text{Li}_2(i\cos(c+dx)-\sin(c+dx))-4idf(e+fx)\text{Li}_2(-\cos(c+dx)-i\sin(c+dx))+4idf(e+fx)\text{Li}_2(\cos(c+dx)+i\sin(c+dx))-2if^2\text{Li}_2(\cos(2(c+dx))+i\sin(2(c+dx)))+4f^2\text{Li}_3(-\cos(c+dx)-i\sin(c+dx))-4f^2\text{Li}_3(\cos(c+dx)+i\sin(c+dx))+\frac{4d^2fx(2e+fx)(-i\cos(c)+\sin(c))}{\cos(c)+i(1+\sin(c))}+d^2(e+fx)^2\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+d^2(e+fx)^2\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+\frac{4d^2(e+fx)^2\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{2ad^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-4*I)*d^2*e*f*x - (2*I)*d^2*f^2*x^2 + 4*d^2*e^2*ArcTanh[Cos[c + d*x] + I*
Sin[c + d*x]] + 8*d^2*e*f*x*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] + 4*d^2*
f^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - 4*d^2*e*f*x*Cot[c] - 2*d^2
*f^2*x^2*Cot[c] + 8*d*f*(e + f*x)*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] +
4*d*e*f*Log[1 - Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]] + 4*d*f^2*x*Log[1 -
Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]] - (8*I)*f^2*PolyLog[2, I*Cos[c + d*x
] - Sin[c + d*x]] - (4*I)*d*f*(e + f*x)*PolyLog[2, -Cos[c + d*x] - I*Sin[c
+ d*x]] + (4*I)*d*f*(e + f*x)*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*x]] - (
2*I)*f^2*PolyLog[2, Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + 4*f^2*PolyLog[
3, -Cos[c + d*x] - I*Sin[c + d*x]] - 4*f^2*PolyLog[3, Cos[c + d*x] + I*Sin[
c + d*x]] + (4*d^2*f*x*(2*e + f*x)*((-I)*Cos[c] + Sin[c]))/(Cos[c] + I*(1 +
 Sin[c])) + d^2*(e + f*x)^2*Csc[c/2]*Csc[(c + d*x)/2]*Sin[(d*x)/2] + d^2*(e
 + f*x)^2*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + (4*d^2*(e + f*x)^2*Sin[(
d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(2*
a*d^3)
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(296) = 592$.
time = 0.20, size = 942, normalized size = 2.88

| method | result |
|---|---|
| risch | $-\frac{2if^2\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(dx+c)}\right)}{a\,d^3}-\frac{2if^2\,\mathrm{polylog}\left(2,\mathrm{e}^{i(dx+c)}\right)}{a\,d^3}-\frac{2f^2c\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{a\,d^3}+\frac{2ef\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{a\,d^2}+\frac{2ef\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{a\,d^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*f^2*polylog(2,exp(I*(d*x+c)))/a/d^3+2*I/a/d^2*e*f*polylog(2,exp(I*(d*x
+c)))-2*I/a/d^2*e*f*polylog(2,-exp(I*(d*x+c)))+2*I/a/d^2*f^2*polylog(2,exp(
I*(d*x+c)))*x-8*I/a/d^2*f^2*c*x-2*I/a/d^2*f^2*polylog(2,-exp(I*(d*x+c)))*x-
2/a/d^3*f^2*c*ln(exp(I*(d*x+c))-1)-4*I/a/d^3*f^2*c^2+2/a/d^2*e*f*ln(exp(I*(
d*x+c))+1)+2/a/d^2*e*f*ln(exp(I*(d*x+c))-1)+2/a/d^2*f^2*ln(1-exp(I*(d*x+c))
)*x+2/a/d^3*f^2*ln(1-exp(I*(d*x+c)))*c+2/a/d^2*f^2*ln(exp(I*(d*x+c))+1)*x-1
/d/a*e^2*ln(exp(I*(d*x+c))-1)+1/d/a*e^2*ln(exp(I*(d*x+c))+1)-2*I/a/d^3*f^2*
polylog(2,-exp(I*(d*x+c)))-4*I/a/d*f^2*x^2-8/a/d^2*f*ln(exp(I*(d*x+c)))*e+4
/a/d^2*f^2*ln(1-I*exp(I*(d*x+c)))*x+4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c-4/
a/d^3*f^2*c*ln(exp(I*(d*x+c))+I)+8/a/d^3*f^2*c*ln(exp(I*(d*x+c)))+4/a/d^2*f
*ln(exp(I*(d*x+c))+I)*e-2/d/a*ln(1-exp(I*(d*x+c)))*e*f*x+2/d/a*ln(exp(I*(d*
```

```
x+c))+1)*e*f*x-2/d^2/a*ln(1-exp(I*(d*x+c)))*c*e*f-2*(-2*x^2*f^2+I*exp(I*(d*
x+c))*f^2*x^2-4*e*f*x+2*I*exp(I*(d*x+c))*e*f*x-2*e^2+I*exp(I*(d*x+c))*e^2+f
^2*x^2*exp(2*I*(d*x+c))+2*e*f*x*exp(2*I*(d*x+c))+e^2*exp(2*I*(d*x+c)))/(exp
(2*I*(d*x+c))-1)/(exp(I*(d*x+c))+I)/d/a+2/d^2/a*e*f*c*ln(exp(I*(d*x+c))-1)+
1/d^3/a*f^2*c^2*ln(1-exp(I*(d*x+c)))-1/d/a*f^2*ln(1-exp(I*(d*x+c)))*x^2+1/d
/a*f^2*ln(exp(I*(d*x+c))+1)*x^2-1/d^3/a*f^2*c^2*ln(exp(I*(d*x+c))-1)+2*f^2*
polylog(3,-exp(I*(d*x+c)))/a/d^3-2*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3-4*I*
f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2577 vs. 2(293) = 586.
time = 0.45, size = 2577, normalized size = 7.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*f^2*x^2 + 4*d^2*f*x*e - 4*(d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2)
*cos(d*x + c)^2 + 2*d^2*e^2 - 2*(d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2)*cos(d
*x + c) - 2*(-I*d*f^2*x + (I*d*f^2*x + I*d*f*e - I*f^2)*cos(d*x + c)^2 - I*
d*f*e + I*f^2 + (-I*d*f^2*x - I*d*f*e + I*f^2 + (-I*d*f^2*x - I*d*f*e + I*f
^2)*cos(d*x + c))*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x + c)) - 2*(I
*d*f^2*x + (-I*d*f^2*x - I*d*f*e + I*f^2)*cos(d*x + c)^2 + I*d*f*e - I*f^2
+ (I*d*f^2*x + I*d*f*e - I*f^2 + (I*d*f^2*x + I*d*f*e - I*f^2)*cos(d*x + c)
)*sin(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x + c)) - 4*(-I*f^2*cos(d*x +
c)^2 + I*f^2 + (I*f^2*cos(d*x + c) + I*f^2)*sin(d*x + c))*dilog(I*cos(d*x +
 c) - sin(d*x + c)) - 4*(I*f^2*cos(d*x + c)^2 - I*f^2 + (-I*f^2*cos(d*x + c
) - I*f^2)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 2*(-I*d*f^
2*x + (I*d*f^2*x + I*d*f*e + I*f^2)*cos(d*x + c)^2 - I*d*f*e - I*f^2 + (-I*
d*f^2*x - I*d*f*e - I*f^2 + (-I*d*f^2*x - I*d*f*e - I*f^2)*cos(d*x + c))*si
n(d*x + c))*dilog(-cos(d*x + c) + I*sin(d*x + c)) - 2*(I*d*f^2*x + (-I*d*f^
2*x - I*d*f*e - I*f^2)*cos(d*x + c)^2 + I*d*f*e + I*f^2 + (I*d*f^2*x + I*d*
f*e + I*f^2 + (I*d*f^2*x + I*d*f*e + I*f^2)*cos(d*x + c))*sin(d*x + c))*dil
og(-cos(d*x + c) - I*sin(d*x + c)) + (d^2*f^2*x^2 + 2*d*f^2*x - (d^2*f^2*x^
```

```
2 + 2*d*f^2*x + d^2*e^2 + 2*(d^2*f*x + d*f)*e)*cos(d*x + c)^2 + d^2*e^2 + 2
*(d^2*f*x + d*f)*e + (d^2*f^2*x^2 + 2*d*f^2*x + d^2*e^2 + (d^2*f^2*x^2 + 2*
d*f^2*x + d^2*e^2 + 2*(d^2*f*x + d*f)*e)*cos(d*x + c) + 2*(d^2*f*x + d*f)*e
)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) - 4*(c*f^2 - (c*f^2
- d*f*e)*cos(d*x + c)^2 - d*f*e + (c*f^2 - d*f*e + (c*f^2 - d*f*e)*cos(d*x
+ c))*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) + (d^2*f^2*x^2 +
 2*d*f^2*x - (d^2*f^2*x^2 + 2*d*f^2*x + d^2*e^2 + 2*(d^2*f*x + d*f)*e)*cos(
d*x + c)^2 + d^2*e^2 + 2*(d^2*f*x + d*f)*e + (d^2*f^2*x^2 + 2*d*f^2*x + d^2
*e^2 + (d^2*f^2*x^2 + 2*d*f^2*x + d^2*e^2 + 2*(d^2*f*x + d*f)*e)*cos(d*x +
c) + 2*(d^2*f*x + d*f)*e)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x + c) +
 1) + 4*(d*f^2*x + c*f^2 - (d*f^2*x + c*f^2)*cos(d*x + c)^2 + (d*f^2*x + c*
f^2 + (d*f^2*x + c*f^2)*cos(d*x + c))*sin(d*x + c))*log(I*cos(d*x + c) + si
n(d*x + c) + 1) + 4*(d*f^2*x + c*f^2 - (d*f^2*x + c*f^2)*cos(d*x + c)^2 + (
d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c))*sin(d*x + c))*log(-I*cos(
d*x + c) + sin(d*x + c) + 1) + (2*(c + 1)*d*f*e - (c^2 + 2*c)*f^2 - (2*(c +
 1)*d*f*e - (c^2 + 2*c)*f^2 - d^2*e^2)*cos(d*x + c)^2 - d^2*e^2 + (2*(c + 1
)*d*f*e - (c^2 + 2*c)*f^2 - d^2*e^2 + (2*(c + 1)*d*f*e - (c^2 + 2*c)*f^2 -
d^2*e^2)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x
+ c) + 1/2) + (2*(c + 1)*d*f*e - (c^2 + 2*c)*f^2 - (2*(c + 1)*d*f*e - (c^2
+ 2*c)*f^2 - d^2*e^2)*cos(d*x + c)^2 - d^2*e^2 + (2*(c + 1)*d*f*e - (c^2 +
2*c)*f^2 - d^2*e^2 + (2*(c + 1)*d*f*e - (c^2 + 2*c)*f^2 - d^2*e^2)*cos(d*x
+ c))*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - (d^
2*f^2*x^2 - 2*d*f^2*x - (c^2 + 2*c)*f^2 - (d^2*f^2*x^2 - 2*d*f^2*x - (c^2 +
 2*c)*f^2 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x + c)^2 + 2*(d^2*f*x + c*d*f)*e +
 (d^2*f^2*x^2 - 2*d*f^2*x - (c^2 + 2*c)*f^2 + (d^2*f^2*x^2 - 2*d*f^2*x - (c
^2 + 2*c)*f^2 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x + c) + 2*(d^2*f*x + c*d*f)*e
)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) - 4*(c*f^2 - (c*f^2
 - d*f*e)*cos(d*x + c)^2 - d*f*e + (c*f^2 - d*f*e + (c*f^2 - d*f*e)*cos(d*x
 + c))*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) - (d^2*f^2*x^2
 - 2*d*f^2*x - (c^2 + 2*c)*f^2 - (d^2*f^2*x^2 - 2*d*f^2*x - (c^2 + 2*c)*f^2
 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x + c)^2 + 2*(d^2*f*x + c*d*f)*e + (d^2*f^2
*x^2 - 2*d*f^2*x - (c^2 + 2*c)*f^2 + (d^2*f^2*x^2 - 2*d*f^2*x - (c^2 + 2*c)
*f^2 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x + c) + 2*(d^2*f*x + c*d*f)*e)*sin(d*x
 + c))*log(-cos(d*x + c) - I*sin(d*x + c) + 1) + 2*(f^2*cos(d*x + c)^2 - f^
2 - (f^2*cos(d*x + c) + f^2)*sin(d*x + c))*polylog(3, cos(d*x + c) + I*sin(
d*x + c)) + 2*(f^2*cos(d*x + c)^2 - f^2 - (f^2*cos(d*x + c) + f^2)*sin(d*x
+ c))*polylog(3, cos(d*x + c) - I*sin(d*x + c)) - 2*(f^2*cos(d*x + c)^2 - f
^2 - (f^2*cos(d*x + c) + f^2)*sin(d*x + c))*polylog(3, -cos(d*x + c) + I*si
n(d*x + c)) - 2*(f^2*cos(d*x + c)^2 - f^2 - (f^2*cos(d*x + c) + f^2)*sin(d*
x + c))*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(d^2*f^2*x^2 + 2*d^2
*f*x*e + d^2*e^2 + 2*(d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2)*cos(d*x + c))*si
n(d*x + c))/(a*d^3*cos(d*x + c)^2 - a*d^3 - (a*d^3*cos(d*x + c) + a*d^3)*si
n(d*x + c))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2 \csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*csc(c + d*x)**2/(
sin(c + d*x) + 1), x))/a
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.205 $\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=169

$$\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{(e+fx)\cot(c+dx)}{ad}+\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+$$

```
[Out] 2*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d-(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d
-(f*x+e)*cot(d*x+c)/a/d+2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2+f*ln(sin(d*
x+c))/a/d^2-I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2+I*f*polylog(2,exp(I*(d*x+c
)))/a/d^2
```

**Rubi** [A]
time = 0.14, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4631, 4269, 3556, 4268, 2317, 2438, 3399}

$$-\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}+\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{f\log(\sin(c+dx))}{ad^2}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{(e+fx)\cot(c+dx)}{ad}+\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) - ((e + f*x)*Cot[c/2 + Pi/4 +
(d*x)/2])/(a*d) - ((e + f*x)*Cot[c + d*x])/(a*d) + (2*f*Log[Sin[c/2 + Pi/4
+ (d*x)/2]])/(a*d^2) + (f*Log[Sin[c + d*x]])/(a*d^2) - (I*f*PolyLog[2, -E^(
I*(c + d*x))])/(a*d^2) + (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\csc^2(c+dx)\,dx}{a} - \int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)\cot(c+dx)}{ad} - \frac{\int (e+fx)\csc(c+dx)\,dx}{a} + \frac{f\int \cot(c+dx)\,dx}{ad} + \int$$

$$= \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{f\log(\sin(c+dx))}{ad^2} + \frac{\int (}{}$$

$$= \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)\cot(c+d}{ad}$$

$$= \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)\cot(c+d}{ad}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 396 vs. 2(169) = 338.
time = 1.05, size = 396, normalized size = 2.34

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-d(e+fx)\cos\left(\frac{1}{2}(c+dx)\right)\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)+4d(e+fx)\sin\left(\frac{1}{2}(c+dx)\right)-2f(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+4f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2f\log(\sin(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2de\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2cf\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2f\left((c+dx)\left(\log\left(1-e^{i(c+dx)}\right)-\log\left(1+e^{i(c+dx)}\right)\right)+i\left(\text{Li}_2\left(-e^{i(c+dx)}\right)-\text{Li}_2\left(e^{i(c+dx)}\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+d(e+fx)\sin\left(\frac{1}{2}(c+dx)\right)\left(1+\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2ad^2(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-(d*(e + f*x)*Cos[(c + d*x)/2]*(1 +
 Cot[(c + d*x)/2])) + 4*d*(e + f*x)*Sin[(c + d*x)/2] - 2*f*(c + d*x)*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]) + 4*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*f*Log[Sin[c + d*x]]*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]) - 2*d*e*Log[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]) + 2*c*f*Log[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]) - 2*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c
 + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])
)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + d*(e + f*x)*Sin[(c + d*x)/2]*(1 +
 Tan[(c + d*x)/2])))/(2*a*d^2*(1 + Sin[c + d*x]))
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(149) = 298$.
time = 0.18, size = 351, normalized size = 2.08

| method | result |
|---|---|
| risch | $-\frac{2\left(-2fx+i\mathrm{e}^{i(dx+c)}fx-2e+i\mathrm{e}^{i(dx+c)}e+fx\,\mathrm{e}^{2i(dx+c)}+e\,\mathrm{e}^{2i(dx+c)}\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)da}+\frac{fc\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d^{2}a}-\frac{e\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{da}+\frac{e\ln\left(\mathrm{e}^{i(dx+c)}+\right.}{da}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-2*f*x+I*exp(I*(d*x+c))*f*x-2*e+I*exp(I*(d*x+c))*e+f*x*exp(2*I*(d*x+c))
+e*exp(2*I*(d*x+c)))/(exp(2*I*(d*x+c))-1)/(exp(I*(d*x+c))+I)/d/a+1/d^2/a*f*
c*ln(exp(I*(d*x+c))-1)-1/d/a*e*ln(exp(I*(d*x+c))-1)+1/d/a*e*ln(exp(I*(d*x+c
))+1)-1/d/a*ln(1-exp(I*(d*x+c)))*f*x-1/d^2/a*ln(1-exp(I*(d*x+c)))*c*f+1/d/a
*ln(exp(I*(d*x+c))+1)*f*x+2*f/a/d^2*ln(exp(I*(d*x+c))+I)-4/d^2/a*f*ln(exp(I
*(d*x+c)))+1/a/d^2*f*ln(exp(I*(d*x+c))-1)+1/a/d^2*f*ln(exp(I*(d*x+c))+1)+I*
f*polylog(2,exp(I*(d*x+c)))/a/d^2-I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(148) = 296$.

time = 0.42, size = 881, normalized size = 5.21

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*f*x - 4*(d*f*x + d*e)*cos(d*x + c)^2 - 2*(d*f*x + d*e)*cos(d*x +
c) + (-I*f*cos(d*x + c)^2 + (I*f*cos(d*x + c) + I*f)*sin(d*x + c) + I*f)*di
log(cos(d*x + c) + I*sin(d*x + c)) + (I*f*cos(d*x + c)^2 + (-I*f*cos(d*x +
c) - I*f)*sin(d*x + c) - I*f)*dilog(cos(d*x + c) - I*sin(d*x + c)) + (-I*f*
cos(d*x + c)^2 + (I*f*cos(d*x + c) + I*f)*sin(d*x + c) + I*f)*dilog(-cos(d*
x + c) + I*sin(d*x + c)) + (I*f*cos(d*x + c)^2 + (-I*f*cos(d*x + c) - I*f)*
sin(d*x + c) - I*f)*dilog(-cos(d*x + c) - I*sin(d*x + c)) + 2*d*e + (d*f*x
- (d*f*x + d*e + f)*cos(d*x + c)^2 + d*e + (d*f*x + (d*f*x + d*e + f)*cos(d
*x + c) + d*e + f)*sin(d*x + c) + f)*log(cos(d*x + c) + I*sin(d*x + c) + 1)
 + (d*f*x - (d*f*x + d*e + f)*cos(d*x + c)^2 + d*e + (d*f*x + (d*f*x + d*e
+ f)*cos(d*x + c) + d*e + f)*sin(d*x + c) + f)*log(cos(d*x + c) - I*sin(d*x
 + c) + 1) - (((c + 1)*f - d*e)*cos(d*x + c)^2 - (c + 1)*f + d*e - ((c + 1)
*f + ((c + 1)*f - d*e)*cos(d*x + c) - d*e)*sin(d*x + c))*log(-1/2*cos(d*x +
 c) + 1/2*I*sin(d*x + c) + 1/2) - (((c + 1)*f - d*e)*cos(d*x + c)^2 - (c +
1)*f + d*e - ((c + 1)*f + ((c + 1)*f - d*e)*cos(d*x + c) - d*e)*sin(d*x + c
))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - (d*f*x - (d*f*x + c*
f)*cos(d*x + c)^2 + c*f + (d*f*x + c*f + (d*f*x + c*f)*cos(d*x + c))*sin(d*
x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) - (d*f*x - (d*f*x + c*f)*co
s(d*x + c)^2 + c*f + (d*f*x + c*f + (d*f*x + c*f)*cos(d*x + c))*sin(d*x + c
))*log(-cos(d*x + c) - I*sin(d*x + c) + 1) - 2*(f*cos(d*x + c)^2 - (f*cos(d
*x + c) + f)*sin(d*x + c) - f)*log(sin(d*x + c) + 1) - 2*(d*f*x + 2*(d*f*x
+ d*e)*cos(d*x + c) + d*e)*sin(d*x + c))/(a*d^2*cos(d*x + c)^2 - a*d^2 - (a
*d^2*cos(d*x + c) + a*d^2)*sin(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f*x*csc(c + d
*x)**2/(sin(c + d*x) + 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.206 $\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=51

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{2\cot(c+dx)}{ad}+\frac{\cot(c+dx)}{d(a+a\sin(c+dx))}$$

```
[Out] arctanh(cos(d*x+c))/a/d-2*cot(d*x+c)/a/d+cot(d*x+c)/d/(a+a*sin(d*x+c))
```

**Rubi** [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2847, 2827, 3852, 8, 3855}

$$-\frac{2\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - (2*Cot[c + d*x])/(a*d) + Cot[c + d*x]/(d*(a +
 a*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cot(c+dx)}{d(a+a\sin(c+dx))}-\frac{\int \csc^2(c+dx)(-2a+a\sin(c+dx))\,dx}{a^2}\\ &= \frac{\cot(c+dx)}{d(a+a\sin(c+dx))}-\frac{\int \csc(c+dx)\,dx}{a}+\frac{2\int \csc^2(c+dx)\,dx}{a}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cot(c+dx)}{d(a+a\sin(c+dx))}-\frac{2\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{ad}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{2\cot(c+dx)}{ad}+\frac{\cot(c+dx)}{d(a+a\sin(c+dx))}\end{aligned}$$

**Mathematica [A]**
time = 0.14, size = 57, normalized size = 1.12

$$\frac{\sec(c+dx)\left(-1+\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)\sqrt{\cos^2(c+dx)}-\csc(c+dx)+2\sin(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x]),x]
[Out] (Sec[c + d*x]*(-1 + ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[c + d*x]^2] - Cs
c[c + d*x] + 2*Sin[c + d*x]))/(a*d)
```

**Maple [A]**
time = 0.09, size = 59, normalized size = 1.16

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$ | 59 |
| default | $\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$ | 59 |
| norman | $\frac{\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{1}{2ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$ | 92 |

| | | |
|---|---|---|
| risch | $-\frac{2\left(-2+i\mathrm{e}^{i(dx+c)}+\mathrm{e}^{2i(dx+c)}\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)da}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{da}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{da}$ | 99 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a*(tan(1/2*d*x+1/2*c)-4/(tan(1/2*d*x+1/2*c)+1)-1/tan(1/2*d*x+1/2*c)-2
*ln(tan(1/2*d*x+1/2*c)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.
time = 0.29, size = 112, normalized size = 2.20

$$-\frac{\frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1}+1}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a*sin(d*x + c)/(cos(d*x + c)
 + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 2*log(sin(d*x + c)/(cos(d*
x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(51) = 102$.
time = 0.35, size = 156, normalized size = 3.06

$$\frac{4\cos(dx+c)^2+(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2(2\cos(dx+c)+1)\sin(dx+c)+2\cos(dx+c)-2}{2\left(ad\cos(dx+c)^2-ad-(ad\cos(dx+c)+ad)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cos(d*x + c)^2 + (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) -
 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(
d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*cos(d*x + c) + 1)*sin(d*x
 + c) + 2*cos(d*x + c) - 2)/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) +
 a*d)*sin(d*x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

**Giac [A]**
time = 5.94, size = 88, normalized size = 1.73

$$-\frac{\frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (tan(1/
2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)^2 + t
an(1/2*d*x + 1/2*c))*a))/d
```

**Mupad [B]**
time = 1.27, size = 83, normalized size = 1.63

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\,\left(2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a*d) - log(tan(c/2 + (d*x)/2))/(a*d) - (5*tan(c/2 + (
d*x)/2) + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2))
```

# 3.207 $\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\mathrm{Int}\left(\frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 14.80, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(dx+c)}{(fx+e)\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^2/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sin\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(e + f*x)*(a + a*sin(c + d*x))),x)
[Out] int(1/(sin(c + d*x)^2*(e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.208 $\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 29.65, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(dx+c)}{(fx+e)^2\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^2/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2 + 2*a*f
*x*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sin \left(c+d\,x\right)^2\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sin \left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(e + f*x)^2*(a + a*sin(c + d*x))),x)
[Out] int(1/(sin(c + d*x)^2*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.209 $\int \frac{(e+fx)^3 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=600

$$\frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3}-\frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{(e+$$

```
[Out] 9*I*f^3*polylog(4,exp(I*(d*x+c)))/a/d^4-6*f^2*(f*x+e)*arctanh(exp(I*(d*x+c)
))/a/d^3-3*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)^3*cot(1/2*c+1/4*Pi
+1/2*d*x)/a/d+(f*x+e)^3*cot(d*x+c)/a/d-3/2*f*(f*x+e)^2*csc(d*x+c)/a/d^2-1/2
*(f*x+e)^3*cot(d*x+c)*csc(d*x+c)/a/d-6*f*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a
/d^2-3*f*(f*x+e)^2*ln(1-exp(2*I*(d*x+c)))/a/d^2-9/2*I*f*(f*x+e)^2*polylog(2
,exp(I*(d*x+c)))/a/d^2+3*I*f^2*(f*x+e)*polylog(2,exp(2*I*(d*x+c)))/a/d^3-3*
I*f^3*polylog(2,exp(I*(d*x+c)))/a/d^4+12*I*f^2*(f*x+e)*polylog(2,I*exp(I*(d
*x+c)))/a/d^3-9*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/d^4+2*I*(f*x+e)^3/a/d-9*
f^2*(f*x+e)*polylog(3,-exp(I*(d*x+c)))/a/d^3-12*f^3*polylog(3,I*exp(I*(d*x+
c)))/a/d^4+9*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a/d^3-3/2*f^3*polylog(3,
exp(2*I*(d*x+c)))/a/d^4+9/2*I*f*(f*x+e)^2*polylog(2,-exp(I*(d*x+c)))/a/d^2+
3*I*f^3*polylog(2,-exp(I*(d*x+c)))/a/d^4
```

**Rubi** [A]
time = 0.74, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {4631, 4271, 4268, 2317, 2438, 2611, 6744, 2320, 6724, 4269, 3798, 2221, 3399}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((2*I)*(e + f*x)^3)/(a*d) - (6*f^2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d
^3) - (3*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)^3*Cot[c/2
 + Pi/4 + (d*x)/2])/(a*d) + ((e + f*x)^3*Cot[c + d*x])/(a*d) - (3*f*(e + f*
x)^2*Csc[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Cot[c + d*x]*Csc[c + d*x])/(2*a
*d) - (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) - (3*f*(e + f*x)
^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) + ((3*I)*f^3*PolyLog[2, -E^(I*(c +
 d*x))])/(a*d^4) + (((9*I)/2)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(
a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - ((3
*I)*f^3*PolyLog[2, E^(I*(c + d*x))])/(a*d^4) - (((9*I)/2)*f*(e + f*x)^2*Pol
yLog[2, E^(I*(c + d*x))])/(a*d^2) + ((3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I
)*(c + d*x))])/(a*d^3) - (9*f^2*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a*
d^3) - (12*f^3*PolyLog[3, I*E^(I*(c + d*x))])/(a*d^4) + (9*f^2*(e + f*x)*Po
lyLog[3, E^(I*(c + d*x))])/(a*d^3) - (3*f^3*PolyLog[3, E^((2*I)*(c + d*x))]
)/(2*a*d^4) - ((9*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((9*I)*f^3
*PolyLog[4, E^(I*(c + d*x))])/(a*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\csc^3(c+dx)\,dx}{a} - \int \frac{(e+fx)^3\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{3f(e+fx)^2\csc(c+dx)}{2ad^2} - \frac{(e+fx)^3\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int (e+fx)$$

$$= -\frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3 c}{a}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} -$$

$$= \frac{i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} -$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1370 vs. 2(600) = 1200.
time = 26.70, size = 1370, normalized size = 2.28

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]

[Out] (3*e^3*Log[Tan[(c + d*x)/2]])/(2*a*d) + (3*e*f^2*Log[Tan[(c + d*x)/2]])/(a*
d^3) + (9*e^2*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*
```

```
x))]) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog
[2, E^(I*(c + d*x))])))/(2*a*d^2) + (3*f^3*((c + d*x)*(Log[1 - E^(I*(c + d*
x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLog[2,
-E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^4) + (f^3*Csc[c]*(2
*d^2*x^2*(2*d*E^((2*I)*c)*x + (3*I)*(-1 + E^((2*I)*c))*Log[1 - E^((2*I)*(c
+ d*x))]) + 6*d*(-1 + E^((2*I)*c))*x*PolyLog[2, E^((2*I)*(c + d*x))] + (3*I
)*(-1 + E^((2*I)*c))*PolyLog[3, E^((2*I)*(c + d*x))]))/(4*a*d^4*E^(I*c)) -
(9*e*f^2*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - I*d*x*PolyLog[2,
 -Cos[c + d*x] - I*Sin[c + d*x]] + I*d*x*PolyLog[2, Cos[c + d*x] + I*Sin[c
+ d*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - PolyLog[3, Cos[c + d
*x] + I*Sin[c + d*x]]))/(a*d^3) + (3*f^3*(-2*d^3*x^3*ArcTanh[Cos[c + d*x] +
 I*Sin[c + d*x]] + (3*I)*d^2*x^2*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]]
 - (3*I)*d^2*x^2*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*x]] - 6*d*x*PolyLog[
3, -Cos[c + d*x] - I*Sin[c + d*x]] + 6*d*x*PolyLog[3, Cos[c + d*x] + I*Sin[
c + d*x]] - (6*I)*PolyLog[4, -Cos[c + d*x] - I*Sin[c + d*x]] + (6*I)*PolyLo
g[4, Cos[c + d*x] + I*Sin[c + d*x]]))/(2*a*d^4) - (3*e^2*f*Csc[c]*(-(d*x*Co
s[c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*d^2*(Cos[c]^2 +
Sin[c]^2)) + (2*f*(-3*d^2*(e + f*x)^2*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]
] + (6*I)*d*f*(e + f*x)*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]] - 6*f^2*P
olyLog[3, I*Cos[c + d*x] - Sin[c + d*x]] + (I*d^3*x*(3*e^2 + 3*e*f*x + f^2*
x^2)*(Cos[c] + I*Sin[c]))/(Cos[c] + I*(1 + Sin[c]))))/(a*d^4) + (Csc[c]*Csc
[c + d*x]^2*(e^3*Sin[d*x] + 3*e^2*f*x*Sin[d*x] + 3*e*f^2*x^2*Sin[d*x] + f^3
*x^3*Sin[d*x]))/(2*a*d) + (Csc[c]*Csc[c + d*x]*(-(d*e^3*Cos[c]) - 3*d*e^2*f
*x*Cos[c] - 3*d*e*f^2*x^2*Cos[c] - d*f^3*x^3*Cos[c] - 3*e^2*f*Sin[c] - 6*e*
f^2*x*Sin[c] - 3*f^3*x^2*Sin[c] - 2*d*e^3*Sin[d*x] - 6*d*e^2*f*x*Sin[d*x] -
 6*d*e*f^2*x^2*Sin[d*x] - 2*d*f^3*x^3*Sin[d*x]))/(2*a*d^2) - (2*(e^3*Sin[(d
*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d
*x)/2]))/(a*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2
])) + (3*e*f^2*Csc[c]*Sec[c]*(d^2*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi +
 2*ArcTan[Tan[c]]) - Pi*Log[1 + E^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*
Log[1 - E^((2*I)*(d*x + ArcTan[Tan[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan
[c]]*Log[Sin[d*x + ArcTan[Tan[c]]]] + I*PolyLog[2, E^((2*I)*(d*x + ArcTan[T
an[c]]))])*Tan[c])/Sqrt[1 + Tan[c]^2]))/(a*d^3*Sqrt[Sec[c]^2*(Cos[c]^2 + Si
n[c]^2)])
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2256 vs. $2(540) = 1080$.
time = 0.27, size = 2257, normalized size = 3.76

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2257 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 9/d^3/a*e*f^2*polylog(3,exp(I*(d*x+c)))-9/d^3/a*e*f^2*polylog(3,-exp(I*(d*x
+c)))-3/2/d^4/a*f^3*c^3*ln(exp(I*(d*x+c))-1)+9/d^3/a*f^3*polylog(3,exp(I*(d
*x+c)))*x-9/d^3/a*f^3*polylog(3,-exp(I*(d*x+c)))*x-6/a/d^2*e*f^2*ln(1-exp(I
*(d*x+c)))*x-6/a/d^3*e*f^2*ln(1-exp(I*(d*x+c)))*c-6/a/d^2*e*f^2*ln(exp(I*(d
*x+c))+1)*x+3/2/d/a*e^3*ln(exp(I*(d*x+c))-1)-3/2/d/a*e^3*ln(exp(I*(d*x+c))+
1)-6*f^3*polylog(3,-exp(I*(d*x+c)))/a/d^4-6*f^3*polylog(3,exp(I*(d*x+c)))/a
/d^4+(-5*d*e^3*exp(2*I*(d*x+c))+3*d*e^3*exp(4*I*(d*x+c))+3*f^3*x^2*exp(3*I*
(d*x+c))+3*e^2*f*exp(3*I*(d*x+c))+4*d*f^3*x^3+3*I*e^2*f*exp(2*I*(d*x+c))+3*
I*f^3*x^2*exp(2*I*(d*x+c))-5*d*f^3*x^3*exp(2*I*(d*x+c))-3*f^3*x^2*exp(I*(d*
x+c))-3*exp(I*(d*x+c))*e^2*f-I*d*e^3*exp(I*(d*x+c))-6*e*f^2*x*exp(I*(d*x+c)
)-I*d*f^3*x^3*exp(I*(d*x+c))+9*I*d*e*f^2*x^2*exp(3*I*(d*x+c))+4*d*e^3+12*d*
e*f^2*x^2+12*d*e^2*f*x+3*d*f^3*x^3*exp(4*I*(d*x+c))-3*I*f^3*x^2*exp(4*I*(d*
x+c))-3*I*e^2*f*exp(4*I*(d*x+c))+3*I*d*e^3*exp(3*I*(d*x+c))+6*e*f^2*x*exp(3
*I*(d*x+c))+9*I*d*e^2*f*x*exp(3*I*(d*x+c))-3*I*d*e*f^2*x^2*exp(I*(d*x+c))-3
*I*d*e^2*f*x*exp(I*(d*x+c))+9*d*e^2*f*x*exp(4*I*(d*x+c))+9*d*e*f^2*x^2*exp(
4*I*(d*x+c))-6*I*e*f^2*x*exp(4*I*(d*x+c))+3*I*d*f^3*x^3*exp(3*I*(d*x+c))-15
*d*e*f^2*x^2*exp(2*I*(d*x+c))-15*d*e^2*f*x*exp(2*I*(d*x+c))+6*I*e*f^2*x*exp
(2*I*(d*x+c)))/(exp(2*I*(d*x+c))-1)^2/d^2/(exp(I*(d*x+c))+I)/a-6/a/d^2*f^3*
ln(1-I*exp(I*(d*x+c)))*x^2+6/a/d^4*f^3*ln(1-I*exp(I*(d*x+c)))*c^2+9*I/a/d^2
*polylog(2,-exp(I*(d*x+c)))*e*f^2*x-9*I/a/d^2*polylog(2,exp(I*(d*x+c)))*e*f
^2*x+24*I/a/d^2*c*e*f^2*x+12/a/d^2*f*ln(exp(I*(d*x+c)))*e^2+12/a/d^4*f^3*c^
2*ln(exp(I*(d*x+c)))-6/a/d^4*f^3*c^2*ln(exp(I*(d*x+c))+I)+3*I*f^3*polylog(2
,-exp(I*(d*x+c)))/a/d^4+9*I*f^3*polylog(4,exp(I*(d*x+c)))/a/d^4-6/a/d^2*f*l
n(exp(I*(d*x+c))+I)*e^2-24/a/d^3*f^2*e*c*ln(exp(I*(d*x+c)))+12*I/a/d^3*f^3*
polylog(2,I*exp(I*(d*x+c)))*x+12*I/a/d^3*f^2*e*polylog(2,I*exp(I*(d*x+c)))+
9/2/d^2/a*ln(1-exp(I*(d*x+c)))*c*e^2*f-9/2/d/a*e*f^2*ln(exp(I*(d*x+c))+1)*x
^2+9/2/d/a*ln(1-exp(I*(d*x+c)))*e^2*f*x-9/2/d/a*ln(exp(I*(d*x+c))+1)*e^2*f*
x+9/2/d^3/a*e*f^2*c^2*ln(exp(I*(d*x+c))-1)-9/2/d^2/a*e^2*f*c*ln(exp(I*(d*x+
c))-1)+3/2/d/a*f^3*ln(1-exp(I*(d*x+c)))*x^3+3/2/d^4/a*f^3*ln(1-exp(I*(d*x+c
)))*c^3-3/2/d/a*f^3*ln(exp(I*(d*x+c))+1)*x^3-9/2/d^3/a*e*f^2*c^2*ln(1-exp(I
*(d*x+c)))+9/2/d/a*e*f^2*ln(1-exp(I*(d*x+c)))*x^2+6*I/a/d^3*f^3*polylog(2,-
exp(I*(d*x+c)))*x+9/2*I/a/d^2*f^3*polylog(2,-exp(I*(d*x+c)))*x^2-9/2*I/a/d^
2*f^3*polylog(2,exp(I*(d*x+c)))*x^2+12*I/a/d*e*f^2*x^2+12*I/a/d^3*c^2*e*f^2
-12*I/a/d^3*f^3*c^2*x+6*I/a/d^3*e*f^2*polylog(2,exp(I*(d*x+c)))+6*I/a/d^3*e
*f^2*polylog(2,-exp(I*(d*x+c)))-3/a/d^3*f^3*ln(exp(I*(d*x+c))+1)*x+3/a/d^3*
f^3*ln(1-exp(I*(d*x+c)))*x+3/a/d^4*f^3*ln(1-exp(I*(d*x+c)))*c-3/a/d^3*e*f^2
*ln(exp(I*(d*x+c))+1)+3/a/d^3*e*f^2*ln(exp(I*(d*x+c))-1)-3/a/d^4*f^3*c*ln(e
xp(I*(d*x+c))-1)-8*I/a/d^4*f^3*c^3+4*I/a/d*f^3*x^3-9/2*I/a/d^2*e^2*f*polylo
g(2,exp(I*(d*x+c)))+9/2*I/a/d^2*e^2*f*polylog(2,-exp(I*(d*x+c)))+6*I/a/d^3*
f^3*polylog(2,exp(I*(d*x+c)))*x-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+12
/a/d^3*f^2*e*c*ln(exp(I*(d*x+c))+I)+6/a/d^3*e*f^2*c*ln(exp(I*(d*x+c))-1)-3*
I*f^3*polylog(2,exp(I*(d*x+c)))/a/d^4-9*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/
d^4-12/a/d^2*f^2*e*ln(1-I*exp(I*(d*x+c)))*x-12/a/d^3*f^2*e*ln(1-I*exp(I*(d*
x+c)))*c-3/a/d^2*e^2*f*ln(exp(I*(d*x+c))-1)-3/a/d^2*e^2*f*ln(exp(I*(d*x+c))
+1)-3/a/d^4*f^3*c^2*ln(exp(I*(d*x+c))-1)-3/a/d^2*f^3*ln(1-exp(I*(d*x+c)))*x
```

```
^2+3/a/d^4*f^3*ln(1-exp(I*(d*x+c)))*c^2-3/a/d^2*f^3*ln(exp(I*(d*x+c))+1)*x^
2
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 12815 vs. $2(522) = 1044$.
time = 12.48, size = 12815, normalized size = 21.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(3*c*e^2*f*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 1)/(a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*d*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3) - (4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(
d*x + c)^2/(cos(d*x + c) + 1)^2)/(a*d) + 12*log(sin(d*x + c)/(cos(d*x + c)
+ 1))/(a*d)) + e^3*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(co
s(d*x + c) + 1)^2)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 1)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3) - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))
/a) + 8*(48*I*c^2*d*e*f^2 - 16*I*c^3*f^3 - 24*(-I*d^2*e^2*f + 2*I*c*d*e*f^2
 - I*c^2*f^3 - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(5*d*x + 5*c) + (-I*d
^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*cos(4*d*x + 4*c) + 2*(d^2*e^2*f - 2*c
*d*e*f^2 + c^2*f^3)*cos(3*d*x + 3*c) + 2*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*c
^2*f^3)*cos(2*d*x + 2*c) - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c)
 + (-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*sin(5*d*x + 5*c) + (d^2*e^2*f
 - 2*c*d*e*f^2 + c^2*f^3)*sin(4*d*x + 4*c) + 2*(I*d^2*e^2*f - 2*I*c*d*e*f^2
 + I*c^2*f^3)*sin(3*d*x + 3*c) - 2*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*sin(
2*d*x + 2*c) + (-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*sin(d*x + c))*arc
tan2(sin(d*x + c) + 1, cos(d*x + c)) - 24*(I*(d*x + c)^2*f^3 + 2*(I*d*e*f^2
 - I*c*f^3)*(d*x + c) + ((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*c
os(5*d*x + 5*c) + (I*(d*x + c)^2*f^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c))*c
os(4*d*x + 4*c) - 2*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(3
*d*x + 3*c) + 2*(-I*(d*x + c)^2*f^3 + 2*(-I*d*e*f^2 + I*c*f^3)*(d*x + c))*c
os(2*d*x + 2*c) + ((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(d*x
 + c) + (I*(d*x + c)^2*f^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c))*sin(5*d*x +
 5*c) - ((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*sin(4*d*x + 4*c)
+ 2*(-I*(d*x + c)^2*f^3 + 2*(-I*d*e*f^2 + I*c*f^3)*(d*x + c))*sin(3*d*x + 3
*c) + 2*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*sin(2*d*x + 2*c)
+ (I*(d*x + c)^2*f^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c))*sin(d*x + c))*arc
tan2(cos(d*x + c), sin(d*x + c) + 1) - 6*(-I*(d*x + c)^3*f^3 - 2*I*d^2*e^2*
f + (-3*I*c^2 + 4*I*c - 2*I)*d*e*f^2 + (I*c^3 - 2*I*c^2 + 2*I*c)*f^3 + (-3*
I*d*e*f^2 + (3*I*c - 2*I)*f^3)*(d*x + c)^2 + (-3*I*d^2*e^2*f + 2*(3*I*c - 2
*I)*d*e*f^2 + (-3*I*c^2 + 4*I*c - 2*I)*f^3)*(d*x + c) - ((d*x + c)^3*f^3 +
2*d^2*e^2*f + (3*c^2 - 4*c + 2)*d*e*f^2 - (c^3 - 2*c^2 + 2*c)*f^3 + (3*d*e*
```

```
f^2 - (3*c - 2)*f^3)*(d*x + c)^2 + (3*d^2*e^2*f - 2*(3*c - 2)*d*e*f^2 + (3*
c^2 - 4*c + 2)*f^3)*(d*x + c))*cos(5*d*x + 5*c) + (-I*(d*x + c)^3*f^3 - 2*I
*d^2*e^2*f + (-3*I*c^2 + 4*I*c - 2*I)*d*e*f^2 + (I*c^3 - 2*I*c^2 + 2*I*c)*f
^3 + (-3*I*d*e*f^2 + (3*I*c - 2*I)*f^3)*(d*x + c)^2 + (-3*I*d^2*e^2*f + 2*(
3*I*c - 2*I)*d*e*f^2 + (-3*I*c^2 + 4*I*c - 2*I)*f^3)*(d*x + c))*cos(4*d*x +
 4*c) + 2*((d*x + c)^3*f^3 + 2*d^2*e^2*f + (3*c^2 - 4*c + 2)*d*e*f^2 - (c^3
 - 2*c^2 + 2*c)*f^3 + (3*d*e*f^2 - (3*c - 2)*f^3)*(d*x + c)^2 + (3*d^2*e^2*
f - 2*(3*c - 2)*d*e*f^2 + (3*c^2 - 4*c + 2)*f^3)*(d*x + c))*cos(3*d*x + 3*c
) + 2*(I*(d*x + c)^3*f^3 + 2*I*d^2*e^2*f + (3*I*c^2 - 4*I*c + 2*I)*d*e*f^2
+ (-I*c^3 + 2*I*c^2 - 2*I*c)*f^3 + (3*I*d*e*f^2 + (-3*I*c + 2*I)*f^3)*(d*x
+ c)^2 + (3*I*d^2*e^2*f + 2*(-3*I*c + 2*I)*d*e*f^2 + (3*I*c^2 - 4*I*c + 2*I
)*f^3)*(d*x + c))*cos(2*d*x + 2*c) - ((d*x + c)^3*f^3 + 2*d^2*e^2*f + (3*c^
2 - 4*c + 2)*d*e*f^2 - (c^3 - 2*c^2 + 2*c)*f^3 + (3*d*e*f^2 - (3*c - 2)*f^3
)*(d*x + c)^2 + (3*d^2*e^2*f - 2*(3*c - 2)*d*e*f^2 + (3*c^2 - 4*c + 2)*f^3)
*(d*x + c))*cos(d*x + c) + (-I*(d*x + c)^3*f^3 - 2*I*d^2*e^2*f + (-3*I*c^2
+ 4*I*c - 2*I)*d*e*f^2 + (I*c^3 - 2*I*c^2 + 2*I*c)*f^3 + (-3*I*d*e*f^2 + (3
*I*c - 2*I)*f^3)*(d*x + c)^2 + (-3*I*d^2*e^2*f + 2*(3*I*c - 2*I)*d*e*f^2 +
(-3*I*c^2 + 4*I*c - 2*I)*f^3)*(d*x + c))*sin(5*d*x + 5*c) + ((d*x + c)^3*f^
3 + 2*d^2*e^2*f + (3*c^2 - 4*c + 2)*d*e*f^2 - (c^3 - 2*c^2 + 2*c)*f^3 + (3*
d*e*f^2 - (3*c - 2)*f^3)*(d*x + c)^2 + (3*d^2*e^2*f - 2*(3*c - 2)*d*e*f^2 +
 (3*c^2 - 4*c + 2)*f^3)*(d*x + c))*sin(4*d*x + 4*c) + 2*(I*(d*x + c)^3*f^3
+ 2*I*d^2*e^2*f + (3*I*c^2 - 4*I*c + 2*I)*d*e*f^2 + (-I*c^3 + 2*I*c^2 - 2*I
*c)*f^3 + (3*I*d*e*f^2 + (-3*I*c + 2*I)*f^3)*(d*x + c)^2 + (3*I*d^2*e^2*f +
 2*(-3*I*c + 2*I)*d*e*f^2 + (3*I*c^2 - 4*I*c + 2*I)*f^3)*(d*x + c))*sin(3*d
*x + 3*c) - 2*((d*x + c)^3*f^3 + 2*d^2*e^2*f + (3*c^2 - 4*c + 2)*d*e*f^2 -
(c^3 - 2*c^2 + 2*c)*f^3 + (3*d*e*f^2 - (3*c - 2)*f^3)*(d*x + c)^2 + (3*d^2*
e^2*f - 2*(3*c - 2)*d*e*f^2 + (3*c^2 - 4*c + 2)*f^3)*(d*x + c))*sin(2*d*x +
 2*c) + (-I*(d*x + c)^3*f^3 - 2*I*d^2*e^2*f + (-3*I*c^2 + 4*I*c - 2*I)*d*e*
f^2 + (I*c^3 - 2*I*c^2 + 2*I*c)*f^3 + (-3*I*d*e*f^2 + (3*I*c - 2*I)*f^3)*(d
*x + c)^2 + (-3*I*d^2*e^2*f + 2*(3*I*c - 2*I)*d*e*f^2 + (-3*I*c^2 + 4*I*c -
 2*I)*f^3)*(d*x + c))*sin(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1)
 - 6*(-2*I*d^2*e^2*f + (3*I*c^2 + 4*I*c + 2*I)*d*e*f^2 + (-I*c^3 - 2*I*c^2
- 2*I*c)*f^3 - (2*d^2*e^2*f - (3*c^2 + 4*c + 2)*d*e*f^2 + (c^3 + 2*c^2 + 2*
c)*f^3)*cos(5*d*x + 5*c) + (-2*I*d^2*e^2*f + (3...
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7847 vs. $2(537) = 1074$.
time = 0.65, size = 7847, normalized size = 13.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d^3*f^3*x^3 - 6*d^2*f^3*x^2 - 8*(d^3*f^3*x^3 + 3*d^3*f^2*x^2*e + 3*
d^3*f*x*e^2 + d^3*e^3)*cos(d*x + c)^3 + 4*d^3*e^3 - 6*(d^3*f^3*x^3 - d^2*f^
```

```
3*x^2 + d^3*e^3 + (3*d^3*f*x - d^2*f)*e^2 + (3*d^3*f^2*x^2 - 2*d^2*f^2*x)*e
)*cos(d*x + c)^2 + 6*(d^3*f^3*x^3 + 3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e
^3)*cos(d*x + c) + 3*(-3*I*d^2*f^3*x^2 + 4*I*d*f^3*x + (3*I*d^2*f^3*x^2 - 4
*I*d*f^3*x + 3*I*d^2*f*e^2 + 2*I*f^3 + 2*I*(3*d^2*f^2*x - 2*d*f^2)*e)*cos(d
*x + c)^3 - 3*I*d^2*f*e^2 - 2*I*f^3 + (3*I*d^2*f^3*x^2 - 4*I*d*f^3*x + 3*I*
d^2*f*e^2 + 2*I*f^3 + 2*I*(3*d^2*f^2*x - 2*d*f^2)*e)*cos(d*x + c)^2 + (-3*I
*d^2*f^3*x^2 + 4*I*d*f^3*x - 3*I*d^2*f*e^2 - 2*I*f^3 - 2*I*(3*d^2*f^2*x - 2
*d*f^2)*e)*cos(d*x + c) - 2*I*(3*d^2*f^2*x - 2*d*f^2)*e + (-3*I*d^2*f^3*x^2
 + 4*I*d*f^3*x - 3*I*d^2*f*e^2 - 2*I*f^3 + (3*I*d^2*f^3*x^2 - 4*I*d*f^3*x +
 3*I*d^2*f*e^2 + 2*I*f^3 + 2*I*(3*d^2*f^2*x - 2*d*f^2)*e)*cos(d*x + c)^2 -
2*I*(3*d^2*f^2*x - 2*d*f^2)*e)*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x
 + c)) + 3*(3*I*d^2*f^3*x^2 - 4*I*d*f^3*x + (-3*I*d^2*f^3*x^2 + 4*I*d*f^3*x
 - 3*I*d^2*f*e^2 - 2*I*f^3 - 2*I*(3*d^2*f^2*x - 2*d*f^2)*e)*cos(d*x + c)^3
+ 3*I*d^2*f*e^2 + 2*I*f^3 + (-3*I*d^2*f^3*x^2 + 4*I*d*f^3*x - 3*I*d^2*f*e^2
 - 2*I*f^3 - 2*I*(3*d^2*f^2*x - 2*d*f^2)*e)*cos(d*x + c)^2 + (3*I*d^2*f^3*x
^2 - 4*I*d*f^3*x + 3*I*d^2*f*e^2 + 2*I*f^3 + 2*I*(3*d^2*f^2*x - 2*d*f^2)*e)
*cos(d*x + c) + 2*I*(3*d^2*f^2*x - 2*d*f^2)*e + (3*I*d^2*f^3*x^2 - 4*I*d*f^
3*x + 3*I*d^2*f*e^2 + 2*I*f^3 + (-3*I*d^2*f^3*x^2 + 4*I*d*f^3*x - 3*I*d^2*f
*e^2 - 2*I*f^3 - 2*I*(3*d^2*f^2*x - 2*d*f^2)*e)*cos(d*x + c)^2 + 2*I*(3*d^2
*f^2*x - 2*d*f^2)*e)*sin(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x + c)) + 2
4*(I*d*f^3*x + (-I*d*f^3*x - I*d*f^2*e)*cos(d*x + c)^3 + I*d*f^2*e + (-I*d*
f^3*x - I*d*f^2*e)*cos(d*x + c)^2 + (I*d*f^3*x + I*d*f^2*e)*cos(d*x + c) +
(I*d*f^3*x + I*d*f^2*e + (-I*d*f^3*x - I*d*f^2*e)*cos(d*x + c)^2)*sin(d*x +
 c))*dilog(I*cos(d*x + c) - sin(d*x + c)) + 24*(-I*d*f^3*x + (I*d*f^3*x + I
*d*f^2*e)*cos(d*x + c)^3 - I*d*f^2*e + (I*d*f^3*x + I*d*f^2*e)*cos(d*x + c)
^2 + (-I*d*f^3*x - I*d*f^2*e)*cos(d*x + c) + (-I*d*f^3*x - I*d*f^2*e + (I*d
*f^3*x + I*d*f^2*e)*cos(d*x + c)^2)*sin(d*x + c))*dilog(-I*cos(d*x + c) - s
in(d*x + c)) + 3*(-3*I*d^2*f^3*x^2 - 4*I*d*f^3*x + (3*I*d^2*f^3*x^2 + 4*I*d
*f^3*x + 3*I*d^2*f*e^2 + 2*I*f^3 + 2*I*(3*d^2*f^2*x + 2*d*f^2)*e)*cos(d*x +
 c)^3 - 3*I*d^2*f*e^2 - 2*I*f^3 + (3*I*d^2*f^3*x^2 + 4*I*d*f^3*x + 3*I*d^2*
f*e^2 + 2*I*f^3 + 2*I*(3*d^2*f^2*x + 2*d*f^2)*e)*cos(d*x + c)^2 + (-3*I*d^2
*f^3*x^2 - 4*I*d*f^3*x - 3*I*d^2*f*e^2 - 2*I*f^3 - 2*I*(3*d^2*f^2*x + 2*d*f
^2)*e)*cos(d*x + c) - 2*I*(3*d^2*f^2*x + 2*d*f^2)*e + (-3*I*d^2*f^3*x^2 - 4
*I*d*f^3*x - 3*I*d^2*f*e^2 - 2*I*f^3 + (3*I*d^2*f^3*x^2 + 4*I*d*f^3*x + 3*I
*d^2*f*e^2 + 2*I*f^3 + 2*I*(3*d^2*f^2*x + 2*d*f^2)*e)*cos(d*x + c)^2 - 2*I*
(3*d^2*f^2*x + 2*d*f^2)*e)*sin(d*x + c))*dilog(-cos(d*x + c) + I*sin(d*x +
c)) + 3*(3*I*d^2*f^3*x^2 + 4*I*d*f^3*x + (-3*I*d^2*f^3*x^2 - 4*I*d*f^3*x -
3*I*d^2*f*e^2 - 2*I*f^3 - 2*I*(3*d^2*f^2*x + 2*d*f^2)*e)*cos(d*x + c)^3 + 3
*I*d^2*f*e^2 + 2*I*f^3 + (-3*I*d^2*f^3*x^2 - 4*I*d*f^3*x - 3*I*d^2*f*e^2 -
2*I*f^3 - 2*I*(3*d^2*f^2*x + 2*d*f^2)*e)*cos(d*x + c)^2 + (3*I*d^2*f^3*x^2
+ 4*I*d*f^3*x + 3*I*d^2*f*e^2 + 2*I*f^3 + 2*I*(3*d^2*f^2*x + 2*d*f^2)*e)*co
s(d*x + c) + 2*I*(3*d^2*f^2*x + 2*d*f^2)*e + (3*I*d^2*f^3*x^2 + 4*I*d*f^3*x
 + 3*I*d^2*f*e^2 + 2*I*f^3 + (-3*I*d^2*f^3*x^2 - 4*I*d*f^3*x - 3*I*d^2*f*e^
2 - 2*I*f^3 - 2*I*(3*d^2*f^2*x + 2*d*f^2)*e)*cos(d*x + c)^2 + 2*I*(3*d^2*f^
2*x + 2*d*f^2)*e)*sin(d*x + c))*dilog(-cos(d*x + c) - I*sin(d*x + c)) + 6*(
```

```
2*d^3*f*x - d^2*f)*e^2 + 12*(d^3*f^2*x^2 - d^2*f^2*x)*e - 3*(d^3*f^3*x^3 +
2*d^2*f^3*x^2 + 2*d*f^3*x - (d^3*f^3*x^3 + 2*d^2*f^3*x^2 + 2*d*f^3*x + d^3*
e^3 + (3*d^3*f*x + 2*d^2*f)*e^2 + (3*d^3*f^2*x^2 + 4*d^2*f^2*x + 2*d*f^2)*e
)*cos(d*x + c)^3 + d^3*e^3 - (d^3*f^3*x^3 + 2*d^2*f^3*x^2 + 2*d*f^3*x + d^3
*e^3 + (3*d^3*f*x + 2*d^2*f)*e^2 + (3*d^3*f^2*x^2 + 4*d^2*f^2*x + 2*d*f^2)*
e)*cos(d*x + c)^2 + (d^3*f^3*x^3 + 2*d^2*f^3*x^2 + 2*d*f^3*x + d^3*e^3 + (3
*d^3*f*x + 2*d^2*f)*e^2 + (3*d^3*f^2*x^2 + 4*d^2*f^2*x + 2*d*f^2)*e)*cos(d*
x + c) + (3*d^3*f*x + 2*d^2*f)*e^2 + (3*d^3*f^2*x^2 + 4*d^2*f^2*x + 2*d*f^2
)*e + (d^3*f^3*x^3 + 2*d^2*f^3*x^2 + 2*d*f^3*x + d^3*e^3 - (d^3*f^3*x^3 + 2
*d^2*f^3*x^2 + 2*d*f^3*x + d^3*e^3 + (3*d^3*f*x + 2*d^2*f)*e^2 + (3*d^3*f^2
*x^2 + 4*d^2*f^2*x + 2*d*f^2)*e)*cos(d*x + c)^2 + (3*d^3*f*x + 2*d^2*f)*e^2
 + (3*d^3*f^2*x^2 + 4*d^2*f^2*x + 2*d*f^2)*e)*sin(d*x + c))*log(cos(d*x + c
) + I*sin(d*x + c) + 1) - 12*(c^2*f^3 - 2*c*d*f^2*e - (c^2*f^3 - 2*c*d*f^2*
e + d^2*f*e^2)*cos(d*x + c)^3 + d^2*f*e^2 - (c^2*f^3 - 2*c*d*f^2*e + d^2*f*
e^2)*cos(d*x + c)^2 + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(d*x + c) + (c
^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 - (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(
d*x + c)^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 3*(d^3*f
^3*x^3 + 2*d^2*f^3*x^2 + 2*d*f^3*x - (d^3*f^3*x^3 + 2*d^2*f^3*x^2 + 2*d*f^3
*x + d^3*e^3 + (3*d^3*f*x + 2*d^2*f)*e^2 + (3*d^3*f^2*x^2 + 4*d^2*f^2*x + 2
*d*f^2)*e)*cos(d*x + c)^3 + d^3*e^3 - (d^3*f^3*...
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3\csc^3(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{f^3x^3\csc^3(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{3ef^2x^2\csc^3(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{3e^2fx\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*
csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*csc(c + d*x
)**3/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*csc(c + d*x)**3/(sin(c +
d*x) + 1), x))/a
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.210 $\int \frac{(e+fx)^2 \csc^3(c+dx)}{a+a\sin(c+dx)} \, dx$

**Optimal**. Leaf size=392

$$\frac{2i(e+fx)^2}{ad}-\frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{(e+fx)^2}{}$$

```
[Out] 2*I*(f*x+e)^2/a/d-3*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d-f^2*arctanh(cos(d
*x+c))/a/d^3+(f*x+e)^2*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d+(f*x+e)^2*cot(d*x+c)/a
/d-f*(f*x+e)*csc(d*x+c)/a/d^2-1/2*(f*x+e)^2*cot(d*x+c)*csc(d*x+c)/a/d-4*f*(
f*x+e)*ln(1-I*exp(I*(d*x+c)))/a/d^2-2*f*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/a/d^
2+3*I*f*(f*x+e)*polylog(2,-exp(I*(d*x+c)))/a/d^2+4*I*f^2*polylog(2,I*exp(I*
(d*x+c)))/a/d^3-3*I*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2+I*f^2*polylog
(2,exp(2*I*(d*x+c)))/a/d^3-3*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3+3*f^2*pol
ylog(3,exp(I*(d*x+c)))/a/d^3
```

**Rubi** [A]
time = 0.50, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {4631, 4271, 3855, 4268, 2611, 2320, 6724, 4269, 3798, 2221, 2317, 2438, 3399}

$$\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}+\frac{if^2\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}-\frac{3f^2\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{3f^2\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}+\frac{3if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{3if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}-\frac{2f(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{ad^2}-\frac{f(e+fx)\csc(c+dx)}{ad^2}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}+\frac{(e+fx)^2\cot(c+dx)}{ad}-\frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^2\cot(c+dx)\csc(c+dx)}{2ad}+\frac{2i(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((2*I)*(e + f*x)^2)/(a*d) - (3*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d)
- (f^2*ArcTanh[Cos[c + d*x]])/(a*d^3) + ((e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)
/2])/(a*d) + ((e + f*x)^2*Cot[c + d*x])/(a*d) - (f*(e + f*x)*Csc[c + d*x])/
(a*d^2) - ((e + f*x)^2*Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - (4*f*(e + f*x)*
Log[1 - I*E^(I*(c + d*x))])/(a*d^2) - (2*f*(e + f*x)*Log[1 - E^((2*I)*(c +
d*x))])/(a*d^2) + ((3*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2)
+ ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - ((3*I)*f*(e + f*x)*Po
lyLog[2, E^(I*(c + d*x))])/(a*d^2) + (I*f^2*PolyLog[2, E^((2*I)*(c + d*x))]
)/(a*d^3) - (3*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) + (3*f^2*PolyLog[3
, E^(I*(c + d*x))])/(a*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\csc^3(c+dx)\,dx}{a} - \int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{f(e+fx)\csc(c+dx)}{ad^2} - \frac{(e+fx)^2\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int (e+fx)^2 \mathrm{cs}}{2a}$$
$$= -\frac{(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+fx)^2\cot(c+}{ad}$$
$$= \frac{i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+}{}$$
$$= \frac{i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+}{}$$
$$= \frac{2i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+}{}$$
$$= \frac{2i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+}{}$$
$$= \frac{2i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+}{}$$
$$= \frac{2i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 916 vs. $2(392) = 784$.
time = 11.53, size = 916, normalized size = 2.34

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (8*((2*I)*d^2*e*f*x + I*d^2*f^2*x^2 - 3*d^2*e^2*ArcTanh[Cos[c + d*x] + I*Si
n[c + d*x]] - 2*f^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - 6*d^2*e*f*x*Ar
cTanh[Cos[c + d*x] + I*Sin[c + d*x]] - 3*d^2*f^2*x^2*ArcTanh[Cos[c + d*x] +
 I*Sin[c + d*x]] + 2*d^2*e*f*x*Cot[c] + d^2*f^2*x^2*Cot[c] - 2*d*e*f*Log[1
- Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]] - 2*d*f^2*x*Log[1 - Cos[2*(c + d*x
)] - I*Sin[2*(c + d*x)]] + (3*I)*d*f*(e + f*x)*PolyLog[2, -Cos[c + d*x] - I
*Sin[c + d*x]] - (3*I)*d*f*(e + f*x)*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*
x]] + I*f^2*PolyLog[2, Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] - 3*f^2*PolyL
```

```
og[3, -Cos[c + d*x] - I*Sin[c + d*x]] + 3*f^2*PolyLog[3, Cos[c + d*x] + I*S
in[c + d*x]]) + (16*I)*f*((2*I)*d*(e + f*x)*Log[1 - I*Cos[c + d*x] + Sin[c
+ d*x]] + 2*f*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]] + (d^2*x*(2*e + f*x
)*(Cos[c] + I*Sin[c]))/(Cos[c] + I*(1 + Sin[c]))) - (d*(e + f*x)*Csc[c]*Csc
[c + d*x]^2*(2*f*Cos[(d*x)/2] + 2*f*Cos[(3*d*x)/2] + 5*d*e*Cos[c - (d*x)/2]
 + 5*d*f*x*Cos[c - (d*x)/2] - d*e*Cos[c + (d*x)/2] - d*f*x*Cos[c + (d*x)/2]
 - 2*f*Cos[2*c + (d*x)/2] + d*e*Cos[c + (3*d*x)/2] + d*f*x*Cos[c + (3*d*x)/
2] - 2*f*Cos[2*c + (3*d*x)/2] - 3*d*e*Cos[3*c + (3*d*x)/2] - 3*d*f*x*Cos[3*
c + (3*d*x)/2] - 4*d*e*Cos[c + (5*d*x)/2] - 4*d*f*x*Cos[c + (5*d*x)/2] + 2*
d*e*Cos[3*c + (5*d*x)/2] + 2*d*f*x*Cos[3*c + (5*d*x)/2] + d*e*Sin[(d*x)/2]
+ d*f*x*Sin[(d*x)/2] + d*e*Sin[(3*d*x)/2] + d*f*x*Sin[(3*d*x)/2] + 2*f*Sin[
c - (d*x)/2] + 2*f*Sin[c + (d*x)/2] + 3*d*e*Sin[2*c + (d*x)/2] + 3*d*f*x*Si
n[2*c + (d*x)/2] + 2*f*Sin[c + (3*d*x)/2] + d*e*Sin[2*c + (3*d*x)/2] + d*f*
x*Sin[2*c + (3*d*x)/2] - 2*f*Sin[3*c + (3*d*x)/2] - 2*d*e*Sin[2*c + (5*d*x)
/2] - 2*d*f*x*Sin[2*c + (5*d*x)/2]))/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])))/(8*a*d^3)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(359) = 718$.
time = 0.25, size = 1215, normalized size = 3.10

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1215 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a/d^3*f^2*c*ln(exp(I*(d*x+c))-1)-2/a/d^2*e*f*ln(exp(I*(d*x+c))+1)-2/a/d^2
*e*f*ln(exp(I*(d*x+c))-1)-2/a/d^2*f^2*ln(1-exp(I*(d*x+c)))*x-2/a/d^3*f^2*ln
(1-exp(I*(d*x+c)))*c-2/a/d^2*f^2*ln(exp(I*(d*x+c))+1)*x+4*I/a/d*f^2*x^2+4*I
/a/d^3*c^2*f^2+2*I/a/d^3*f^2*polylog(2,exp(I*(d*x+c)))+4*I*f^2*polylog(2,I*
exp(I*(d*x+c)))/a/d^3+3/2/d/a*e^2*ln(exp(I*(d*x+c))-1)-3/2/d/a*e^2*ln(exp(I
*(d*x+c))+1)+(3*d*f^2*x^2*exp(4*I*(d*x+c))+6*d*e*f*x*exp(4*I*(d*x+c))+3*d*e
^2*exp(4*I*(d*x+c))-5*d*f^2*x^2*exp(2*I*(d*x+c))-I*d*f^2*x^2*exp(I*(d*x+c))
-10*d*e*f*x*exp(2*I*(d*x+c))+2*f^2*x*exp(3*I*(d*x+c))-2*I*f^2*x*exp(4*I*(d*
x+c))-2*I*d*e*f*x*exp(I*(d*x+c))-5*d*e^2*exp(2*I*(d*x+c))+4*d*f^2*x^2+2*e*f
*exp(3*I*(d*x+c))+3*I*d*f^2*x^2*exp(3*I*(d*x+c))-2*I*e*f*exp(4*I*(d*x+c))+3
*I*d*e^2*exp(3*I*(d*x+c))+8*d*e*f*x-2*f^2*x*exp(I*(d*x+c))-I*d*e^2*exp(I*(d
*x+c))+2*I*e*f*exp(2*I*(d*x+c))+4*d*e^2-2*exp(I*(d*x+c))*e*f+6*I*d*e*f*x*ex
p(3*I*(d*x+c))+2*I*f^2*x*exp(2*I*(d*x+c)))/(exp(2*I*(d*x+c))-1)^2/d^2/(exp(
I*(d*x+c))+I)/a-3*I/a/d^2*e*f*polylog(2,exp(I*(d*x+c)))+3*I/a/d^2*e*f*polyl
og(2,-exp(I*(d*x+c)))+8*I/a/d^2*c*f^2*x-3*I/a/d^2*polylog(2,exp(I*(d*x+c)))
*f^2*x+3*I/a/d^2*polylog(2,-exp(I*(d*x+c)))*f^2*x+2*I*f^2*polylog(2,-exp(I*
(d*x+c)))/a/d^3+8/a/d^2*f*ln(exp(I*(d*x+c)))*e-4/a/d^2*f^2*ln(1-I*exp(I*(d*
x+c)))*x-4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c+4/a/d^3*f^2*c*ln(exp(I*(d*x+c
```

```
))+I)-8/a/d^3*f^2*c*ln(exp(I*(d*x+c)))-4/a/d^2*f*ln(exp(I*(d*x+c))+I)*e+3/d
/a*ln(1-exp(I*(d*x+c)))*e*f*x-3/d/a*ln(exp(I*(d*x+c))+1)*e*f*x+3/d^2/a*ln(1
-exp(I*(d*x+c)))*c*e*f-3/d^2/a*e*f*c*ln(exp(I*(d*x+c))-1)-3/2/d^3/a*f^2*c^2
*ln(1-exp(I*(d*x+c)))+3/2/d/a*f^2*ln(1-exp(I*(d*x+c)))*x^2-3/2/d/a*f^2*ln(e
xp(I*(d*x+c))+1)*x^2+3/2/d^3/a*f^2*c^2*ln(exp(I*(d*x+c))-1)-3*f^2*polylog(3
,-exp(I*(d*x+c)))/a/d^3+3*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3+1/a/d^3*f^2*l
n(exp(I*(d*x+c))-1)-1/a/d^3*f^2*ln(exp(I*(d*x+c))+1)
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6160 vs. 2(348) = 696.
time = 2.63, size = 6160, normalized size = 15.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(2*c*e*f*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1)/(a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*d*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3) - (4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)/(a*d) + 12*log(sin(d*x + c)/(cos(d*x + c) +
1))/(a*d)) + e^2*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 - 1)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3) - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a
) + 8*(16*I*c^2*f^2 - 16*(-I*d*e*f + I*c*f^2 - (d*e*f - c*f^2)*cos(5*d*x +
5*c) + (-I*d*e*f + I*c*f^2)*cos(4*d*x + 4*c) + 2*(d*e*f - c*f^2)*cos(3*d*x
+ 3*c) + 2*(I*d*e*f - I*c*f^2)*cos(2*d*x + 2*c) - (d*e*f - c*f^2)*cos(d*x +
 c) + (-I*d*e*f + I*c*f^2)*sin(5*d*x + 5*c) + (d*e*f - c*f^2)*sin(4*d*x + 4
*c) + 2*(I*d*e*f - I*c*f^2)*sin(3*d*x + 3*c) - 2*(d*e*f - c*f^2)*sin(2*d*x
+ 2*c) + (-I*d*e*f + I*c*f^2)*sin(d*x + c))*arctan2(sin(d*x + c) + 1, cos(d
*x + c)) - 16*((d*x + c)*f^2*cos(5*d*x + 5*c) + I*(d*x + c)*f^2*cos(4*d*x +
 4*c) - 2*(d*x + c)*f^2*cos(3*d*x + 3*c) - 2*I*(d*x + c)*f^2*cos(2*d*x + 2*
c) + (d*x + c)*f^2*cos(d*x + c) + I*(d*x + c)*f^2*sin(5*d*x + 5*c) - (d*x +
 c)*f^2*sin(4*d*x + 4*c) - 2*I*(d*x + c)*f^2*sin(3*d*x + 3*c) + 2*(d*x + c)
*f^2*sin(2*d*x + 2*c) + I*(d*x + c)*f^2*sin(d*x + c) + I*(d*x + c)*f^2)*arc
tan2(cos(d*x + c), sin(d*x + c) + 1) - 2*(-3*I*(d*x + c)^2*f^2 - 4*I*d*e*f
+ (-3*I*c^2 + 4*I*c - 2*I)*f^2 + 2*(-3*I*d*e*f + (3*I*c - 2*I)*f^2)*(d*x +
c) - (3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f - (3
*c - 2)*f^2)*(d*x + c))*cos(5*d*x + 5*c) + (-3*I*(d*x + c)^2*f^2 - 4*I*d*e*
f + (-3*I*c^2 + 4*I*c - 2*I)*f^2 + 2*(-3*I*d*e*f + (3*I*c - 2*I)*f^2)*(d*x
+ c))*cos(4*d*x + 4*c) + 2*(3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)
*f^2 + 2*(3*d*e*f - (3*c - 2)*f^2)*(d*x + c))*cos(3*d*x + 3*c) + 2*(3*I*(d*
x + c)^2*f^2 + 4*I*d*e*f + (3*I*c^2 - 4*I*c + 2*I)*f^2 + 2*(3*I*d*e*f + (-3
*I*c + 2*I)*f^2)*(d*x + c))*cos(2*d*x + 2*c) - (3*(d*x + c)^2*f^2 + 4*d*e*f
```

```
 + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c - 2)*f^2)*(d*x + c))*cos(d*x +
 c) + (-3*I*(d*x + c)^2*f^2 - 4*I*d*e*f + (-3*I*c^2 + 4*I*c - 2*I)*f^2 + 2*
(-3*I*d*e*f + (3*I*c - 2*I)*f^2)*(d*x + c))*sin(5*d*x + 5*c) + (3*(d*x + c)
^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c - 2)*f^2)*(d*x
 + c))*sin(4*d*x + 4*c) + 2*(3*I*(d*x + c)^2*f^2 + 4*I*d*e*f + (3*I*c^2 - 4
*I*c + 2*I)*f^2 + 2*(3*I*d*e*f + (-3*I*c + 2*I)*f^2)*(d*x + c))*sin(3*d*x +
 3*c) - 2*(3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f
 - (3*c - 2)*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (-3*I*(d*x + c)^2*f^2 - 4*I
*d*e*f + (-3*I*c^2 + 4*I*c - 2*I)*f^2 + 2*(-3*I*d*e*f + (3*I*c - 2*I)*f^2)*
(d*x + c))*sin(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1) - 2*(-4*I*
d*e*f + (3*I*c^2 + 4*I*c + 2*I)*f^2 - (4*d*e*f - (3*c^2 + 4*c + 2)*f^2)*cos
(5*d*x + 5*c) + (-4*I*d*e*f + (3*I*c^2 + 4*I*c + 2*I)*f^2)*cos(4*d*x + 4*c)
 + 2*(4*d*e*f - (3*c^2 + 4*c + 2)*f^2)*cos(3*d*x + 3*c) + 2*(4*I*d*e*f + (-
3*I*c^2 - 4*I*c - 2*I)*f^2)*cos(2*d*x + 2*c) - (4*d*e*f - (3*c^2 + 4*c + 2)
*f^2)*cos(d*x + c) + (-4*I*d*e*f + (3*I*c^2 + 4*I*c + 2*I)*f^2)*sin(5*d*x +
 5*c) + (4*d*e*f - (3*c^2 + 4*c + 2)*f^2)*sin(4*d*x + 4*c) + 2*(4*I*d*e*f +
 (-3*I*c^2 - 4*I*c - 2*I)*f^2)*sin(3*d*x + 3*c) - 2*(4*d*e*f - (3*c^2 + 4*c
 + 2)*f^2)*sin(2*d*x + 2*c) + (-4*I*d*e*f + (3*I*c^2 + 4*I*c + 2*I)*f^2)*si
n(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) - 1) - 2*(-3*I*(d*x + c)^2*f
^2 + 2*(-3*I*d*e*f + (3*I*c + 2*I)*f^2)*(d*x + c) - (3*(d*x + c)^2*f^2 + 2*
(3*d*e*f - (3*c + 2)*f^2)*(d*x + c))*cos(5*d*x + 5*c) + (-3*I*(d*x + c)^2*f
^2 + 2*(-3*I*d*e*f + (3*I*c + 2*I)*f^2)*(d*x + c))*cos(4*d*x + 4*c) + 2*(3*
(d*x + c)^2*f^2 + 2*(3*d*e*f - (3*c + 2)*f^2)*(d*x + c))*cos(3*d*x + 3*c) +
 2*(3*I*(d*x + c)^2*f^2 + 2*(3*I*d*e*f + (-3*I*c - 2*I)*f^2)*(d*x + c))*cos
(2*d*x + 2*c) - (3*(d*x + c)^2*f^2 + 2*(3*d*e*f - (3*c + 2)*f^2)*(d*x + c))
*cos(d*x + c) + (-3*I*(d*x + c)^2*f^2 + 2*(-3*I*d*e*f + (3*I*c + 2*I)*f^2)*
(d*x + c))*sin(5*d*x + 5*c) + (3*(d*x + c)^2*f^2 + 2*(3*d*e*f - (3*c + 2)*f
^2)*(d*x + c))*sin(4*d*x + 4*c) + 2*(3*I*(d*x + c)^2*f^2 + 2*(3*I*d*e*f + (
-3*I*c - 2*I)*f^2)*(d*x + c))*sin(3*d*x + 3*c) - 2*(3*(d*x + c)^2*f^2 + 2*(
3*d*e*f - (3*c + 2)*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (-3*I*(d*x + c)^2*f^
2 + 2*(-3*I*d*e*f + (3*I*c + 2*I)*f^2)*(d*x + c))*sin(d*x + c))*arctan2(sin
(d*x + c), -cos(d*x + c) + 1) - 16*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*
x + c))*cos(5*d*x + 5*c) - 4*(I*(d*x + c)^2*f^2 - 2*d*e*f + (-3*I*c^2 + 2*c
)*f^2 + 2*(I*d*e*f + (-I*c - 1)*f^2)*(d*x + c))*cos(4*d*x + 4*c) + 4*(5*(d*
x + c)^2*f^2 + 2*I*d*e*f - (3*c^2 + 2*I*c)*f^2 + 2*(5*d*e*f - (5*c - I)*f^2
)*(d*x + c))*cos(3*d*x + 3*c) - 4*(-3*I*(d*x + c)^2*f^2 + 2*d*e*f + (5*I*c^
2 - 2*c)*f^2 + 2*(-3*I*d*e*f + (3*I*c + 1)*f^2)*(d*x + c))*cos(2*d*x + 2*c)
 - 4*(3*(d*x + c)^2*f^2 + 2*I*d*e*f - (c^2 + 2*I*c)*f^2 + 2*(3*d*e*f - (3*c
 - I)*f^2)*(d*x + c))*cos(d*x + c) - 16*(f^2*co...
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4060 vs. $2(358) = 716$.
time = 0.51, size = 4060, normalized size = 10.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d^2*f^2*x^2 - 4*d*f^2*x - 8*(d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2)*c
os(d*x + c)^3 - 2*(3*d^2*f^2*x^2 - 2*d*f^2*x + 3*d^2*e^2 + 2*(3*d^2*f*x - d
*f)*e)*cos(d*x + c)^2 + 4*d^2*e^2 + 6*(d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2)
*cos(d*x + c) + 2*(-3*I*d*f^2*x + (3*I*d*f^2*x + 3*I*d*f*e - 2*I*f^2)*cos(d
*x + c)^3 + (3*I*d*f^2*x + 3*I*d*f*e - 2*I*f^2)*cos(d*x + c)^2 - 3*I*d*f*e
+ 2*I*f^2 + (-3*I*d*f^2*x - 3*I*d*f*e + 2*I*f^2)*cos(d*x + c) + (-3*I*d*f^2
*x + (3*I*d*f^2*x + 3*I*d*f*e - 2*I*f^2)*cos(d*x + c)^2 - 3*I*d*f*e + 2*I*f
^2)*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x + c)) + 2*(3*I*d*f^2*x + (
-3*I*d*f^2*x - 3*I*d*f*e + 2*I*f^2)*cos(d*x + c)^3 + (-3*I*d*f^2*x - 3*I*d*
f*e + 2*I*f^2)*cos(d*x + c)^2 + 3*I*d*f*e - 2*I*f^2 + (3*I*d*f^2*x + 3*I*d*
f*e - 2*I*f^2)*cos(d*x + c) + (3*I*d*f^2*x + (-3*I*d*f^2*x - 3*I*d*f*e + 2*
I*f^2)*cos(d*x + c)^2 + 3*I*d*f*e - 2*I*f^2)*sin(d*x + c))*dilog(cos(d*x +
c) - I*sin(d*x + c)) + 8*(-I*f^2*cos(d*x + c)^3 - I*f^2*cos(d*x + c)^2 + I*
f^2*cos(d*x + c) + I*f^2 + (-I*f^2*cos(d*x + c)^2 + I*f^2)*sin(d*x + c))*di
log(I*cos(d*x + c) - sin(d*x + c)) + 8*(I*f^2*cos(d*x + c)^3 + I*f^2*cos(d*
x + c)^2 - I*f^2*cos(d*x + c) - I*f^2 + (I*f^2*cos(d*x + c)^2 - I*f^2)*sin(
d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) + 2*(-3*I*d*f^2*x + (3*I*d*
f^2*x + 3*I*d*f*e + 2*I*f^2)*cos(d*x + c)^3 + (3*I*d*f^2*x + 3*I*d*f*e + 2*
I*f^2)*cos(d*x + c)^2 - 3*I*d*f*e - 2*I*f^2 + (-3*I*d*f^2*x - 3*I*d*f*e - 2
*I*f^2)*cos(d*x + c) + (-3*I*d*f^2*x + (3*I*d*f^2*x + 3*I*d*f*e + 2*I*f^2)*
cos(d*x + c)^2 - 3*I*d*f*e - 2*I*f^2)*sin(d*x + c))*dilog(-cos(d*x + c) + I
*sin(d*x + c)) + 2*(3*I*d*f^2*x + (-3*I*d*f^2*x - 3*I*d*f*e - 2*I*f^2)*cos(
d*x + c)^3 + (-3*I*d*f^2*x - 3*I*d*f*e - 2*I*f^2)*cos(d*x + c)^2 + 3*I*d*f*
e + 2*I*f^2 + (3*I*d*f^2*x + 3*I*d*f*e + 2*I*f^2)*cos(d*x + c) + (3*I*d*f^2
*x + (-3*I*d*f^2*x - 3*I*d*f*e - 2*I*f^2)*cos(d*x + c)^2 + 3*I*d*f*e + 2*I*
f^2)*sin(d*x + c))*dilog(-cos(d*x + c) - I*sin(d*x + c)) + 4*(2*d^2*f*x - d
*f)*e - (3*d^2*f^2*x^2 + 4*d*f^2*x - (3*d^2*f^2*x^2 + 4*d*f^2*x + 3*d^2*e^2
 + 2*f^2 + 2*(3*d^2*f*x + 2*d*f)*e)*cos(d*x + c)^3 - (3*d^2*f^2*x^2 + 4*d*f
^2*x + 3*d^2*e^2 + 2*f^2 + 2*(3*d^2*f*x + 2*d*f)*e)*cos(d*x + c)^2 + 3*d^2*
e^2 + 2*f^2 + (3*d^2*f^2*x^2 + 4*d*f^2*x + 3*d^2*e^2 + 2*f^2 + 2*(3*d^2*f*x
 + 2*d*f)*e)*cos(d*x + c) + 2*(3*d^2*f*x + 2*d*f)*e + (3*d^2*f^2*x^2 + 4*d*
f^2*x - (3*d^2*f^2*x^2 + 4*d*f^2*x + 3*d^2*e^2 + 2*f^2 + 2*(3*d^2*f*x + 2*d
*f)*e)*cos(d*x + c)^2 + 3*d^2*e^2 + 2*f^2 + 2*(3*d^2*f*x + 2*d*f)*e)*sin(d*
x + c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) - 8*((c*f^2 - d*f*e)*cos(d*x
 + c)^3 - c*f^2 + (c*f^2 - d*f*e)*cos(d*x + c)^2 + d*f*e - (c*f^2 - d*f*e)*
cos(d*x + c) - (c*f^2 - (c*f^2 - d*f*e)*cos(d*x + c)^2 - d*f*e)*sin(d*x + c
))*log(cos(d*x + c) + I*sin(d*x + c) + I) - (3*d^2*f^2*x^2 + 4*d*f^2*x - (3
*d^2*f^2*x^2 + 4*d*f^2*x + 3*d^2*e^2 + 2*f^2 + 2*(3*d^2*f*x + 2*d*f)*e)*cos
(d*x + c)^3 - (3*d^2*f^2*x^2 + 4*d*f^2*x + 3*d^2*e^2 + 2*f^2 + 2*(3*d^2*f*x
 + 2*d*f)*e)*cos(d*x + c)^2 + 3*d^2*e^2 + 2*f^2 + (3*d^2*f^2*x^2 + 4*d*f^2*
x + 3*d^2*e^2 + 2*f^2 + 2*(3*d^2*f*x + 2*d*f)*e)*cos(d*x + c) + 2*(3*d^2*f*
```

```
x + 2*d*f)*e + (3*d^2*f^2*x^2 + 4*d*f^2*x - (3*d^2*f^2*x^2 + 4*d*f^2*x + 3*
d^2*e^2 + 2*f^2 + 2*(3*d^2*f*x + 2*d*f)*e)*cos(d*x + c)^2 + 3*d^2*e^2 + 2*f
^2 + 2*(3*d^2*f*x + 2*d*f)*e)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x +
c) + 1) - 8*(d*f^2*x - (d*f^2*x + c*f^2)*cos(d*x + c)^3 + c*f^2 - (d*f^2*x
+ c*f^2)*cos(d*x + c)^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2
 - (d*f^2*x + c*f^2)*cos(d*x + c)^2)*sin(d*x + c))*log(I*cos(d*x + c) + sin
(d*x + c) + 1) - 8*(d*f^2*x - (d*f^2*x + c*f^2)*cos(d*x + c)^3 + c*f^2 - (d
*f^2*x + c*f^2)*cos(d*x + c)^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x
+ c*f^2 - (d*f^2*x + c*f^2)*cos(d*x + c)^2)*sin(d*x + c))*log(-I*cos(d*x +
c) + sin(d*x + c) + 1) + ((2*(3*c + 2)*d*f*e - (3*c^2 + 4*c + 2)*f^2 - 3*d^
2*e^2)*cos(d*x + c)^3 - 2*(3*c + 2)*d*f*e + (3*c^2 + 4*c + 2)*f^2 + (2*(3*c
 + 2)*d*f*e - (3*c^2 + 4*c + 2)*f^2 - 3*d^2*e^2)*cos(d*x + c)^2 + 3*d^2*e^2
 - (2*(3*c + 2)*d*f*e - (3*c^2 + 4*c + 2)*f^2 - 3*d^2*e^2)*cos(d*x + c) - (
2*(3*c + 2)*d*f*e - (3*c^2 + 4*c + 2)*f^2 - (2*(3*c + 2)*d*f*e - (3*c^2 + 4
*c + 2)*f^2 - 3*d^2*e^2)*cos(d*x + c)^2 - 3*d^2*e^2)*sin(d*x + c))*log(-1/2
*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + ((2*(3*c + 2)*d*f*e - (3*c^2 +
4*c + 2)*f^2 - 3*d^2*e^2)*cos(d*x + c)^3 - 2*(3*c + 2)*d*f*e + (3*c^2 + 4*c
 + 2)*f^2 + (2*(3*c + 2)*d*f*e - (3*c^2 + 4*c + 2)*f^2 - 3*d^2*e^2)*cos(d*x
 + c)^2 + 3*d^2*e^2 - (2*(3*c + 2)*d*f*e - (3*c^2 + 4*c + 2)*f^2 - 3*d^2*e^
2)*cos(d*x + c) - (2*(3*c + 2)*d*f*e - (3*c^2 + 4*c + 2)*f^2 - (2*(3*c + 2)
*d*f*e - (3*c^2 + 4*c + 2)*f^2 - 3*d^2*e^2)*cos(d*x + c)^2 - 3*d^2*e^2)*sin
(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + (3*d^2*f^2*x
^2 - 4*d*f^2*x - (3*d^2*f^2*x^2 - 4*d*f^2*x - (3*c^2 + 4*c)*f^2 + 6*(d^2*f*
x + c*d*f)*e)*cos(d*x + c)^3 - (3*c^2 + 4*c)*f^2 - (3*d^2*f^2*x^2 - 4*d*f^2
*x - (3*c^2 + 4*c)*f^2 + 6*(d^2*f*x + c*d*f)*e)*cos(d*x + c)^2 + (3*d^2*f^2
*x^2 - 4*d*f^2*x - (3*c^2 + 4*c)*f^2 + 6*(d^2*f...
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2 \csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*csc(c + d*x)**3/(
sin(c + d*x) + 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csc(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.211 $\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=216

$$-\frac{3(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{(e+fx)\cot(c+dx)}{ad}-\frac{f\csc(c+dx)}{2ad^2}-\frac{(e+fx)\,c}{}$$

```
[Out] -3*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)/a/
d+(f*x+e)*cot(d*x+c)/a/d-1/2*f*csc(d*x+c)/a/d^2-1/2*(f*x+e)*cot(d*x+c)*csc(
d*x+c)/a/d-2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2-f*ln(sin(d*x+c))/a/d^2+3
/2*I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2-3/2*I*f*polylog(2,exp(I*(d*x+c)))/a
/d^2
```

**Rubi** [A]
time = 0.20, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4631, 4270, 4268, 2317, 2438, 4269, 3556, 3399}

$$\frac{3if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{2ad^2}-\frac{3if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{2ad^2}-\frac{f\csc(c+dx)}{2ad^2}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}-\frac{f\log(\sin(c+dx))}{ad^2}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}+\frac{(e+fx)\cot(c+dx)}{ad}-\frac{3(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) + ((e + f*x)*Cot[c + d*x])/(a*d) - (f*Csc[c + d*x])/(2*a*d
^2) - ((e + f*x)*Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - (2*f*Log[Sin[c/2 + Pi
/4 + (d*x)/2]])/(a*d^2) - (f*Log[Sin[c + d*x]])/(a*d^2) + (((3*I)/2)*f*Poly
Log[2, -E^(I*(c + d*x))])/(a*d^2) - (((3*I)/2)*f*PolyLog[2, E^(I*(c + d*x))
])/(a*d^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\csc^3(c+dx)\,dx}{a} - \int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{f\csc(c+dx)}{2ad^2} - \frac{(e+fx)\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int (e+fx)\csc(c+dx)}{2a}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot(c+dx)}{ad} - \frac{f\csc(c+dx)}{2ad^2} - \frac{(e+}{}$$

$$= -\frac{3(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot(c+dx)}{ad} - \frac{f\csc(c+dx)}{2ad^2} - \frac{(e+}{}$$

$$= -\frac{3(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(e+fx)\cot(c-}{ad}$$

$$= -\frac{3(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(e+fx)\cot(c-}{ad}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 484 vs. 2(216) = 432.
time = 2.16, size = 484, normalized size = 2.24

---

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-(d*(e + f*x)*(1 + Cot[(c + d*x)/2]
)*Csc[(c + d*x)/2]) - 16*d*(e + f*x)*Sin[(c + d*x)/2] + 8*f*(c + d*x)*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*(-f + 2*d*(e + f*x))*Cot[(c + d*x)/2]*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 16*f*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 8*f*Log[Sin[c + d*x]]*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]) + 12*d*e*Log[Tan[(c + d*x)/2]]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]) - 12*c*f*Log[Tan[(c + d*x)/2]]*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]) + 12*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1
 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c
 + d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 2*(f + 2*d*(e + f*x))*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Tan[(c + d*x)/2] + d*(e + f*x)*Sec[(c
+ d*x)/2]*(1 + Tan[(c + d*x)/2])))/(8*a*d^2*(1 + Sin[c + d*x]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(188) = 376.
time = 0.23, size = 468, normalized size = 2.17

| method | result |
| --- | --- |
| risch | $\frac{3dfx\,e^{4i(dx+c)}+3de\,e^{4i(dx+c)}-5dfx\,e^{2i(dx+c)}+3idfx\,e^{3i(dx+c)}-5de\,e^{2i(dx+c)}+f\,e^{3i(dx+c)}+3ide\,e^{3i(dx+c)}-if\,e^{4i(dx+c)}+4dxf-idfx\,e}{\left(e^{2i(dx+c)}-1\right)^2 d^2\left(e^{i(dx+c)}+i\right)a}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (3*d*f*x*exp(4*I*(d*x+c))+3*d*e*exp(4*I*(d*x+c))-5*d*f*x*exp(2*I*(d*x+c))+3
*I*d*f*x*exp(3*I*(d*x+c))-5*d*e*exp(2*I*(d*x+c))+f*exp(3*I*(d*x+c))+3*I*d*e
*exp(3*I*(d*x+c))-I*f*exp(4*I*(d*x+c))+4*d*x*f-I*d*f*x*exp(I*(d*x+c))+4*d*e
-exp(I*(d*x+c))*f-I*d*e*exp(I*(d*x+c))+I*f*exp(2*I*(d*x+c)))/(exp(2*I*(d*x+
c))-1)^2/d^2/(exp(I*(d*x+c))+I)/a-3/2/d^2/a*f*c*ln(exp(I*(d*x+c))-1)+3/2/d/
a*e*ln(exp(I*(d*x+c))-1)-3/2/d/a*e*ln(exp(I*(d*x+c))+1)+3/2/d/a*ln(1-exp(I*
(d*x+c)))*f*x+3/2/d^2/a*ln(1-exp(I*(d*x+c)))*c*f-3/2/d/a*ln(exp(I*(d*x+c))+
1)*f*x-2*f/a/d^2*ln(exp(I*(d*x+c))+I)+4/d^2/a*f*ln(exp(I*(d*x+c)))-1/a/d^2*
f*ln(exp(I*(d*x+c))-1)-1/a/d^2*f*ln(exp(I*(d*x+c))+1)-3/2*I*f*polylog(2,exp
(I*(d*x+c)))/a/d^2+3/2*I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2080 vs. 2(184) = 368.
time = 0.90, size = 2080, normalized size = 9.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (16*d*f*x*cos(5*d*x + 5*c) + 16*I*d*f*x*sin(5*d*x + 5*c) - 16*I*d*e - 8*(f*
cos(5*d*x + 5*c) + I*f*cos(4*d*x + 4*c) - 2*f*cos(3*d*x + 3*c) - 2*I*f*cos(
2*d*x + 2*c) + f*cos(d*x + c) + I*f*sin(5*d*x + 5*c) - f*sin(4*d*x + 4*c) -
 2*I*f*sin(3*d*x + 3*c) + 2*f*sin(2*d*x + 2*c) + I*f*sin(d*x + c) + I*f)*ar
ctan2(cos(c) + sin(d*x), cos(d*x) + sin(c)) + 2*(-3*I*d*f*x - 3*I*d*e - (3*
d*f*x + 3*d*e + 2*f)*cos(5*d*x + 5*c) + (-3*I*d*f*x - 3*I*d*e - 2*I*f)*cos(
4*d*x + 4*c) + 2*(3*d*f*x + 3*d*e + 2*f)*cos(3*d*x + 3*c) + 2*(3*I*d*f*x +
3*I*d*e + 2*I*f)*cos(2*d*x + 2*c) - (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c) +
(-3*I*d*f*x - 3*I*d*e - 2*I*f)*sin(5*d*x + 5*c) + (3*d*f*x + 3*d*e + 2*f)*s
in(4*d*x + 4*c) + 2*(3*I*d*f*x + 3*I*d*e + 2*I*f)*sin(3*d*x + 3*c) - 2*(3*d
*f*x + 3*d*e + 2*f)*sin(2*d*x + 2*c) + (-3*I*d*f*x - 3*I*d*e - 2*I*f)*sin(d
*x + c) - 2*I*f)*arctan2(sin(d*x + c), cos(d*x + c) + 1) + 2*(3*I*d*e + (3*
d*e - 2*f)*cos(5*d*x + 5*c) + (3*I*d*e - 2*I*f)*cos(4*d*x + 4*c) - 2*(3*d*e
 - 2*f)*cos(3*d*x + 3*c) + 2*(-3*I*d*e + 2*I*f)*cos(2*d*x + 2*c) + (3*d*e -
 2*f)*cos(d*x + c) + (3*I*d*e - 2*I*f)*sin(5*d*x + 5*c) - (3*d*e - 2*f)*sin
(4*d*x + 4*c) + 2*(-3*I*d*e + 2*I*f)*sin(3*d*x + 3*c) + 2*(3*d*e - 2*f)*sin
(2*d*x + 2*c) + (3*I*d*e - 2*I*f)*sin(d*x + c) - 2*I*f)*arctan2(sin(d*x + c
```

```
), cos(d*x + c) - 1) - 6*(d*f*x*cos(5*d*x + 5*c) + I*d*f*x*cos(4*d*x + 4*c)
 - 2*d*f*x*cos(3*d*x + 3*c) - 2*I*d*f*x*cos(2*d*x + 2*c) + d*f*x*cos(d*x +
c) + I*d*f*x*sin(5*d*x + 5*c) - d*f*x*sin(4*d*x + 4*c) - 2*I*d*f*x*sin(3*d*
x + 3*c) + 2*d*f*x*sin(2*d*x + 2*c) + I*d*f*x*sin(d*x + c) + I*d*f*x)*arcta
n2(sin(d*x + c), -cos(d*x + c) + 1) + 4*(I*d*f*x - 3*I*d*e - f)*cos(4*d*x +
 4*c) - 4*(5*d*f*x - 3*d*e + I*f)*cos(3*d*x + 3*c) + 4*(-3*I*d*f*x + 5*I*d*
e + f)*cos(2*d*x + 2*c) + 4*(3*d*f*x - d*e + I*f)*cos(d*x + c) + 6*(f*cos(5
*d*x + 5*c) + I*f*cos(4*d*x + 4*c) - 2*f*cos(3*d*x + 3*c) - 2*I*f*cos(2*d*x
 + 2*c) + f*cos(d*x + c) + I*f*sin(5*d*x + 5*c) - f*sin(4*d*x + 4*c) - 2*I*
f*sin(3*d*x + 3*c) + 2*f*sin(2*d*x + 2*c) + I*f*sin(d*x + c) + I*f)*dilog(-
e^(I*d*x + I*c)) - 6*(f*cos(5*d*x + 5*c) + I*f*cos(4*d*x + 4*c) - 2*f*cos(3
*d*x + 3*c) - 2*I*f*cos(2*d*x + 2*c) + f*cos(d*x + c) + I*f*sin(5*d*x + 5*c
) - f*sin(4*d*x + 4*c) - 2*I*f*sin(3*d*x + 3*c) + 2*f*sin(2*d*x + 2*c) + I*
f*sin(d*x + c) + I*f)*dilog(e^(I*d*x + I*c)) - (3*d*f*x + 3*d*e + (-3*I*d*f
*x - 3*I*d*e - 2*I*f)*cos(5*d*x + 5*c) + (3*d*f*x + 3*d*e + 2*f)*cos(4*d*x
+ 4*c) - 2*(-3*I*d*f*x - 3*I*d*e - 2*I*f)*cos(3*d*x + 3*c) - 2*(3*d*f*x + 3
*d*e + 2*f)*cos(2*d*x + 2*c) + (-3*I*d*f*x - 3*I*d*e - 2*I*f)*cos(d*x + c)
+ (3*d*f*x + 3*d*e + 2*f)*sin(5*d*x + 5*c) + (3*I*d*f*x + 3*I*d*e + 2*I*f)*
sin(4*d*x + 4*c) - 2*(3*d*f*x + 3*d*e + 2*f)*sin(3*d*x + 3*c) - 2*(3*I*d*f*
x + 3*I*d*e + 2*I*f)*sin(2*d*x + 2*c) + (3*d*f*x + 3*d*e + 2*f)*sin(d*x + c
) + 2*f)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1) + (3*d*f
*x + 3*d*e - (3*I*d*f*x + 3*I*d*e - 2*I*f)*cos(5*d*x + 5*c) + (3*d*f*x + 3*
d*e - 2*f)*cos(4*d*x + 4*c) + 2*(3*I*d*f*x + 3*I*d*e - 2*I*f)*cos(3*d*x + 3
*c) - 2*(3*d*f*x + 3*d*e - 2*f)*cos(2*d*x + 2*c) - (3*I*d*f*x + 3*I*d*e - 2
*I*f)*cos(d*x + c) + (3*d*f*x + 3*d*e - 2*f)*sin(5*d*x + 5*c) - (-3*I*d*f*x
 - 3*I*d*e + 2*I*f)*sin(4*d*x + 4*c) - 2*(3*d*f*x + 3*d*e - 2*f)*sin(3*d*x
+ 3*c) + 2*(-3*I*d*f*x - 3*I*d*e + 2*I*f)*sin(2*d*x + 2*c) + (3*d*f*x + 3*d
*e - 2*f)*sin(d*x + c) - 2*f)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d
*x + c) + 1) + 4*(I*f*cos(5*d*x + 5*c) - f*cos(4*d*x + 4*c) - 2*I*f*cos(3*d
*x + 3*c) + 2*f*cos(2*d*x + 2*c) + I*f*cos(d*x + c) - f*sin(5*d*x + 5*c) -
I*f*sin(4*d*x + 4*c) + 2*f*sin(3*d*x + 3*c) + 2*I*f*sin(2*d*x + 2*c) - f*si
n(d*x + c) - f)*log(cos(d*x)^2 + cos(c)^2 + 2*cos(c)*sin(d*x) + sin(d*x)^2
+ 2*cos(d*x)*sin(c) + sin(c)^2) - 4*(d*f*x - 3*d*e + I*f)*sin(4*d*x + 4*c)
+ 4*(-5*I*d*f*x + 3*I*d*e + f)*sin(3*d*x + 3*c) + 4*(3*d*f*x - 5*d*e + I*f)
*sin(2*d*x + 2*c) + 4*(3*I*d*f*x - I*d*e - f)*sin(d*x + c))/(-4*I*a*d^2*cos
(5*d*x + 5*c) + 4*a*d^2*cos(4*d*x + 4*c) + 8*I*a*d^2*cos(3*d*x + 3*c) - 8*a
*d^2*cos(2*d*x + 2*c) - 4*I*a*d^2*cos(d*x + c) + 4*a*d^2*sin(5*d*x + 5*c) +
 4*I*a*d^2*sin(4*d*x + 4*c) - 8*a*d^2*sin(3*d*x + 3*c) - 8*I*a*d^2*sin(2*d*
x + 2*c) + 4*a*d^2*sin(d*x + c) + 4*a*d^2)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1388 vs. $2(188) = 376$.
time = 0.44, size = 1388, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*(d*f*x + d*e)*cos(d*x + c)^3 - 4*d*f*x + 2*(3*d*f*x + 3*d*e - f)*cos
(d*x + c)^2 - 6*(d*f*x + d*e)*cos(d*x + c) - 3*(I*f*cos(d*x + c)^3 + I*f*co
s(d*x + c)^2 - I*f*cos(d*x + c) + (I*f*cos(d*x + c)^2 - I*f)*sin(d*x + c) -
 I*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) - 3*(-I*f*cos(d*x + c)^3 - I*f*c
os(d*x + c)^2 + I*f*cos(d*x + c) + (-I*f*cos(d*x + c)^2 + I*f)*sin(d*x + c)
 + I*f)*dilog(cos(d*x + c) - I*sin(d*x + c)) - 3*(I*f*cos(d*x + c)^3 + I*f*
cos(d*x + c)^2 - I*f*cos(d*x + c) + (I*f*cos(d*x + c)^2 - I*f)*sin(d*x + c)
 - I*f)*dilog(-cos(d*x + c) + I*sin(d*x + c)) - 3*(-I*f*cos(d*x + c)^3 - I*
f*cos(d*x + c)^2 + I*f*cos(d*x + c) + (-I*f*cos(d*x + c)^2 + I*f)*sin(d*x +
 c) + I*f)*dilog(-cos(d*x + c) - I*sin(d*x + c)) - 4*d*e - ((3*d*f*x + 3*d*
e + 2*f)*cos(d*x + c)^3 - 3*d*f*x + (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c)^2
- (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c) - 3*d*e - (3*d*f*x - (3*d*f*x + 3*d*
e + 2*f)*cos(d*x + c)^2 + 3*d*e + 2*f)*sin(d*x + c) - 2*f)*log(cos(d*x + c)
 + I*sin(d*x + c) + 1) - ((3*d*f*x + 3*d*e + 2*f)*cos(d*x + c)^3 - 3*d*f*x
+ (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c)^2 - (3*d*f*x + 3*d*e + 2*f)*cos(d*x
+ c) - 3*d*e - (3*d*f*x - (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c)^2 + 3*d*e +
2*f)*sin(d*x + c) - 2*f)*log(cos(d*x + c) - I*sin(d*x + c) + 1) - (((3*c +
2)*f - 3*d*e)*cos(d*x + c)^3 + ((3*c + 2)*f - 3*d*e)*cos(d*x + c)^2 - (3*c
+ 2)*f - ((3*c + 2)*f - 3*d*e)*cos(d*x + c) + 3*d*e + (((3*c + 2)*f - 3*d*e
)*cos(d*x + c)^2 - (3*c + 2)*f + 3*d*e)*sin(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2*I*sin(d*x + c) + 1/2) - (((3*c + 2)*f - 3*d*e)*cos(d*x + c)^3 + ((3*
c + 2)*f - 3*d*e)*cos(d*x + c)^2 - (3*c + 2)*f - ((3*c + 2)*f - 3*d*e)*cos(
d*x + c) + 3*d*e + (((3*c + 2)*f - 3*d*e)*cos(d*x + c)^2 - (3*c + 2)*f + 3*
d*e)*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + 3*((
d*f*x + c*f)*cos(d*x + c)^3 - d*f*x + (d*f*x + c*f)*cos(d*x + c)^2 - c*f -
(d*f*x + c*f)*cos(d*x + c) - (d*f*x - (d*f*x + c*f)*cos(d*x + c)^2 + c*f)*s
in(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + 3*((d*f*x + c*f)*cos
(d*x + c)^3 - d*f*x + (d*f*x + c*f)*cos(d*x + c)^2 - c*f - (d*f*x + c*f)*co
s(d*x + c) - (d*f*x - (d*f*x + c*f)*cos(d*x + c)^2 + c*f)*sin(d*x + c))*log
(-cos(d*x + c) - I*sin(d*x + c) + 1) - 4*(f*cos(d*x + c)^3 + f*cos(d*x + c)
^2 - f*cos(d*x + c) + (f*cos(d*x + c)^2 - f)*sin(d*x + c) - f)*log(sin(d*x
+ c) + 1) + 2*(2*d*f*x - 4*(d*f*x + d*e)*cos(d*x + c)^2 - (d*f*x + d*e - f)
*cos(d*x + c) + 2*d*e + f)*sin(d*x + c) + 2*f)/(a*d^2*cos(d*x + c)^3 + a*d^
2*cos(d*x + c)^2 - a*d^2*cos(d*x + c) - a*d^2 + (a*d^2*cos(d*x + c)^2 - a*d
^2)*sin(d*x + c))
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e\csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f*x*csc(c + d
*x)**3/(sin(c + d*x) + 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csc(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.212 $\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=82

$$-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{2\cot(c+dx)}{ad}-\frac{3\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{d(a+a\sin(c+dx))}$$

```
[Out] -3/2*arctanh(cos(d*x+c))/a/d+2*cot(d*x+c)/a/d-3/2*cot(d*x+c)*csc(d*x+c)/a/d
+cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))
```

**Rubi** [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2847, 2827, 3853, 3855, 3852, 8}

$$\frac{2\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{3\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (2*Cot[c + d*x])/(a*d) - (3*Cot[c + d*
x]*Csc[c + d*x])/(2*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(d*(a + a*Sin[c + d*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cot(c+dx)\csc(c+dx)}{d(a+a\sin(c+dx))}-\frac{\int \csc^3(c+dx)(-3a+2a\sin(c+dx))\,dx}{a^2}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{d(a+a\sin(c+dx))}-\frac{2\int \csc^2(c+dx)\,dx}{a}+\frac{3\int \csc^3(c+dx)\,dx}{a}\\ &= -\frac{3\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{d(a+a\sin(c+dx))}+\frac{3\int \csc(c+dx)\,dx}{2a}+\frac{2S\cdots}{\cdots}\\ &= -\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{2\cot(c+dx)}{ad}-\frac{3\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\cot(c+a\cdots}{d(a+a\cdots}\end{aligned}$$

**Mathematica [A]**
time = 0.36, size = 85, normalized size = 1.04

$$-\frac{-4\csc(2(c+dx))-3\sec(c+dx)+3\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)\sqrt{\cos^2(c+dx)}\,\sec(c+dx)+\csc^2(c+dx)\sec(c+dx)+4\tan(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(-4*Csc[2*(c + d*x)] - 3*Sec[c + d*x] + 3*ArcTanh[Sqrt[Cos[c + d*x]^2]
]*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x] + Csc[c + d*x]^2*Sec[c + d*x] + 4*Tan[c
 + d*x])/(a*d)
```

**Maple [A]**
time = 0.12, size = 87, normalized size = 1.06

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$ | 87 |
| default | $\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$ | 87 |
| risch | $\frac{3\,\mathrm{e}^{4i(dx+c)}-5\,\mathrm{e}^{2i(dx+c)}+3i\mathrm{e}^{3i(dx+c)}+4-i\mathrm{e}^{i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}-1\right)^2\left(\mathrm{e}^{i(dx+c)}+i\right)da}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{2da}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{2da}$ | 124 |
| norman | $\frac{-\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$ | 128 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a*(1/2*tan(1/2*d*x+1/2*c)^2-2*tan(1/2*d*x+1/2*c)+8/(tan(1/2*d*x+1/2*c
)+1)-1/2/tan(1/2*d*x+1/2*c)^2+2/tan(1/2*d*x+1/2*c)+6*ln(tan(1/2*d*x+1/2*c))
)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(78) = 156$.
time = 0.28, size = 157, normalized size = 1.91

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a}-\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 1)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3) - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(78) = 156$.
time = 0.35, size = 232, normalized size = 2.83

$$\frac{8\cos(dx+c)^3+6\cos(dx+c)^2-3\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\left(4\cos(dx+c)^2+\cos(dx+c)-2\right)\sin(dx+c)-6\cos(dx+c)-4}{4\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad+\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*cos(d*x + c)^3 + 6*cos(d*x + c)^2 - 3*(cos(d*x + c)^3 + cos(d*x + c)
^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*log(1/2*cos(d*x
+ c) + 1/2) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin
(d*x + c) - cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*(4*cos(d*x +
 c)^2 + cos(d*x + c) - 2)*sin(d*x + c) - 6*cos(d*x + c) - 4)/(a*d*cos(d*x +
 c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d*cos(d*x + c)^2 -
 a*d)*sin(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

**Giac [A]**
time = 6.51, size = 112, normalized size = 1.37

$$\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}+\frac{16}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x + 1/2*c)^2 - 4*a*
tan(1/2*d*x + 1/2*c))/a^2 + 16/(a*(tan(1/2*d*x + 1/2*c) + 1)) - (18*tan(1/2
*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*c) + 1)/(a*tan(1/2*d*x + 1/2*c)^2))/d
```

**Mupad [B]**
time = 1.39, size = 116, normalized size = 1.41

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8ad}+\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2ad}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad}+\frac{10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}-\frac{1}{2}}{d\left(4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) + (3*log(tan(c/2 + (d*x)/2)))/(2*a*d) - tan(c/
2 + (d*x)/2)/(2*a*d) + ((3*tan(c/2 + (d*x)/2))/2 + 10*tan(c/2 + (d*x)/2)^2
- 1/2)/(d*(4*a*tan(c/2 + (d*x)/2)^2 + 4*a*tan(c/2 + (d*x)/2)^3))
```

# 3.213 $\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))} \, dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))} \, dx = \int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))} \, dx$$

**Mathematica** [A]
time = 53.95, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(dx+c)}{(fx+e)(a+a\sin(dx+c))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (f*cos(4*d*x + 4*c)^2 + 2*f*cos(3*d*x + 3*c)^2 + 2*f*cos(2*d*x + 2*c)^2 + f
*cos(d*x + c)^2 + f*sin(4*d*x + 4*c)^2 + 2*f*sin(3*d*x + 3*c)^2 + 2*f*sin(2
*d*x + 2*c)^2 + f*sin(d*x + c)^2 + (4*d*f*x + 4*d*e + 3*(d*f*x + d*e)*cos(4
*d*x + 4*c) - f*cos(3*d*x + 3*c) - 5*(d*f*x + d*e)*cos(2*d*x + 2*c) + f*cos
(d*x + c) - f*sin(4*d*x + 4*c) - 3*(d*f*x + d*e)*sin(3*d*x + 3*c) + f*sin(2
*d*x + 2*c) + (d*f*x + d*e)*sin(d*x + c))*cos(5*d*x + 5*c) - (3*(d*f*x + d*
e)*cos(3*d*x + 3*c) + 3*f*cos(2*d*x + 2*c) - 2*(d*f*x + d*e)*cos(d*x + c) +
 3*f*sin(3*d*x + 3*c) - (d*f*x + d*e)*sin(2*d*x + 2*c) - 2*f*sin(d*x + c) -
 f)*cos(4*d*x + 4*c) - (5*d*f*x + 5*d*e - 4*(d*f*x + d*e)*cos(2*d*x + 2*c)
+ 3*f*cos(d*x + c) + 4*f*sin(2*d*x + 2*c) - (d*f*x + d*e)*sin(d*x + c))*cos
(3*d*x + 3*c) - (3*(d*f*x + d*e)*cos(d*x + c) + 3*f*sin(d*x + c) + f)*cos(2
*d*x + 2*c) + 3*(d*f*x + d*e)*cos(d*x + c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x
 + a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(5*d*x + 5*c)
^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(4*d*x + 4*c)^2 + 4*(a*
d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(3*d*x + 3*c)^2 + 4*(a*d^2*f^2*
x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(2*d*x + 2*c)^2 + (a*d^2*f^2*x^2 + 2*a*
d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*
d^2*e^2)*sin(5*d*x + 5*c)^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*s
in(4*d*x + 4*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(3*d*x
 + 3*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)*sin(
2*d*x + 2*c) + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2*d*x + 2*
c)^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c)^2 - 2*(2*(a
*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(3*d*x + 3*c) - (a*d^2*f^2*x^2
 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x
 + a*d^2*e^2)*sin(4*d*x + 4*c) - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e
^2)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x +
 a*d^2*e^2 - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(2*d*x + 2*c)
 - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(3*d*x + 3*c) + (a*d^2*
f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*cos(4*d*x + 4*c) - 4*((a
*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c) + 2*(a*d^2*f^2*x^2 +
 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2*d*x + 2*c))*cos(3*d*x + 3*c) - 4*(a*d^2*f
^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2
```

```
*e^2)*sin(d*x + c))*cos(2*d*x + 2*c) + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a
*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(4*d*x + 4*c) - 2
*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(2*d*x + 2*c) - 2*(a*d^2*f^
2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(3*d*x + 3*c) + (a*d^2*f^2*x^2 + 2*a*
d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*sin(5*d*x + 5*c) + 2*(2*(a*d^2*f^2*x^2
 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(3*d*x + 3*c) - (a*d^2*f^2*x^2 + 2*a*d^2*e
*f*x + a*d^2*e^2)*cos(d*x + c) - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e
^2)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x +
 a*d^2*e^2 - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(2*d*x + 2*c)
 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*sin(3*d*x + 3*
c) + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*integrate(
1/2*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 2*d*e*f + 2*f^2 + 2*(3*d^2*e*f + d*f^2)*x)
*sin(d*x + c)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*
e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos
(d*x + c)^2 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*
e^3)*sin(d*x + c)^2 - 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*
x + a*d^2*e^3)*cos(d*x + c)), x) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e
^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(5*d*x + 5*c)^2 + (a*d^
2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(4*d*x + 4*c)^2 + 4*(a*d^2*f^2*x^
2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(3*d*x + 3*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*
d^2*e*f*x + a*d^2*e^2)*cos(2*d*x + 2*c)^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x
+ a*d^2*e^2)*cos(d*x + c)^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*s
in(5*d*x + 5*c)^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(4*d*x +
 4*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(3*d*x + 3*c)^2
+ 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)*sin(2*d*x + 2*
c) + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2*d*x + 2*c)^2 + (a*
d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c)^2 - 2*(2*(a*d^2*f^2*x
^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(3*d*x + 3*c) - (a*d^2*f^2*x^2 + 2*a*d^2
*e*f*x + a*d^2*e^2)*cos(d*x + c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e
^2)*sin(4*d*x + 4*c) - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2*
d*x + 2*c))*cos(5*d*x + 5*c) + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2
 - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(2*d*x + 2*c) - 2*(a*d^
2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(3*d*x + 3*c) + (a*d^2*f^2*x^2 +
2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*cos(4*d*x + 4*c) - 4*((a*d^2*f^2*x
^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c) + ...
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^3/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sin(c+d\,x)^3\,(e+f\,x)\,(a+a\,\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(e + f*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] int(1/(sin(c + d*x)^3*(e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.214 $\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 110.10, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(dx+c)}{(fx+e)^2\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*f*cos(4*d*x + 4*c)^2 + 4*f*cos(3*d*x + 3*c)^2 + 4*f*cos(2*d*x + 2*c)^2 +
 2*f*cos(d*x + c)^2 + 2*f*sin(4*d*x + 4*c)^2 + 4*f*sin(3*d*x + 3*c)^2 + 4*f
*sin(2*d*x + 2*c)^2 + 2*f*sin(d*x + c)^2 + (4*d*f*x + 4*d*e + 3*(d*f*x + d*
e)*cos(4*d*x + 4*c) - 2*f*cos(3*d*x + 3*c) - 5*(d*f*x + d*e)*cos(2*d*x + 2*
c) + 2*f*cos(d*x + c) - 2*f*sin(4*d*x + 4*c) - 3*(d*f*x + d*e)*sin(3*d*x +
3*c) + 2*f*sin(2*d*x + 2*c) + (d*f*x + d*e)*sin(d*x + c))*cos(5*d*x + 5*c)
- (3*(d*f*x + d*e)*cos(3*d*x + 3*c) + 6*f*cos(2*d*x + 2*c) - 2*(d*f*x + d*e
)*cos(d*x + c) + 6*f*sin(3*d*x + 3*c) - (d*f*x + d*e)*sin(2*d*x + 2*c) - 4*
f*sin(d*x + c) - 2*f)*cos(4*d*x + 4*c) - (5*d*f*x + 5*d*e - 4*(d*f*x + d*e)
*cos(2*d*x + 2*c) + 6*f*cos(d*x + c) + 8*f*sin(2*d*x + 2*c) - (d*f*x + d*e)
*sin(d*x + c))*cos(3*d*x + 3*c) - (3*(d*f*x + d*e)*cos(d*x + c) + 6*f*sin(d
*x + c) + 2*f)*cos(2*d*x + 2*c) + 3*(d*f*x + d*e)*cos(d*x + c) + (a*d^2*f^3
*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3
*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(5*d*x + 5*c)^2 + (a*d^2
*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(4*d*x + 4*c
)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*c
os(3*d*x + 3*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x
+ a*d^2*e^3)*cos(2*d*x + 2*c)^2 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*
d^2*e^2*f*x + a*d^2*e^3)*cos(d*x + c)^2 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^
2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(5*d*x + 5*c)^2 + (a*d^2*f^3*x^3 + 3*a*
d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(4*d*x + 4*c)^2 + 4*(a*d^2*
f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(3*d*x + 3*c)
^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*co
s(d*x + c)*sin(2*d*x + 2*c) + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^
2*e^2*f*x + a*d^2*e^3)*sin(2*d*x + 2*c)^2 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*
x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)^2 - 2*(2*(a*d^2*f^3*x^3 + 3
*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(3*d*x + 3*c) - (a*d^2*f
^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(d*x + c) + (a
*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(4*d*x +
 4*c) - 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)
*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) + 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2
+ 3*a*d^2*e^2*f*x + a*d^2*e^3 - 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*
```

```
d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2*c) - 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^
2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(3*d*x + 3*c) + (a*d^2*f^3*x^3 + 3*
a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*cos(4*d*x + 4*
c) - 4*((a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*c
os(d*x + c) + 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^
2*e^3)*sin(2*d*x + 2*c))*cos(3*d*x + 3*c) - 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^
2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*cos(2*d*x + 2*c) + 2*(a*d^2*f^3*
x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*
a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(4*d*x + 4*c) - 2*(a*d^2*
f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2*c)
 - 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(
3*d*x + 3*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2
*e^3)*sin(d*x + c))*sin(5*d*x + 5*c) + 2*(2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*
x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(3*d*x + 3*c) - (a*d^2*f^3*x^3 + 3*a*
d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(d*x + c) - 2*(a*d^2*f^3*x^
3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(2*d*x + 2*c))*sin(
4*d*x + 4*c) - 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d
^2*e^3 - 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3
)*cos(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x +
 a*d^2*e^3)*sin(d*x + c))*sin(3*d*x + 3*c) + 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f
^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*integrate(1/2*(3*d^2*f^
2*x^2 + 3*d^2*e^2 + 4*d*e*f + 6*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*sin(d*x +
c)/(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f
*x + a*d^2*e^4 + (a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 +
 4*a*d^2*e^3*f*x + a*d^2*e^4)*cos(d*x + c)^2 + (a*d^2*f^4*x^4 + 4*a*d^2*e*f
^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*sin(d*x + c)^2
- 2*(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*
f*x + a*d^2*e^4)*cos(d*x + c)), x) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3
*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e
^2*f*x + a*d^2*e^3)*cos(5*d*x + 5*c)^2 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2
 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(4*d*x + 4*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a
*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(3*d*x + 3*c)^2 + 4*(a*d^2
*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2*c
)^2 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*...
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^3/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2 + 2*a*f
*x*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sin(c+d\,x)^3\,(e+f\,x)^2\,(a+a\,\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(e + f*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] int(1/(sin(c + d*x)^3*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.215 $\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=31

$$\mathrm{Int}\left(\frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 4.55, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m \left(\sin^2(dx+c)\right)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
[Out] int((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(f*x + e)^m/(a*sin(d*x + c) + a), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m \sin^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*sin(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(c+d\,x)^2\,(e+f\,x)^m}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
[Out] int((sin(c + d*x)^2*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

# 3.216 $\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=29

$$\mathrm{Int}\left(\frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m \sin(dx+c)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] int((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m \sin(c+dx)}{\sin(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*sin(c + d*x)/(sin(c + d*x) + 1), x)/a
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(c+d\,x)\,{(e+f\,x)}^m}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
[Out] int((sin(c + d*x)*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

# 3.217 $\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=23

$$\operatorname{Int}\left(\frac{(e+fx)^m}{a+a\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
[Out] Defer[Int][(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
[Out] Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m/(sin(c + d*x) + 1), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e+f\,x)^m}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(a + a*sin(c + d*x)),x)
[Out] int((e + f*x)^m/(a + a*sin(c + d*x)), x)
```

# 3.218 $\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)} \, dx$

**Optimal**. Leaf size=29

$$\text{Int}\left(\frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)} \, dx = \int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)} \, dx$$

**Mathematica** [A]
time = 20.81, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m \csc(dx+c)}{a+a\sin(dx+c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] int((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m \csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(e+f\,x)^m}{\sin(c+d\,x)\,(a+a\,\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
[Out] int((e + f*x)^m/(sin(c + d*x)*(a + a*sin(c + d*x))), x)
```

# 3.219 $\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=31

$$\operatorname{Int}\left(\frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 20.58, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m \left(\csc^2(dx+c)\right)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
[Out] int((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m \csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(e+f\,x)^m}{\sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
[Out] int((e + f*x)^m/(sin(c + d*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.220 $\int \frac{(e+fx)^3 \sin(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=544

$$\frac{(e+fx)^4}{4bf}+\frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}-\frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}+\frac{3af(e+fx)^2\text{Li}_2\left(\cdots\right.}{b\sqrt{a^2-b\cdots}}$$

```
[Out] 1/4*(f*x+e)^4/b/f+I*a*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2))
)/b/d/(a^2-b^2)^(1/2)-I*a*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1
/2)))/b/d/(a^2-b^2)^(1/2)+3*a*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(
a^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)-3*a*f*(f*x+e)^2*polylog(2,I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)+6*I*a*f^2*(f*x+e)*polyl
og(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^3/(a^2-b^2)^(1/2)-6*I*a*f^
2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^3/(a^2-b^2)
^(1/2)-6*a*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^4/(a^2
-b^2)^(1/2)+6*a*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^4
/(a^2-b^2)^(1/2)
```

**Rubi** [A]

time = 0.66, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4611, 32, 3404, 2296, 2221, 2611, 6744, 2320, 6724}

$$-\frac{6af^3\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4\sqrt{a^2-b^2}}+\frac{6af^3\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^4\sqrt{a^2-b^2}}+\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{3af(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{3af(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}-\frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd\sqrt{a^2-b^2}}+\frac{(e+fx)^4}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*b*f) + (I*a*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a*(e + f*x)^3*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) + (3*a*f*(e + f
*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2
- b^2]*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3,
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((6
*I)*a*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(b*Sqrt[a^2 - b^2]*d^3) - (6*a*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a -
Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) + (6*a*f^3*PolyLog[4, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\,dx}{b} - \frac{a\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{(2a)\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b}$$

$$= \frac{(e+fx)^4}{4bf} + \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)^3}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)^3}{2a+2\sqrt{a^2-b^2}-2ibe\cdots}}{\sqrt{a^2-b^2}}$$

$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)^3\log\left(1-\frac{\cdots}{a+\cdots}\right.}{b\sqrt{a^2-b^2}\,d}$$

$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)^3\log\left(1-\frac{\cdots}{a+\cdots}\right.}{b\sqrt{a^2-b^2}\,d}$$

$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)^3\log\left(1-\frac{\cdots}{a+\cdots}\right.}{b\sqrt{a^2-b^2}\,d}$$

$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)^3\log\left(1-\frac{\cdots}{a+\cdots}\right.}{b\sqrt{a^2-b^2}\,d}$$

$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)^3\log\left(1-\frac{\cdots}{a+\cdots}\right.}{b\sqrt{a^2-b^2}\,d}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1528 vs. 2(544) = 1088.
time = 2.50, size = 1528, normalized size = 2.81

---

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) - (I*a*((3*I)*Sqrt[a^
2 - b^2]*d^3*e^2*f*x*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*C
os[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*S
in[c]) + (3*I)*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*(Cos[2*c + d*x] + I
*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] -
a*Sin[c])]*(Cos[c] + I*Sin[c]) + I*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*(
Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c]
 + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^2*f*
(e + f*x)^2*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c
] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[
c]) - 3*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*(Cos[2*c + d*x] + I
*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2]
 + a*Sin[c])]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3
, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)
*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2
- b^2]*d*f^3*x*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Co
s[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*S
in[c]) - 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -((b*(Cos[2*c + d*x] + I*Sin[2*c
+ d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])
)]*(Cos[c] + I*Sin[c]) + 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*(Cos[2*c + d*x
] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c
])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[
1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b
^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 3*Sqrt[a^2
 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)
*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos
[c] + Sin[c]) + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*(Cos[2*c + d*x] + I*
Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2]
+ a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3,
(b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*
(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b
^2]*d*f^3*x*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[
c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + S
in[c]) - (2*I)*d^3*e^3*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqr
t[a^2 - b^2]]*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))/(b*Sqrt[a^2 - b^
2]*d^4*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])])
```

**Maple [F]**
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \sin(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2317 vs. 2(475) = 950.
time = 0.57, size = 2317, normalized size = 4.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - b^2)*d^4*f^3*x^4 + 4*(a^2 - b^2)*d^4*f^2*x^3*e + 6*(a^2 - b^2)*
d^4*f*x^2*e^2 + 4*(a^2 - b^2)*d^4*x*e^3 + 12*I*a*b*f^3*sqrt(-(a^2 - b^2)/b^
2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*a*b*f^3*sqrt(-(a^2 - b^2)/b^
2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*a*b*f^3*sqrt(-(a^2 - b^2)/b^
2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*I*a*b*f^3*sqrt(-(a^2 - b^2)/b
^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(-I*a*b*d^2*f^3*x^2 - 2*I*a*b
*d^2*f^2*x*e - I*a*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x +
 c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
```

```
)/b^2) - b)/b + 1) + 6*(I*a*b*d^2*f^3*x^2 + 2*I*a*b*d^2*f^2*x*e + I*a*b*d^2
*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(
I*a*b*d^2*f^3*x^2 + 2*I*a*b*d^2*f^2*x*e + I*a*b*d^2*f*e^2)*sqrt(-(a^2 - b^2
)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(-I*a*b*d^2*f^3*x^2 - 2*
I*a*b*d^2*f^2*x*e - I*a*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos
(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b + 1) + 2*(a*b*c^3*f^3 - 3*a*b*c^2*d*f^2*e + 3*a*b*c*d^2
*f*e^2 - a*b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(a*b*c^3*f^3 - 3*a*b*
c^2*d*f^2*e + 3*a*b*c*d^2*f*e^2 - a*b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2
*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)
- 2*(a*b*c^3*f^3 - 3*a*b*c^2*d*f^2*e + 3*a*b*c*d^2*f*e^2 - a*b*d^3*e^3)*sqr
t(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-
(a^2 - b^2)/b^2) + 2*I*a) - 2*(a*b*c^3*f^3 - 3*a*b*c^2*d*f^2*e + 3*a*b*c*d^
2*f*e^2 - a*b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b
*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(a*b*d^3*f^3*x^3 +
a*b*c^3*f^3 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*e^2 + 3*(a*b*d^3*f^2*x^2 - a*b*
c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c
) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*
(a*b*d^3*f^3*x^3 + a*b*c^3*f^3 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*e^2 + 3*(a*b
*d^3*f^2*x^2 - a*b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x +
 c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b) + 2*(a*b*d^3*f^3*x^3 + a*b*c^3*f^3 + 3*(a*b*d^3*f*x + a*b*c*
d^2*f)*e^2 + 3*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*
log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*(a*b*d^3*f^3*x^3 + a*b*c^3*f^3 + 3*(
a*b*d^3*f*x + a*b*c*d^2*f)*e^2 + 3*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2)*e)*sqr
t(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 12*(a*b*d*f^3*x +
a*b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d
*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) +
12*(a*b*d*f^3*x + a*b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(
d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2))/b) - 12*(a*b*d*f^3*x + a*b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*pol
ylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a*b*d*f^3*x + a*b*d*f^2*e)*sqrt(-(
a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b))/((a^2*b - b^3)*d^4)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sin(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)\,{\left(e+f\,x\right)}^3}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] int((sin(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)), x)
```

# 3.221 $\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=408

$$\frac{(e+fx)^3}{3bf}+\frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}-\frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}+\frac{2af(e+fx)\mathrm{Li}_2\left(\frac{i}{a-}\right.}{b\sqrt{a^2-b^2}}$$

```
[Out] 1/3*(f*x+e)^3/b/f+I*a*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2))
)/b/d/(a^2-b^2)^(1/2)-I*a*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1
/2)))/b/d/(a^2-b^2)^(1/2)+2*a*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^
2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)-2*a*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x
+c))/(a+(a^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)+2*I*a*f^2*polylog(3,I*b*exp
(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^3/(a^2-b^2)^(1/2)-2*I*a*f^2*polylog(3,
I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^3/(a^2-b^2)^(1/2)
```

**Rubi** [A]

time = 0.56, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4611, 32, 3404, 2296, 2221, 2611, 2320, 6724}

$$\frac{2iaf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{2iaf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{2af(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{2af(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}-\frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd\sqrt{a^2-b^2}}+\frac{(e+fx)^3}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(3*b*f) + (I*a*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a*(e + f*x)^2*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) + (2*a*f*(e + f
*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
b^2]*d^2) - (2*a*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + ((2*I)*a*f^2*PolyLog[3, (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a*f^2*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3
)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\,dx}{b} - \frac{a\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{(2a)\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c}}}{\sqrt{a^2-b^2}}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)^2\log\left(1-\frac{ibe}{a+\sqrt{}}\right.}{b\sqrt{a^2-b^2}\,d}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)^2\log\left(1-\frac{ibe}{a+\sqrt{}}\right.}{b\sqrt{a^2-b^2}\,d}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)^2\log\left(1-\frac{ibe}{a+\sqrt{}}\right.}{b\sqrt{a^2-b^2}\,d}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)^2\log\left(1-\frac{ibe}{a+\sqrt{}}\right.}{b\sqrt{a^2-b^2}\,d}$$

**Mathematica** [**A**]
time = 1.98, size = 702, normalized size = 1.72

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) - (I*a*(2*Sqrt[a^2 - b^2]*d*f*(e + f*
x)*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[
(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 2*S
qrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*
x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]
*(Cos[c] + I*Sin[c]) - I*(-2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*(Cos[2*c +
```

```
d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[
c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3,
 (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)
*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c]) + d^2*(Sqrt[a^2 -
b^2]*f*x*(2*e + f*x)*(-Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a
*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])] + Log[1 - (
b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(
Cos[c] + I*Sin[c])^2] + a*Sin[c])])*(Cos[c] + I*Sin[c]) + 2*e^2*ArcTan[(b*C
os[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]]*Sqrt[(-a^2 + b^2)*(C
os[2*c] + I*Sin[2*c])]))))/(b*Sqrt[a^2 - b^2]*d^3*Sqrt[(-a^2 + b^2)*(Cos[2*
c] + I*Sin[2*c])])
```

**Maple [F]**
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \sin(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1654 vs. 2(353) = 706.
time = 0.51, size = 1654, normalized size = 4.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^2 - b^2)*d^3*f^2*x^3 + 6*(a^2 - b^2)*d^3*f*x^2*e + 6*(a^2 - b^2)*
d^3*x*e^2 - 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c)
```

```
+ a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2))/b) + 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))
/b) - 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*s
in(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
) + 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin
(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
+ 6*(-I*a*b*d*f^2*x - I*a*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*
x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) - b)/b + 1) + 6*(I*a*b*d*f^2*x + I*a*b*d*f*e)*sqrt(-(a^2 - b^2)/b
^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(I*a*b*d*f^2*x + I*a*b*d*f*e
)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos
(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(-I*a*
b*d*f^2*x - I*a*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) -
a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 - b)/b + 1) - 3*(a*b*c^2*f^2 - 2*a*b*c*d*f*e + a*b*d^2*e^2)*sqrt(-(a^2 - b
^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/
b^2) + 2*I*a) - 3*(a*b*c^2*f^2 - 2*a*b*c*d*f*e + a*b*d^2*e^2)*sqrt(-(a^2 -
b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) - 2*I*a) + 3*(a*b*c^2*f^2 - 2*a*b*c*d*f*e + a*b*d^2*e^2)*sqrt(-(a^2 -
 b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) + 2*I*a) + 3*(a*b*c^2*f^2 - 2*a*b*c*d*f*e + a*b*d^2*e^2)*sqrt(-(a^2
 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
b^2)/b^2) - 2*I*a) + 3*(a*b*d^2*f^2*x^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*x + a*
b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c)
+ (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*(a
*b*d^2*f^2*x^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*x + a*b*c*d*f)*e)*sqrt(-(a^2 -
b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 3*(a*b*d^2*f^2*x^2 - a*b*c^2*
f^2 + 2*(a*b*d^2*f*x + a*b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(
d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b) - 3*(a*b*d^2*f^2*x^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*x + a
*b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c
) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a
^2*b - b^3)*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sin(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)\,{\left(e+f\,x\right)}^2}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
[Out] int((sin(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)), x)
```

# 3.222 $\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=267

$$\frac{ex}{b}+\frac{fx^2}{2b}+\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}-\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}+\frac{af\operatorname{Li}_2\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}-$$

```
[Out] e*x/b+1/2*f*x^2/b+I*a*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/
b/d/(a^2-b^2)^(1/2)-I*a*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2))
)/b/d/(a^2-b^2)^(1/2)+a*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))
/b/d^2/(a^2-b^2)^(1/2)-a*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2))
)/b/d^2/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4611, 3404, 2296, 2221, 2317, 2438}

$$\frac{af\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{af\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}-\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd\sqrt{a^2-b^2}}+\frac{ex}{b}+\frac{fx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (e*x)/b + (f*x^2)/(2*b) + (I*a*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a -
 Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a*(e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) + (a*f*PolyLog[
2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) -
(a*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
 b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\,dx}{b} - \frac{a\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{(2a)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+}}}{\sqrt{a^2-b^2}}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+}}{a+\sqrt{a^2}}\right.}{b\sqrt{a^2-b^2}\,d}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+}}{a+\sqrt{a^2}}\right.}{b\sqrt{a^2-b^2}\,d}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+}}{a+\sqrt{a^2}}\right.}{b\sqrt{a^2-b^2}\,d}$$

__Mathematica__ [A]
time = 2.85, size = 343, normalized size = 1.28

$$\frac{x(2e+fx)}{2b} + \frac{a\left(-\frac{2de\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{e^{ic}f\left(dx\left(\log\left(1+\frac{be^{i(2c+dx)}}{iae^{ic}-\sqrt{(-a^2+b^2)e^{2ic}}}\right)-\log\left(1+\frac{be^{i(2c+dx)}}{iae^{ic}+\sqrt{(-a^2+b^2)e^{2ic}}}\right)\right)-i\text{Li}_2\left(\frac{ibe^{i(2c+dx)}}{ae^{ic}+i\sqrt{(-a^2+b^2)e^{2ic}}}\right)+i\text{Li}_2\left(-\frac{be^{i(2c+dx)}}{iae^{ic}+\sqrt{(-a^2+b^2)e^{2ic}}}\right)\right)}{\sqrt{(-a^2+b^2)e^{2ic}}}\right)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (x*(2*e + f*x))/(2*b) + (a*((-2*d*e*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a
^2 - b^2]])/Sqrt[a^2 - b^2] - (E^(I*c)*f*(d*x*(Log[1 + (b*E^(I*(2*c + d*x))
)/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - Log[1 + (b*E^(I*(2*c +
d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]) - I*PolyLog[2, (I*b
*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + I*Pol
yLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c
)]))]))/Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))/(b*d^2)
```

__Maple__ [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(235) = 470.
time = 0.11, size = 548, normalized size = 2.05

| method | result |
|---|---|
| risch | $\frac{f\,x^2}{2b}+\frac{ex}{b}-\frac{2iae\arctan\left(\frac{2ib\,\mathrm{e}^{i(dx+c)}-2a}{2\sqrt{-a^2+b^2}}\right)}{db\sqrt{-a^2+b^2}}-\frac{af\ln\left(\frac{ia+b\,\mathrm{e}^{i(dx+c)}-\sqrt{-a^2+b^2}}{ia-\sqrt{-a^2+b^2}}\right)x}{db\sqrt{-a^2+b^2}}-\frac{af\ln\left(\frac{ia+b\,\mathrm{e}^{i(dx+c)}-\sqrt{-a^2+}}{ia-\sqrt{-a^2+b^2}}\right.}{d^2b\sqrt{-a^2+b^2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/b+e*x/b-2*I/d/b*a*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x
+c))-2*a)/(-a^2+b^2)^(1/2))-1/d/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x
+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-1/d^2/b*a*f/(-a^2+b^2)^(1/
2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1/d
/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a
^2+b^2)^(1/2)))*x+1/d^2/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a
^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-I/d^2/b*a*f/(-a^2+b^2)^(1/2)*dilog
((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+I/d^2/b*a*
f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2
+b^2)^(1/2)))+2*I/d^2/b*a*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x
+c))-2*a)/(-a^2+b^2)^(1/2))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(230) = 460$.
time = 0.54, size = 1057, normalized size = 3.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - b^2)*d^2*f*x^2 + 2*(a^2 - b^2)*d^2*x*e - I*a*b*f*sqrt(-(a^2 - b
^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*s
```

```
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*a*b*f*sqrt(-(a^2 - b^2)
/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*a*b*f*sqrt(-(a^2 - b^2)/b^
2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*a*b*f*sqrt(-(a^2 - b^2)/b^2)
*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (a*b*c*f - a*b*d*e)*sqrt(-(a^2 -
 b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a) + (a*b*c*f - a*b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*
x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (a*b*c*
f - a*b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (a*b*c*f - a*b*d*e)*sqrt(-(a^2
- b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b
^2)/b^2) - 2*I*a) + (a*b*d*f*x + a*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b) - (a*b*d*f*x + a*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(
-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b) + (a*b*d*f*x + a*b*c*f)*sqrt(-(a^2 - b^2)/b^2
)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a*b*d*f*x + a*b*c*f)*sqrt(-(a^2 - b
^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^2*b - b^3)*d^2)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)*sin(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)\,(e+f\,x)}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)
[Out] int((sin(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)), x)
```

# 3.223 $\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=57

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}$$

```
[Out] x/b-2*a*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/d/(a^2-b^2)^(1/2
)
```

**Rubi** [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2814, 2739, 632, 210}

$$\frac{x}{b}-\frac{2a\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] x/b - (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 -
b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sin(c+dx)}\,dx}{b}\\ &= \frac{x}{b}-\frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\ &= \frac{x}{b}+\frac{(4a)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\ &= \frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}\end{aligned}$$

**Mathematica [A]**
time = 0.08, size = 59, normalized size = 1.04

$$\frac{\frac{c}{d}+x-\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]),x]
[Out] (c/d + x - (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2
 - b^2]*d))/b
```

**Maple [A]**
time = 0.06, size = 68, normalized size = 1.19

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$ | 68 |
| default | $\frac{-\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$ | 68 |

| | | |
|---|---|---|
| risch | $\frac{x}{b}-\frac{ia\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,db}+\frac{ia\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,db}$ | 149 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))+2/b*arctan(tan(1/2*d*x+1/2*c)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**A**]
time = 0.38, size = 237, normalized size = 4.16

$$\left[\frac{2\,(a^2-b^2)dx-\sqrt{-a^2+b^2}\,a\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)}{2\,(a^2b-b^3)d},\ \frac{(a^2-b^2)dx+\sqrt{a^2-b^2}\,a\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)}{(a^2b-b^3)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*a*log(-((2*a^2 - b^2)*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*
cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2)))/((a^2*b - b^3)*d), ((a^2 - b^2)*d*x + sqrt(a^2 - b^2)*a*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/((a^2*b - b^3)*d)]
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(44) = 88$.

time = 38.05, size = 335, normalized size = 5.88

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ -\frac{\cos(c+dx)}{ad} & \text{for } b = 0 \\ \frac{x\sin(c)}{a+b\sin(c)} & \text{for } d = 0 \\ \frac{bdx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-bd\sqrt{b^2}} + \frac{2b}{b^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-bd\sqrt{b^2}} - \frac{dx\sqrt{b^2}}{b^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-bd\sqrt{b^2}} & \text{for } a = -\sqrt{b^2} \\ \frac{bdx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+bd\sqrt{b^2}} + \frac{2b}{b^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+bd\sqrt{b^2}} + \frac{dx\sqrt{b^2}}{b^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+bd\sqrt{b^2}} & \text{for } a = \sqrt{b^2} \\ -\frac{a\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{bd\sqrt{-a^2+b^2}} + \frac{a\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{bd\sqrt{-a^2+b^2}} + \frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b, Eq(a, 0)), (-cos(c
 + d*x)/(a*d), Eq(b, 0)), (x*sin(c)/(a + b*sin(c)), Eq(d, 0)), (b*d*x*tan(c
/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2) - b*d*sqrt(b**2)) + 2*b/(b**2*d*tan(c/
2 + d*x/2) - b*d*sqrt(b**2)) - d*x*sqrt(b**2)/(b**2*d*tan(c/2 + d*x/2) - b*
d*sqrt(b**2)), Eq(a, -sqrt(b**2))), (b*d*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2
 + d*x/2) + b*d*sqrt(b**2)) + 2*b/(b**2*d*tan(c/2 + d*x/2) + b*d*sqrt(b**2)
) + d*x*sqrt(b**2)/(b**2*d*tan(c/2 + d*x/2) + b*d*sqrt(b**2)), Eq(a, sqrt(b
**2))), (-a*log(tan(c/2 + d*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(b*d*sqrt(-a
**2 + b**2)) + a*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 + b**2)/a)/(b*d*sq
rt(-a**2 + b**2)) + x/b, True))
```

**Giac** [A]
time = 5.11, size = 77, normalized size = 1.35

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}\,b} - \frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a/(sqrt(a^2 - b^2)*b) - (d*x + c)/b)/d
```

**Mupad** [B]

time = 2.00, size = 139, normalized size = 2.44

$$\frac{x}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b-3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^3+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^4}{\left(b^2-a^2\right)^{3/2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{b\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*sin(c + d*x)),x)
```

```
[Out] x/b - (2*a*atanh((a^4*sin(c/2 + (d*x)/2) + 2*b^4*sin(c/2 + (d*x)/2) + a*b^3
*cos(c/2 + (d*x)/2) - a^3*b*cos(c/2 + (d*x)/2) - 3*a^2*b^2*sin(c/2 + (d*x)/
2))/((b^2 - a^2)^(3/2)*(a*cos(c/2 + (d*x)/2) + 2*b*sin(c/2 + (d*x)/2)))))/(
b*d*(b^2 - a^2)^(1/2))
```

# 3.224 $\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=643

$$-\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cos(c+dx)}{bd^3}-\frac{(e+fx)^3\cos(c+dx)}{bd}-\frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}+ ia^2$$

```
[Out] -1/4*a*(f*x+e)^4/b^2/f+6*f^2*(f*x+e)*cos(d*x+c)/b/d^3-(f*x+e)^3*cos(d*x+c)/
b/d-6*f^3*sin(d*x+c)/b/d^4+3*f*(f*x+e)^2*sin(d*x+c)/b/d^2-I*a^2*(f*x+e)^3*l
n(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d/(a^2-b^2)^(1/2)+I*a^2*(f*
x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d/(a^2-b^2)^(1/2)-3
*a^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d^2/
(a^2-b^2)^(1/2)+3*a^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)
^(1/2)))/b^2/d^2/(a^2-b^2)^(1/2)-6*I*a^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d
*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d^3/(a^2-b^2)^(1/2)+6*I*a^2*f^2*(f*x+e)*pol
ylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d^3/(a^2-b^2)^(1/2)+6*a^
2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d^4/(a^2-b^2)^(
1/2)-6*a^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d^4/(a
^2-b^2)^(1/2)
```

**Rubi** [A]

time = 0.78, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {4611, 3377, 2717, 32, 3404, 2296, 2221, 2611, 6744, 2320, 6724}

$$\frac{6a^2f^3\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^4\sqrt{a^2-b^2}}-\frac{6a^2f^3\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^4\sqrt{a^2-b^2}}-\frac{6ia^2f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3\sqrt{a^2-b^2}}+\frac{6ia^2f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^3\sqrt{a^2-b^2}}-\frac{3a^2f(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2\sqrt{a^2-b^2}}+\frac{3a^2f(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2\sqrt{a^2-b^2}}-\frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}+\frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{a(e+fx)^4}{4b^2f}-\frac{6f^3\sin(c+dx)}{bd^4}+\frac{6f^2(e+fx)\cos(c+dx)}{bd^3}+\frac{3f(e+fx)^2\sin(c+dx)}{bd^2}-\frac{(e+fx)^3\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(a*(e + f*x)^4)/(b^2*f) + (6*f^2*(e + f*x)*Cos[c + d*x])/(b*d^3) - ((e
 + f*x)^3*Cos[c + d*x])/(b*d) - (I*a^2*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) + (I*a^2*(e + f*x)^3*
Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*
d) - (3*a^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) - ((6*I)*a
^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
b^2*Sqrt[a^2 - b^2]*d^3) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^3) + (6*a^2*f^3*Po
lyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]
*d^4) - (6*a^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])
/(b^2*Sqrt[a^2 - b^2]*d^4) - (6*f^3*Sin[c + d*x])/(b*d^4) + (3*f*(e + f*x)^
2*Sin[c + d*x])/(b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sin(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{a\int (e+fx)^3\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{b^2} + \frac{(3f)\int (e$$

$$= -\frac{a(e+fx)^4}{4b^2f} - \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{3f(e+fx)^2\sin(c+dx)}{bd^2} + \frac{(2a^2)\int \frac{}{ib}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{3f(e+f$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1590 vs. $2(643) = 1286$.
time = 2.70, size = 1590, normalized size = 2.47

---

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)) - 4*b*d*(e + f*x)*(
-6*f^2 + d^2*(e + f*x)^2)*Cos[c + d*x] + ((4*I)*a^2*((3*I)*Sqrt[a^2 - b^2]*
d^3*e^2*f*x*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + S
```

```
qrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) +
(3*I)*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c
+ d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])
]*(Cos[c] + I*Sin[c]) + I*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*(Cos[2*c +
 d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[
c])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)
^2*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[
(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 3*S
qrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*(Cos[2*c + d*x] + I*Sin[2*c
+ d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[
c])]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*(C
os[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c]
+ I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*
f^3*x*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sq
rt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) -
6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/
(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c
] + I*Sin[c]) + 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*(Cos[2*c + d*x] + I*Sin
[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a
*Sin[c])]*(Cos[c] + I*Sin[c]) - (2*I)*d^3*e^3*ArcTan[(b*Cos[c + d*x] + I*(a
 + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]]*Sqrt[-((a^2 - b^2)*(Cos[c] + I*Sin[c])
^2)] + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c
 + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin
[c])]*((-I)*Cos[c] + Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*(
Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos
[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + Sqrt[a^2 - b^2]*d^
3*f^3*x^3*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] +
Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]
) + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d
*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])
]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, (b*(Cos[2*c
 + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I
*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c])))/(Sqrt[a^2 - b^2]*Sqrt[-((
a^2 - b^2)*(Cos[c] + I*Sin[c])^2)]) + 12*b*f*(-2*f^2 + d^2*(e + f*x)^2)*Sin
[c + d*x])/(4*b^2*d^4)
```

**Maple [F]**
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\sin^2(dx+c)\right)}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2665 vs. 2(577) = 1154.
time = 0.62, size = 2665, normalized size = 4.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((a^3 - a*b^2)*d^4*f^3*x^4 + 4*(a^3 - a*b^2)*d^4*f^2*x^3*e + 6*(a^3 -
a*b^2)*d^4*f*x^2*e^2 + 12*I*a^2*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I
*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))/b) - 12*I*a^2*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -
(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2))/b) - 12*I*a^2*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4,
 -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b) + 12*I*a^2*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog
(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*(a^3 - a*b^2)*d^4*x*e^3 - 6*(I*a^2*b*d^2
*f^3*x^2 + 2*I*a^2*b*d^2*f^2*x*e + I*a^2*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2
)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a^2*b*d^2*f^3*x^2 - 2*I*a^
2*b*d^2*f^2*x*e - I*a^2*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - 6*(-I*a^2*b*d^2*f^3*x^2 - 2*I*a^2*b*d^2*f^2*x*e -
 I*a^2*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin
(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)
/b + 1) - 6*(I*a^2*b*d^2*f^3*x^2 + 2*I*a^2*b*d^2*f^2*x*e + I*a^2*b*d^2*f*e^
2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(a^2*
b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*e + 3*a^2*b*c*d^2*f*e^2 - a^2*b*d^3*e^3)*sqrt
(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a
```

```
^2 - b^2)/b^2) + 2*I*a) + 2*(a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*e + 3*a^2*b*
c*d^2*f*e^2 - a^2*b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) -
2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(a^2*b*c^3*f^3
 - 3*a^2*b*c^2*d*f^2*e + 3*a^2*b*c*d^2*f*e^2 - a^2*b*d^3*e^3)*sqrt(-(a^2 -
b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a) - 2*(a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*e + 3*a^2*b*c*d^2*f*
e^2 - a^2*b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(a^2*b*d^3*f^3*x^3 +
a^2*b*c^3*f^3 + 3*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*e^2 + 3*(a^2*b*d^3*f^2*x^
2 - a^2*b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*s
in(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b) - 2*(a^2*b*d^3*f^3*x^3 + a^2*b*c^3*f^3 + 3*(a^2*b*d^3*f*x + a^2*b*c*d
^2*f)*e^2 + 3*(a^2*b*d^3*f^2*x^2 - a^2*b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^
2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*(a^2*b*d^3*f^3*x^3 + a^2*b*c^3*f^3
 + 3*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*e^2 + 3*(a^2*b*d^3*f^2*x^2 - a^2*b*c^2
*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c)
+ (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*(a
^2*b*d^3*f^3*x^3 + a^2*b*c^3*f^3 + 3*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*e^2 +
3*(a^2*b*d^3*f^2*x^2 - a^2*b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*
a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) - b)/b) - 12*(a^2*b*d*f^3*x + a^2*b*d*f^2*e)*sqrt(-(a^2 -
 b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c)
 - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a^2*b*d*f^3*x + a^2*b
*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x
+ c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*
(a^2*b*d*f^3*x + a^2*b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*co
s(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2))/b) + 12*(a^2*b*d*f^3*x + a^2*b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^
2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*((a^2*b - b^3)*d^3*f^3*x^3 + 3
*(a^2*b - b^3)*d^3*f*x*e^2 - 6*(a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d^3*e^
3 + 3*((a^2*b - b^3)*d^3*f^2*x^2 - 2*(a^2*b - b^3)*d*f^2)*e)*cos(d*x + c) -
 12*((a^2*b - b^3)*d^2*f^3*x^2 + 2*(a^2*b - b^3)*d^2*f^2*x*e + (a^2*b - b^3
)*d^2*f*e^2 - 2*(a^2*b - b^3)*f^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d^4)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.225 $\int \frac{(e+fx)^2 \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx$

**Optimal**. Leaf size=479

$$-\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cos(c+dx)}{bd^3}-\frac{(e+fx)^2\cos(c+dx)}{bd}-\frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}+\frac{ia^2(e+fx)}{}$$

```
[Out] -1/3*a*(f*x+e)^3/b^2/f+2*f^2*cos(d*x+c)/b/d^3-(f*x+e)^2*cos(d*x+c)/b/d+2*f*
(f*x+e)*sin(d*x+c)/b/d^2-I*a^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/b^2/d/(a^2-b^2)^(1/2)+I*a^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a
+(a^2-b^2)^(1/2)))/b^2/d/(a^2-b^2)^(1/2)-2*a^2*f*(f*x+e)*polylog(2,I*b*exp(
I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d^2/(a^2-b^2)^(1/2)+2*a^2*f*(f*x+e)*pol
ylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d^2/(a^2-b^2)^(1/2)-2*I*
a^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d^3/(a^2-b^2)
^(1/2)+2*I*a^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d^
3/(a^2-b^2)^(1/2)
```

**Rubi** [A]

time = 0.65, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4611, 3377, 2718, 32, 3404, 2296, 2221, 2611, 2320, 6724}

$$-\frac{2ia^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3\sqrt{a^2-b^2}}+\frac{2ia^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^3\sqrt{a^2-b^2}}-\frac{2a^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2\sqrt{a^2-b^2}}+\frac{2a^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2\sqrt{a^2-b^2}}-\frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}+\frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cos(c+dx)}{bd^3}+\frac{2f(e+fx)\sin(c+dx)}{bd^2}-\frac{(e+fx)^2\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*(e + f*x)^3)/(b^2*f) + (2*f^2*Cos[c + d*x])/(b*d^3) - ((e + f*x)^2*
Cos[c + d*x])/(b*d) - (I*a^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a -
 Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) + (I*a^2*(e + f*x)^2*Log[1 - (I
*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) - (2*a^
2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2
*Sqrt[a^2 - b^2]*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))
/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) - ((2*I)*a^2*f^2*PolyLog
[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^3)
 + ((2*I)*a^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(b^2*Sqrt[a^2 - b^2]*d^3) + (2*f*(e + f*x)*Sin[c + d*x])/(b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
```

```
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sin(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{a\int (e+fx)^2\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{b^2} + \frac{(2f)\int (e$$

$$= -\frac{a(e+fx)^3}{3b^2f} - \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{2f(e+fx)\sin(c+dx)}{bd^2} + \frac{(2a^2)\int \frac{}{i}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{2f(e+fx)\sin(c}{bd^2}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^2\log}{b^2\sqrt{}}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^2\log}{b^2\sqrt{}}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^2\log}{b^2\sqrt{}}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^2\log}{b^2\sqrt{}}$$

**Mathematica** [**A**]
time = 1.82, size = 786, normalized size = 1.64

$$\frac{-ax\left(3e^2+3efx+f^2x^2\right)+\frac{3ia^2\left(-i\left(d^2\left(\sqrt{a^2-b^2}fx(2e+fx)\left(-\log\left(1+\frac{b(\cos(2c+dx)+i\sin(2c+dx))}{ia\cos(c)+\sqrt{(-a^2+b^2)(\cos(c)+i\sin(c))^2}-a\sin(c)}\right)+\log\left(1-\frac{b(\cos(2c+dx)+i\sin(2c+dx))}{-ia\cos(c)+\sqrt{(-a^2+b^2)(\cos(c)+i\sin(c))^2}+a\sin(c)}\right)\right)(\cos(c)+i\sin(c))+2e^2\tan^{-1}\left(\frac{b\cos(c+dx)+i(a+b\sin(c+dx))}{\sqrt{a^2-b^2}}\right)\sqrt{-\left((a^2-b^2)(\cos(c)+i\sin(c))^2\right)}\right)-2\sqrt{a^2-b^2}f^2\text{Li}_3\left(-\frac{b(\cos(2c+dx)+i\sin(2c+dx))}{ia\cos(c)+\sqrt{(-a^2+b^2)(\cos(c)+i\sin(c))^2}-a\sin(c)}\right)(\cos(c)+i\sin(c))+2\sqrt{a^2-b^2}f^2\text{Li}_3\left(\frac{b(\cos(2c+dx)+i\sin(2c+dx))}{-ia\cos(c)+\sqrt{(-a^2+b^2)(\cos(c)+i\sin(c))^2}+a\sin(c)}\right)(\cos(c)+i\sin(c))+2\sqrt{a^2-b^2}df(e+fx)\text{Li}_2\left(-\frac{b(\cos(2c+dx)+i\sin(2c+dx))}{ia\cos(c)+\sqrt{(-a^2+b^2)(\cos(c)+i\sin(c))^2}-a\sin(c)}\right)(\cos(c)+i\sin(c))-2\sqrt{a^2-b^2}df(e+fx)\text{Li}_2\left(\frac{b(\cos(2c+dx)+i\sin(2c+dx))}{-ia\cos(c)+\sqrt{(-a^2+b^2)(\cos(c)+i\sin(c))^2}+a\sin(c)}\right)(\cos(c)+i\sin(c))\right)\right)}{\sqrt{a^2-b^2}d^3\sqrt{-\left((a^2-b^2)(\cos(c)+i\sin(c))^2\right)}}-\frac{3b\cos(dx)\left(\left(-2f^2+d^2(e+fx)^2\right)\cos(c)-2df(e+fx)\sin(c)\right)}{d^3}+\frac{3b\left(2df(e+fx)\cos(c)+\left(-2f^2+d^2(e+fx)^2\right)\sin(c)\right)\sin(dx)}{d^3}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + ((3*I)*a^2*((-I)*(d^2*(Sqrt[a^2 - b^2
]*f*x*(2*e + f*x)*(-Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Co
s[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])] + Log[1 - (b*(
Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos
[c] + I*Sin[c])^2] + a*Sin[c])])*(Cos[c] + I*Sin[c]) + 2*e^2*ArcTan[(b*Cos[
c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]]*Sqrt[-((a^2 - b^2)*(Cos
[c] + I*Sin[c])^2)]) - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*(Cos[2*c + d*x
] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^
2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*
(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Co
s[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c])) + 2*Sqrt[a^2 - b^2]*d
*f*(e + f*x)*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[
c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin
[c]) - 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*(Cos[2*c + d*x] + I*Si
n[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] +
a*Sin[c])]*(Cos[c] + I*Sin[c])))/(Sqrt[a^2 - b^2]*d^3*Sqrt[-((a^2 - b^2)*(C
os[c] + I*Sin[c])^2)]) - (3*b*Cos[d*x]*((-2*f^2 + d^2*(e + f*x)^2)*Cos[c] -
 2*d*f*(e + f*x)*Sin[c]))/d^3 + (3*b*(2*d*f*(e + f*x)*Cos[c] + (-2*f^2 + d^
2*(e + f*x)^2)*Sin[c])*Sin[d*x])/d^3)/(3*b^2)
```

**Maple** [**F**]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{2} \left(\sin^{2}\left(dx+c\right)\right)}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1864 vs. 2(426) = 852.
time = 0.56, size = 1864, normalized size = 3.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(a^3 - a*b^2)*d^3*f^2*x^3 + 6*(a^3 - a*b^2)*d^3*f*x^2*e - 6*a^2*b*f
^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) +
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*a^2*b*f^
2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*a^2*b*f^2
*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*a^2*b*f^2
*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(a^3 - a*
b^2)*d^3*x*e^2 - 6*(I*a^2*b*d*f^2*x + I*a^2*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)
*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a^2*b*d*f^2*x - I*a^2*b*d*f
*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a
^2*b*d*f^2*x - I*a^2*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x +
c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) - b)/b + 1) - 6*(I*a^2*b*d*f^2*x + I*a^2*b*d*f*e)*sqrt(-(a^2 - b^2)/b
^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(a^2*b*c^2*f^2 - 2*a^2*b*c*
d*f*e + a^2*b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*
sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 3*(a^2*b*c^2*f^2 - 2*a
^2*b*c*d*f*e + a^2*b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) -
 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 3*(a^2*b*c^2*f^
2 - 2*a^2*b*c*d*f*e + a^2*b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*
x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 3*(a^2*
b*c^2*f^2 - 2*a^2*b*c*d*f*e + a^2*b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*
b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) +
 3*(a^2*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*e)*
sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*(a^2*b*d^2*f^2
```

```
*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*e)*sqrt(-(a^2 - b^2)
/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 3*(a^2*b*d^2*f^2*x^2 - a^2*b*c^2*
f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b) - 3*(a^2*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d
^2*f*x + a^2*b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b) + 6*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*d^2*f*x*e + (a^2*b
 - b^3)*d^2*e^2 - 2*(a^2*b - b^3)*f^2)*cos(d*x + c) - 12*((a^2*b - b^3)*d*f
^2*x + (a^2*b - b^3)*d*f*e)*sin(d*x + c))/((a^2*b^2 - b^4)*d^3)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.226 $\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=311

$$-\frac{aex}{b^2}-\frac{afx^2}{2b^2}-\frac{(e+fx)\cos(c+dx)}{bd}-\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}+\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-}}\right.}{b^2\sqrt{a^2-b^2}\,d}$$

```
[Out] -a*e*x/b^2-1/2*a*f*x^2/b^2-(f*x+e)*cos(d*x+c)/b/d+f*sin(d*x+c)/b/d^2-I*a^2*
(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d/(a^2-b^2)^(1/2)+
I*a^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d/(a^2-b^2)^
(1/2)-a^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d^2/(a^2-
b^2)^(1/2)+a^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d^2/
(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.38, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4611, 3377, 2717, 3404, 2296, 2221, 2317, 2438}

$$-\frac{a^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2\sqrt{a^2-b^2}}+\frac{a^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2\sqrt{a^2-b^2}}-\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}+\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{f\sin(c+dx)}{bd^2}-\frac{(e+fx)\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*e*x)/b^2) - (a*f*x^2)/(2*b^2) - ((e + f*x)*Cos[c + d*x])/(b*d) - (I*a^
2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt
[a^2 - b^2]*d) + (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) - (a^2*f*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) + (a^2*f*PolyLog[2, (
I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) + (f
*Sin[c + d*x])/(b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\sin(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)\cos(c+dx)}{bd} - \frac{a\int (e+fx)\,dx}{b^2} + \frac{a^2\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{b^2} + \frac{f\int \cos(c+ \ldots}{bd}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} + \frac{f\sin(c+dx)}{bd^2} + \frac{(2a^2)\int \frac{e^{i(c+dx)}(\ldots}{ib+2ae^{i(c+dx)}-\ldots}}{b^2}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} + \frac{f\sin(c+dx)}{bd^2} - \frac{(2ia^2)\int \frac{e^{i(c+\ldots}}{2a-2\sqrt{a^2-\ldots}}}{b\sqrt{a^2-\ldots}}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d} \ldots$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d} \ldots$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d} \ldots$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 709 vs. 2(311) = 622.
time = 4.60, size = 709, normalized size = 2.28

$$\frac{a(c+dx)(cf-d(2e+fx)) - 2bd(e+fx)\cos(c+dx) + 2a^2d(e+fx)\left(\ldots\text{[illegible]}\ldots\right) + 2bf\sin(c+dx)}{2b^2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x)*(c*f - d*(2*e + f*x)) - 2*b*d*(e + f*x)*Cos[c + d*x] + (2*a^2*
d*(e + f*x)*((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 +
 b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (
a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b
^2] + (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(
c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c +
d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1
- I*Tan[(c + d*x)/2]]*Log[(-b + Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a
```

```
- b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b
+ Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]
*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2
])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))
]))/Sqrt[-a^2 + b^2]))/(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*L
og[1 + I*Tan[(c + d*x)/2]]) + 2*b*f*Sin[c + d*x])/(2*b^2*d^2)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(279) = 558$.
time = 0.41, size = 625, normalized size = 2.01

| method | result |
|---|---|
| risch | $-\frac{af\,x^2}{2b^2}-\frac{aex}{b^2}-\frac{(dxf+de+if)\mathrm{e}^{i(dx+c)}}{2b\,d^2}-\frac{(dxf+de-if)\mathrm{e}^{-i(dx+c)}}{2b\,d^2}+\frac{2ia^2e\arctan\left(\frac{2ib\,\mathrm{e}^{i(dx+c)}-2a}{2\sqrt{-a^2+b^2}}\right)}{b^2d\sqrt{-a^2+b^2}}+\frac{a^2f\ln\left(\frac{ia+b\,\mathrm{e}^{i(dx+}}{ia-\sqrt{}}\right.}{b^2d\sqrt{-}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*f*x^2/b^2-a*e*x/b^2-1/2*(d*x*f+I*f+d*e)/b/d^2*exp(I*(d*x+c))-1/2*(d*
x*f-I*f+d*e)/b/d^2*exp(-I*(d*x+c))+2*I*a^2/b^2/d*e/(-a^2+b^2)^(1/2)*arctan(
1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+a^2/b^2/d*f/(-a^2+b^2)^(1/
2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+a^2
/b^2/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a
-(-a^2+b^2)^(1/2)))*c-a^2/b^2/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))
+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)
*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-I*a^2
/b^2/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(
I*a-(-a^2+b^2)^(1/2)))+I*a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*
(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I*a^2/b^2/d^2*f*c/(-a^
2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(275) = 550$.
time = 0.55, size = 1160, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((a^3 - a*b^2)*d^2*f*x^2 - I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a
*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1) + I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*
cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) - I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*
cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + 2*(a^3 - a*b^2)*d^2*x*e - 2*(a^2*b - b^3)*f*s
in(d*x + c) + (a^2*b*c*f - a^2*b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*
x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (a^2*b*
c*f - a^2*b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*
x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (a^2*b*c*f - a^2*b*d*e)*sqrt
(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(
a^2 - b^2)/b^2) + 2*I*a) - (a^2*b*c*f - a^2*b*d*e)*sqrt(-(a^2 - b^2)/b^2)*l
og(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) + (a^2*b*d*f*x + a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x +
 c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b) - (a^2*b*d*f*x + a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a
*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b) + (a^2*b*d*f*x + a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)
*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a^2*b*d*f*x + a^2*b*c*f)*sqrt(-(a^2
- b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*((a^2*b - b^3)*d*f*x + (
a^2*b - b^3)*d*e)*cos(d*x + c))/((a^2*b^2 - b^4)*d^2)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^2\,\left(e+f\,x\right)}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] int((sin(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)), x)
```

# 3.227 $\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=75

$$-\frac{ax}{b^2}+\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}-\frac{\cos(c+dx)}{bd}$$

```
[Out] -a*x/b^2-cos(d*x+c)/b/d+2*a^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/
2))/b^2/d/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {2825, 12, 2814, 2739, 632, 210}

$$\frac{2a^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^
2*Sqrt[a^2 - b^2]*d) - Cos[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{\cos(c+dx)}{bd} - \frac{\int \frac{a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b} \\
&= -\frac{\cos(c+dx)}{bd} - \frac{a\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b} \\
&= -\frac{ax}{b^2} - \frac{\cos(c+dx)}{bd} + \frac{a^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2} \\
&= -\frac{ax}{b^2} - \frac{\cos(c+dx)}{bd} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d} \\
&= -\frac{ax}{b^2} - \frac{\cos(c+dx)}{bd} - \frac{(4a^2)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d} \\
&= -\frac{ax}{b^2} + \frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d} - \frac{\cos(c+dx)}{bd}
\end{aligned}$$

**Mathematica** [A]
time = 0.13, size = 71, normalized size = 0.95

$$-\frac{a(c+dx) - \frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + b\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*(c + d*x) - (2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + b*Cos[c + d*x])/(b^2*d))
```

**Maple** [A]
time = 0.10, size = 90, normalized size = 1.20

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)a^2}{b^2\sqrt{a^2-b^2}}-\frac{2\left(\frac{b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}}{d}$ |
| default | $\frac{\frac{2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)a^2}{b^2\sqrt{a^2-b^2}}-\frac{2\left(\frac{b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}}{d}$ |
| risch | $-\frac{ax}{b^2}-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}+\frac{a^2\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d\,b^2}-\frac{a^2\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b}}{b\sqrt{-a^2}}\right.}{\sqrt{-a^2+b^2}\,d\,b^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2
)^(1/2))*a^2-2/b^2*(b/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1/2*d*x+1/2*c))
))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [A]
time = 0.36, size = 283, normalized size = 3.77

$$\left[-\frac{\sqrt{-a^2+b^2}\,a^2\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2\,(a^3-ab^2)dx+2\,(a^2b-b^3)\cos(dx+c)}{2\,(a^2b^2-b^4)d},\ -\frac{\sqrt{a^2-b^2}\,a^2\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+(a^3-ab^2)dx+(a^2b-b^3)\cos(dx+c)}{(a^2b^2-b^4)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^2*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt
(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a
^3 - a*b^2)*d*x + 2*(a^2*b - b^3)*cos(d*x + c))/((a^2*b^2 - b^4)*d), -(sqrt
(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))
) + (a^3 - a*b^2)*d*x + (a^2*b - b^3)*cos(d*x + c))/((a^2*b^2 - b^4)*d)]
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(61) = 122$.
time = 175.77, size = 1690, normalized size = 22.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-cos(c + d*x)/(b
*d), Eq(a, 0)), (-b*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)**2 + b
**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2
)) - b*d*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt
(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 2*b*tan(c/2
 + d*x/2)**2/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2
+ d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 4*b/(b**2*d*tan(c/2 + d*x/
2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/
2 + d*x/2)) + d*x*sqrt(b**2)*tan(c/2 + d*x/2)**2/(b**2*d*tan(c/2 + d*x/2)**
2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 +
d*x/2)) + d*x*sqrt(b**2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b*
*2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2*sqrt(b**2)*t
an(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c
/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)), Eq(a, -sqrt(b**2))), (-b
*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b*
*2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - b*d*x*tan(c/2
+ d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*
x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 2*b*tan(c/2 + d*x/2)**2/(b**2*
d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*s
qrt(b**2)*tan(c/2 + d*x/2)) - 4*b/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*
d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - d*x*s
qrt(b**2)*tan(c/2 + d*x/2)**2/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sq
rt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - d*x*sqrt(
b**2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*x/2
)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 2*sqrt(b**2)*tan(c/2 + d*x/2)/(b*
*2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*
d*sqrt(b**2)*tan(c/2 + d*x/2)), Eq(a, sqrt(b**2))), ((x*sin(c + d*x)**2/2 +
 x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))/a, Eq(b, 0)), (x*si
```

```
n(c)**2/(a + b*sin(c)), Eq(d, 0)), (a**2*log(tan(c/2 + d*x/2) + b/a - sqrt(
-a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d
*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + a**2*log(tan(c/2 + d*x/2) + b/a - s
qrt(-a**2 + b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*
d*sqrt(-a**2 + b**2)) - a**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 + b**2
)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b
**2*d*sqrt(-a**2 + b**2)) - a**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 +
b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2
 + b**2)) - a*d*x*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2/(b**2*d*sqrt(-a**2
 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) - a*d*x*sqrt(-a**
2 + b**2)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**
2 + b**2)) - 2*b*sqrt(-a**2 + b**2)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*
x/2)**2 + b**2*d*sqrt(-a**2 + b**2)), True))
```

**Giac [A]**
time = 3.14, size = 99, normalized size = 1.32

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\sqrt{a^2-b^2}\,b^2}-\frac{(dx+c)a}{b^2}-\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a^2/(sqrt(a^2 - b^2)*b^2) - (d*x + c)*a/b^2 - 2/((
tan(1/2*d*x + 1/2*c)^2 + 1)*b))/d
```

**Mupad [B]**
time = 2.50, size = 127, normalized size = 1.69

$$-\frac{\cos\left(c+d\,x\right)}{b\,d}-\frac{a\,x}{b^2}-\frac{a^2\,\mathrm{atan}\left(\frac{\left(-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)\,1{}\mathrm{i}}{\sqrt{b^2-a^2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)\,2{}\mathrm{i}}{b^2\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

```
[Out] - cos(c + d*x)/(b*d) - (a*x)/b^2 - (a^2*atan(((2*b^2*sin(c/2 + (d*x)/2) - a
^2*sin(c/2 + (d*x)/2) + a*b*cos(c/2 + (d*x)/2))*1i)/((b^2 - a^2)^(1/2)*(a*c
os(c/2 + (d*x)/2) + 2*b*sin(c/2 + (d*x)/2))))*2i)/(b^2*d*(b^2 - a^2)^(1/2))
```

# 3.228 $\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=802

$$-\frac{3ef^2x}{4bd^2}-\frac{3f^3x^2}{8bd^2}+\frac{a^2(e+fx)^4}{4b^3f}+\frac{(e+fx)^4}{8bf}-\frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3}+\frac{a(e+fx)^3\cos(c+dx)}{b^2d}+\frac{ia^3(e+fx)}{\quad}$$

```
[Out] -3/4*e*f^2*x/b/d^2-3/8*f^3*x^2/b/d^2+1/4*a^2*(f*x+e)^4/b^3/f+1/8*(f*x+e)^4/
b/f-6*a*f^2*(f*x+e)*cos(d*x+c)/b^2/d^3+a*(f*x+e)^3*cos(d*x+c)/b^2/d+6*a*f^3
*sin(d*x+c)/b^2/d^4-3*a*f*(f*x+e)^2*sin(d*x+c)/b^2/d^2+3/4*f^2*(f*x+e)*cos(
d*x+c)*sin(d*x+c)/b/d^3-1/2*(f*x+e)^3*cos(d*x+c)*sin(d*x+c)/b/d-3/8*f^3*sin
(d*x+c)^2/b/d^4+3/4*f*(f*x+e)^2*sin(d*x+c)^2/b/d^2+6*I*a^3*f^2*(f*x+e)*poly
log(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^3/(a^2-b^2)^(1/2)-6*I*a
^3*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d^3/(a
^2-b^2)^(1/2)+3*a^3*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(
1/2)))/b^3/d^2/(a^2-b^2)^(1/2)-3*a^3*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/b^3/d^2/(a^2-b^2)^(1/2)+I*a^3*(f*x+e)^3*ln(1-I*b*ex
p(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d/(a^2-b^2)^(1/2)-I*a^3*(f*x+e)^3*ln(
1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d/(a^2-b^2)^(1/2)-6*a^3*f^3*p
olylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^4/(a^2-b^2)^(1/2)+6*
a^3*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d^4/(a^2-b^2)
^(1/2)
```

**Rubi** [A]

time = 0.91, antiderivative size = 802, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {4611, 3392, 32, 3391, 3377, 2717, 3404, 2296, 2221, 2611, 6744, 2320, 6724}

$$\frac{(e+fx)^4}{8bf}+\frac{a^2(e+fx)^4}{4b^3f}+\frac{a\cos(c+dx)(e+fx)^3}{b^2d}+\frac{ia^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)(e+fx)^3}{b^3\sqrt{a^2-b^2}\,d}-\frac{ia^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)(e+fx)^3}{b^3\sqrt{a^2-b^2}\,d}-\frac{\cos(c+dx)\sin(c+dx)(e+fx)^3}{2bd}+\frac{3f\sin^2(c+dx)(e+fx)^2}{4bd^2}+\frac{3a^3f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)(e+fx)^2}{b^3\sqrt{a^2-b^2}\,d^2}-\frac{3a^3f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)(e+fx)^2}{b^3\sqrt{a^2-b^2}\,d^2}-\frac{3af\sin(c+dx)(e+fx)^2}{b^2d^2}-\frac{6af^2\cos(c+dx)(e+fx)}{b^2d^3}+\frac{6ia^3f^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)(e+fx)}{b^3\sqrt{a^2-b^2}\,d^3}-\frac{6ia^3f^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)(e+fx)}{b^3\sqrt{a^2-b^2}\,d^3}+\frac{3f^2\cos(c+dx)\sin(c+dx)(e+fx)}{4bd^3}-\frac{3f^3x^2}{8bd^2}-\frac{3f^3\sin^2(c+dx)}{8bd^4}-\frac{3ef^2x}{4bd^2}-\frac{6a^3f^3\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^4}+\frac{6a^3f^3\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^4}+\frac{6af^3\sin(c+dx)}{b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*e*f^2*x)/(4*b*d^2) - (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f
) + (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cos[c + d*x])/(b^2*d^3) + (a*(
e + f*x)^3*Cos[c + d*x])/(b^2*d) + (I*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c
+ d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) - (I*a^3*(e + f*x)
^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^
2]*d) + (3*a^3*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^2) - (3*a^3*f*(e + f*x)^2*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^2) + ((6*I
)*a^3*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]
)/(b^3*Sqrt[a^2 - b^2]*d^3) - ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I
```

```
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^3) - (6*a^3*f^3
*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b
^2]*d^4) + (6*a^3*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]
)])/(b^3*Sqrt[a^2 - b^2]*d^4) + (6*a*f^3*Sin[c + d*x])/(b^2*d^4) - (3*a*f*(
e + f*x)^2*Sin[c + d*x])/(b^2*d^2) + (3*f^2*(e + f*x)*Cos[c + d*x]*Sin[c +
d*x])/(4*b*d^3) - ((e + f*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*b*d) - (3*f^3*
Sin[c + d*x]^2)/(8*b*d^4) + (3*f*(e + f*x)^2*Sin[c + d*x]^2)/(4*b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sin^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3\cos(c+dx)\sin(c+dx)}{2bd} + \frac{3f(e+fx)^2\sin^2(c+dx)}{4bd^2} - \frac{a\int(e+f}{}$$

$$= \frac{(e+fx)^4}{8bf} + \frac{a(e+fx)^3\cos(c+dx)}{b^2d} + \frac{3f^2(e+fx)\cos(c+dx)\sin(c+dx)}{4bd^3}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} + \frac{a(e+fx)^3\cos(c+dx)}{b^2d} - 3$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} +$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} +$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} +$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} +$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} +$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} +$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than

twice the leaf count of optimal. 1851 vs. 2(802) = 1604.
time = 3.08, size = 1851, normalized size = 2.31

---

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (16*(2*a^2 + b^2)*e^3*x + 24*(2*a^2 + b^2)*e^2*f*x^2 + 16*(2*a^2 + b^2)*e*f
^2*x^3 + 4*(2*a^2 + b^2)*f^3*x^4 - ((32*I)*a^3*((3*I)*Sqrt[a^2 - b^2]*d^3*e
^2*f*x*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(
-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + (3*I)
*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x
]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Co
s[c] + I*Sin[c]) + I*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*(Cos[2*c + d*x]
 + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2
] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*Po
lyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2
 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 3*Sqrt[a
^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x
]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*
(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*(Cos[2*
c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*S
in[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*f^3*x
*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-
a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 6*Sqr
t[a^2 - b^2]*f^3*PolyLog[4, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*
Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I
*Sin[c]) + 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*(Cos[2*c + d*x] + I*Sin[2*c
+ d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[
c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*(Cos[2*
c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] +
I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e*
f^2*x^2*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sq
rt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c])
+ Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]
))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(
(-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*(Cos[2*c +
d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Si
n[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*f^3*x*Po
lyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^
2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) - (2*I)
*d^3*e^3*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]]*
Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))/(Sqrt[a^2 - b^2]*d^4*Sqrt[(-a^
2 + b^2)*(Cos[2*c] + I*Sin[2*c])]) + (16*a*b*((6*I)*f^3 - 6*d*f^2*(e + f*x)
```

```
 - (3*I)*d^2*f*(e + f*x)^2 + d^3*(e + f*x)^3)*(Cos[c + d*x] - I*Sin[c + d*x
]))/d^4 + (16*a*b*((-6*I)*f^3 - 6*d*f^2*(e + f*x) + (3*I)*d^2*f*(e + f*x)^2
 + d^3*(e + f*x)^3)*(Cos[c + d*x] + I*Sin[c + d*x]))/d^4 + (b^2*(3*f^3 + (6
*I)*d*f^2*(e + f*x) - 6*d^2*f*(e + f*x)^2 - (4*I)*d^3*(e + f*x)^3)*(Cos[2*(
c + d*x)] - I*Sin[2*(c + d*x)]))/d^4 + (b^2*(3*f^3 - (6*I)*d*f^2*(e + f*x)
- 6*d^2*f*(e + f*x)^2 + (4*I)*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] + I*Sin[2*
(c + d*x)]))/d^4)/(32*b^3)
```

**Maple** [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\sin^3 (dx+c))}{a+b\sin (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3002 vs. 2(727) = 1454.
time = 0.68, size = 3002, normalized size = 3.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^4 - a^2*b^2 - b^4)*d^4*f^3*x^4 + 6*(2*a^4 - a^2*b^2 - b^4)*d^4*f*
x^2*e^2 + 24*I*a^3*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x +
c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b) - 24*I*a^3*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x
+ c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
```

```
2)/b^2))/b) - 24*I*a^3*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d
*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2))/b) + 24*I*a^3*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*co
s(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2))/b) + 3*(a^2*b^2 - b^4)*d^2*f^3*x^2 + 4*(2*a^4 - a^2*b^2 - b^
4)*d^4*x*e^3 - 3*(2*(a^2*b^2 - b^4)*d^2*f^3*x^2 + 4*(a^2*b^2 - b^4)*d^2*f^2
*x*e + 2*(a^2*b^2 - b^4)*d^2*f*e^2 - (a^2*b^2 - b^4)*f^3)*cos(d*x + c)^2 +
12*(-I*a^3*b*d^2*f^3*x^2 - 2*I*a^3*b*d^2*f^2*x*e - I*a^3*b*d^2*f*e^2)*sqrt(
-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 12*(I*a^3*b*d^2*
f^3*x^2 + 2*I*a^3*b*d^2*f^2*x*e + I*a^3*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)
*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 12*(I*a^3*b*d^2*f^3*x^2 + 2*I*a^3
*b*d^2*f^2*x*e + I*a^3*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(
d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) + 12*(-I*a^3*b*d^2*f^3*x^2 - 2*I*a^3*b*d^2*f^2*x*e
- I*a^3*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*si
n(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b
)/b + 1) + 4*(a^3*b*c^3*f^3 - 3*a^3*b*c^2*d*f^2*e + 3*a^3*b*c*d^2*f*e^2 - a
^3*b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 4*(a^3*b*c^3*f^3 - 3*a^3*b*c^2*
d*f^2*e + 3*a^3*b*c*d^2*f*e^2 - a^3*b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2
*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)
- 4*(a^3*b*c^3*f^3 - 3*a^3*b*c^2*d*f^2*e + 3*a^3*b*c*d^2*f*e^2 - a^3*b*d^3*
e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 4*(a^3*b*c^3*f^3 - 3*a^3*b*c^2*d*f^2*e
+ 3*a^3*b*c*d^2*f*e^2 - a^3*b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(
d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 4*(a^
3*b*d^3*f^3*x^3 + a^3*b*c^3*f^3 + 3*(a^3*b*d^3*f*x + a^3*b*c*d^2*f)*e^2 + 3
*(a^3*b*d^3*f^2*x^2 - a^3*b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b) - 4*(a^3*b*d^3*f^3*x^3 + a^3*b*c^3*f^3 + 3*(a^3*b*d
^3*f*x + a^3*b*c*d^2*f)*e^2 + 3*(a^3*b*d^3*f^2*x^2 - a^3*b*c^2*d*f^2)*e)*sq
rt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x +
 c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 4*(a^3*b*d^3*f^3*x
^3 + a^3*b*c^3*f^3 + 3*(a^3*b*d^3*f*x + a^3*b*c*d^2*f)*e^2 + 3*(a^3*b*d^3*f
^2*x^2 - a^3*b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c)
 - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b) - 4*(a^3*b*d^3*f^3*x^3 + a^3*b*c^3*f^3 + 3*(a^3*b*d^3*f*x + a^3
*b*c*d^2*f)*e^2 + 3*(a^3*b*d^3*f^2*x^2 - a^3*b*c^2*d*f^2)*e)*sqrt(-(a^2 - b
^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 24*(a^3*b*d*f^3*x + a^3*b*d*f
^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c)
 + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*(a^3
*b*d*f^3*x + a^3*b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x
```

```
 + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2))/b) - 24*(a^3*b*d*f^3*x + a^3*b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*po
lylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*(a^3*b*d*f^3*x + a^3*b*d*f^2*e)*sq
rt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 8*((a^3*b - a*
b^3)*d^3*f^3*x^3 + 3*(a^3*b - a*b^3)*d^3*f*x*e^2 - 6*(a^3*b - a*b^3)*d*f^3*
x + (a^3*b - a*b^3)*d^3*e^3 + 3*((a^3*b - a*b^3)*d^3*f^2*x^2 - 2*(a^3*b - a
*b^3)*d*f^2)*e)*cos(d*x + c) + 2*(2*(2*a^4 - a^2*b^2 - b^4)*d^4*f^2*x^3 + 3
*(a^2*b^2 - b^4)*d^2*f^2*x)*e - 2*(12*(a^3*b - a*b^3)*d^2*f^3*x^2 + 24*(a^3
*b - a*b^3)*d^2*f^2*x*e + 12*(a^3*b - a*b^3)*d^2*f*e^2 - 24*(a^3*b - a*b^3)
*f^3 + (2*(a^2*b^2 - b^4)*d^3*f^3*x^3 + 6*(a^2*b^2 - b^4)*d^3*f*x*e^2 - 3*(
a^2*b^2 - b^4)*d*f^3*x + 2*(a^2*b^2 - b^4)*d^3*e^3 + 3*(2*(a^2*b^2 - b^4)*d
^3*f^2*x^2 - (a^2*b^2 - b^4)*d*f^2)*e)*cos(d*x + c))*sin(d*x + c))/((a^2*b^
3 - b^5)*d^4)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.229 $\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=592

$$-\frac{f^2x}{4bd^2}+\frac{a^2(e+fx)^3}{3b^3f}+\frac{(e+fx)^3}{6bf}-\frac{2af^2\cos(c+dx)}{b^2d^3}+\frac{a(e+fx)^2\cos(c+dx)}{b^2d}+\frac{ia^3(e+fx)^2\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2-}}\right.}{b^3\sqrt{a^2-b^2}\,d}$$

```
[Out] -1/4*f^2*x/b/d^2+1/3*a^2*(f*x+e)^3/b^3/f+1/6*(f*x+e)^3/b/f-2*a*f^2*cos(d*x+
c)/b^2/d^3+a*(f*x+e)^2*cos(d*x+c)/b^2/d-2*a*f*(f*x+e)*sin(d*x+c)/b^2/d^2+1/
4*f^2*cos(d*x+c)*sin(d*x+c)/b/d^3-1/2*(f*x+e)^2*cos(d*x+c)*sin(d*x+c)/b/d+1
/2*f*(f*x+e)*sin(d*x+c)^2/b/d^2+I*a^3*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-
(a^2-b^2)^(1/2)))/b^3/d/(a^2-b^2)^(1/2)-I*a^3*(f*x+e)^2*ln(1-I*b*exp(I*(d*x
+c))/(a+(a^2-b^2)^(1/2)))/b^3/d/(a^2-b^2)^(1/2)+2*a^3*f*(f*x+e)*polylog(2,I
*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^2/(a^2-b^2)^(1/2)-2*a^3*f*(f*x
+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d^2/(a^2-b^2)^(1/
2)+2*I*a^3*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^3/(a
^2-b^2)^(1/2)-2*I*a^3*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))
/b^3/d^3/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.77, antiderivative size = 592, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {4611, 3392, 32, 2715, 8, 3377, 2718, 3404, 2296, 2221, 2611, 2320, 6724}

$$\frac{2ia^3f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3\sqrt{a^2-b^2}}-\frac{2ia^3f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^3\sqrt{a^2-b^2}}+\frac{2a^3f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2\sqrt{a^2-b^2}}-\frac{2a^3f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2\sqrt{a^2-b^2}}+\frac{a^2(e+fx)^3}{3b^3f}+\frac{ia^3(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}-\frac{ia^3(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d\sqrt{a^2-b^2}}-\frac{2af^2\cos(c+dx)}{b^2d^3}-\frac{2af(e+fx)\sin(c+dx)}{b^2d^2}+\frac{a(e+fx)^2\cos(c+dx)}{b^2d}+\frac{f^2\sin(c+dx)\cos(c+dx)}{4bd^3}+\frac{f(e+fx)\sin^2(c+dx)}{2bd^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2bd}-\frac{f^2x}{4bd^2}+\frac{(e+fx)^3}{6bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(f^2*x)/(b*d^2) + (a^2*(e + f*x)^3)/(3*b^3*f) + (e + f*x)^3/(6*b*f) -
(2*a*f^2*Cos[c + d*x])/(b^2*d^3) + (a*(e + f*x)^2*Cos[c + d*x])/(b^2*d) + (
I*a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^
3*Sqrt[a^2 - b^2]*d) - (I*a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) + (2*a^3*f*(e + f*x)*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^2) -
 (2*a^3*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(b^3*Sqrt[a^2 - b^2]*d^2) + ((2*I)*a^3*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a^3*f^2*PolyL
og[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^
3) - (2*a*f*(e + f*x)*Sin[c + d*x])/(b^2*d^2) + (f^2*Cos[c + d*x]*Sin[c + d
*x])/(4*b*d^3) - ((e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*b*d) + (f*(e +
f*x)*Sin[c + d*x]^2)/(2*b*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sin^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^2\cos(c+dx)\sin(c+dx)}{2bd} + \frac{f(e+fx)\sin^2(c+dx)}{2bd^2} - \frac{a\int (e+fx)}{}$$

$$= \frac{(e+fx)^3}{6bf} + \frac{a(e+fx)^2\cos(c+dx)}{b^2d} + \frac{f^2\cos(c+dx)\sin(c+dx)}{4bd^3} - \frac{(e+fx}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} + \frac{a(e+fx)^2\cos(c+dx)}{b^2d} - \frac{2af(e+fx)}{b^2}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+}{b^2d}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+}{b^2d}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+}{b^2d}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+}{b^2d}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+}{b^2d}$$

**Mathematica [A]**
time = 2.21, size = 909, normalized size = 1.54

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (24*a^2*d^3*e^2*x + 12*b^2*d^3*e^2*x + 24*a^2*d^3*e*f*x^2 + 12*b^2*d^3*e*f*
x^2 + 8*a^2*d^3*f^2*x^3 + 4*b^2*d^3*f^2*x^3 + 24*a*b*(-2*f^2 + d^2*(e + f*x
)^2)*Cos[c + d*x] - 6*b^2*d*f*(e + f*x)*Cos[2*(c + d*x)] - ((24*I)*a^3*((-I
)*(d^2*(Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(-Log[1 + (b*(Cos[2*c + d*x] + I*Si
n[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*S
```

```
in[c])] + Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] +
Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])])*(Cos[c] + I*Sin[c])
+ 2*e^2*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]]*S
qrt[-((a^2 - b^2)*(Cos[c] + I*Sin[c])^2)]) - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[
3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2
)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + 2*Sqrt[a^2 - b
^2]*f^2*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] +
 Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c]))
 + 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[
2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin
[c]))]*(Cos[c] + I*Sin[c]) - 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*
(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Co
s[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c])))/(Sqrt[a^2 - b^2]*Sqr
t[-((a^2 - b^2)*(Cos[c] + I*Sin[c])^2)]) - 48*a*b*d*e*f*Sin[c + d*x] - 48*a
*b*d*f^2*x*Sin[c + d*x] - 6*b^2*d^2*e^2*Sin[2*(c + d*x)] + 3*b^2*f^2*Sin[2*
(c + d*x)] - 12*b^2*d^2*e*f*x*Sin[2*(c + d*x)] - 6*b^2*d^2*f^2*x^2*Sin[2*(c
 + d*x)])/(24*b^3*d^3)
```

**Maple [F]**
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 (\sin^3 (dx+c))}{a+b\sin (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2059 vs. 2(532) = 1064.
time = 0.59, size = 2059, normalized size = 3.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")

```
[Out] 1/12*(2*(2*a^4 - a^2*b^2 - b^4)*d^3*f^2*x^3 + 6*(2*a^4 - a^2*b^2 - b^4)*d^3
*f*x^2*e - 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c
) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) + 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c
) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) - 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x +
c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b) + 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x +
 c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))/b) + 6*(2*a^4 - a^2*b^2 - b^4)*d^3*x*e^2 + 3*(a^2*b^2 - b^4)*d*f^2*
x - 6*((a^2*b^2 - b^4)*d*f^2*x + (a^2*b^2 - b^4)*d*f*e)*cos(d*x + c)^2 + 12
*(-I*a^3*b*d*f^2*x - I*a^3*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d
*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b + 1) + 12*(I*a^3*b*d*f^2*x + I*a^3*b*d*f*e)*sqrt(-(a^2 -
b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 12*(I*a^3*b*d*f^2*x + I*
a^3*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x +
c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)
 + 12*(-I*a^3*b*d*f^2*x - I*a^3*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a
*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1) - 6*(a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*e + a^3*b*d
^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 6*(a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*e +
a^3*b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 6*(a^3*b*c^2*f^2 - 2*a^3*b*c*d
*f*e + a^3*b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*
sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 6*(a^3*b*c^2*f^2 - 2*a
^3*b*c*d*f*e + a^3*b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c)
- 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 6*(a^3*b*d^2*f
^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*e)*sqrt(-(a^2 - b^
2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 6*(a^3*b*d^2*f^2*x^2 - a^3*b*c^
2*f^2 + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a
*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b) + 6*(a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*
d^2*f*x + a^3*b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) -
a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 - b)/b) - 6*(a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*d^2*f*x + a^3*b*
c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) -
 (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 12*((
a^3*b - a*b^3)*d^2*f^2*x^2 + 2*(a^3*b - a*b^3)*d^2*f*x*e + (a^3*b - a*b^3)*
```

```
d^2*e^2 - 2*(a^3*b - a*b^3)*f^2)*cos(d*x + c) - 3*(8*(a^3*b - a*b^3)*d*f^2*
x + 8*(a^3*b - a*b^3)*d*f*e + (2*(a^2*b^2 - b^4)*d^2*f^2*x^2 + 4*(a^2*b^2 -
 b^4)*d^2*f*x*e + 2*(a^2*b^2 - b^4)*d^2*e^2 - (a^2*b^2 - b^4)*f^2)*cos(d*x
+ c))*sin(d*x + c))/((a^2*b^3 - b^5)*d^3)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.230 $\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=382

$$\frac{a^2ex}{b^3}+\frac{ex}{2b}+\frac{a^2fx^2}{2b^3}+\frac{fx^2}{4b}+\frac{a(e+fx)\cos(c+dx)}{b^2d}+\frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d}-\frac{ia^3(e+fx)\log\left(1\right.}{b^3\sqrt{a^2-}}$$

```
[Out] a^2*e*x/b^3+1/2*e*x/b+1/2*a^2*f*x^2/b^3+1/4*f*x^2/b+a*(f*x+e)*cos(d*x+c)/b^
2/d-a*f*sin(d*x+c)/b^2/d^2-1/2*(f*x+e)*cos(d*x+c)*sin(d*x+c)/b/d+1/4*f*sin(
d*x+c)^2/b/d^2+I*a^3*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b
^3/d/(a^2-b^2)^(1/2)-I*a^3*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/
2)))/b^3/d/(a^2-b^2)^(1/2)+a^3*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^
(1/2)))/b^3/d^2/(a^2-b^2)^(1/2)-a^3*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-
b^2)^(1/2)))/b^3/d^2/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.44, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4611, 3391, 3377, 2717, 3404, 2296, 2221, 2317, 2438}

$$\frac{a^3f\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2\sqrt{a^2-b^2}}-\frac{a^3f\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2\sqrt{a^2-b^2}}+\frac{a^2ex}{b^3}+\frac{a^2fx^2}{2b^3}+\frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}-\frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d\sqrt{a^2-b^2}}-\frac{af\sin(c+dx)}{b^2d^2}+\frac{a(e+fx)\cos(c+dx)}{b^2d}+\frac{f\sin^2(c+dx)}{4bd^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2bd}+\frac{ex}{2b}+\frac{fx^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a^2*e*x)/b^3 + (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) + (f*x^2)/(4*b) + (a*(e +
 f*x)*Cos[c + d*x])/(b^2*d) + (I*a^3*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x))
)/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) - (I*a^3*(e + f*x)*Log[1
- (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) + (
a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^
2 - b^2]*d^2) - (a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b^3*Sqrt[a^2 - b^2]*d^2) - (a*f*Sin[c + d*x])/(b^2*d^2) - ((e + f*x)*
Cos[c + d*x]*Sin[c + d*x])/(2*b*d) + (f*Sin[c + d*x]^2)/(4*b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4611

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
```

```
d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a +
 b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\sin^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2bd} + \frac{f\sin^2(c+dx)}{4bd^2} - \frac{a\int (e+fx)\sin(c+d}{b^2}$$

$$= \frac{ex}{2b} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2 d} - \frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2bd} + \frac{f\,\mathrm{si}}{}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2 d} - \frac{af\sin(c+dx)}{b^2d^2} - \frac{(e+}{}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2 d} - \frac{af\sin(c+dx)}{b^2d^2} - \frac{(e+}{}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2 d} + \frac{ia^3(e+fx)\log\left(1-\right.}{b^3\sqrt{a^2-b}}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2 d} + \frac{ia^3(e+fx)\log\left(1-\right.}{b^3\sqrt{a^2-b}}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2 d} + \frac{ia^3(e+fx)\log\left(1-\right.}{b^3\sqrt{a^2-b}}$$

**Mathematica** [A]
time = 4.92, size = 752, normalized size = 1.97

$$-\frac{2(2a^2+b^2)(c+dx)(cf-d(2e+fx)) - 8abd(e+fx)\cos(c+dx) + b^2f\cos(2(c+dx)) + \frac{8a^3d(e+fx)\left(\frac{2(de-cf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{-ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}} + \frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}} + \frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{-b+\sqrt{-a^2+b^2}-a\tan\left(\frac{1}{2}(c+dx)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(i+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}} - \frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b-\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b-\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a+ia\tan\left(\frac{1}{2}(c+dx)\right)}{a+i\left(-b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}\right)}{de-cf+if\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)-if\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)} + 8abf\sin(c+dx) + 2b^2d(e+fx)\sin(2(c+dx))}{8b^3d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/8*(2*(2*a^2 + b^2)*(c + d*x)*(c*f - d*(2*e + f*x)) - 8*a*b*d*(e + f*x)*C
os[c + d*x] + b^2*f*Cos[2*(c + d*x)] + (8*a^3*d*(e + f*x)*((2*(d*e - c*f)*A
```

```
rcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Lo
g[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(
(-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(
a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c
+ d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[
-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a
^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-b
 + Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] + P
olyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-
a^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] +
a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan
[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/(d*e -
 c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]]) +
 8*a*b*f*Sin[c + d*x] + 2*b^2*d*(e + f*x)*Sin[2*(c + d*x)])/(b^3*d^2)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(342) = 684$.
time = 0.69, size = 686, normalized size = 1.80

| method | result |
|---|---|
| risch | $\frac{a^2 f x^2}{2b^3} + \frac{f x^2}{4b} + \frac{a^2 e x}{b^3} + \frac{ex}{2b} + \frac{a(dxf+de+if)e^{i(dx+c)}}{2b^2d^2} + \frac{a(dxf+de-if)e^{-i(dx+c)}}{2b^2d^2} - \frac{2ia^3 e \arctan\left(\frac{2ib\,e^{i(dx+c)}-2a}{2\sqrt{-a^2+b^2}}\right)}{b^3 d\sqrt{-a^2+b^2}} -$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*f*x^2/b^3+1/4*f*x^2/b+a^2*e*x/b^3+1/2*e*x/b+1/2*a*(d*x*f+I*f+d*e)/b
^2/d^2*exp(I*(d*x+c))+1/2*a*(d*x*f-I*f+d*e)/b^2/d^2*exp(-I*(d*x+c))-2*I*a^3
/b^3/d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^
(1/2))-a^3/b^3/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/
2))/(I*a-(-a^2+b^2)^(1/2)))*x-a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(
I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+a^3/b^3/d*f/(-a^2+b^
2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))
*x+a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2)
)/(I*a+(-a^2+b^2)^(1/2)))*c-I*a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*e
xp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+I*a^3/b^3/d^2*f/(-a
^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)
^(1/2)))+2*I*a^3/b^3/d^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+
c))-2*a)/(-a^2+b^2)^(1/2))-1/8*f/b/d^2*cos(2*d*x+2*c)-1/4*(f*x+e)/d/b*sin(2
*d*x+2*c)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(340) = 680$.
time = 0.56, size = 1254, normalized size = 3.28

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*
x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
+ 1) - 2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d
*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
 + 1) - 2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin
(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)
/b + 1) + 2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*s
in(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) - (2*a^4 - a^2*b^2 - b^4)*d^2*f*x^2 - 2*(2*a^4 - a^2*b^2 - b^4)*d
^2*x*e + (a^2*b^2 - b^4)*f*cos(d*x + c)^2 - 2*(a^3*b*c*f - a^3*b*d*e)*sqrt(
-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^
2 - b^2)/b^2) + 2*I*a) - 2*(a^3*b*c*f - a^3*b*d*e)*sqrt(-(a^2 - b^2)/b^2)*l
og(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I
*a) + 2*(a^3*b*c*f - a^3*b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c
) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(a^3*b*c*f
 - a^3*b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(a^3*b*d*f*x + a^3*b*c*f)*sq
rt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x +
 c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*(a^3*b*d*f*x + a
^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) -
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*(a^3
*b*d*f*x + a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*si
n(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b
)/b) + 2*(a^3*b*d*f*x + a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*
x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) - b)/b) - 4*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*d*e)*cos(d*x
 + c) + 2*(2*(a^3*b - a*b^3)*f + ((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*d
*e)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d^2)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(e + f*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.231 $\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=107

$$\frac{(2a^2+b^2)\,x}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d}+\frac{a\cos(c+dx)}{b^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

```
[Out] 1/2*(2*a^2+b^2)*x/b^3+a*cos(d*x+c)/b^2/d-1/2*cos(d*x+c)*sin(d*x+c)/b/d-2*a^
3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^3/d/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2872, 3102, 2814, 2739, 632, 210}

$$\frac{x(2a^2+b^2)}{2b^3}-\frac{2a^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(b^3*Sqrt[a^2 - b^2]*d) + (a*Cos[c + d*x])/(b^2*d) - (Cos[c + d*x
]*Sin[c + d*x])/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{a+b\sin(c+dx)-2a\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{ab+(2a^2+b^2)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b^2}$$

$$= \frac{(2a^2+b^2)\,x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{a^3\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^3}$$

$$= \frac{(2a^2+b^2)\,x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(2a^3)\,\text{Subst}\Big(\int \frac{1}{a+2bx+ax}$$

$$= \frac{(2a^2+b^2)\,x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{(4a^3)\,\text{Subst}\Big(\int \frac{1}{-4(a^2-b^2)}$$

$$= \frac{(2a^2+b^2)\,x}{2b^3} - \frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+}{2bd}$$

**Mathematica [A]**
time = 0.18, size = 97, normalized size = 0.91

$$\frac{2(2a^2+b^2)(c+dx) - \frac{8a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 4ab\cos(c+dx) - b^2\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(2*a^2 + b^2)*(c + d*x) - (8*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*b*Cos[c + d*x] - b^2*Sin[2*(c + d*x)])/(4*
b^3*d)
```

**Maple [A]**
time = 0.13, size = 146, normalized size = 1.36

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2a^3\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}} + \frac{2\left(\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \left(2a^2+b^2\right)\arctan\left(\tan\left(\right.\right.}{b^3}}{d}$ |

| | |
|---|---|
| default | $\frac{-\frac{2a^3 \arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{\frac{2\left(\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(2a^2+b^2\right)\arctan\left(\tan\left(\frac{dx}{2}\right.\right.}{b^3}}{d}$ |
| risch | $\frac{x\,a^2}{b^3}+\frac{x}{2b}+\frac{a\,\mathrm{e}^{i(dx+c)}}{2b^2d}+\frac{a\,\mathrm{e}^{-i(dx+c)}}{2b^2d}+\frac{ia^3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,d\,b^3}-\frac{ia^3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\left(\sqrt{\right.}{}\right.}{\sqrt{a^2-}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))+2/b^3*((1/2*b^2*tan(1/2*d*x+1/2*c)^3+a*b*tan(1/2*d*x+1/2*c)^2
-1/2*b^2*tan(1/2*d*x+1/2*c)+a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(2*a^2+b^2)
*arctan(tan(1/2*d*x+1/2*c))))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.38, size = 359, normalized size = 3.36

$$\left[-\frac{\sqrt{-a^2+b^2}\,a^3\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)-(2\,a^4-a^2b^2-b^4)dx+(a^2b^2-b^4)\cos(dx+c)\sin(dx+c)-2\,(a^3b-ab^3)\cos(dx+c)}{2\,(a^2b^3-b^5)d},\ \frac{2\sqrt{a^2-b^2}\,a^3\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+(2\,a^4-a^2b^2-b^4)dx-(a^2b^2-b^4)\cos(dx+c)\sin(dx+c)+2\,(a^3b-ab^3)\cos(dx+c)}{2\,(a^2b^3-b^5)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^3*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - (2*
a^4 - a^2*b^2 - b^4)*d*x + (a^2*b^2 - b^4)*cos(d*x + c)*sin(d*x + c) - 2*(a
^3*b - a*b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d), 1/2*(2*sqrt(a^2 - b^2)*a^3
*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (2*a^4 - a^
```

```
2*b^2 - b^4)*d*x - (a^2*b^2 - b^4)*cos(d*x + c)*sin(d*x + c) + 2*(a^3*b - a
*b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

**Giac** [**A**]
time = 3.68, size = 151, normalized size = 1.41

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3}{\sqrt{a^2-b^2}\,b^3}-\frac{(2a^2+b^2)(dx+c)}{b^3}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a^3/(sqrt(a^2 - b^2)*b^3) - (2*a^2 + b^2)*(d*
x + c)/b^3 - 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

**Mupad** [**B**]
time = 3.01, size = 199, normalized size = 1.86

$$\frac{\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{b\,d}-\frac{\sin(2c+2dx)}{4bd}+\frac{2a^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{b^3d}+\frac{a\cos(c+dx)}{b^2d}+\frac{a^3\operatorname{atan}\left(\frac{\left(-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab+2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^2\right)1\mathrm{i}}{\sqrt{b^2-a^2}\left(a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+2b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}\right)2\mathrm{i}}{b^3d\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*sin(c + d*x)),x)
[Out] atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(b*d) - sin(2*c + 2*d*x)/(4*b*d
) + (2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d) + (a*cos(c
+ d*x))/(b^2*d) + (a^3*atan(((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2 + (d*x
)/2) + a*b*cos(c/2 + (d*x)/2))*1i)/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d*x)/2)
 + 2*b*sin(c/2 + (d*x)/2))))*2i)/(b^3*d*(b^2 - a^2)^(1/2))
```

# 3.232 $\int \frac{(e+fx)^3 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=732

$$-\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}-\frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}+3$$

```
[Out] -2*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a/d+3*I*f*(f*x+e)^2*polylog(2,-exp(I*(
d*x+c)))/a/d^2-3*I*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a/d^2-6*f^2*(f*x+e
)*polylog(3,-exp(I*(d*x+c)))/a/d^3+6*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/
a/d^3-6*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/d^4+6*I*f^3*polylog(4,exp(I*(d*x
+c)))/a/d^4+I*b*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/d/
(a^2-b^2)^(1/2)-I*b*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/
a/d/(a^2-b^2)^(1/2)+3*b*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/a/d^2/(a^2-b^2)^(1/2)-3*b*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/a/d^2/(a^2-b^2)^(1/2)+6*I*b*f^2*(f*x+e)*polylog(3,I
*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/d^3/(a^2-b^2)^(1/2)-6*I*b*f^2*(f*x
+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/d^3/(a^2-b^2)^(1/2)
-6*b*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/d^4/(a^2-b^2)^
(1/2)+6*b*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/d^4/(a^2-
b^2)^(1/2)
```

**Rubi** [A]
time = 0.76, antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4631, 4268, 2611, 6744, 2320, 6724, 3404, 2296, 2221}

$$-\frac{6bf^3\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^4\sqrt{a^2-b^2}}+\frac{6bf^3\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^4\sqrt{a^2-b^2}}+\frac{6ibf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^3\sqrt{a^2-b^2}}-\frac{6ibf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^3\sqrt{a^2-b^2}}+\frac{3bf(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{3bf(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{6if^3\text{PolyLog}\left(4,-e^{i(c+dx)}\right)}{ad^4}+\frac{6if^3\text{PolyLog}\left(4,e^{i(c+dx)}\right)}{ad^4}-\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}+\frac{3if(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad\sqrt{a^2-b^2}}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) + (I*b*(e + f*x)^3*Log[1 -
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (I*b*
(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a
^2 - b^2]*d) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) -
 ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*
x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 -
 b^2]*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^2) - (6*f^2*(e + f*x)*PolyLog[3, -E^(I*(
c + d*x))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a*d^3)
 + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(a*Sqrt[a^2 - b^2]*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (I*b*E^
```

```
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^3) - ((6*I)*f^3
*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*PolyLog[4, E^(I*(c + d*
x))])/(a*d^4) - (6*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*Sqrt[a^2 - b^2]*d^4) + (6*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/
(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\csc(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(2b)\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{a} - \frac{(3f)\int (e+$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{3if(e+fx)^2\text{Li}_2\left(-e^{i(c+dx)}\right)}{ad^2} - \frac{3if(e+fx)^2}{ad}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e}{}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2189 vs. $2(732) = 1464$.
time = 8.85, size = 2189, normalized size = 2.99

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*d^3*e^3*ArcTanh[E^(I*(c + d*x))] + 3*d^3*e^2*f*x*Log[1 - E^(I*(c + d*x)
)] + 3*d^3*e*f^2*x^2*Log[1 - E^(I*(c + d*x))] + d^3*f^3*x^3*Log[1 - E^(I*(c
 + d*x))] - 3*d^3*e^2*f*x*Log[1 + E^(I*(c + d*x))] - 3*d^3*e*f^2*x^2*Log[1
+ E^(I*(c + d*x))] - d^3*f^3*x^3*Log[1 + E^(I*(c + d*x))] + (3*I)*d^2*f*(e
+ f*x)^2*PolyLog[2, -E^(I*(c + d*x))] - (3*I)*d^2*f*(e + f*x)^2*PolyLog[2,
E^(I*(c + d*x))] - 6*d*e*f^2*PolyLog[3, -E^(I*(c + d*x))] - 6*d*f^3*x*PolyL
```

```
og[3, -E^(I*(c + d*x))] + 6*d*e*f^2*PolyLog[3, E^(I*(c + d*x))] + 6*d*f^3*x
*PolyLog[3, E^(I*(c + d*x))] - (6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))] + (6*
I)*f^3*PolyLog[4, E^(I*(c + d*x))])/(a*d^4) + (b*((2*I)*d^3*e^3*Sqrt[-((a^2
 - b^2)^2*E^((4*I)*c))]*ArcTanh[(-a + I*b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]]
 + (3*I)*Sqrt[a^2 - b^2]*d^3*e^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f*x
*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)]
)] + I*Sqrt[a^2 - b^2]*d^3*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f^3*x^3*L
og[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
 - (3*I)*Sqrt[a^2 - b^2]*d^3*e^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f*x
*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]
)] - I*Sqrt[a^2 - b^2]*d^3*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f^3*x^3*L
og[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
 - 3*Sqrt[a^2 - b^2]*d^3*e*E^(I*c)*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*f^2*x^2*Lo
g[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
 + 3*Sqrt[a^2 - b^2]*d^3*e*E^(I*c)*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*f^2*x^2*Lo
g[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
 + 3*Sqrt[a^2 - b^2]*d^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f*(e^2 + f^
2*x^2)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^(
(2*I)*c)])] - 3*Sqrt[a^2 - b^2]*d^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*
f*(e^2 + f^2*x^2)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2
 - b^2)*E^((2*I)*c)])] + (6*I)*Sqrt[a^2 - b^2]*d^2*e*E^(I*c)*Sqrt[(a^2 - b^
2)*E^((2*I)*c)]*f^2*x*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqr
t[(-a^2 + b^2)*E^((2*I)*c)])] - (6*I)*Sqrt[a^2 - b^2]*d^2*e*E^(I*c)*Sqrt[(a
^2 - b^2)*E^((2*I)*c)]*f^2*x*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c
) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (6*I)*Sqrt[a^2 - b^2]*d*E^(I*c)*Sqr
t[(-a^2 + b^2)*E^((2*I)*c)]*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(
I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (6*I)*Sqrt[a^2 - b^2]*d*E^(I*c)*Sq
rt[(-a^2 + b^2)*E^((2*I)*c)]*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^
(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 6*Sqrt[a^2 - b^2]*d*e*E^(I*c)*Sqr
t[(a^2 - b^2)*E^((2*I)*c)]*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c
) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*Sqrt[a^2 - b^2]*d*e*E^(I*c)*Sqrt
[(a^2 - b^2)*E^((2*I)*c)]*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*
c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 6*Sqrt[a^2 - b^2]*E^(I*c)*Sqrt[(-a
^2 + b^2)*E^((2*I)*c)]*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) -
Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 6*Sqrt[a^2 - b^2]*E^(I*c)*Sqrt[(-a^2 + b^
2)*E^((2*I)*c)]*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a
^2 - b^2)*E^((2*I)*c)])]))/(a*Sqrt[a^2 - b^2]*d^4*Sqrt[-((a^2 - b^2)^2*E^((
4*I)*c))])
```

**Maple** [**F**]

time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \csc(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3580 vs. 2(645) = 1290.
time = 0.70, size = 3580, normalized size = 4.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*b^2*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*s
in(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
) - 6*I*b^2*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*si
n(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 - 6*I*b^2*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*si
n(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 + 6*I*b^2*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*si
n(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 + 6*I*(a^2 - b^2)*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c)) - 6*I*(a^2
 - b^2)*f^3*polylog(4, cos(d*x + c) - I*sin(d*x + c)) + 6*I*(a^2 - b^2)*f^3
*polylog(4, -cos(d*x + c) + I*sin(d*x + c)) - 6*I*(a^2 - b^2)*f^3*polylog(4
, -cos(d*x + c) - I*sin(d*x + c)) + 3*(-I*b^2*d^2*f^3*x^2 - 2*I*b^2*d^2*f^2
*x*e - I*b^2*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*
sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
 b)/b + 1) + 3*(I*b^2*d^2*f^3*x^2 + 2*I*b^2*d^2*f^2*x*e + I*b^2*d^2*f*e^2)*
sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(I*b^2*d^
2*f^3*x^2 + 2*I*b^2*d^2*f^2*x*e + I*b^2*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*d
ilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(-I*b^2*d^2*f^3*x^2 - 2*I*b^2*d^
```

```
2*f^2*x*e - I*b^2*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c
) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) + (b^2*c^3*f^3 - 3*b^2*c^2*d*f^2*e + 3*b^2*c*d^2*f*e^2 - b
^2*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c
) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b^2*c^3*f^3 - 3*b^2*c^2*d*f^2*e
+ 3*b^2*c*d^2*f*e^2 - b^2*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x +
 c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b^2*c^3*f
^3 - 3*b^2*c^2*d*f^2*e + 3*b^2*c*d^2*f*e^2 - b^2*d^3*e^3)*sqrt(-(a^2 - b^2)
/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) + 2*I*a) - (b^2*c^3*f^3 - 3*b^2*c^2*d*f^2*e + 3*b^2*c*d^2*f*e^2 - b^2*d^
3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b^2*d^3*f^3*x^3 + b^2*c^3*f^3 + 3*(b
^2*d^3*f*x + b^2*c*d^2*f)*e^2 + 3*(b^2*d^3*f^2*x^2 - b^2*c^2*d*f^2)*e)*sqrt
(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b^2*d^3*f^3*x^3 + b
^2*c^3*f^3 + 3*(b^2*d^3*f*x + b^2*c*d^2*f)*e^2 + 3*(b^2*d^3*f^2*x^2 - b^2*c
^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c)
 - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b^
2*d^3*f^3*x^3 + b^2*c^3*f^3 + 3*(b^2*d^3*f*x + b^2*c*d^2*f)*e^2 + 3*(b^2*d^
3*f^2*x^2 - b^2*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c
) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b) - (b^2*d^3*f^3*x^3 + b^2*c^3*f^3 + 3*(b^2*d^3*f*x + b^2*c*d^2*
f)*e^2 + 3*(b^2*d^3*f^2*x^2 - b^2*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(
-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) - b)/b) - 6*(b^2*d*f^3*x + b^2*d*f^2*e)*sqrt(-(a^2 -
 b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c)
 - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(b^2*d*f^3*x + b^2*d*f^
2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c)
- (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(b^2*d
*f^3*x + b^2*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c)
 + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2))/b) + 6*(b^2*d*f^3*x + b^2*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -
(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2))/b) + 3*(-I*(a^2 - b^2)*d^2*f^3*x^2 - 2*I*(a^2 - b^2)
*d^2*f^2*x*e - I*(a^2 - b^2)*d^2*f*e^2)*dilog(cos(d*x + c) + I*sin(d*x + c)
) + 3*(I*(a^2 - b^2)*d^2*f^3*x^2 + 2*I*(a^2 - b^2)*d^2*f^2*x*e + I*(a^2 - b
^2)*d^2*f*e^2)*dilog(cos(d*x + c) - I*sin(d*x + c)) + 3*(-I*(a^2 - b^2)*d^2
*f^3*x^2 - 2*I*(a^2 - b^2)*d^2*f^2*x*e - I*(a^2 - b^2)*d^2*f*e^2)*dilog(-co
s(d*x + c) + I*sin(d*x + c)) + 3*(I*(a^2 - b^2)*d^2*f^3*x^2 + 2*I*(a^2 - b^
2)*d^2*f^2*x*e + I*(a^2 - b^2)*d^2*f*e^2)*dilog(-cos(d*x + c) - I*sin(d*x +
 c)) - ((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*f^2*x^2*e + 3*(a^2 - b^
2)*d^3*f*x*e^2 + (a^2 - b^2)*d^3*e^3)*log(cos(d*x + c) + I*sin(d*x + c) + 1
) - ((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*f^2*x^2*e + 3*(a^2 - b^2)*
d^3*f*x*e^2 + (a^2 - b^2)*d^3*e^3)*log(cos(d*x + c) - I*sin(d*x + c) + 1) -
 ((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e
```

```
^2 - (a^2 - b^2)*d^3*e^3)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)
 - ((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f...
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.233 $\int \frac{(e+fx)^2 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=528

$$-\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}-\frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}+2$$

```
[Out] -2*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d+2*I*f*(f*x+e)*polylog(2,-exp(I*(d*
x+c)))/a/d^2-2*I*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2-2*f^2*polylog(3,
-exp(I*(d*x+c)))/a/d^3+2*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3+I*b*(f*x+e)^2*
ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/d/(a^2-b^2)^(1/2)-I*b*(f*x+e
)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/d/(a^2-b^2)^(1/2)+2*b*f*
(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/d^2/(a^2-b^2)^(
1/2)-2*b*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/d^2/
(a^2-b^2)^(1/2)+2*I*b*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))
/a/d^3/(a^2-b^2)^(1/2)-2*I*b*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/a/d^3/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.63, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4631, 4268, 2611, 2320, 6724, 3404, 2296, 2221}

$$\frac{2ibf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^3\sqrt{a^2-b^2}}-\frac{2ibf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^3\sqrt{a^2-b^2}}+\frac{2bf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{2bf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{2f^2\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{2f^2\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}+\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad\sqrt{a^2-b^2}}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) + (I*b*(e + f*x)^2*Log[1 -
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (I*b*
(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a
^2 - b^2]*d) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - (
(2*I)*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*P
olyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*
d^2) - (2*b*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])])/(a*Sqrt[a^2 - b^2]*d^2) - (2*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3
) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) + ((2*I)*b*f^2*PolyLog[3, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^3) - ((2*
I)*b*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a
^2 - b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
```

```
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\csc(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(2b)\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{a} - \frac{(2f)\int (e+ \ldots$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{2if(e+fx)\text{Li}_2\left(-e^{i(c+dx)}\right)}{ad^2} - \frac{2if(e+fx)\text{Li}_2 \ldots}{ad^2}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+ \ldots}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+ \ldots}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+ \ldots}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+ \ldots}{}$$

**Mathematica [A]**
time = 2.49, size = 982, normalized size = 1.86

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*d^2*e^2*ArcTanh[E^(I*(c + d*x))] + 2*d^2*e*f*x*Log[1 - E^(I*(c + d*x))]
 + d^2*f^2*x^2*Log[1 - E^(I*(c + d*x))] - 2*d^2*e*f*x*Log[1 + E^(I*(c + d*x
))] - d^2*f^2*x^2*Log[1 + E^(I*(c + d*x))] + (2*I)*d*f*(e + f*x)*PolyLog[2,
 -E^(I*(c + d*x))] - (2*I)*d*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))] - 2*f^
2*PolyLog[3, -E^(I*(c + d*x))] + 2*f^2*PolyLog[3, E^(I*(c + d*x))] + (b*(2*
Sqrt[a^2 - b^2]*d*E^(I*c)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a
*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + I*(2*d^2*e^2*Sqrt[(a^2 - b^2)*
E^((2*I)*c)]*ArcTanh[(-a + I*b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 2*Sqrt[a
^2 - b^2]*d^2*e*E^(I*c)*f*x*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sq
rt[(a^2 - b^2)*E^((2*I)*c)])] + Sqrt[a^2 - b^2]*d^2*E^(I*c)*f^2*x^2*Log[1 -
 (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 2*S
qrt[a^2 - b^2]*d^2*e*E^(I*c)*f*x*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c)
 + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - Sqrt[a^2 - b^2]*d^2*E^(I*c)*f^2*x^2*Lo
g[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
+ (2*I)*Sqrt[a^2 - b^2]*d*E^(I*c)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(2*c + d
*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 2*Sqrt[a^2 - b^2]*E^(I
*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^
((2*I)*c)])] - 2*Sqrt[a^2 - b^2]*E^(I*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*
x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])])))/(Sqrt[a^2 - b^2]*Sqrt[
(a^2 - b^2)*E^((2*I)*c)]))/(a*d^3)
```

**Maple [F]**
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \csc(dx+c)}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2426 vs. $2(461) = 922$.
time = 0.61, size = 2426, normalized size = 4.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*si
n(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 - 2*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d
*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) +
2*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x
 + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*
b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x +
 c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(a
^2 - b^2)*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c)) - 2*(a^2 - b^2)*f^2
*polylog(3, cos(d*x + c) - I*sin(d*x + c)) + 2*(a^2 - b^2)*f^2*polylog(3, -
cos(d*x + c) + I*sin(d*x + c)) + 2*(a^2 - b^2)*f^2*polylog(3, -cos(d*x + c)
 - I*sin(d*x + c)) - 2*(-I*b^2*d*f^2*x - I*b^2*d*f*e)*sqrt(-(a^2 - b^2)/b^2
)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*b^2*d*f^2*x + I*b^2*d*f*e)*
sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*b^2*d*
f^2*x + I*b^2*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*si
n(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b
)/b + 1) - 2*(-I*b^2*d*f^2*x - I*b^2*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-
I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b + 1) + (b^2*c^2*f^2 - 2*b^2*c*d*f*e + b^2*d^2*e^
2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*s
qrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b^2*c^2*f^2 - 2*b^2*c*d*f*e + b^2*d^2*e^2
)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) - 2*I*a) - (b^2*c^2*f^2 - 2*b^2*c*d*f*e + b^2*d^2*e^2)
*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) + 2*I*a) - (b^2*c^2*f^2 - 2*b^2*c*d*f*e + b^2*d^2*e^2)
*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) - 2*I*a) - (b^2*d^2*f^2*x^2 - b^2*c^2*f^2 + 2*(b^2*d^2
*f*x + b^2*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(
d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/
b) + (b^2*d^2*f^2*x^2 - b^2*c^2*f^2 + 2*(b^2*d^2*f*x + b^2*c*d*f)*e)*sqrt(-
(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c)
+ I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b^2*d^2*f^2*x^2 - b^2
*c^2*f^2 + 2*(b^2*d^2*f*x + b^2*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a
*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
```

```
(a^2 - b^2)/b^2) - b)/b) + (b^2*d^2*f^2*x^2 - b^2*c^2*f^2 + 2*(b^2*d^2*f*x
+ b^2*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x
+ c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) -
 2*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*d*f*e)*dilog(cos(d*x + c) + I*si
n(d*x + c)) - 2*(I*(a^2 - b^2)*d*f^2*x + I*(a^2 - b^2)*d*f*e)*dilog(cos(d*x
 + c) - I*sin(d*x + c)) - 2*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*d*f*e)*
dilog(-cos(d*x + c) + I*sin(d*x + c)) - 2*(I*(a^2 - b^2)*d*f^2*x + I*(a^2 -
 b^2)*d*f*e)*dilog(-cos(d*x + c) - I*sin(d*x + c)) + ((a^2 - b^2)*d^2*f^2*x
^2 + 2*(a^2 - b^2)*d^2*f*x*e + (a^2 - b^2)*d^2*e^2)*log(cos(d*x + c) + I*si
n(d*x + c) + 1) + ((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*f*x*e + (a^2
 - b^2)*d^2*e^2)*log(cos(d*x + c) - I*sin(d*x + c) + 1) - ((a^2 - b^2)*c^2*
f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^2)*d^2*e^2)*log(-1/2*cos(d*x + c) +
1/2*I*sin(d*x + c) + 1/2) - ((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e +
(a^2 - b^2)*d^2*e^2)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - ((
a^2 - b^2)*d^2*f^2*x^2 - (a^2 - b^2)*c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x + (a^
2 - b^2)*c*d*f)*e)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) - ((a^2 - b^2)*d
^2*f^2*x^2 - (a^2 - b^2)*c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d
*f)*e)*log(-cos(d*x + c) - I*sin(d*x + c) + 1))/((a^3 - a*b^2)*d^3)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**2*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.234 $\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=325

$$-\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}-\frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}+\frac{ifL}{}$$

```
[Out] -2*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d+I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2
-I*f*polylog(2,exp(I*(d*x+c)))/a/d^2+I*b*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a
-(a^2-b^2)^(1/2)))/a/d/(a^2-b^2)^(1/2)-I*b*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/a/d/(a^2-b^2)^(1/2)+b*f*polylog(2,I*b*exp(I*(d*x+c))/(
a-(a^2-b^2)^(1/2)))/a/d^2/(a^2-b^2)^(1/2)-b*f*polylog(2,I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/a/d^2/(a^2-b^2)^(1/2)
```

**Rubi** [A]

time = 0.36, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4631, 4268, 2317, 2438, 3404, 2296, 2221}

$$\frac{bf\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{bf\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^2\sqrt{a^2-b^2}}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad\sqrt{a^2-b^2}}-\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + (I*b*(e + f*x)*Log[1 - (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (I*b*(e +
 f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b
^2]*d) + (I*f*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - (I*f*PolyLog[2, E^(I*
(c + d*x))])/(a*d^2) + (b*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(a*Sqrt[a^2 - b^2]*d^2) - (b*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
```

```
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\csc(c+dx)\,dx}{a} - \frac{b\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(2b)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{a} - \frac{f\int \log\left(1-e\right.}{ad}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(2ib^2)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{a\sqrt{a^2-b^2}} - \frac{(2ib^2)\int}{}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+fx}{}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+fx}{}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+fx}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 764 vs. 2(325) = 650.
time = 3.88, size = 764, normalized size = 2.35

$$\frac{de\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)-cf\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)+f\left((c+dx)\left(\log\left(1-e^{i(c+dx)}\right)-\log\left(1+e^{i(c+dx)}\right)\right)+i\left(\mathrm{Li}_2\left(-e^{i(c+dx)}\right)-\mathrm{Li}_2\left(e^{i(c+dx)}\right)\right)\right)-\frac{bd(e+fx)\left(\frac{2(de-cf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{-ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}+\frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}+\frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{-b+\sqrt{-a^2+b^2}-a\tan\left(\frac{1}{2}(c+dx)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(i+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}}-\frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b-\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b-\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a+ia\tan\left(\frac{1}{2}(c+dx)\right)}{a+i\left(-b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}\right)}{de-cf+if\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)-if\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}}{ad^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (d*e*Log[Tan[(c + d*x)/2]] - c*f*Log[Tan[(c + d*x)/2]] + f*((c + d*x)*(Log[
1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c +
 d*x))] - PolyLog[2, E^(I*(c + d*x))])) - (b*d*(e + f*x)*((2*(d*e - c*f)*Ar
cTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log
[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((
-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a
 + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c +
 d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-
a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^
2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-b
+ Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] + Po
lyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a
```

^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a *Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[ (c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]]))/( a*d^2)

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(287) = 574$.
time = 0.15, size = 660, normalized size = 2.03

| method | result |
|---|---|
| risch | $-\frac{2ieb\arctan\left(\frac{2ib\,\mathrm{e}^{i(dx+c)}-2a}{2\sqrt{-a^2+b^2}}\right)}{da\sqrt{-a^2+b^2}}-\frac{fc\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d^2a}+\frac{if\,\mathrm{dilog}\left(\mathrm{e}^{i(dx+c)}\right)}{d^2a}+\frac{e\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{da}-\frac{e\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{da}-\frac{fb}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d*e/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b
^2)^(1/2))-1/d^2/a*f*c*ln(exp(I*(d*x+c))-1)+I/d^2*f*dilog(exp(I*(d*x+c)))/a
+1/d/a*e*ln(exp(I*(d*x+c))-1)-1/d/a*e*ln(exp(I*(d*x+c))+1)-1/d*f*b/a/(-a^2+
b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)
))*x-1/d^2*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2)
)/(I*a-(-a^2+b^2)^(1/2)))*c+1/d*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x
+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2*f*b/a/(-a^2+b^2)^(1/
2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/d
^2*f*b/a/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*
a-(-a^2+b^2)^(1/2)))-I/d^2*f*b/a/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c
))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-1/d/a*ln(exp(I*(d*x+c))+1)*f*x
+I/d^2*f/a*dilog(exp(I*(d*x+c))+1)+2*I/d^2*f*c/a*b/(-a^2+b^2)^(1/2)*arctan(
1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1438 vs. 2(278) = 556.
time = 0.64, size = 1438, normalized size = 4.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)
 + I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c)
- (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) +
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*
b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*(a
^2 - b^2)*f*dilog(cos(d*x + c) + I*sin(d*x + c)) + I*(a^2 - b^2)*f*dilog(co
s(d*x + c) - I*sin(d*x + c)) - I*(a^2 - b^2)*f*dilog(-cos(d*x + c) + I*sin(
d*x + c)) + I*(a^2 - b^2)*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) + (b^2*c*
f - b^2*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x +
c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b^2*c*f - b^2*d*e)*sqrt(-(a^2 -
 b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) - 2*I*a) - (b^2*c*f - b^2*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d
*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b^2*c
*f - b^2*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b^2*d*f*x + b^2*c*f)*sqrt(-(a
^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b^2*d*f*x + b^2*c*f)*sq
rt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x +
 c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b^2*d*f*x + b^2*c
*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b^2*d*f*x
+ b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c)
- (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - ((a^
2 - b^2)*d*f*x + (a^2 - b^2)*d*e)*log(cos(d*x + c) + I*sin(d*x + c) + 1) -
((a^2 - b^2)*d*f*x + (a^2 - b^2)*d*e)*log(cos(d*x + c) - I*sin(d*x + c) + 1
) - ((a^2 - b^2)*c*f - (a^2 - b^2)*d*e)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d
*x + c) + 1/2) - ((a^2 - b^2)*c*f - (a^2 - b^2)*d*e)*log(-1/2*cos(d*x + c)
- 1/2*I*sin(d*x + c) + 1/2) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-co
s(d*x + c) + I*sin(d*x + c) + 1) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*lo
g(-cos(d*x + c) - I*sin(d*x + c) + 1))/((a^3 - a*b^2)*d^2)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.235 $\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=67

$$-\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d-2*b*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/a/d/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2826, 3855, 2739, 632, 210}

$$-\frac{2b\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]*
d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\int \csc(c+dx)\,dx}{a}-\frac{b\int \frac{1}{a+b\sin(c+dx)}\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{(2b)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{(4b)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\\ &= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}\end{aligned}$$

**Mathematica** [A]
time = 0.06, size = 77, normalized size = 1.15

$$\frac{-\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(a*d)
```

**Maple** [A]
time = 0.14, size = 67, normalized size = 1.00

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$ |
| default | $\frac{-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$ |
| risch | $\frac{ib\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,da}-\frac{ib\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,da}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{da}-$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))+1/a*ln(tan(1/2*d*x+1/2*c)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.41, size = 297, normalized size = 4.43

$$\left[-\frac{\sqrt{-a^2+b^2}\,b\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+(a^2-b^2)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(a^2-b^2)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2(a^3-ab^2)d},\ \frac{2\sqrt{a^2-b^2}\,b\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-(a^2-b^2)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(a^2-b^2)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2(a^3-ab^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*
x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(
-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + (a^2
- b^2)*log(1/2*cos(d*x + c) + 1/2) - (a^2 - b^2)*log(-1/2*cos(d*x + c) + 1/
2))/((a^3 - a*b^2)*d), 1/2*(2*sqrt(a^2 - b^2)*b*arctan(-(a*sin(d*x + c) + b
```

```
)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^2 - b^2)*log(1/2*cos(d*x + c) + 1/2)
 + (a^2 - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral(csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 5.62, size = 83, normalized size = 1.24

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a) - log(abs(tan(1/2*d*x + 1/2
*c)))/a)/d
```

**Mupad** [B]
time = 2.68, size = 173, normalized size = 2.58

$$\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{b^2-a^2}\,\left(-1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+4\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)}{1\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+3\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b-2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2-4\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)}{a\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) + (2*b*atanh(((b^2 - a^2)^
(1/2)*(b^2*sin(c/2 + (d*x)/2)*4i - a^2*sin(c/2 + (d*x)/2)*1i + a*b*cos(c/2
+ (d*x)/2)*2i))/(a^3*cos(c/2 + (d*x)/2)*1i - b^3*sin(c/2 + (d*x)/2)*4i - a*
b^2*cos(c/2 + (d*x)/2)*2i + a^2*b*sin(c/2 + (d*x)/2)*3i)))/(a*d*(b^2 - a^2)
^(1/2))
```

# 3.236 $\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)} \, dx$

**Optimal**. Leaf size=882

$$-\frac{i(e+fx)^3}{ad}+\frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d}-\frac{(e+fx)^3\cot(c+dx)}{ad}-\frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d}+$$

```
[Out] -6*I*b^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/
d^3/(a^2-b^2)^(1/2)+2*b*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a^2/d-(f*x+e)^3*c
ot(d*x+c)/a/d+3*f*(f*x+e)^2*ln(1-exp(2*I*(d*x+c)))/a/d^2-3*I*b*f*(f*x+e)^2*
polylog(2,-exp(I*(d*x+c)))/a^2/d^2-I*b^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/
(a-(a^2-b^2)^(1/2)))/a^2/d/(a^2-b^2)^(1/2)-6*I*b*f^3*polylog(4,exp(I*(d*x+c
)))/a^2/d^4+6*b*f^2*(f*x+e)*polylog(3,-exp(I*(d*x+c)))/a^2/d^3-6*b*f^2*(f*x
+e)*polylog(3,exp(I*(d*x+c)))/a^2/d^3+3/2*f^3*polylog(3,exp(2*I*(d*x+c)))/a
/d^4-3*I*f^2*(f*x+e)*polylog(2,exp(2*I*(d*x+c)))/a/d^3+I*b^2*(f*x+e)^3*ln(1
-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/d/(a^2-b^2)^(1/2)+3*I*b*f*(f*x
+e)^2*polylog(2,exp(I*(d*x+c)))/a^2/d^2-I*(f*x+e)^3/a/d-3*b^2*f*(f*x+e)^2*p
olylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/d^2/(a^2-b^2)^(1/2)+3*
b^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/d^2/(
a^2-b^2)^(1/2)+6*I*b^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2
)^(1/2)))/a^2/d^3/(a^2-b^2)^(1/2)+6*I*b*f^3*polylog(4,-exp(I*(d*x+c)))/a^2/
d^4+6*b^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/d^4/(a^
2-b^2)^(1/2)-6*b^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^
2/d^4/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.97, antiderivative size = 882, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {4631, 4269, 3798, 2221, 2611, 2320, 6724, 4268, 6744, 3404, 2296}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^3)/(a*d) + (2*b*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a^2*
d) - ((e + f*x)^3*Cot[c + d*x])/(a*d) - (I*b^2*(e + f*x)^3*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d) + (I*b^2*(e +
 f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2
 - b^2]*d) + (3*f*(e + f*x)^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((3*I
)*b*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + ((3*I)*b*f*(e +
 f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - (3*b^2*f*(e + f*x)^2*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^
```

```
2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I
)*(c + d*x))])/(a*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/(
a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a^2*d^3) - ((6*
I)*b^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])
])/(a^2*Sqrt[a^2 - b^2]*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^3) + (3*f^3*Po
lyLog[3, E^((2*I)*(c + d*x))])/(2*a*d^4) + ((6*I)*b*f^3*PolyLog[4, -E^(I*(c
 + d*x))])/(a^2*d^4) - ((6*I)*b*f^3*PolyLog[4, E^(I*(c + d*x))])/(a^2*d^4)
+ (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*
Sqrt[a^2 - b^2]*d^4) - (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3 \csc^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3 \csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3 \cot(c+dx)}{ad} - \frac{b\int (e+fx)^3 \csc(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{a^2}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2 d} - \frac{(e+fx)^3 \cot(c+dx)}{ad} + \frac{(2b}{}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2 d} - \frac{(e+fx)^3 \cot(c+dx)}{ad} + \frac{3f}{}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2 d} - \frac{(e+fx)^3 \cot(c+dx)}{ad} - \frac{ib^2}{}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2 d} - \frac{(e+fx)^3 \cot(c+dx)}{ad} - \frac{ib^2}{}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2 d} - \frac{(e+fx)^3 \cot(c+dx)}{ad} - \frac{ib^2}{}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2 d} - \frac{(e+fx)^3 \cot(c+dx)}{ad} - \frac{ib^2}{}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2 d} - \frac{(e+fx)^3 \cot(c+dx)}{ad} - \frac{ib^2}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2452 vs. $2(882) = 1764$.
time = 41.17, size = 2452, normalized size = 2.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*e^3*Log[Tan[(c + d*x)/2]])/(a^2*d)) - (3*b*e^2*f*((c + d*x)*(Log[1 - E
^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(
PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a^2*d^2) - (
f^3*Csc[c]*(2*d^2*x^2*(2*d*E^((2*I)*c)*x + (3*I)*(-1 + E^((2*I)*c))*Log[1 -
 E^((2*I)*(c + d*x))]) + 6*d*(-1 + E^((2*I)*c))*x*PolyLog[2, E^((2*I)*(c +
d*x))] + (3*I)*(-1 + E^((2*I)*c))*PolyLog[3, E^((2*I)*(c + d*x))]))/(4*a*d^
4*E^(I*c)) + (6*b*e*f^2*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - I
*d*x*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + I*d*x*PolyLog[2, Cos[c +
d*x] + I*Sin[c + d*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - PolyL
og[3, Cos[c + d*x] + I*Sin[c + d*x]]))/(a^2*d^3) - (b*f^3*(-2*d^3*x^3*ArcTa
nh[Cos[c + d*x] + I*Sin[c + d*x]] + (3*I)*d^2*x^2*PolyLog[2, -Cos[c + d*x]
- I*Sin[c + d*x]] - (3*I)*d^2*x^2*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*x]]
 - 6*d*x*PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] + 6*d*x*PolyLog[3, Cos[
c + d*x] + I*Sin[c + d*x]] - (6*I)*PolyLog[4, -Cos[c + d*x] - I*Sin[c + d*x
]] + (6*I)*PolyLog[4, Cos[c + d*x] + I*Sin[c + d*x]]))/(a^2*d^4) + (3*e^2*f
*Csc[c]*(-(d*x*Cos[c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a
*d^2*(Cos[c]^2 + Sin[c]^2)) + (I*b^2*((3*I)*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log
[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2
)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + (3*I)*Sqrt[a^2
- b^2]*d^3*e*f^2*x^2*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*C
os[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*S
in[c]) + I*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2
*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[
c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -
((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(C
os[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 3*Sqrt[a^2 - b^2]*
d^2*f*(e + f*x)^2*PolyLog[2, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*
a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] +
I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*(Cos[2*c + d*x] +
 I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2]
- a*Sin[c]))]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3
, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)
*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 6*Sqrt[a^2 - b^
2]*f^3*PolyLog[4, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + S
qrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) +
 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(
(-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[
c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*(Cos[2*c + d*x] +
 I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^
2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Lo
g[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 +
 b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + Sqrt[a^2
 - b^2]*d^3*f^3*x^3*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a
*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c
] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*(Cos[2*c + d*x] + I*S
```

```
in[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] +
 a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, (
b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(
Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) - (2*I)*d^3*e^3*A
rcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]]*Sqrt[(-a^2
 + b^2)*(Cos[2*c] + I*Sin[2*c])]))/(a^2*Sqrt[a^2 - b^2]*d^4*Sqrt[(-a^2 + b^
2)*(Cos[2*c] + I*Sin[2*c])]) + (Csc[c/2]*Csc[c/2 + (d*x)/2]*(e^3*Sin[(d*x)/
2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/
2]))/(2*a*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(e^3*Sin[(d*x)/2] + 3*e^2*f*x*S
in[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2]))/(2*a*d) - (
3*e*f^2*Csc[c]*Sec[c]*(d^2*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi + 2*ArcT
an[Tan[c]]) - Pi*Log[1 + E^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*Log[1 -
 E^((2*I)*(d*x + ArcTan[Tan[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan[c]]*Lo
g[Sin[d*x + ArcTan[Tan[c]]]] + I*PolyLog[2, E^((2*I)*(d*x + ArcTan[Tan[c]])
)])*Tan[c])/Sqrt[1 + Tan[c]^2]))/(a*d^3*Sqrt[Sec[c]^2*(Cos[c]^2 + Sin[c]^2)
])
```

**Maple** [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\csc^2 (dx+c))}{a+b\sin (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4553 vs. 2(786) = 1572.
time = 0.77, size = 4553, normalized size = 5.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(-6*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*
sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b)*sin(d*x + c) + 6*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d
*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2))/b)*sin(d*x + c) + 6*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4,
 -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 6*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^
2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 6*I*(a^2*b - b^3)*f
^3*polylog(4, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 6*I*(a^2*b - b^
3)*f^3*polylog(4, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 6*I*(a^2*b
- b^3)*f^3*polylog(4, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 6*I*(a
^2*b - b^3)*f^3*polylog(4, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 3
*(I*b^3*d^2*f^3*x^2 + 2*I*b^3*d^2*f^2*x*e + I*b^3*d^2*f*e^2)*sqrt(-(a^2 - b
^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 3*(-I*b^3*d^
2*f^3*x^2 - 2*I*b^3*d^2*f^2*x*e - I*b^3*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*d
ilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 3*(-I*b^3*d^2*f^3*x^2
- 2*I*b^3*d^2*f^2*x*e - I*b^3*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a
*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 3*(I*b^3*d^2*f^3*x^2 + 2*I*b^3*
d^2*f^2*x*e + I*b^3*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x +
 c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b + 1)*sin(d*x + c) - (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*e + 3*b^3*
c*d^2*f*e^2 - b^3*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*
I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - (b^3*
c^3*f^3 - 3*b^3*c^2*d*f^2*e + 3*b^3*c*d^2*f*e^2 - b^3*d^3*e^3)*sqrt(-(a^2 -
 b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) - 2*I*a)*sin(d*x + c) + (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*e + 3*b^3*c*d
^2*f*e^2 - b^3*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*
b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + (b^3*c^
3*f^3 - 3*b^3*c^2*d*f^2*e + 3*b^3*c*d^2*f*e^2 - b^3*d^3*e^3)*sqrt(-(a^2 - b
^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) - 2*I*a)*sin(d*x + c) - (b^3*d^3*f^3*x^3 + b^3*c^3*f^3 + 3*(b^3*d^3*f
*x + b^3*c*d^2*f)*e^2 + 3*(b^3*d^3*f^2*x^2 - b^3*c^2*d*f^2)*e)*sqrt(-(a^2 -
 b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + (b^3*d^3*f^3*x^
3 + b^3*c^3*f^3 + 3*(b^3*d^3*f*x + b^3*c*d^2*f)*e^2 + 3*(b^3*d^3*f^2*x^2 -
b^3*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x
 + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*
```

```
sin(d*x + c) - (b^3*d^3*f^3*x^3 + b^3*c^3*f^3 + 3*(b^3*d^3*f*x + b^3*c*d^2*
f)*e^2 + 3*(b^3*d^3*f^2*x^2 - b^3*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(
-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + (b^3*d^3*f^3*x^3 + b^3*c^3*f^
3 + 3*(b^3*d^3*f*x + b^3*c*d^2*f)*e^2 + 3*(b^3*d^3*f^2*x^2 - b^3*c^2*d*f^2)
*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c)
+ 6*(b^3*d*f^3*x + b^3*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos
(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2))/b)*sin(d*x + c) - 6*(b^3*d*f^3*x + b^3*d*f^2*e)*sqrt(-(a^2 -
b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c)
- I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 6*(b^3*d*f^3*
x + b^3*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*
sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b)*sin(d*x + c) - 6*(b^3*d*f^3*x + b^3*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*poly
log(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 3*(I*(a^2*b - b^3)*d^2*f^3
*x^2 - 2*I*(a^3 - a*b^2)*d*f^3*x + I*(a^2*b - b^3)*d^2*f*e^2 + 2*I*((a^2*b
- b^3)*d^2*f^2*x - (a^3 - a*b^2)*d*f^2)*e)*dilog(cos(d*x + c) + I*sin(d*x +
 c))*sin(d*x + c) + 3*(-I*(a^2*b - b^3)*d^2*f^3*x^2 + 2*I*(a^3 - a*b^2)*d*f
^3*x - I*(a^2*b - b^3)*d^2*f*e^2 - 2*I*((a^2*b - b^3)*d^2*f^2*x - (a^3 - a*
b^2)*d*f^2)*e)*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 3*(I*(a^
2*b - b^3)*d^2*f^3*x^2 + 2*I*(a^3 - a*b^2)*d*f^3*x + I*(a^2*b - b^3)*d^2*f*
e^2 + 2*I*((a^2*b - b^3)*d^2*f^2*x + (a^3 - a*b^2)*d*f^2)*e)*dilog(-cos(d*x
 + c) + I*sin(d*x + c))*sin(d*x + c) + 3*(-I*(a^2*b - b^3)*d^2*f^3*x^2 - 2*
I*(a^3 - a*b^2)*d*f^3*x - I*(a^2*b - b^3)*d^2*f...
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.237 $\int \frac{(e+fx)^2 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=639

$$-\frac{i(e+fx)^2}{ad}+\frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d}-\frac{(e+fx)^2\cot(c+dx)}{ad}-\frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d}+$$

```
[Out] -I*(f*x+e)^2/a/d+2*b*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a^2/d-(f*x+e)^2*cot(
d*x+c)/a/d+2*f*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/a/d^2-2*I*b*f*(f*x+e)*polylog
(2,-exp(I*(d*x+c)))/a^2/d^2+2*I*b*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a^2/d
^2-I*f^2*polylog(2,exp(2*I*(d*x+c)))/a/d^3+2*b*f^2*polylog(3,-exp(I*(d*x+c)
))/a^2/d^3-2*b*f^2*polylog(3,exp(I*(d*x+c)))/a^2/d^3-I*b^2*(f*x+e)^2*ln(1-I
*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/d/(a^2-b^2)^(1/2)+I*b^2*(f*x+e)^
2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/d/(a^2-b^2)^(1/2)-2*b^2*
f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/d^2/(a^2-b^
2)^(1/2)+2*b^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/
a^2/d^2/(a^2-b^2)^(1/2)-2*I*b^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/a^2/d^3/(a^2-b^2)^(1/2)+2*I*b^2*f^2*polylog(3,I*b*exp(I*(d*x+c))
/(a+(a^2-b^2)^(1/2)))/a^2/d^3/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.78, antiderivative size = 639, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4631, 4269, 3798, 2221, 2317, 2438, 4268, 2611, 2320, 6724, 3404, 2296}

$$-\frac{2ib^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d^3\sqrt{a^2-b^2}}+\frac{2ib^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2d^3\sqrt{a^2-b^2}}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d^2\sqrt{a^2-b^2}}+\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2d^2\sqrt{a^2-b^2}}+\frac{2bf^2\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{a^2d^3}-\frac{2bf^2\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{a^2d^3}-\frac{2ibf(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}+\frac{2ibf(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2}-\frac{if^2\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}-\frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}+\frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2d\sqrt{a^2-b^2}}+\frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d}+\frac{2f(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{ad^2}-\frac{(e+fx)^2\cot(c+dx)}{ad}-\frac{i(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^2)/(a*d) + (2*b*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a^2*
d) - ((e + f*x)^2*Cot[c + d*x])/(a*d) - (I*b^2*(e + f*x)^2*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d) + (I*b^2*(e +
 f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2
 - b^2]*d) + (2*f*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((2*I)*
b*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + ((2*I)*b*f*(e + f*x
)*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2) + (2*
b^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a
^2*Sqrt[a^2 - b^2]*d^2) - (I*f^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^3) +
 (2*b*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a^2*d^3) - (2*b*f^2*PolyLog[3, E^(
I*(c + d*x))])/(a^2*d^3) - ((2*I)*b^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/
(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^3) + ((2*I)*b^2*f^2*PolyLog[
3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
```

```
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\csc^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{b\int (e+fx)^2\csc(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{a^2}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} + \frac{(2b^2)}{}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} + \frac{2f(e}{}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{ib^2(e}{}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{ib^2(e}{}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{ib^2(e}{}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{ib^2(e}{}$$

**Mathematica [A]**
time = 21.17, size = 1276, normalized size = 2.00

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*e^2*Log[Tan[(c + d*x)/2]])/(a^2*d)) - (2*b*e*f*((c + d*x)*(Log[1 - E^(
I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(Po
lyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a^2*d^2) + (2*
b*f^2*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - I*d*x*PolyLog[2, -C
os[c + d*x] - I*Sin[c + d*x]] + I*d*x*PolyLog[2, Cos[c + d*x] + I*Sin[c + d
*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - PolyLog[3, Cos[c + d*x]
 + I*Sin[c + d*x]]))/(a^2*d^3) + (2*e*f*Csc[c]*(-(d*x*Cos[c]) + Log[Cos[d*x
```

```
]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*d^2*(Cos[c]^2 + Sin[c]^2)) + (I*b^2
*(2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2
*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[
c]))]*(Cos[c] + I*Sin[c]) - 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*(
Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos
[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c]) - I*(-2*Sqrt[a^2 - b^2]
*f^2*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqr
t[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + 2
*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-
I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c]
 + I*Sin[c]) + d^2*(Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(-Log[1 + (b*(Cos[2*c +
 d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[
c])^2] - a*Sin[c])] + Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)
*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])])*(Cos[c]
+ I*Sin[c]) + 2*e^2*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a
^2 - b^2]]*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))))/(a^2*Sqrt[a^2 - b
^2]*d^3*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]) + (Csc[c/2]*Csc[c/2 + (
d*x)/2]*(e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2]))/(
2*a*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d*x)
/2] + f^2*x^2*Sin[(d*x)/2]))/(2*a*d) - (f^2*Csc[c]*Sec[c]*(d^2*E^(I*ArcTan[
Tan[c]])*x^2 + ((I*d*x*(-Pi + 2*ArcTan[Tan[c]]) - Pi*Log[1 + E^((-2*I)*d*x)
] - 2*(d*x + ArcTan[Tan[c]])*Log[1 - E^((2*I)*(d*x + ArcTan[Tan[c]]))] + Pi
*Log[Cos[d*x]] + 2*ArcTan[Tan[c]]*Log[Sin[d*x + ArcTan[Tan[c]]]] + I*PolyLo
g[2, E^((2*I)*(d*x + ArcTan[Tan[c]]))])*Tan[c])/Sqrt[1 + Tan[c]^2]))/(a*d^3
*Sqrt[Sec[c]^2*(Cos[c]^2 + Sin[c]^2)])
```

**Maple [F]**
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 (\csc^2(dx+c))}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2984 vs. 2(566) = 1132.
time = 0.69, size = 2984, normalized size = 4.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin
(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*
sin(d*x + c) - 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x +
c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b)*sin(d*x + c) + 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a
*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*polylo
g(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 2*(a^2*b - b^3)*f^2*polylog(
3, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 2*(a^2*b - b^3)*f^2*polylo
g(3, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*(a^2*b - b^3)*f^2*poly
log(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 2*(a^2*b - b^3)*f^2*p
olylog(3, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*(I*b^3*d*f^2*x +
 I*b^3*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)
*sin(d*x + c) + 2*(-I*b^3*d*f^2*x - I*b^3*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dil
og((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 2*(-I*b^3*d*f^2*x - I*b^
3*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) +
 (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin
(d*x + c) + 2*(I*b^3*d*f^2*x + I*b^3*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-
I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + (b^3*c^2*f^2 - 2*b^3*c*d*f*e
 + b^3*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + (b^3*c^2*f^2 - 2
*b^3*c*d*f*e + b^3*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2
*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - (b^3
*c^2*f^2 - 2*b^3*c*d*f*e + b^3*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos
(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*
x + c) - (b^3*c^2*f^2 - 2*b^3*c*d*f*e + b^3*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)
*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
2*I*a)*sin(d*x + c) - (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3
*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) +
```

```
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x
 + c) + (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*sqr
t(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x +
c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) - (b^3*d
^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*sqrt(-(a^2 - b^2)
/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + (b^3*d^2*f^2*x^2 -
b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-
I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + 2*(I*(a^2*b - b^3)*d*f^2*x + I*(
a^2*b - b^3)*d*f*e - I*(a^3 - a*b^2)*f^2)*dilog(cos(d*x + c) + I*sin(d*x +
c))*sin(d*x + c) + 2*(-I*(a^2*b - b^3)*d*f^2*x - I*(a^2*b - b^3)*d*f*e + I*
(a^3 - a*b^2)*f^2)*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*(I
*(a^2*b - b^3)*d*f^2*x + I*(a^2*b - b^3)*d*f*e + I*(a^3 - a*b^2)*f^2)*dilog
(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 2*(-I*(a^2*b - b^3)*d*f^2*x
 - I*(a^2*b - b^3)*d*f*e - I*(a^3 - a*b^2)*f^2)*dilog(-cos(d*x + c) - I*sin
(d*x + c))*sin(d*x + c) + ((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^3 - a*b^2)*d*f^
2*x + (a^2*b - b^3)*d^2*e^2 + 2*((a^2*b - b^3)*d^2*f*x + (a^3 - a*b^2)*d*f)
*e)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + ((a^2*b - b^3)*d^
2*f^2*x^2 + 2*(a^3 - a*b^2)*d*f^2*x + (a^2*b - b^3)*d^2*e^2 + 2*((a^2*b - b
^3)*d^2*f*x + (a^3 - a*b^2)*d*f)*e)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*
sin(d*x + c) - ((a^2*b - b^3)*d^2*e^2 - 2*(a^3 - a*b^2 + (a^2*b - b^3)*c)*d
*f*e + ((a^2*b - b^3)*c^2 + 2*(a^3 - a*b^2)*c)*f^2)*log(-1/2*cos(d*x + c) +
 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) - ((a^2*b - b^3)*d^2*e^2 - 2*(a^3 -
 a*b^2 + (a^2*b - b^3)*c)*d*f*e + ((a^2*b - b^3)*c^2 + 2*(a^3 - a*b^2)*c)*f
^2)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) - ((a^2*
b - b^3)*d^2*f^2*x^2 - 2*(a^3 - a*b^2)*d*f^2*x - ((a^2*b - b^3)*c^2 + 2*(a^
3 - a*b^2)*c)*f^2 + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*c*d*f)*e)*log(
-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) - ((a^2*b - b^3)*d^2*f^2*x
^2 - 2*(a^3 - a*b^2)*d*f^2*x - ((a^2*b - b^3)*c^2 + 2*(a^3 - a*b^2)*c)*f^2
+ 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*c*d*f)*e)*log(-cos(d*x + c) - I*
sin(d*x + c) + 1)*sin(d*x + c) - 2*((a^3 - a*b^2)*d^2*f^2*x^2 + 2*(a^3 - a*
b^2)*d^2*f*x*e + (a^3 - a*b^2)*d^2*e^2)*cos(d*x + c))/((a^4 - a^2*b^2)*d^3*
sin(d*x + c))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.238 $\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=370

$$\frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d}-\frac{(e+fx)\cot(c+dx)}{ad}-\frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d}+\frac{ib^2(e+fx)\log\left(\right.}{a^2\sqrt{a}}$$

```
[Out] 2*b*(f*x+e)*arctanh(exp(I*(d*x+c)))/a^2/d-(f*x+e)*cot(d*x+c)/a/d+f*ln(sin(d
*x+c))/a/d^2-I*b*f*polylog(2,-exp(I*(d*x+c)))/a^2/d^2+I*b*f*polylog(2,exp(I
*(d*x+c)))/a^2/d^2-I*b^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)
))/a^2/d/(a^2-b^2)^(1/2)+I*b^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)
^(1/2)))/a^2/d/(a^2-b^2)^(1/2)-b^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b
^2)^(1/2)))/a^2/d^2/(a^2-b^2)^(1/2)+b^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(
a^2-b^2)^(1/2)))/a^2/d^2/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.41, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4631, 4269, 3556, 4268, 2317, 2438, 3404, 2296, 2221}

$$-\frac{b^2f\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d^2\sqrt{a^2-b^2}}+\frac{b^2f\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2d^2\sqrt{a^2-b^2}}-\frac{ibf\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}+\frac{ibf\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2}-\frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}+\frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2d\sqrt{a^2-b^2}}+\frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d}+\frac{f\log(\sin(c+dx))}{ad^2}-\frac{(e+fx)\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a^2*d) - ((e + f*x)*Cot[c + d*x])
/(a*d) - (I*b^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2
])])/(a^2*Sqrt[a^2 - b^2]*d) + (I*b^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d) + (f*Log[Sin[c + d*x]])/
(a*d^2) - (I*b*f*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + (I*b*f*PolyLog[2
, E^(I*(c + d*x))])/(a^2*d^2) - (b^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2) + (b^2*f*PolyLog[2, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4631

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*Sin
```

```
[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\csc^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\cot(c+dx)}{ad} - \frac{b\int (e+fx)\csc(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{a^2} +$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{f\log(\sin(c+dx))}{ad^2} +$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{f\log(\sin(c+dx))}{ad^2} -$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} - \frac{ib^2(e+fx)\log\left(1-\right.}{a^2\sqrt{a^2-b}}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} - \frac{ib^2(e+fx)\log\left(1-\right.}{a^2\sqrt{a^2-b}}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} - \frac{ib^2(e+fx)\log\left(1-\right.}{a^2\sqrt{a^2-b}}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 822 vs. $2(370) = 740$.
time = 7.61, size = 822, normalized size = 2.22

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*d*(e + f*x)*Cot[(c + d*x)/2]) + 2*a*f*Log[Sin[c + d*x]] - 2*b*d*e*Log[
Tan[(c + d*x)/2]] + 2*b*c*f*Log[Tan[(c + d*x)/2]] - 2*b*f*((c + d*x)*(Log[1
 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c +
d*x))] - PolyLog[2, E^(I*(c + d*x))])) + (2*b^2*d*(e + f*x)*((2*(d*e - c*f)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(
```

Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])
/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))
/(a + I*(b + Sqrt[-a^2 + b^2]))]]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(
c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqr
t[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[
-a^2 + b^2]))]]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(
-b + Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] +
 PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]]))/Sqrt
[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2]
+ a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*T
an[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]]))/Sqrt[-a^2 + b^2]))/(d*e
 - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]])
 + a*d*(e + f*x)*Tan[(c + d*x)/2])/(2*a^2*d^2)

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(332) = 664$.
time = 0.16, size = 766, normalized size = 2.07

| method | result |
|---|---|
| risch | $\frac{i b^2 f \operatorname{dilog}\left(\frac{ia+b\,e^{i(dx+c)}+\sqrt{-a^2+b^2}}{ia+\sqrt{-a^2+b^2}}\right)}{a^2 d^2 \sqrt{-a^2+b^2}}+\frac{b^2 f \ln\left(\frac{ia+b\,e^{i(dx+c)}-\sqrt{-a^2+b^2}}{ia-\sqrt{-a^2+b^2}}\right) x}{a^2 d \sqrt{-a^2+b^2}}+\frac{b^2 f \ln\left(\frac{ia+b\,e^{i(dx+c)}-\sqrt{-a^2+b^2}}{ia-\sqrt{-a^2+b^2}}\right)}{a^2 d^2 \sqrt{-a^2+b^2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -I/a^2/d^2*b*f*dilog(exp(I*(d*x+c))+1)+1/a^2/d*b^2*f/(-a^2+b^2)^(1/2)*ln((I
*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/a^2/d^2*b
^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2
+b^2)^(1/2)))*c+1/a^2/d*b*f*ln(exp(I*(d*x+c))+1)*x-I/a^2/d^2*b*f*dilog(exp(
I*(d*x+c)))-1/a^2/d*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b
^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/a^2/d^2*b^2*f/(-a^2+b^2)^(1/2)*ln((I
*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2/d^2/a*f*l
n(exp(I*(d*x+c)))-1/a^2/d*b*e*ln(exp(I*(d*x+c))-1)+1/a^2/d*b*e*ln(exp(I*(d*
x+c))+1)+1/a^2/d^2*b*f*c*ln(exp(I*(d*x+c))-1)-I/a^2/d^2*b^2*f/(-a^2+b^2)^(1
/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2
*I*(f*x+e)/d/a/(exp(2*I*(d*x+c))-1)-2*I/a^2/d^2*b^2*f*c/(-a^2+b^2)^(1/2)*ar
ctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/a^2/d^2*b^2*f/(-a^2
+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(
1/2)))+2*I/a^2/d*b^2*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*
a)/(-a^2+b^2)^(1/2))+1/a/d^2*f*ln(exp(I*(d*x+c))-1)+1/a/d^2*f*ln(exp(I*(d*x
+c))+1)

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1695 vs. $2(324) = 648$.
time = 0.63, size = 1695, normalized size = 4.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c
) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*
sin(d*x + c) - I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*s
in(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1)*sin(d*x + c) - I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x
 + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) - b)/b + 1)*sin(d*x + c) + I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-
I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + I*(a^2*b - b^3)*f*dilog(cos(
d*x + c) + I*sin(d*x + c))*sin(d*x + c) - I*(a^2*b - b^3)*f*dilog(cos(d*x +
 c) - I*sin(d*x + c))*sin(d*x + c) + I*(a^2*b - b^3)*f*dilog(-cos(d*x + c)
+ I*sin(d*x + c))*sin(d*x + c) - I*(a^2*b - b^3)*f*dilog(-cos(d*x + c) - I*
sin(d*x + c))*sin(d*x + c) - (b^3*c*f - b^3*d*e)*sqrt(-(a^2 - b^2)/b^2)*log
(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a
)*sin(d*x + c) - (b^3*c*f - b^3*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x
 + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x +
c) + (b^3*c*f - b^3*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I
*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + (b^3*c
*f - b^3*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - (b^3*d*f*x + b^3*
c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c)
+ (b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*s
in(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b)*sin(d*x + c) - (b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*
a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + (b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 -
 b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b
```

```
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + ((a^2*b - b^3)
*d*f*x + (a^2*b - b^3)*d*e + (a^3 - a*b^2)*f)*log(cos(d*x + c) + I*sin(d*x
+ c) + 1)*sin(d*x + c) + ((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*d*e + (a^3 -
a*b^2)*f)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) - ((a^2*b - b
^3)*d*e - (a^3 - a*b^2 + (a^2*b - b^3)*c)*f)*log(-1/2*cos(d*x + c) + 1/2*I*
sin(d*x + c) + 1/2)*sin(d*x + c) - ((a^2*b - b^3)*d*e - (a^3 - a*b^2 + (a^2
*b - b^3)*c)*f)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x +
 c) - ((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*log(-cos(d*x + c) + I*sin(d
*x + c) + 1)*sin(d*x + c) - ((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*log(-
cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) - 2*((a^3 - a*b^2)*d*f*x +
(a^3 - a*b^2)*d*e)*cos(d*x + c))/((a^4 - a^2*b^2)*d^2*sin(d*x + c))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.239 $\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=83

$$\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d+2*b^2*arctan((b+a*tan(1/2*d*x+1/
2*c))/(a^2-b^2)^(1/2))/a^2/d/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2881, 12, 2826, 3855, 2739, 632, 210}

$$\frac{2b^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^
2]*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)}{ad} - \frac{\int \frac{b\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{b\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+\right.\right.}{a^2d}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{(4b^2)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\right.}{a^2d}$$

$$= \frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

**Mathematica [A]**
time = 0.32, size = 111, normalized size = 1.34

$$\frac{\frac{4b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - a\cot\left(\frac{1}{2}(c+dx)\right) + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + a\tan\left(\frac{1}{2}(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
 a*Cot[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]]
 + a*Tan[(c + d*x)/2])/(2*a^2*d)
```

**Maple [A]**
time = 0.14, size = 101, normalized size = 1.22

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$ |
| default | $\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$ |

| | |
|---|---|
| risch | $-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}-\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^{2}d}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^{2}d}+\frac{b^{2}\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,d\,a^{2}}-\frac{b^{2}\ln}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tan(1/2*d*x+1/2*c)+2*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/2/a/tan(1/2*d*x+1/2*c)-1/a^2*b*ln(t
an(1/2*d*x+1/2*c)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(78) = 156$.
time = 0.41, size = 400, normalized size = 4.82

$$\left[-\frac{\sqrt{-a^{2}+b^{2}}\,b^{2}\log\left(\frac{(2a^{2}-b^{2})\cos(dx+c)^{2}-2ab\sin(dx+c)-a^{2}-b^{2}+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(dx+c)^{2}-2ab\sin(dx+c)-a^{2}-b^{2}}\right)\sin(dx+c)-(a^{2}b-b^{3})\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+(a^{2}b-b^{3})\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+2(a^{3}-ab^{2})\cos(dx+c)}{2(a^{4}-a^{2}b^{2})d\sin(dx+c)},\ -\frac{2\sqrt{a^{2}-b^{2}}\,b^{2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^{2}-b^{2}}\cos(dx+c)}\right)\sin(dx+c)-(a^{2}b-b^{3})\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+(a^{2}b-b^{3})\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+2(a^{3}-ab^{2})\cos(dx+c)}{2(a^{4}-a^{2}b^{2})d\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b^2*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt
(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*
x + c) - (a^2*b - b^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (a^2*b -
b^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*(a^3 - a*b^2)*cos(d*x +
c))/((a^4 - a^2*b^2)*d*sin(d*x + c)), -1/2*(2*sqrt(a^2 - b^2)*b^2*arctan(-(
a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - (a^2*b -
 b^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (a^2*b - b^3)*log(-1/2*cos
(d*x + c) + 1/2)*sin(d*x + c) + 2*(a^3 - a*b^2)*cos(d*x + c))/((a^4 - a^2*b
^2)*d*sin(d*x + c))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac [A]**
time = 4.53, size = 130, normalized size = 1.57

$$\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\sqrt{a^2-b^2}\,a^2}-\frac{2b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*b^2/(sqrt(a^2 - b^2)*a^2) - 2*b*log(abs(tan(1/
2*d*x + 1/2*c)))/a^2 + tan(1/2*d*x + 1/2*c)/a + (2*b*tan(1/2*d*x + 1/2*c) -
 a)/(a^2*tan(1/2*d*x + 1/2*c)))/d
```

**Mupad [B]**
time = 2.96, size = 222, normalized size = 2.67

$$\frac{a\,b^2-a^3}{a^4\,d\tan\left(c+d\,x\right)-a^2\,b^2\,d\tan\left(c+d\,x\right)}+\frac{b^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-a^2\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+b^2\,\mathrm{atan}\left(\frac{-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}+b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+a\,b\,\sqrt{b^2-a^2}\,2\mathrm{i}}{-a^3-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b+2\,a\,b^2+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^3}\right)\sqrt{b^2-a^2}\,2\mathrm{i}}{a^4\,d-a^2\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (a*b^2 - a^3)/(a^4*d*tan(c + d*x) - a^2*b^2*d*tan(c + d*x)) + (b^3*log(tan(
c/2 + (d*x)/2)) + b^2*atan((b^2*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i - a
^2*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + a*b*(b^2 - a^2)^(1/2)*2i)/(2*a
*b^2 - a^3 + 4*b^3*tan(c/2 + (d*x)/2) - 3*a^2*b*tan(c/2 + (d*x)/2)))*(b^2 -
 a^2)^(1/2)*2i - a^2*b*log(tan(c/2 + (d*x)/2)))/(a^4*d - a^2*b^2*d)
```

# 3.240 $\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 4.59, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m\left(\sin^2(dx+c)\right)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(f*x + e)^m/(b*sin(d*x + c) + a), x)
```

**Sympy** [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*sin(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(c+d\,x)^2\,(e+f\,x)^m}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(e + f*x)^m)/(a + b*sin(c + d*x)),x)
[Out] int((sin(c + d*x)^2*(e + f*x)^m)/(a + b*sin(c + d*x)), x)
```

# 3.241 $\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=29

$$\text{Int}\left(\frac{(e+fx)^m\sin(c+dx)}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m\sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m\sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m\sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m\sin(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)^m \sin(c + dx)}{a + b \sin(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*sin(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(c + d\,x)\,{(e + f\,x)}^m}{a + b\,\sin(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^m)/(a + b*sin(c + d*x)),x)
[Out] int((sin(c + d*x)*(e + f*x)^m)/(a + b*sin(c + d*x)), x)
```

# 3.242 $\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
[Out] Defer[Int][(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
[Out] Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e+f\,x)^m}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(a + b*sin(c + d*x)),x)
[Out] int((e + f*x)^m/(a + b*sin(c + d*x)), x)
```

# 3.243 $\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=29

$$\mathrm{Int}\left(\frac{(e+fx)^m\csc(c+dx)}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 11.94, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m\csc(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*csc(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*csc(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*csc(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(e+f\,x)^m}{\sin(c+d\,x)\,\left(a+b\,\sin(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
[Out] int((e + f*x)^m/(sin(c + d*x)*(a + b*sin(c + d*x))), x)
```

# 3.244 $\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 22.99, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m \left(\csc^2(dx+c)\right)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*csc(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*csc(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \csc^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*csc(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(e+f\,x)^m}{\sin\left(c+d\,x\right)^2\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
[Out] int((e + f*x)^m/(sin(c + d*x)^2*(a + b*sin(c + d*x))), x)
```

# 3.245 $\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

**Optimal**. Leaf size=574

$$\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d}-\frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}-\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d}$$

```
[Out] a*f*ln(a+b*sin(d*x+c))/b/(a^2-b^2)/d^2+I*a^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c)
)/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d-I*a^2*(f*x+e)*ln(1-I*b*exp(I*(d*
x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d+a^2*f*polylog(2,I*b*exp(I*(d
*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-a^2*f*polylog(2,I*b*exp(I
*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-a*(f*x+e)*cos(d*x+c)/(
a^2-b^2)/d/(a+b*sin(d*x+c))-I*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^
(1/2)))/b/d/(a^2-b^2)^(1/2)+I*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/b/d/(a^2-b^2)^(1/2)-f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/
2)))/b/d^2/(a^2-b^2)^(1/2)+f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2
)))/b/d^2/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 1.11, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6874, 3405, 3404, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{a^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}-\frac{a^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}-\frac{f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{af\log(a+b\sin(c+dx))}{bd^2\left(a^2-b^2\right)}+\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{3/2}}-\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd\left(a^2-b^2\right)^{3/2}}-\frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}+\frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd\sqrt{a^2-b^2}}-\frac{a(e+fx)\cos(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(
a^2 - b^2)^(3/2)*d) - (I*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*(e + f*x)*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) +
 (a*f*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d^2) + (a^2*f*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (f*Po
lyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d
^2) - (a^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a
^2 - b^2)^(3/2)*d^2) + (f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (a*(e + f*x)*Cos[c + d*x])/((a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
```

```
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{a(e+fx)}{b(a+b\sin(c+dx))^2}+\frac{e+fx}{b(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{b} - \frac{a\int \frac{e+fx}{(a+b\sin(c+dx))^2}\,dx}{b}$$

$$= -\frac{a(e+fx)\cos(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{2\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b} - \frac{a^2\int \frac{e+fx}{a+b\sin(c+}}{b\,(a^2-b}$$

$$= -\frac{a(e+fx)\cos(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{(2a^2)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b\,(a^2-b^2)} - \frac{(2i)\int}{2a-}$$

$$= -\frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} + \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} + \frac{a}{}$$

$$= \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d} - \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia}{}$$

$$= \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d} - \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia}{}$$

$$= \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d} - \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} - \frac{ia}{}$$

**Mathematica** [A]
time = 8.58, size = 833, normalized size = 1.45

$$-\frac{\frac{(-bd(e+fx)+af\cos(c+dx))\left(\frac{2af\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{2(-bde+af+bcf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{af\log\left(\sec^2\left(\frac{1}{2}(c+dx)\right)\right)}{b}-\frac{af\log\left(\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\sin(c+dx))\right)}{b}-\frac{ibf\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{-ia+b+\sqrt{-a^2+b^2}}\right)+\text{Li}_2\left(\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}+\frac{ibf\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b+\sqrt{-a^2+b^2}}\right)+\text{Li}_2\left(\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}+\frac{ibf\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{-b+\sqrt{-a^2+b^2}-a\tan\left(\frac{1}{2}(c+dx)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)+\text{Li}_2\left(\frac{a\left(i+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}}-\frac{ibf\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b-\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b-\sqrt{-a^2+b^2}}\right)+\text{Li}_2\left(\frac{a+ia\tan\left(\frac{1}{2}(c+dx)\right)}{a+i\left(-b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\left(af\cos(c+dx)+b\left(-de+cf-if\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)+if\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}+\frac{ad(e+fx)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))}}{d^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]

[Out] -((((-(b*d*(e + f*x)) + a*f*Cos[c + d*x])*((2*a*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (2*(-(b*d*e) + a*f + b*c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (a*f*Log[Sec[(c + d*x)/2]^2])/b - (a*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])])/b - (I*b*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*b*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*b*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-b + Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*b*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/((a^2 - b^2)*(a*f*Cos[c + d*x] + b*(-(d*e) + c*f - I*f*Log[1 - I*Tan[(c + d*x)/2]] + I*f*Log[1 + I*Tan[(c + d*x)/2]]))) + (a*d*(e + f*x)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])))/d^2)

**Maple** [A]
time = 1.42, size = 750, normalized size = 1.31

| method | result |
|---|---|
| risch | $\frac{2ia(fx+e)\left(b-ia\,\mathrm{e}^{i(dx+c)}\right)}{b(-a^2+b^2)d\left(b\,\mathrm{e}^{2i(dx+c)}-b+2ia\,\mathrm{e}^{i(dx+c)}\right)}-\frac{2af\ln\left(\mathrm{e}^{i(dx+c)}\right)}{b\,d^2(a^2-b^2)}+\frac{af\ln\left(ib\,\mathrm{e}^{2i(dx+c)}-2a\,\mathrm{e}^{i(dx+c)}-ib\right)}{b\,d^2(a^2-b^2)}+\frac{bf\ln\left(\frac{ia+b\,\mathrm{e}^{i(dx+c)}+\sqrt{-\cdots}}{ia+\sqrt{-a^2+\cdots}}\right.}{d(a^2-b^2)\sqrt{-a^2\cdots}}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 2*I*a*(f*x+e)*(b-I*a*exp(I*(d*x+c)))/b/(-a^2+b^2)/d/(b*exp(2*I*(d*x+c))-b+2*I*a*exp(I*(d*x+c)))-2/b/d^2/(a^2-b^2)*a*f*ln(exp(I*(d*x+c)))+1/b/d^2/(a^2-b^2)*a*f*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+b/d/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+b/d^2/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+2*I*b/d^2/(a^2-b^2)*c*f/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-b/d/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-b/d^2/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-I*b/d^2/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I*b/d/(a^2-b^2)*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-

```
a^2+b^2)^(1/2))+I*b/d^2/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*
x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1515 vs. 2(508) = 1016.
time = 0.57, size = 1515, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((-I*b^4*f*sin(d*x + c) - I*a*b^3*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + (I*b^4*f*sin(d*x + c) + I*a*b^3*f)*sqrt(-(a^2
 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (I*b^4*f*sin(d*x + c)
 + I*a*b^3*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x +
 c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) + (-I*b^4*f*sin(d*x + c) - I*a*b^3*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*
cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + (a*b^3*d*f*x + a*b^3*c*f + (b^4*d*f*x + b^4*c
*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x
 + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)
- (a*b^3*d*f*x + a*b^3*c*f + (b^4*d*f*x + b^4*c*f)*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a*b^3*d*f*x + a*b^3*c*f +
 (b^4*d*f*x + b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(
d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b) - (a*b^3*d*f*x + a*b^3*c*f + (b^4*d*f*x + b^4*c*f)*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) -
 (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*((a
^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*d*e)*cos(d*x + c) + ((a^3*b - a*b^3)*
```

```
f*sin(d*x + c) + (a^4 - a^2*b^2)*f + (a*b^3*c*f - a*b^3*d*e + (b^4*c*f - b^
4*d*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + ((a^3*b - a*b^3)*f*sin(
d*x + c) + (a^4 - a^2*b^2)*f + (a*b^3*c*f - a*b^3*d*e + (b^4*c*f - b^4*d*e)
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + ((a^3*b - a*b^3)*f*sin(d*x +
c) + (a^4 - a^2*b^2)*f - (a*b^3*c*f - a*b^3*d*e + (b^4*c*f - b^4*d*e)*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
 + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + ((a^3*b - a*b^3)*f*sin(d*x + c) +
(a^4 - a^2*b^2)*f - (a*b^3*c*f - a*b^3*d*e + (b^4*c*f - b^4*d*e)*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d^2*sin(d*x
 + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d^2)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)*sin(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x))/(a + b*sin(c + d*x))^2,x)
[Out] \text{Hanged}
```

# 3.246 $\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

**Optimal**. Leaf size=1106

$$-\frac{ia(e+fx)^2}{b\,(a^2-b^2)\,d}+\frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2}+\frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d}-\frac{i(e+fx)^2\log}{b}$$

```
[Out] -I*a^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(
3/2)/d+2*a*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^
2)/d^2-2*I*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^3/(a^2
-b^2)^(1/2)+2*a*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a
^2-b^2)/d^2-2*I*a*f^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(
a^2-b^2)/d^3-I*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d/(
a^2-b^2)^(1/2)+2*a^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1
/2)))/b/(a^2-b^2)^(3/2)/d^2+I*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2
)^(1/2)))/b/d/(a^2-b^2)^(1/2)-2*a^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2+2*I*a^2*f^2*polylog(3,I*b*exp(I*
(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3+I*a^2*(f*x+e)^2*ln(1-I*
b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d-a*(f*x+e)^2*cos(d
*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))-I*a*(f*x+e)^2/b/(a^2-b^2)/d-2*I*a^2*f^2*
polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-2*f
*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^
(1/2)+2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^2/(
a^2-b^2)^(1/2)+2*I*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/
d^3/(a^2-b^2)^(1/2)-2*I*a*f^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/
2)))/b/(a^2-b^2)/d^3
```

**Rubi** [A]
time = 1.67, antiderivative size = 1106, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6874, 3405, 3404, 2296, 2221, 2611, 2320, 6724, 4615, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-I)*a*(e + f*x)^2)/(b*(a^2 - b^2)*d) + (2*a*f*(e + f*x)*Log[1 - (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (I*a^2*(e + f*x)
^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/
2)*d) - (I*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]
)/(b*Sqrt[a^2 - b^2]*d) + (2*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (I*a^2*(e + f*x)^2*Log[1 - (I*b
```

```
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*(e +
 f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
 b^2]*d) - ((2*I)*a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(b*(a^2 - b^2)*d^3) + (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (2*f*(e + f*x)*Pol
yLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^
2) - ((2*I)*a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(b*(a^2 - b^2)*d^3) - (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) + (2*f*(e + f*x)*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + (
(2*I)*a^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(
a^2 - b^2)^(3/2)*d^3) - ((2*I)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a^2*f^2*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + ((2*I)*
f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
b^2]*d^3) - (a*(e + f*x)^2*Cos[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]
))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{a(e+fx)^2}{b(a+b\sin(c+dx))^2}+\frac{(e+fx)^2}{b(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{b} - \frac{a\int \frac{(e+fx)^2}{(a+b\sin(c+dx))^2}\,dx}{b}$$

$$= -\frac{a(e+fx)^2\cos(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{2\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b} - \frac{a^2\int \frac{(e+fx)^2}{a+b\sin(c+}}{b\,(a^2-b^2}$$

$$= -\frac{ia(e+fx)^2}{b\,(a^2-b^2)\,d} - \frac{a(e+fx)^2\cos(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{(2a^2)\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}}{b\,(a^2-b^2)}$$

$$= -\frac{ia(e+fx)^2}{b\,(a^2-b^2)\,d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} - \frac{i(e+fx)^2\log\left(1-\right.}{b\sqrt{a^2-b}}$$

$$= -\frac{ia(e+fx)^2}{b\,(a^2-b^2)\,d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^2\log\left(1-\right.}{b\,(a^2-b^2}$$

$$= -\frac{ia(e+fx)^2}{b\,(a^2-b^2)\,d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^2\log\left(1-\right.}{b\,(a^2-b^2}$$

$$= -\frac{ia(e+fx)^2}{b\,(a^2-b^2)\,d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^2\log\left(1-\right.}{b\,(a^2-b^2}$$

$$= -\frac{ia(e+fx)^2}{b\,(a^2-b^2)\,d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^2\log\left(1-\right.}{b\,(a^2-b^2}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2332 vs. 2(1106) = 2212.

time = 21.43, size = 2332, normalized size = 2.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*I)*E^(I*c)*(2*a*e*E^(I*c)*f*x + a*E^(I*c)*f^2*x^2 + (I*b^2*e^2*ArcTan[(
I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*E^(I*c)) - (I*b
^2*e^2*E^(I*c)*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/Sqrt[a^2
- b^2] + (2*a^2*e*f*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqr
t[a^2 - b^2]*d*E^(I*c)) + (a*e*f*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((
2*I)*(c + d*x))))])/(d*E^(I*c)) - (a*e*E^(I*c)*f*ArcTan[(2*a*E^(I*(c + d*x)
))/(b*(-1 + E^((2*I)*(c + d*x))))])/d + ((2*I)*a^2*e*f*ArcTanh[(-a + I*b*E^
(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*d*E^(I*c)) - ((I/2)*a*e*f
*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2])/(d*E^(I
*c)) + ((I/2)*a*e*E^(I*c)*f*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2
*I)*(c + d*x)))^2])/d + (I*b^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(
I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (
I*b^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(
-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (I*a*f^2*x*Log
[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])
/(d*E^(I*c)) + (I*a*E^(I*c)*f^2*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c
) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/d + ((I/2)*b^2*f^2*x^2*Log[1 + (b*E^(
I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2
 + b^2)*E^((2*I)*c)] - ((I/2)*b^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c
+ d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)
*E^((2*I)*c)] - (I*b^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + S
qrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + (I*b^2*e*
E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b
^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (I*a*f^2*x*Log[1 + (b*
E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/(d*E^(I
*c)) + (I*a*E^(I*c)*f^2*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt
[(-a^2 + b^2)*E^((2*I)*c)])])/d - ((I/2)*b^2*f^2*x^2*Log[1 + (b*E^(I*(2*c +
 d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*
E^((2*I)*c)] + ((I/2)*b^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))
/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I
)*c)] - ((-1 + E^((2*I)*c))*f*(-(a*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f) + b^2*
d*E^(I*c)*(e + f*x))*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt
[(-a^2 + b^2)*E^((2*I)*c)])])/(d^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
+ ((-1 + E^((2*I)*c))*f*(a*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f + b^2*d*E^(I*c)
*(e + f*x))*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 +
b^2)*E^((2*I)*c)]))])/(d^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) + (I*b^2
*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^
((2*I)*c)])])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) - (I*b^2*E^((2*I)*c)*f^2
```

```
*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*
I)*c)])])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) - (I*b^2*f^2*PolyLog[3, -((b
*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))])/(d^2*
Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) + (I*b^2*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^
(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))])/(d^2*Sqr
t[(-a^2 + b^2)*E^((2*I)*c)])))/(b*(-a^2 + b^2)*d*(-1 + E^((2*I)*c))) + (Csc
[c/2]*Sec[c/2]*(a^2*e^2*Cos[c] + 2*a^2*e*f*x*Cos[c] + a^2*f^2*x^2*Cos[c] +
a*b*e^2*Sin[d*x] + 2*a*b*e*f*x*Sin[d*x] + a*b*f^2*x^2*Sin[d*x]))/(2*(a - b)
*b*(a + b)*d*(a + b*Sin[c + d*x]))
```

**Maple** [**F**]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \sin(dx+c)}{(a+b\sin(dx+c))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3142 vs. 2(970) = 1940.
time = 0.61, size = 3142, normalized size = 2.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^4*f^2*sin(d*x + c) + a*b^3*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3
, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(b^4*f^2*sin(d*x + c) + a*b^3*f^2)*sqrt(-(a
```

```
^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(b^4*f^2*sin(d*x +
c) + a*b^3*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*s
in(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
) - 2*(b^4*f^2*sin(d*x + c) + a*b^3*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3,
-(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2))/b) + 2*((a^3*b - a*b^3)*d^2*f^2*x^2 + 2*(a^3*b - a*
b^3)*d^2*f*x*e + (a^3*b - a*b^3)*d^2*e^2)*cos(d*x + c) + 2*(I*(a^3*b - a*b^
3)*f^2*sin(d*x + c) + I*(a^4 - a^2*b^2)*f^2 + (I*a*b^3*d*f^2*x + I*a*b^3*d*
f*e + (I*b^4*d*f^2*x + I*b^4*d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*d
ilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(I*(a^3*b - a*b^3)*f^2*sin(d*x +
c) + I*(a^4 - a^2*b^2)*f^2 + (-I*a*b^3*d*f^2*x - I*a*b^3*d*f*e + (-I*b^4*d*
f^2*x - I*b^4*d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a*cos(d
*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b + 1) + 2*(-I*(a^3*b - a*b^3)*f^2*sin(d*x + c) - I*(a^4 -
a^2*b^2)*f^2 + (-I*a*b^3*d*f^2*x - I*a*b^3*d*f*e + (-I*b^4*d*f^2*x - I*b^4*
d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x + c) - a*s
in(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) + 2*(-I*(a^3*b - a*b^3)*f^2*sin(d*x + c) - I*(a^4 - a^2*b^2)*f^2
+ (I*a*b^3*d*f^2*x + I*a*b^3*d*f*e + (I*b^4*d*f^2*x + I*b^4*d*f*e)*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (2*
(a^4 - a^2*b^2)*c*f^2 - 2*(a^4 - a^2*b^2)*d*f*e + 2*((a^3*b - a*b^3)*c*f^2
- (a^3*b - a*b^3)*d*f*e)*sin(d*x + c) + (a*b^3*c^2*f^2 - 2*a*b^3*c*d*f*e +
a*b^3*d^2*e^2 + (b^4*c^2*f^2 - 2*b^4*c*d*f*e + b^4*d^2*e^2)*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt
(-(a^2 - b^2)/b^2) + 2*I*a) + (2*(a^4 - a^2*b^2)*c*f^2 - 2*(a^4 - a^2*b^2)*
d*f*e + 2*((a^3*b - a*b^3)*c*f^2 - (a^3*b - a*b^3)*d*f*e)*sin(d*x + c) + (a
*b^3*c^2*f^2 - 2*a*b^3*c*d*f*e + a*b^3*d^2*e^2 + (b^4*c^2*f^2 - 2*b^4*c*d*f
*e + b^4*d^2*e^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c
) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (2*(a^4 - a^
2*b^2)*c*f^2 - 2*(a^4 - a^2*b^2)*d*f*e + 2*((a^3*b - a*b^3)*c*f^2 - (a^3*b
- a*b^3)*d*f*e)*sin(d*x + c) - (a*b^3*c^2*f^2 - 2*a*b^3*c*d*f*e + a*b^3*d^2
*e^2 + (b^4*c^2*f^2 - 2*b^4*c*d*f*e + b^4*d^2*e^2)*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
 b^2)/b^2) + 2*I*a) + (2*(a^4 - a^2*b^2)*c*f^2 - 2*(a^4 - a^2*b^2)*d*f*e +
2*((a^3*b - a*b^3)*c*f^2 - (a^3*b - a*b^3)*d*f*e)*sin(d*x + c) - (a*b^3*c^2
*f^2 - 2*a*b^3*c*d*f*e + a*b^3*d^2*e^2 + (b^4*c^2*f^2 - 2*b^4*c*d*f*e + b^4
*d^2*e^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I
*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (2*(a^4 - a^2*b^2)*
d*f^2*x + 2*(a^4 - a^2*b^2)*c*f^2 + 2*((a^3*b - a*b^3)*d*f^2*x + (a^3*b - a
*b^3)*c*f^2)*sin(d*x + c) + (a*b^3*d^2*f^2*x^2 - a*b^3*c^2*f^2 + 2*(a*b^3*d
^2*f*x + a*b^3*c*d*f)*e + (b^4*d^2*f^2*x^2 - b^4*c^2*f^2 + 2*(b^4*d^2*f*x +
 b^4*c*d*f)*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-(I*a*cos(d*x + c)
```

```
 - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b) - (2*(a^4 - a^2*b^2)*d*f^2*x + 2*(a^4 - a^2*b^2)*c*f^2 + 2*((a^
3*b - a*b^3)*d*f^2*x + (a^3*b - a*b^3)*c*f^2)*sin(d*x + c) - (a*b^3*d^2*f^2
*x^2 - a*b^3*c^2*f^2 + 2*(a*b^3*d^2*f*x + a*b^3*c*d*f)*e + (b^4*d^2*f^2*x^2
 - b^4*c^2*f^2 + 2*(b^4*d^2*f*x + b^4*c*d*f)*e)*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (2*(a^4 - a^2*b^2)*d*f^2*x +
 2*(a^4 - a^2*b^2)*c*f^2 + 2*((a^3*b - a*b^3)*d*f^2*x + (a^3*b - a*b^3)*c*f
^2)*sin(d*x + c) + (a*b^3*d^2*f^2*x^2 - a*b^3*c^2*f^2 + 2*(a*b^3*d^2*f*x +
a*b^3*c*d*f)*e + (b^4*d^2*f^2*x^2 - b^4*c^2*f^2 + 2*(b^4*d^2*f*x + b^4*c*d*
f)*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-(-I*a*cos(d*x + c) - a*sin
(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)
/b) - (2*(a^4 - a^2*b^2)*d*f^2*x + 2*(a^4 - a^2*b^2)*c*f^2 + 2*((a^3*b - a*
b^3)*d*f^2*x + (a^3*b - a*b^3)*c*f^2)*sin(d*x + c) - (a*b^3*d^2*f^2*x^2 - a
*b^3*c^2*f^2 + 2*(a*b^3*d^2*f*x + a*b^3*c*d*f)*e + (b^4*d^2*f^2*x^2 - b^4*c
^2*f^2 + 2*(b^4*d^2*f*x + b^4*c*d*f)*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
))*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d^3*sin
(d*x + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d^3)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)^2*sin(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x))^2,x)
[Out] \text{Hanged}
```

# 3.247 $\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

**Optimal**. Leaf size=1512

$$-\frac{ia(e+fx)^3}{b\,(a^2-b^2)\,d}+\frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2}+\frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d}-\frac{i(e+fx)^3\,\mathrm{lo}}{b}$$

```
[Out] -I*a^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(
3/2)/d+3*a*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-
b^2)/d^2-6*I*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/
b/d^3/(a^2-b^2)^(1/2)+3*a*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/b/(a^2-b^2)/d^2-6*I*a*f^2*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(
a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^3+I*a^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a
-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d+3*a^2*f*(f*x+e)^2*polylog(2,I*b*exp(
I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-6*I*a*f^2*(f*x+e)*pol
ylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^3-3*a^2*f*(f*x
+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d
^2+6*a*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^
4+I*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d/(a^2-b^2)^(1
/2)+6*a*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d
^4+6*I*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^3/
(a^2-b^2)^(1/2)-6*a^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))
/b/(a^2-b^2)^(3/2)/d^4+6*a^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/b/(a^2-b^2)^(3/2)/d^4-a*(f*x+e)^3*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d
*x+c))-6*I*a^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2))
)/b/(a^2-b^2)^(3/2)/d^3-I*a*(f*x+e)^3/b/(a^2-b^2)/d-3*f*(f*x+e)^2*polylog(2
,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)+3*f*(f*x+e)^
2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)-I
*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d/(a^2-b^2)^(1/2)
+6*I*a^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a
^2-b^2)^(3/2)/d^3+6*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b
/d^4/(a^2-b^2)^(1/2)-6*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2))
)/b/d^4/(a^2-b^2)^(1/2)
```

**Rubi** [A]

time = 2.03, antiderivative size = 1512, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6874, 3405, 3404, 2296, 2221, 2611, 6744, 2320, 6724, 4615}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-I)*a*(e + f*x)^3)/(b*(a^2 - b^2)*d) + (3*a*f*(e + f*x)^2*Log[1 - (I*b*E^
(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (I*a^2*(e + f*
x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(
3/2)*d) - (I*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]
)])/(b*Sqrt[a^2 - b^2]*d) + (3*a*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (I*a^2*(e + f*x)^3*Log[1 -
(I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*
(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a
^2 - b^2]*d) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a -
 Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (3*a^2*f*(e + f*x)^2*PolyLog[2, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (3
*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
Sqrt[a^2 - b^2]*d^2) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (3*a^2*f*(e + f*x)^2*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
^2) + (3*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b*Sqrt[a^2 - b^2]*d^2) + (6*a*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a
 - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + ((6*I)*a^2*f^2*(e + f*x)*PolyLo
g[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3
) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(b*Sqrt[a^2 - b^2]*d^3) + (6*a*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/
(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) - ((6*I)*a^2*f^2*(e + f*x)*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
^3) + ((6*I)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - (6*a^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) + (6*f^3*PolyLog[4, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) + (6*a
^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b
^2)^(3/2)*d^4) - (6*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])])/(b*Sqrt[a^2 - b^2]*d^4) - (a*(e + f*x)^3*Cos[c + d*x])/((a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{a(e+fx)^3}{b(a+b\sin(c+dx))^2}+\frac{(e+fx)^3}{b(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{b} - \frac{a\int \frac{(e+fx)^3}{(a+b\sin(c+dx))^2}\,dx}{b}$$

$$= -\frac{a(e+fx)^3\cos(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{2\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b} - \frac{a^2\int \frac{(e+fx}{a+b\sin(c}}{b\,(a^2-}$$

$$= -\frac{ia(e+fx)^3}{b\,(a^2-b^2)\,d} - \frac{a(e+fx)^3\cos(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{(2a^2)\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+da}}}{b\,(a^2-b^2)}$$

$$= -\frac{ia(e+fx)^3}{b\,(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} - \frac{i(e+fx)^3\log\left(1-\right.}{b\sqrt{a^2-}}$$

$$= -\frac{ia(e+fx)^3}{b\,(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(\right.}{b\,(a^2-}$$

$$= -\frac{ia(e+fx)^3}{b\,(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(\right.}{b\,(a^2-}$$

$$= -\frac{ia(e+fx)^3}{b\,(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(\right.}{b\,(a^2-}$$

$$= -\frac{ia(e+fx)^3}{b\,(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(\right.}{b\,(a^2-}$$

$$= -\frac{ia(e+fx)^3}{b\,(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(\right.}{b\,(a^2-}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6310 vs. 2(1512) = 3024.
time = 23.99, size = 6310, normalized size = 4.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] Result too large to show
```

**Maple [F]**
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \sin(dx+c)}{(a+b\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5191 vs. 2(1338) = 2676.
time = 0.75, size = 5191, normalized size = 3.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(6*(-I*b^4*f^3*sin(d*x + c) - I*a*b^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*poly
log(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(I*b^4*f^3*sin(d*x + c) + I*a*b^3*f^3)
*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(I*b^4*f^3
*sin(d*x + c) + I*a*b^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d
*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2))/b) + 6*(-I*b^4*f^3*sin(d*x + c) - I*a*b^3*f^3)*sqrt(-(a^2 - b^2
```

)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*((a^3*b - a*b^3)*d^3*f^3*x
^3 + 3*(a^3*b - a*b^3)*d^3*f^2*x^2*e + 3*(a^3*b - a*b^3)*d^3*f*x*e^2 + (a^3
*b - a*b^3)*d^3*e^3)*cos(d*x + c) + 3*(2*I*(a^4 - a^2*b^2)*d*f^3*x + 2*I*(a
^4 - a^2*b^2)*d*f^2*e + 2*(I*(a^3*b - a*b^3)*d*f^3*x + I*(a^3*b - a*b^3)*d*
f^2*e)*sin(d*x + c) + (I*a*b^3*d^2*f^3*x^2 + 2*I*a*b^3*d^2*f^2*x*e + I*a*b^
3*d^2*f*e^2 + (I*b^4*d^2*f^3*x^2 + 2*I*b^4*d^2*f^2*x*e + I*b^4*d^2*f*e^2)*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a*cos(d*x + c) - a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)
 + 3*(2*I*(a^4 - a^2*b^2)*d*f^3*x + 2*I*(a^4 - a^2*b^2)*d*f^2*e + 2*(I*(a^3
*b - a*b^3)*d*f^3*x + I*(a^3*b - a*b^3)*d*f^2*e)*sin(d*x + c) + (-I*a*b^3*d
^2*f^3*x^2 - 2*I*a*b^3*d^2*f^2*x*e - I*a*b^3*d^2*f*e^2 + (-I*b^4*d^2*f^3*x^
2 - 2*I*b^4*d^2*f^2*x*e - I*b^4*d^2*f*e^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(-2*I*(a^4 - a^2*b^2)*d*f^
3*x - 2*I*(a^4 - a^2*b^2)*d*f^2*e + 2*(-I*(a^3*b - a*b^3)*d*f^3*x - I*(a^3*
b - a*b^3)*d*f^2*e)*sin(d*x + c) + (-I*a*b^3*d^2*f^3*x^2 - 2*I*a*b^3*d^2*f^
2*x*e - I*a*b^3*d^2*f*e^2 + (-I*b^4*d^2*f^3*x^2 - 2*I*b^4*d^2*f^2*x*e - I*b
^4*d^2*f*e^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x + c
) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) + 3*(-2*I*(a^4 - a^2*b^2)*d*f^3*x - 2*I*(a^4 - a^2*b^2)*d*
f^2*e + 2*(-I*(a^3*b - a*b^3)*d*f^3*x - I*(a^3*b - a*b^3)*d*f^2*e)*sin(d*x
+ c) + (I*a*b^3*d^2*f^3*x^2 + 2*I*a*b^3*d^2*f^2*x*e + I*a*b^3*d^2*f*e^2 + (
I*b^4*d^2*f^3*x^2 + 2*I*b^4*d^2*f^2*x*e + I*b^4*d^2*f*e^2)*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (3*(a^4 - a
^2*b^2)*c^2*f^3 - 6*(a^4 - a^2*b^2)*c*d*f^2*e + 3*(a^4 - a^2*b^2)*d^2*f*e^2
 + 3*((a^3*b - a*b^3)*c^2*f^3 - 2*(a^3*b - a*b^3)*c*d*f^2*e + (a^3*b - a*b^
3)*d^2*f*e^2)*sin(d*x + c) + (a*b^3*c^3*f^3 - 3*a*b^3*c^2*d*f^2*e + 3*a*b^3
*c*d^2*f*e^2 - a*b^3*d^3*e^3 + (b^4*c^3*f^3 - 3*b^4*c^2*d*f^2*e + 3*b^4*c*d
^2*f*e^2 - b^4*d^3*e^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d
*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (3*(a^
4 - a^2*b^2)*c^2*f^3 - 6*(a^4 - a^2*b^2)*c*d*f^2*e + 3*(a^4 - a^2*b^2)*d^2*
f*e^2 + 3*((a^3*b - a*b^3)*c^2*f^3 - 2*(a^3*b - a*b^3)*c*d*f^2*e + (a^3*b -
 a*b^3)*d^2*f*e^2)*sin(d*x + c) + (a*b^3*c^3*f^3 - 3*a*b^3*c^2*d*f^2*e + 3*
a*b^3*c*d^2*f*e^2 - a*b^3*d^3*e^3 + (b^4*c^3*f^3 - 3*b^4*c^2*d*f^2*e + 3*b^
4*c*d^2*f*e^2 - b^4*d^3*e^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*
cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (
3*(a^4 - a^2*b^2)*c^2*f^3 - 6*(a^4 - a^2*b^2)*c*d*f^2*e + 3*(a^4 - a^2*b^2)
*d^2*f*e^2 + 3*((a^3*b - a*b^3)*c^2*f^3 - 2*(a^3*b - a*b^3)*c*d*f^2*e + (a^
3*b - a*b^3)*d^2*f*e^2)*sin(d*x + c) - (a*b^3*c^3*f^3 - 3*a*b^3*c^2*d*f^2*e
 + 3*a*b^3*c*d^2*f*e^2 - a*b^3*d^3*e^3 + (b^4*c^3*f^3 - 3*b^4*c^2*d*f^2*e +
 3*b^4*c*d^2*f*e^2 - b^4*d^3*e^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log
(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*
a) - (3*(a^4 - a^2*b^2)*c^2*f^3 - 6*(a^4 - a^2*b^2)*c*d*f^2*e + 3*(a^4 - a^

```
2*b^2)*d^2*f*e^2 + 3*((a^3*b - a*b^3)*c^2*f^3 - 2*(a^3*b - a*b^3)*c*d*f^2*e
 + (a^3*b - a*b^3)*d^2*f*e^2)*sin(d*x + c) - (a*b^3*c^3*f^3 - 3*a*b^3*c^2*d
*f^2*e + 3*a*b^3*c*d^2*f*e^2 - a*b^3*d^3*e^3 + (b^4*c^3*f^3 - 3*b^4*c^2*d*f
^2*e + 3*b^4*c*d^2*f*e^2 - b^4*d^3*e^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
- 2*I*a) - (3*(a^4 - a^2*b^2)*d^2*f^3*x^2 - 3*(a^4 - a^2*b^2)*c^2*f^3 + 6*(
(a^4 - a^2*b^2)*d^2*f^2*x + (a^4 - a^2*b^2)*c*d*f^2)*e + 3*((a^3*b - a*b^3)
*d^2*f^3*x^2 - (a^3*b - a*b^3)*c^2*f^3 + 2*((a^3*b - a*b^3)*d^2*f^2*x + (a^
3*b - a*b^3)*c*d*f^2)*e)*sin(d*x + c) + (a*b^3*d^3*f^3*x^3 + a*b^3*c^3*f^3
+ 3*(a*b^3*d^3*f*x + a*b^3*c*d^2*f)*e^2 + 3*(a*b^3*d^3*f^2*x^2 - a*b^3*c^2*
d*f^2)*e + (b^4*d^3*f^3*x^3 + b^4*c^3*f^3 + 3*(b^4*d^3*f*x + b^4*c*d^2*f)*e
^2 + 3*(b^4*d^3*f^2*x^2 - b^4*c^2*d*f^2)*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (3*(...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] \text{Hanged}
```

# 3.248 $\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

**Optimal**. Leaf size=751

$$\frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{5/2}d}-\frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d}-\frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c\cdots}}{a+\sqrt{a^2\cdots}}\right.}{2b\left(a^2-b^2\right)^{5/2}d}$$

```
[Out] 3/2*a^2*f*ln(a+b*sin(d*x+c))/b/(a^2-b^2)^2/d^2-f*ln(a+b*sin(d*x+c))/b/(a^2-
b^2)/d^2+3/2*I*a^3*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(
a^2-b^2)^(5/2)/d-3/2*I*a*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)
))/b/(a^2-b^2)^(3/2)/d-3/2*I*a^3*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^
2)^(1/2)))/b/(a^2-b^2)^(5/2)/d+3/2*I*a*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(
a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d+3/2*a^3*f*polylog(2,I*b*exp(I*(d*x+c))
/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^2-3/2*a*f*polylog(2,I*b*exp(I*(d*
x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-3/2*a^3*f*polylog(2,I*b*ex
p(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^2+3/2*a*f*polylog(2,I
*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-1/2*a*(f*x+e)*
cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-1/2*a*f/b/(a^2-b^2)/d^2/(a+b*sin(
d*x+c))-3/2*a^2*(f*x+e)*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))+(f*x+e)*c
os(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

**Rubi** [A]
time = 1.97, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6874, 3406, 3405, 3404, 2296, 2221, 2317, 2438, 2747, 31, 32}

$$-\frac{3af\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{3/2}}+\frac{3af\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd^2\left(a^2-b^2\right)^{3/2}}+\frac{3a^3f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{5/2}}-\frac{3a^3f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd^2\left(a^2-b^2\right)^{5/2}}-\frac{af}{2bd^2\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3a^2f\log(a+b\sin(c+dx))}{2bd^2\left(a^2-b^2\right)^2}-\frac{f\log(a+b\sin(c+dx))}{bd^2\left(a^2-b^2\right)}-\frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd\left(a^2-b^2\right)^{3/2}}+\frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd\left(a^2-b^2\right)^{3/2}}-\frac{3a^2(e+fx)\cos(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{a(e+fx)\cos(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}+\frac{(e+fx)\cos(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd\left(a^2-b^2\right)^{5/2}}-\frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((3*I)/2)*a^3*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]
)])/(b*(a^2 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (((3*I)/2)*a^3*(e
 + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2
)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (3*a^2*f*Log[a + b*Sin[c + d*x]])/
(2*b*(a^2 - b^2)^2*d^2) - (f*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d^2) +
 (3*a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^
2 - b^2)^(5/2)*d^2) - (3*a*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) - (3*a^3*f*PolyLog[2, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(5/2)*d^2) + (3*a*f*PolyLo
g[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d
```

```
^2) - (a*(e + f*x)*Cos[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) -
 (a*f)/(2*b*(a^2 - b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a^2*(e + f*x)*Cos[c
+ d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e + f*x)*Cos[c + d*x])
/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3406

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(-b)*(c + d*x)^m*Cos[e + f*x]*((a + b*Sin[e + f*x])^(n +
 1)/(f*(n + 1)*(a^2 - b^2))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a
+ b*Sin[e + f*x])^(n + 1), x], x] - Dist[b*((n + 2)/((n + 1)*(a^2 - b^2))),
 Int[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[b
*d*(m/(f*(n + 1)*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{a(e+fx)}{b(a+b\sin(c+dx))^3}+\frac{e+fx}{b(a+b\sin(c+dx))^2}\right)dx$$

$$= \frac{\int \frac{e+fx}{(a+b\sin(c+dx))^2}\,dx}{b} - \frac{a\int \frac{e+fx}{(a+b\sin(c+dx))^3}\,dx}{b}$$

$$= -\frac{a(e+fx)\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{a\int \frac{(e+fx)}{(a+b\sin}}{2\left(a^2\right.}$$

$$= -\frac{a(e+fx)\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{a^2(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{(e+}{(a^2-b^2)}$$

$$= -\frac{f\log(a+b\sin(c+dx))}{b\left(a^2-b^2\right)d^2} - \frac{a(e+fx)\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{}{2b\left(a^2-b^2\right)d^2(}$$

$$= -\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} + \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} + \frac{a}{}$$

$$= \frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} - \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} - \frac{ia}{}$$

$$= \frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} - \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d} - $$

$$= \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{5/2}d} - \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d} - $$

$$= \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{5/2}d} - \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d} - $$

$$= \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{5/2}d} - \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d} - $$

**Mathematica [A]**

time = 11.33, size = 959, normalized size = 1.28

$$\frac{\frac{(-3abd(e+fx)+(a^2+2b^2)f\cos(c+dx))\left(-\frac{2(a^2+2b^2)f\tan^{-1}\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{2(a^2f+2b^2f+ab(-3de+3cf))\tan^{-1}\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{(a^2+2b^2)f\log(\sec^2(\frac{1}{2}(c+dx)))}{b}+\frac{(a^2+2b^2)f\log(\sec^2(\frac{1}{2}(c+dx))(a+b\sin(c+dx)))}{b}+\frac{3iabf\left(\log(1-i\tan(\frac{1}{2}(c+dx)))\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan(\frac{1}{2}(c+dx))}{-ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a(1-i\tan(\frac{1}{2}(c+dx)))}{a+i(b+\sqrt{-a^2+b^2})}\right)\right)}{\sqrt{-a^2+b^2}}-\frac{3iabf\left(\log(1+i\tan(\frac{1}{2}(c+dx)))\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan(\frac{1}{2}(c+dx))}{ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a(1+i\tan(\frac{1}{2}(c+dx)))}{a-i(b+\sqrt{-a^2+b^2})}\right)\right)}{\sqrt{-a^2+b^2}}-\frac{3iabf\left(\log(1-i\tan(\frac{1}{2}(c+dx)))\log\left(\frac{-b+\sqrt{-a^2+b^2}-a\tan(\frac{1}{2}(c+dx))}{ia-b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a(i+\tan(\frac{1}{2}(c+dx)))}{ia-b+\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}}+\frac{3iabf\left(\log(1+i\tan(\frac{1}{2}(c+dx)))\log\left(\frac{b-\sqrt{-a^2+b^2}+a\tan(\frac{1}{2}(c+dx))}{ia+b-\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a+ia\tan(\frac{1}{2}(c+dx))}{a+i(-b+\sqrt{-a^2+b^2})}\right)\right)}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)^2((a^2+2b^2)f\cos(c+dx)+3ab(-de+cf-if\log(1-i\tan(\frac{1}{2}(c+dx)))+if\log(1+i\tan(\frac{1}{2}(c+dx)))))}-\frac{ad(e+fx)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}+\frac{a(-a^2+b^2)f-b(a^2+2b^2)d(e+fx)\cos(c+dx)}{(a-b)^2b(a+b)^2(a+b\sin(c+dx))}}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((-3*a*b*d*(e + f*x) + (a^2 + 2*b^2)*f*Cos[c + d*x])*((-2*(a^2 + 2*b^2)*f*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (2*(a^2
*f + 2*b^2*f + a*b*(-3*d*e + 3*c*f))*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/Sqrt[a^2 - b^2] - ((a^2 + 2*b^2)*f*Log[Sec[(c + d*x)/2]^2])/b +
 ((a^2 + 2*b^2)*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])])/b + ((3*I)*
a*b*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d
*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*
x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] - ((3*I)*a*b*f*(
Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])
/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a
 - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] - ((3*I)*a*b*f*(Log[1 - I*
Tan[(c + d*x)/2]]*Log[(-b + Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a - b
 + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sq
rt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] + ((3*I)*a*b*f*(Log[1 + I*Tan[(c + d*x)
/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 +
 b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^
2]))]))/Sqrt[-a^2 + b^2]))/((a^2 - b^2)^2*((a^2 + 2*b^2)*f*Cos[c + d*x] + 3
*a*b*(-(d*e) + c*f - I*f*Log[1 - I*Tan[(c + d*x)/2]] + I*f*Log[1 + I*Tan[(c
 + d*x)/2]]))) - (a*d*(e + f*x)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c
 + d*x])^2) + (a*(-a^2 + b^2)*f - b*(a^2 + 2*b^2)*d*(e + f*x)*Cos[c + d*x])
/((a - b)^2*b*(a + b)^2*(a + b*Sin[c + d*x])))/(2*d^2)
```

**Maple [A]**
time = 2.34, size = 1084, normalized size = 1.44

| method | result |
|---|---|
| risch | $\frac{i\left(4iba^3dfx\,\mathrm{e}^{i(dx+c)}-3ib^3ade\,\mathrm{e}^{3i(dx+c)}+5ib^3ade\,\mathrm{e}^{i(dx+c)}-3ib^3adfx\,\mathrm{e}^{3i(dx+c)}+2a^4dfx\,\mathrm{e}^{2i(dx+c)}+5b^2a^2dfx\,\mathrm{e}^{2i(dx+c)}+2b^4dfx\,\mathrm{e}^{2i(dx}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] I*(4*I*b*a^3*d*f*x*exp(I*(d*x+c))-3*I*b^3*a*d*e*exp(3*I*(d*x+c))+5*I*b^3*a*
d*e*exp(I*(d*x+c))-3*I*b^3*a*d*f*x*exp(3*I*(d*x+c))+2*a^4*d*f*x*exp(2*I*(d*
x+c))+5*b^2*a^2*d*f*x*exp(2*I*(d*x+c))+2*b^4*d*f*x*exp(2*I*(d*x+c))+5*I*b^3
*a*d*f*x*exp(I*(d*x+c))+2*I*a^4*f*exp(2*I*(d*x+c))-2*I*b^2*f*exp(2*I*(d*x+c
```

```
))*a^2+4*I*b*a^3*d*e*exp(I*(d*x+c))+2*a^4*d*e*exp(2*I*(d*x+c))+b*a^3*f*exp(
3*I*(d*x+c))+5*b^2*a^2*d*e*exp(2*I*(d*x+c))-b^3*a*f*exp(3*I*(d*x+c))+2*b^4*
d*e*exp(2*I*(d*x+c))-a^2*b^2*d*f*x-2*b^4*d*f*x-b*a^3*f*exp(I*(d*x+c))-a^2*b
^2*d*e+b^3*a*f*exp(I*(d*x+c))-2*b^4*d*e)/(-I*b*exp(2*I*(d*x+c))+I*b+2*a*exp
(I*(d*x+c)))^2/(a^2-b^2)^2/d^2/b-1/b/d^2/(-a^2+b^2)^2*a^2*f*ln(exp(I*(d*x+c
)))+1/2/b/d^2/(-a^2+b^2)^2*a^2*f*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))
-I*b)-2*b/d^2/(-a^2+b^2)^2*f*ln(exp(I*(d*x+c)))+b/d^2/(-a^2+b^2)^2*f*ln(I*b
*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-3/2*b/d/(-a^2+b^2)^(5/2)*a*f*ln((
I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-3/2*b/d^2/
(-a^2+b^2)^(5/2)*a*f*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+
b^2)^(1/2)))*c+3*I*b/d^2/(-a^2+b^2)^(5/2)*a*f*c*arctan(1/2*(2*I*b*exp(I*(d*
x+c))-2*a)/(-a^2+b^2)^(1/2))-3*I*b/d/(-a^2+b^2)^(5/2)*a*e*arctan(1/2*(2*I*b
*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+3/2*b/d/(-a^2+b^2)^(5/2)*a*f*ln((I*a
+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+3/2*b/d^2/(-a
^2+b^2)^(5/2)*a*f*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2
)^(1/2)))*c-3/2*I*b/d^2/(-a^2+b^2)^(5/2)*a*f*dilog((I*a+b*exp(I*(d*x+c))+(-
a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+3/2*I*b/d^2/(-a^2+b^2)^(5/2)*a*f*di
log((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2443 vs. 2(658) = 1316.
time = 0.67, size = 2443, normalized size = 3.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(I*a*b^5*f*cos(d*x + c)^2 - 2*I*a^2*b^4*f*sin(d*x + c) - I*(a^3*b^3
 + a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c
) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)
+ 3*(-I*a*b^5*f*cos(d*x + c)^2 + 2*I*a^2*b^4*f*sin(d*x + c) + I*(a^3*b^3 +
a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) -
```

```
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3
*(-I*a*b^5*f*cos(d*x + c)^2 + 2*I*a^2*b^4*f*sin(d*x + c) + I*(a^3*b^3 + a*b
^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(
I*a*b^5*f*cos(d*x + c)^2 - 2*I*a^2*b^4*f*sin(d*x + c) - I*(a^3*b^3 + a*b^5)
*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*c
os(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*((a^
3*b^3 + a*b^5)*d*f*x + (a^3*b^3 + a*b^5)*c*f - (a*b^5*d*f*x + a*b^5*c*f)*co
s(d*x + c)^2 + 2*(a^2*b^4*d*f*x + a^2*b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*((a^3*b^3 + a*b^5)*d*f*x + (
a^3*b^3 + a*b^5)*c*f - (a*b^5*d*f*x + a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^
4*d*f*x + a^2*b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d
*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b) + 3*((a^3*b^3 + a*b^5)*d*f*x + (a^3*b^3 + a*b^5)*c*f - (
a*b^5*d*f*x + a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f*x + a^2*b^4*c*f)*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c
) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*
((a^3*b^3 + a*b^5)*d*f*x + (a^3*b^3 + a*b^5)*c*f - (a*b^5*d*f*x + a*b^5*c*f
)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f*x + a^2*b^4*c*f)*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*(a^6 - 2*a^4*b^2 + a^2*
b^4)*f - 2*((2*a^5*b - a^3*b^3 - a*b^5)*d*f*x + (2*a^5*b - a^3*b^3 - a*b^5)
*d*e)*cos(d*x + c) - ((a^4*b^2 + a^2*b^4 - 2*b^6)*f*cos(d*x + c)^2 - 2*(a^5
*b + a^3*b^3 - 2*a*b^5)*f*sin(d*x + c) - (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6
)*f - 3*((a^3*b^3 + a*b^5)*c*f - (a*b^5*c*f - a*b^5*d*e)*cos(d*x + c)^2 - (
a^3*b^3 + a*b^5)*d*e + 2*(a^2*b^4*c*f - a^2*b^4*d*e)*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) + 2*I*a) - ((a^4*b^2 + a^2*b^4 - 2*b^6)*f*cos(d*x + c)^2 - 2*(a
^5*b + a^3*b^3 - 2*a*b^5)*f*sin(d*x + c) - (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b
^6)*f - 3*((a^3*b^3 + a*b^5)*c*f - (a*b^5*c*f - a*b^5*d*e)*cos(d*x + c)^2 -
 (a^3*b^3 + a*b^5)*d*e + 2*(a^2*b^4*c*f - a^2*b^4*d*e)*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^
2 - b^2)/b^2) - 2*I*a) - ((a^4*b^2 + a^2*b^4 - 2*b^6)*f*cos(d*x + c)^2 - 2*
(a^5*b + a^3*b^3 - 2*a*b^5)*f*sin(d*x + c) - (a^6 + 2*a^4*b^2 - a^2*b^4 - 2
*b^6)*f + 3*((a^3*b^3 + a*b^5)*c*f - (a*b^5*c*f - a*b^5*d*e)*cos(d*x + c)^2
 - (a^3*b^3 + a*b^5)*d*e + 2*(a^2*b^4*c*f - a^2*b^4*d*e)*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-
(a^2 - b^2)/b^2) + 2*I*a) - ((a^4*b^2 + a^2*b^4 - 2*b^6)*f*cos(d*x + c)^2 -
 2*(a^5*b + a^3*b^3 - 2*a*b^5)*f*sin(d*x + c) - (a^6 + 2*a^4*b^2 - a^2*b^4
- 2*b^6)*f + 3*((a^3*b^3 + a*b^5)*c*f - (a*b^5*c*f - a*b^5*d*e)*cos(d*x + c
)^2 - (a^3*b^3 + a*b^5)*d*e + 2*(a^2*b^4*c*f - a^2*b^4*d*e)*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqr
t(-(a^2 - b^2)/b^2) - 2*I*a) - 2*((a^5*b - 2*a^3*b^3 + a*b^5)*f + ((a^4*b^2
 + a^2*b^4 - 2*b^6)*d*f*x + (a^4*b^2 + a^2*b^4 - 2*b^6)*d*e)*cos(d*x + c))*
```

```
sin(d*x + c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d^2*cos(d*x + c)^2 -
 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d^2*sin(d*x + c) - (a^8*b - 2*
a^6*b^3 + 2*a^2*b^7 - b^9)*d^2)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x))/(a + b*sin(c + d*x))^3,x)
```

```
[Out] \text{Hanged}
```

# 3.249 $\int \frac{(e+fx)^2 \sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

**Optimal**. Leaf size=1584

$$-\frac{3ia^2(e+fx)^2}{2b\left(a^2-b^2\right)^2 d}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right) d}+\frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2} d^3}+\frac{3a^2f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^2 d^2}-\frac{2f(\cdots}{\cdots}$$

```
[Out] -a*f*(f*x+e)/b/(a^2-b^2)/d^2/(a+b*sin(d*x+c))+3*a^2*f*(f*x+e)*ln(1-I*b*exp(
I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^2+3*a^3*f*(f*x+e)*polylog(2
,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^2-3*a*f*(f*x+e
)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-3
*a^3*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2
)^(5/2)/d^2+3*a*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))
/b/(a^2-b^2)^(3/2)/d^2-3/2*I*a*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/b/(a^2-b^2)^(3/2)/d-3/2*I*a^3*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d-3*I*a^2*f^2*polylog(2,I*b*exp(I*(d
*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^3-3*I*a^2*f^2*polylog(2,I*b*exp
(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^3-3*I*a*f^2*polylog(3,I*b*
exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-3*I*a^3*f^2*polyl
og(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^3+3/2*I*a^
3*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/
d+3/2*I*a*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2
)^(3/2)/d+3*I*a^3*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(
a^2-b^2)^(5/2)/d^3+3*I*a*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2
)))/b/(a^2-b^2)^(3/2)/d^3+3*a^2*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b
^2)^(1/2)))/b/(a^2-b^2)^2/d^2+2*I*f^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-
b^2)^(1/2)))/b/(a^2-b^2)/d^3+2*I*f^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b
^2)^(1/2)))/b/(a^2-b^2)/d^3+2*a*f^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^
2)^(1/2))/b/(a^2-b^2)^(3/2)/d^3-2*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2
-b^2)^(1/2)))/b/(a^2-b^2)/d^2-2*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b
^2)^(1/2)))/b/(a^2-b^2)/d^2+I*(f*x+e)^2/b/(a^2-b^2)/d+(f*x+e)^2*cos(d*x+c)/
(a^2-b^2)/d/(a+b*sin(d*x+c))-1/2*a*(f*x+e)^2*cos(d*x+c)/(a^2-b^2)/d/(a+b*si
n(d*x+c))^2-3/2*a^2*(f*x+e)^2*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-3/2
*I*a^2*(f*x+e)^2/b/(a^2-b^2)^2/d
```

**Rubi** [A]

time = 3.94, antiderivative size = 1584, normalized size of antiderivative = 1.00, number of steps used = 73, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6874, 3406, 3405, 3404, 2296, 2221, 2611, 2320, 6724, 4615, 2317, 2438, 4507, 2739, 632, 210}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((-3*I)/2)*a^2*(e + f*x)^2)/(b*(a^2 - b^2)^2*d) + (I*(e + f*x)^2)/(b*(a^2
- b^2)*d) + (2*a*f^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(
a^2 - b^2)^(3/2)*d^3) + (3*a^2*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
 - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^2) - (2*f*(e + f*x)*Log[1 - (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (((3*I)/2)*a^
3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2
 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (3*a^2*f*(e + f*x)*Log[1 -
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^2) - (2*f
*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 -
b^2)*d^2) - (((3*I)/2)*a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)^2*Log[1
- (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (
(3*I)*a^2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(
a^2 - b^2)^2*d^3) + ((2*I)*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a
^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (3*a^3*f*(e + f*x)*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^2) - (3*a*f*(e +
 f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^
2)^(3/2)*d^2) - ((3*I)*a^2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(b*(a^2 - b^2)^2*d^3) + ((2*I)*f^2*PolyLog[2, (I*b*E^(I*(c + d
*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (3*a^3*f*(e + f*x)*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d
^2) + (3*a*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b*(a^2 - b^2)^(3/2)*d^2) + ((3*I)*a^3*f^2*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^3) - ((3*I)*a*f^2*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
^3) - ((3*I)*a^3*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])
])/(b*(a^2 - b^2)^(5/2)*d^3) + ((3*I)*a*f^2*PolyLog[3, (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) - (a*(e + f*x)^2*Cos[c
+ d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (a*f*(e + f*x))/(b*(a^2
- b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a^2*(e + f*x)^2*Cos[c + d*x])/(2*(a^2
 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e + f*x)^2*Cos[c + d*x])/((a^2 - b^2)
*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3406

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(-b)*(c + d*x)^m*Cos[e + f*x]*((a + b*Sin[e + f*x])^(n +
 1)/(f*(n + 1)*(a^2 - b^2))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a
+ b*Sin[e + f*x])^(n + 1), x], x] - Dist[b*((n + 2)/((n + 1)*(a^2 - b^2))),
 Int[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[b
*d*(m/(f*(n + 1)*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
```

```
- I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
&& PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{a(e+fx)^2}{b(a+b\sin(c+dx))^3} + \frac{(e+fx)^2}{b(a+b\sin(c+dx))^2}\right)\,dx$$

$$= \frac{\int \frac{(e+fx)^2}{(a+b\sin(c+dx))^2}\,dx}{b} - \frac{a\int \frac{(e+fx)^2}{(a+b\sin(c+dx))^3}\,dx}{b}$$

$$= -\frac{a(e+fx)^2\cos(c+dx)}{2(a^2-b^2)d(a+b\sin(c+dx))^2} + \frac{(e+fx)^2\cos(c+dx)}{(a^2-b^2)d(a+b\sin(c+dx))} + \frac{a\int \frac{(e+fx}{(a+b}}{2(a}$$

$$= \frac{i(e+fx)^2}{b(a^2-b^2)d} - \frac{a(e+fx)^2\cos(c+dx)}{2(a^2-b^2)d(a+b\sin(c+dx))^2} - \frac{af(e+fx)}{b(a^2-b^2)d^2(a+b\sin(c+}$$

$$= -\frac{ia^2(e+fx)^2}{b(a^2-b^2)^2 d} + \frac{i(e+fx)^2}{b(a^2-b^2)d} - \frac{2f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^2} - \frac{2f(e+}{}$$

$$= -\frac{ia^2(e+fx)^2}{b(a^2-b^2)^2 d} + \frac{i(e+fx)^2}{b(a^2-b^2)d} + \frac{2a^2f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^2 d^2} - \frac{2f(}{}$$

$$= -\frac{3ia^2(e+fx)^2}{2b(a^2-b^2)^2 d} + \frac{i(e+fx)^2}{b(a^2-b^2)d} + \frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} + \frac{2a^2f(e+f}{}$$

$$= -\frac{3ia^2(e+fx)^2}{2b(a^2-b^2)^2 d} + \frac{i(e+fx)^2}{b(a^2-b^2)d} + \frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} + \frac{3a^2f(e+f}{}$$

$$= -\frac{3ia^2(e+fx)^2}{2b(a^2-b^2)^2 d} + \frac{i(e+fx)^2}{b(a^2-b^2)d} + \frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} + \frac{3a^2f(e+f}{}$$

$$= -\frac{3ia^2(e+fx)^2}{2b(a^2-b^2)^2 d} + \frac{i(e+fx)^2}{b(a^2-b^2)d} + \frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} + \frac{3a^2f(e+f}{}$$

$$= -\frac{3ia^2(e+fx)^2}{2b(a^2-b^2)^2 d} + \frac{i(e+fx)^2}{b(a^2-b^2)d} + \frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} + \frac{3a^2f(e+f}{}$$

$$= -\frac{3ia^2(e+fx)^2}{2b(a^2-b^2)^2 d} + \frac{i(e+fx)^2}{b(a^2-b^2)d} + \frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} + \frac{3a^2f(e+f}{}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 13567 vs. $2(1584) = 3168$.
time = 21.93, size = 13567, normalized size = 8.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

**Maple** [**F**]
time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \sin(dx+c)}{(a+b\sin(dx+c))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5793 vs. $2(1403) = 2806$.
time = 0.83, size = 5793, normalized size = 3.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a^6 - 2*a^4*b^2 + a^2*b^4)*d*f^2*x + 4*(a^6 - 2*a^4*b^2 + a^2*b^4)*
d*f*e - 6*(a*b^5*f^2*cos(d*x + c)^2 - 2*a^2*b^4*f^2*sin(d*x + c) - (a^3*b^3
```

+ a*b^5)*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin (d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(a*b^5*f^2*cos(d*x + c)^2 - 2*a^2*b^4*f^2*sin(d*x + c) - (a^3*b^3 + a*b ^5)*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(a *b^5*f^2*cos(d*x + c)^2 - 2*a^2*b^4*f^2*sin(d*x + c) - (a^3*b^3 + a*b^5)*f^ 2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(a*b^5* f^2*cos(d*x + c)^2 - 2*a^2*b^4*f^2*sin(d*x + c) - (a^3*b^3 + a*b^5)*f^2)*sq rt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*c os(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*((2*a^5*b - a^3*b^3 - a*b^5)*d^2*f^2*x^2 + 2*(2*a^5*b - a^3*b^3 - a*b^5)*d^2*f*x*e + (2 *a^5*b - a^3*b^3 - a*b^5)*d^2*e^2)*cos(d*x + c) - 2*(I*(a^4*b^2 + a^2*b^4 - 2*b^6)*f^2*cos(d*x + c)^2 - 2*I*(a^5*b + a^3*b^3 - 2*a*b^5)*f^2*sin(d*x + c) - I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f^2 + 3*(-I*(a^3*b^3 + a*b^5)*d* f^2*x - I*(a^3*b^3 + a*b^5)*d*f*e + (I*a*b^5*d*f^2*x + I*a*b^5*d*f*e)*cos(d *x + c)^2 + 2*(-I*a^2*b^4*d*f^2*x - I*a^2*b^4*d*f*e)*sin(d*x + c))*sqrt(-(a ^2 - b^2)/b^2))*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*(a^4*b^2 + a^ 2*b^4 - 2*b^6)*f^2*cos(d*x + c)^2 - 2*I*(a^5*b + a^3*b^3 - 2*a*b^5)*f^2*sin (d*x + c) - I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f^2 + 3*(I*(a^3*b^3 + a*b ^5)*d*f^2*x + I*(a^3*b^3 + a*b^5)*d*f*e + (-I*a*b^5*d*f^2*x - I*a*b^5*d*f*e )*cos(d*x + c)^2 + 2*(I*a^2*b^4*d*f^2*x + I*a^2*b^4*d*f*e)*sin(d*x + c))*sq rt(-(a^2 - b^2)/b^2))*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*(a^4*b ^2 + a^2*b^4 - 2*b^6)*f^2*cos(d*x + c)^2 + 2*I*(a^5*b + a^3*b^3 - 2*a*b^5)* f^2*sin(d*x + c) + I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f^2 + 3*(I*(a^3*b^ 3 + a*b^5)*d*f^2*x + I*(a^3*b^3 + a*b^5)*d*f*e + (-I*a*b^5*d*f^2*x - I*a*b^ 5*d*f*e)*cos(d*x + c)^2 + 2*(I*a^2*b^4*d*f^2*x + I*a^2*b^4*d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b *cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(- I*(a^4*b^2 + a^2*b^4 - 2*b^6)*f^2*cos(d*x + c)^2 + 2*I*(a^5*b + a^3*b^3 - 2 *a*b^5)*f^2*sin(d*x + c) + I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f^2 + 3*(- I*(a^3*b^3 + a*b^5)*d*f^2*x - I*(a^3*b^3 + a*b^5)*d*f*e + (I*a*b^5*d*f^2*x + I*a*b^5*d*f*e)*cos(d*x + c)^2 + 2*(-I*a^2*b^4*d*f^2*x - I*a^2*b^4*d*f*e)* sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f^2 - 2*(a^6 + 2*a^4*b^2 - a^ 2*b^4 - 2*b^6)*d*f*e - 2*((a^4*b^2 + a^2*b^4 - 2*b^6)*c*f^2 - (a^4*b^2 + a^ 2*b^4 - 2*b^6)*d*f*e)*cos(d*x + c)^2 + 4*((a^5*b + a^3*b^3 - 2*a*b^5)*c*f^2 - (a^5*b + a^3*b^3 - 2*a*b^5)*d*f*e)*sin(d*x + c) - (6*(a^3*b^3 + a*b^5)*c *d*f*e - 3*(a^3*b^3 + a*b^5)*d^2*e^2 + (2*a^5*b - 2*a*b^5 - 3*(a^3*b^3 + a* b^5)*c^2)*f^2 - (6*a*b^5*c*d*f*e - 3*a*b^5*d^2*e^2 - (3*a*b^5*c^2 - 2*a^3*b ^3 + 2*a*b^5)*f^2)*cos(d*x + c)^2 + 2*(6*a^2*b^4*c*d*f*e - 3*a^2*b^4*d^2*e^ 2 - (3*a^2*b^4*c^2 - 2*a^4*b^2 + 2*a^2*b^4)*f^2)*sin(d*x + c))*sqrt(-(a^2 -

```
 b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) + 2*I*a) + (2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f^2 - 2*(a^6 +
2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*e - 2*((a^4*b^2 + a^2*b^4 - 2*b^6)*c*f^2 -
 (a^4*b^2 + a^2*b^4 - 2*b^6)*d*f*e)*cos(d*x + c)^2 + 4*((a^5*b + a^3*b^3 -
2*a*b^5)*c*f^2 - (a^5*b + a^3*b^3 - 2*a*b^5)*d*f*e)*sin(d*x + c) - (6*(a^3*
b^3 + a*b^5)*c*d*f*e - 3*(a^3*b^3 + a*b^5)*d^2*e^2 + (2*a^5*b - 2*a*b^5 - 3
*(a^3*b^3 + a*b^5)*c^2)*f^2 - (6*a*b^5*c*d*f*e - 3*a*b^5*d^2*e^2 - (3*a*b^5
*c^2 - 2*a^3*b^3 + 2*a*b^5)*f^2)*cos(d*x + c)^2 + 2*(6*a^2*b^4*c*d*f*e - 3*
a^2*b^4*d^2*e^2 - (3*a^2*b^4*c^2 - 2*a^4*b^2 + 2*a^2*b^4)*f^2)*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*s
qrt(-(a^2 - b^2)/b^2) - 2*I*a) + (2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f
^2 - 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*e - 2*((a^4*b^2 + a^2*b^4 -
2*b^6)*c*f^2 - (a^4*b^2 + a^2*b^4 - 2*b^6)*d*f*e)*cos(d*x + c)^2 + 4*((a^5*
b + a^3*b^3 - 2*a*b^5)*c*f^2 - (a^5*b + a^3*b^3 - 2*a*b^5)*d*f*e)*sin(d*x +
 c) + (6*(a^3*b^3 + a*b^5)*c*d*f*e - 3*(a^3*b^3 + a*b^5)*d^2*e^2 + (2*a^5*b
 - 2*a*b^5 - 3*(a^3*b^3 + a*b^5)*c^2)*f^2 - (6*a*b^5*c*d*f*e - 3*a*b^5*d^2*
e^2 - (3*a*b^5*c^2 - 2*a^3*b^3 + 2*a*b^5)*f^2)*cos(d*x + c)^2 + 2*(6*a^2*b^
4*c*d*f*e - 3*a^2*b^4*d^2*e^2 - (3*a^2*b^4*c^2 - 2*a^4*b^2 + 2*a^2*b^4)*f^2
)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d
*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) +...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)^2*sin(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x))^3,x)
[Out] \text{Hanged}
```

# 3.250 $\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

**Optimal**. Leaf size=2348

result too large to display

```
[Out] 9/2*a^2*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2
)^2/d^2+9/2*a^2*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/
(a^2-b^2)^2/d^2+9/2*a^3*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^2-9/2*a*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x
+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-9/2*a^3*f*(f*x+e)^2*polylog
(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^2+3/2*I*a^3*
(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d+
3/2*I*a*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^
(3/2)/d+6*I*f^2*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b
/(a^2-b^2)/d^3+6*I*f^2*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1
/2)))/b/(a^2-b^2)/d^3+9/2*a*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^
2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-3*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/
(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^2-3*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^2-3*a*f^3*polylog(2,I*b*exp(I*(d*x+c))/(
a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^4+3*a*f^3*polylog(2,I*b*exp(I*(d*x+
c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^4+9*a^2*f^3*polylog(3,I*b*exp(
I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^4+9*a^2*f^3*polylog(3,I*b*e
xp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^4-9*a^3*f^3*polylog(4,I*
b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^4+9*a*f^3*polylog
(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^4+9*a^3*f^3*
polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^4-9*a
*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^
4+I*(f*x+e)^3/b/(a^2-b^2)/d+(f*x+e)^3*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c
))-6*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^4-
6*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^4-1/2
*a*(f*x+e)^3*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-3/2*a^2*(f*x+e)^3*co
s(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-3/2*I*a^2*(f*x+e)^3/b/(a^2-b^2)^2/d
-3/2*I*a*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)
^(3/2)/d-3/2*I*a^3*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b
/(a^2-b^2)^(5/2)/d-3/2*a*f*(f*x+e)^2/b/(a^2-b^2)/d^2/(a+b*sin(d*x+c))+3*I*a
*f^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)
/d^3+9*I*a^3*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/
b/(a^2-b^2)^(5/2)/d^3+9*I*a*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^
2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-3*I*a*f^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+
c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-9*I*a^2*f^2*(f*x+e)*polylog(
2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^3-9*I*a^2*f^2*(f*
x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^3-9*
I*a*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^
2)^(3/2)/d^3-9*I*a^3*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
```

```
(1/2)))/b/(a^2-b^2)^(5/2)/d^3
```

**Rubi** [A]
time = 5.10, antiderivative size = 2348, normalized size of antiderivative = 1.00, number of steps used = 92, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6874, 3406, 3405, 3404, 2296, 2221, 2611, 6744, 2320, 6724, 4615, 4507, 2317, 2438}

---

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]

[Out] (((-3*I)/2)*a^2*(e + f*x)^3)/(b*(a^2 - b^2)^2*d) + (I*(e + f*x)^3)/(b*(a^2
- b^2)*d) - ((3*I)*a*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f*(e + f*x)^2*Log[1 - (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^2*d^2) - (3*f*(e
 + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b
^2)*d^2) + (((3*I)/2)*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)^3*Log[1 -
 (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + ((
3*I)*a*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^2*d^2) - (3*f*(e + f*x)^2*Log[
1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (((
3*I)/2)*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])
])/(b*(a^2 - b^2)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)^3*Log[1 - (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (3*a*f^3*PolyLog
[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4)
 - ((9*I)*a^2*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(b*(a^2 - b^2)^2*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (9*a^3*f*(e + f*
x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^2 - b
^2)^(5/2)*d^2) - (9*a*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) + (3*a*f^3*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) - ((9*I)*a
^2*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
b*(a^2 - b^2)^2*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (9*a^3*f*(e + f*x)^2*PolyLo
g[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(5/2)*d
^2) + (9*a*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) + (9*a^2*f^3*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^4) - (6*f^3*PolyLog[3, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + ((9*I)*a^3
*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
```

```
(a^2 - b^2)^(5/2)*d^3) - ((9*I)*a*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f^3*PolyLo
g[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^4) -
(6*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b
^2)*d^4) - ((9*I)*a^3*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^3) + ((9*I)*a*f^2*(e + f*x)*PolyLo
g[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3
) - (9*a^3*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
(a^2 - b^2)^(5/2)*d^4) + (9*a*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) + (9*a^3*f^3*PolyLog[4, (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^4) - (9*a*f^3*P
olyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2
)*d^4) - (a*(e + f*x)^3*Cos[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
^2) - (3*a*f*(e + f*x)^2)/(2*b*(a^2 - b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a
^2*(e + f*x)^3*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e
 + f*x)^3*Cos[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3406

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(-b)*(c + d*x)^m*Cos[e + f*x]*((a + b*Sin[e + f*x])^(n +
 1)/(f*(n + 1)*(a^2 - b^2))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a
+ b*Sin[e + f*x])^(n + 1), x], x] - Dist[b*((n + 2)/((n + 1)*(a^2 - b^2))),
 Int[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[b
*d*(m/(f*(n + 1)*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
```

```
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{a(e+fx)^3}{b(a+b\sin(c+dx))^3}+\frac{(e+fx)^3}{b(a+b\sin(c+dx))^2}\right)dx$$

$$= \frac{\int \frac{(e+fx)^3}{(a+b\sin(c+dx))^2}\,dx}{b} - \frac{a\int \frac{(e+fx)^3}{(a+b\sin(c+dx))^3}\,dx}{b}$$

$$= -\frac{a(e+fx)^3\cos(c+dx)}{2(a^2-b^2)d(a+b\sin(c+dx))^2}+\frac{(e+fx)^3\cos(c+dx)}{(a^2-b^2)d(a+b\sin(c+dx))}+\frac{a\int \frac{(e+}{(a+}}{2}$$

$$= \frac{i(e+fx)^3}{b(a^2-b^2)d}-\frac{a(e+fx)^3\cos(c+dx)}{2(a^2-b^2)d(a+b\sin(c+dx))^2}-\frac{3af(e+fx)^2}{2b(a^2-b^2)d^2(a+b\sin(c}$$

$$= -\frac{ia^2(e+fx)^3}{b(a^2-b^2)^2d}+\frac{i(e+fx)^3}{b(a^2-b^2)d}-\frac{3f(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^2}-\frac{3f(}{}$$

$$= -\frac{ia^2(e+fx)^3}{b(a^2-b^2)^2d}+\frac{i(e+fx)^3}{b(a^2-b^2)d}+\frac{3a^2f(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^2d^2}-\frac{3}{}$$

$$= -\frac{3ia^2(e+fx)^3}{2b(a^2-b^2)^2d}+\frac{i(e+fx)^3}{b(a^2-b^2)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3}+$$

$$= -\frac{3ia^2(e+fx)^3}{2b(a^2-b^2)^2d}+\frac{i(e+fx)^3}{b(a^2-b^2)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3}+$$

$$= -\frac{3ia^2(e+fx)^3}{2b(a^2-b^2)^2d}+\frac{i(e+fx)^3}{b(a^2-b^2)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3}+$$

$$= -\frac{3ia^2(e+fx)^3}{2b(a^2-b^2)^2d}+\frac{i(e+fx)^3}{b(a^2-b^2)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3}+$$

$$= -\frac{3ia^2(e+fx)^3}{2b(a^2-b^2)^2d}+\frac{i(e+fx)^3}{b(a^2-b^2)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3}+$$

$$= -\frac{3ia^2(e+fx)^3}{2b(a^2-b^2)^2d}+\frac{i(e+fx)^3}{b(a^2-b^2)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3}+$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 12936 vs. 2(2348) = 4696.
time = 20.84, size = 12936, normalized size = 5.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

**Maple** [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \sin(dx+c)}{(a+b\sin(dx+c))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10616 vs. 2(2074) = 4148.
time = 1.21, size = 10616, normalized size = 4.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^2*f^3*x^2 + 12*(a^6 - 2*a^4*b^2 + a^2*
b^4)*d^2*f^2*x*e + 6*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^2*f*e^2 - 18*(-I*a*b^5*f
```

```
^3*cos(d*x + c)^2 + 2*I*a^2*b^4*f^3*sin(d*x + c) + I*(a^3*b^3 + a*b^5)*f^3)
*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 18*(I*a*b^5*
f^3*cos(d*x + c)^2 - 2*I*a^2*b^4*f^3*sin(d*x + c) - I*(a^3*b^3 + a*b^5)*f^3
)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 18*(I*a*b^5
*f^3*cos(d*x + c)^2 - 2*I*a^2*b^4*f^3*sin(d*x + c) - I*(a^3*b^3 + a*b^5)*f^
3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) +
 (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 18*(-I*a*
b^5*f^3*cos(d*x + c)^2 + 2*I*a^2*b^4*f^3*sin(d*x + c) + I*(a^3*b^3 + a*b^5)
*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c
) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*((2*
a^5*b - a^3*b^3 - a*b^5)*d^3*f^3*x^3 + 3*(2*a^5*b - a^3*b^3 - a*b^5)*d^3*f^
2*x^2*e + 3*(2*a^5*b - a^3*b^3 - a*b^5)*d^3*f*x*e^2 + (2*a^5*b - a^3*b^3 -
a*b^5)*d^3*e^3)*cos(d*x + c) - 3*(-2*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*
d*f^3*x - 2*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f^2*e + 2*(I*(a^4*b^2 +
 a^2*b^4 - 2*b^6)*d*f^3*x + I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^2*e)*cos(d*x
+ c)^2 + 4*(-I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*f^3*x - I*(a^5*b + a^3*b^3 - 2
*a*b^5)*d*f^2*e)*sin(d*x + c) + (-3*I*(a^3*b^3 + a*b^5)*d^2*f^3*x^2 - 6*I*(
a^3*b^3 + a*b^5)*d^2*f^2*x*e - 3*I*(a^3*b^3 + a*b^5)*d^2*f*e^2 + 2*I*(a^5*b
 - a*b^5)*f^3 + (3*I*a*b^5*d^2*f^3*x^2 + 6*I*a*b^5*d^2*f^2*x*e + 3*I*a*b^5*
d^2*f*e^2 - 2*I*(a^3*b^3 - a*b^5)*f^3)*cos(d*x + c)^2 + 2*(-3*I*a^2*b^4*d^2
*f^3*x^2 - 6*I*a^2*b^4*d^2*f^2*x*e - 3*I*a^2*b^4*d^2*f*e^2 + 2*I*(a^4*b^2 -
 a^2*b^4)*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a*cos(d*x + c
) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) - 3*(-2*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f^3*x - 2*
I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f^2*e + 2*(I*(a^4*b^2 + a^2*b^4 - 2
*b^6)*d*f^3*x + I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^2*e)*cos(d*x + c)^2 + 4*(
-I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*f^3*x - I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*f^
2*e)*sin(d*x + c) + (3*I*(a^3*b^3 + a*b^5)*d^2*f^3*x^2 + 6*I*(a^3*b^3 + a*b
^5)*d^2*f^2*x*e + 3*I*(a^3*b^3 + a*b^5)*d^2*f*e^2 - 2*I*(a^5*b - a*b^5)*f^3
 + (-3*I*a*b^5*d^2*f^3*x^2 - 6*I*a*b^5*d^2*f^2*x*e - 3*I*a*b^5*d^2*f*e^2 +
2*I*(a^3*b^3 - a*b^5)*f^3)*cos(d*x + c)^2 + 2*(3*I*a^2*b^4*d^2*f^3*x^2 + 6*
I*a^2*b^4*d^2*f^2*x*e + 3*I*a^2*b^4*d^2*f*e^2 - 2*I*(a^4*b^2 - a^2*b^4)*f^3
)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a*cos(d*x + c) - a*sin(d*x
 + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
 1) - 3*(2*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f^3*x + 2*I*(a^6 + 2*a^4
*b^2 - a^2*b^4 - 2*b^6)*d*f^2*e + 2*(-I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^3*x
 - I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^2*e)*cos(d*x + c)^2 + 4*(I*(a^5*b + a^
3*b^3 - 2*a*b^5)*d*f^3*x + I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*f^2*e)*sin(d*x +
 c) + (3*I*(a^3*b^3 + a*b^5)*d^2*f^3*x^2 + 6*I*(a^3*b^3 + a*b^5)*d^2*f^2*x*
e + 3*I*(a^3*b^3 + a*b^5)*d^2*f*e^2 - 2*I*(a^5*b - a*b^5)*f^3 + (-3*I*a*b^5
*d^2*f^3*x^2 - 6*I*a*b^5*d^2*f^2*x*e - 3*I*a*b^5*d^2*f*e^2 + 2*I*(a^3*b^3 -
 a*b^5)*f^3)*cos(d*x + c)^2 + 2*(3*I*a^2*b^4*d^2*f^3*x^2 + 6*I*a^2*b^4*d^2*
f^2*x*e + 3*I*a^2*b^4*d^2*f*e^2 - 2*I*(a^4*b^2 - a^2*b^4)*f^3)*sin(d*x + c)
```

```
)*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(2*I*
(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f^3*x + 2*I*(a^6 + 2*a^4*b^2 - a^2*b^
4 - 2*b^6)*d*f^2*e + 2*(-I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^3*x - I*(a^4*b^2
 + a^2*b^4 - 2*b^6)*d*f^2*e)*cos(d*x + c)^2 + 4*(I*(a^5*b + a^3*b^3 - 2*a*b
^5)*d*f^3*x + I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*f^2*e)*sin(d*x + c) + (-3*I*(
a^3*b^3 + a*b^5)*d^2*f^3*x^2 - 6*I*(a^3*b^3 + a*b^5)*d^2*f^2*x*e - 3*I*(a^3
*b^3 + a*b^5)*d^2*f*e^2 + 2*I*(a^5*b - a*b^5)*f^3 + (3*I*a*b^5*d^2*f^3*x^2
+ 6*I*a*b^5*d^2*f^2*x*e + 3*I*a*b^5*d^2*f*e^2 - 2*I*(a^3*b^3 - a*b^5)*f^3)*
cos(d*x + c)^2 + 2*(-3*I*a^2*b^4*d^2*f^3*x^2 - 6*I*a^2*b^4*d^2*f^2*x*e - 3*
I*a^2*b^4*d^2*f*e^2 + 2*I*(a^4*b^2 - a^2*b^4)*f^3)*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2))*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*((a^6 + 2*a^4*b^2
 - a^2*b^4 - 2*b^6)*c^2*f^3 - 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*d*f^2
*e + (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d^2*f*e^2 - ((a^4*b^2 + a^2*b^4 -
2*b^6)*c^2*f^3 - 2*(a^4*b^2 + a^2*b^4 - 2*b^6)*c*d*f^2*e + (a^4*b^2 + a^2*b
^4 - 2*b^6)*d^2*f*e^2)*cos(d*x + c)^2 + 2*((a^5*b + a^3*b^3 - 2*a*b^5)*c^2*
f^3 - 2*(a^5*b + a^3*b^3 - 2*a*b^5)*c*d*f^2*e + (a^5*b + a^3*b^3 - 2*a*b^5)
*d^2*f*e^2)*sin(d*x + c) + (3*(a^3*b^3 + a*b^5)*c*d^2*f*e^2 - (a^3*b^3 + a*
b^5)*d^3*e^3 + (2*a^5*b - 2*a*b^5 - 3*(a^3*b^3 ...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)^3*sin(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x))^3,x)
[Out] \text{Hanged}
```

# 3.251 $\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=151

$$-\frac{i(e+fx)^4}{4af}+\frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{6if(e+fx)^2\text{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+\frac{12f^2(e+fx)\text{Li}_3\left(ie^{i(c+dx)}\right)}{ad^3}+\frac{12if}{}$$

```
[Out] -1/4*I*(f*x+e)^4/a/f+2*(f*x+e)^3*ln(1-I*exp(I*(d*x+c)))/a/d-6*I*f*(f*x+e)^2
*polylog(2,I*exp(I*(d*x+c)))/a/d^2+12*f^2*(f*x+e)*polylog(3,I*exp(I*(d*x+c)
))/a/d^3+12*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4
```

**Rubi** [A]
time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4613, 2221, 2611, 6744, 2320, 6724}

$$\frac{12if^3\text{PolyLog}\left(4,ie^{i(c+dx)}\right)}{ad^4}+\frac{12f^2(e+fx)\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}-\frac{6if(e+fx)^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}+\frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{i(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-1/4*I)*(e + f*x)^4)/(a*f) + (2*(e + f*x)^3*Log[1 - I*E^(I*(c + d*x))])/(
a*d) - ((6*I)*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) + (12*f^
2*(e + f*x)*PolyLog[3, I*E^(I*(c + d*x))])/(a*d^3) + ((12*I)*f^3*PolyLog[4,
 I*E^(I*(c + d*x))])/(a*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4613

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + Dist[2, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - I*b*E^(I*(c + d*x))
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 - b^2,
0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{i(e+fx)^4}{4af}+2\int \frac{e^{i(c+dx)}(e+fx)^3}{a-iae^{i(c+dx)}}\,dx \\
&= -\frac{i(e+fx)^4}{4af}+\frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{(6f)\int(e+fx)^2\log\left(1-ie^{i(c+
\\
&= -\frac{i(e+fx)^4}{4af}+\frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{6if(e+fx)^2\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+ \\
&= -\frac{i(e+fx)^4}{4af}+\frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{6if(e+fx)^2\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+ \\
&= -\frac{i(e+fx)^4}{4af}+\frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{6if(e+fx)^2\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+ \\
&= -\frac{i(e+fx)^4}{4af}+\frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{6if(e+fx)^2\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+
\end{aligned}
$$

**Mathematica [A]**
time = 1.05, size = 275, normalized size = 1.82

$$-\frac{i(4d^4e^3x+6d^4e^2fx^2+4d^4ef^2x^3+d^4f^3x^4+8id^3e^3\log(1-i\cos(c+dx)+\sin(c+dx))+24id^3e^2fx\log(1-i\cos(c+dx)+\sin(c+dx))+24id^3ef^2x^2\log(1-i\cos(c+dx)+\sin(c+dx))+8id^3f^3x^3\log(1-i\cos(c+dx)+\sin(c+dx))+24d^2f(e+fx)^2\mathrm{Li}_2(i\cos(c+dx)-\sin(c+dx))+48idf^2(e+fx)\mathrm{Li}_3(i\cos(c+dx)-\sin(c+dx))-48f^3\mathrm{Li}_4(i\cos(c+dx)-\sin(c+dx)))}{4ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-1/4*I)*(4*d^4*e^3*x + 6*d^4*e^2*f*x^2 + 4*d^4*e*f^2*x^3 + d^4*f^3*x^4 +
(8*I)*d^3*e^3*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] + (24*I)*d^3*e^2*f*x*L
og[1 - I*Cos[c + d*x] + Sin[c + d*x]] + (24*I)*d^3*e*f^2*x^2*Log[1 - I*Cos[
c + d*x] + Sin[c + d*x]] + (8*I)*d^3*f^3*x^3*Log[1 - I*Cos[c + d*x] + Sin[c
 + d*x]] + 24*d^2*f*(e + f*x)^2*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]] +
 (48*I)*d*f^2*(e + f*x)*PolyLog[3, I*Cos[c + d*x] - Sin[c + d*x]] - 48*f^3*
PolyLog[4, I*Cos[c + d*x] - Sin[c + d*x]]))/(a*d^4)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(134) = 268.
time = 0.18, size = 691, normalized size = 4.58

| method | result |
|---|---|
| risch | $\frac{6ief^2c^2x}{d^2a}-\frac{12ief^2\,\mathrm{polylog}(2,ie^{i(dx+c)})x}{d^2a}+\frac{12ef^2\,\mathrm{polylog}(3,ie^{i(dx+c)})}{d^3a}-\frac{if^2ex^3}{a}-\frac{3ife^2x^2}{2a}+\frac{12if^3\,\mathrm{polylog}(4,ie^{i(dx+c)})}{a\,d^4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*f^2/a*e*x^3-3/2*I*f/a*e^2*x^2+2/d/a*ln(exp(I*(d*x+c))+I)*e^3-2/d/a*ln(ex
p(I*(d*x+c)))*e^3-1/4*I*f^3/a*x^4+6/d/a*e*f^2*ln(1-I*exp(I*(d*x+c)))*x^2-6/
d^3/a*e*f^2*ln(1-I*exp(I*(d*x+c)))*c^2-6*I/d/a*e^2*f*c*x+6*I/d^2/a*e*f^2*c^
2*x-12*I/d^2/a*e*f^2*polylog(2,I*exp(I*(d*x+c)))*x+2/d/a*f^3*ln(1-I*exp(I*(
d*x+c)))*x^3+2/d^4/a*f^3*ln(1-I*exp(I*(d*x+c)))*c^3+6/d/a*e^2*f*ln(1-I*exp(
I*(d*x+c)))*x+6/d^2/a*e^2*f*ln(1-I*exp(I*(d*x+c)))*c-6*I/d^2/a*e^2*f*polylo
g(2,I*exp(I*(d*x+c)))-3*I/d^2/a*e^2*f*c^2-2*I/d^3/a*f^3*c^3*x+4*I/d^3/a*e*f
^2*c^3-6*I/d^2/a*f^3*polylog(2,I*exp(I*(d*x+c)))*x^2-6/d^2/a*e^2*f*c*ln(exp
(I*(d*x+c))+I)+6/d^3/a*e*f^2*c^2*ln(exp(I*(d*x+c))+I)-6/d^3/a*e*f^2*c^2*ln(
exp(I*(d*x+c)))+6/d^2/a*e^2*f*c*ln(exp(I*(d*x+c)))+I/a*e^3*x+1/4*I/f/a*e^4+
12/d^3/a*e*f^2*polylog(3,I*exp(I*(d*x+c)))+2/d^4/a*f^3*c^3*ln(exp(I*(d*x+c)
))-2/d^4/a*f^3*c^3*ln(exp(I*(d*x+c))+I)+12/d^3/a*f^3*polylog(3,I*exp(I*(d*x
+c)))*x-3/2*I/d^4/a*f^3*c^4+12*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(126) = 252.
time = 0.35, size = 519, normalized size = 3.44

$$-\frac{\frac{12ce^2f\log(ad\sin(dx+c)+ad)}{ad}-\frac{4e^3\log(a\sin(dx+c)+a)}{a}-\frac{-i(dx+c)^4f^3-4(idef^2-icf^3)(dx+c)^3+48if^3\mathrm{Li}_4(ie^{(idx+ic)})-6(id^2e^2f-2icdef^2+ic^2f^3)(dx+c)^2-4(3ic^2def^2-ic^3f^3)(dx+c)-8(-3ic^2def^2+ic^3f^3)\arctan(\sin(dx+c)+1,\cos(dx+c))-8\left(i(dx+c)^3f^3+3(idef^2-icf^3)(dx+c)^2+3(id^2e^2f-2icdef^2+ic^2f^3)(dx+c)\right)\arctan(\cos(dx+c),\sin(dx+c)+1)-24\left(id^2e^2f-2icdef^2+i(dx+c)^2f^3+ic^2f^3+2(idef^2-icf^3)(dx+c)\right)\mathrm{Li}_2(ie^{(idx+ic)})+4\left(3c^2def^2+(dx+c)^3f^3-c^3f^3+3(def^2-cf^3)(dx+c)^2+3(d^2e^2f-2cdef^2+c^2f^3)(dx+c)\right)\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)+48(def^2+(dx+c)f^3-cf^3)\mathrm{Li}_3(ie^{(idx+ic)})}{ad^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(12*c*e^2*f*log(a*d*sin(d*x + c) + a*d)/(a*d) - 4*e^3*log(a*sin(d*x +
c) + a)/a - (-I*(d*x + c)^4*f^3 - 4*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^3 + 48*
I*f^3*polylog(4, I*e^(I*d*x + I*c)) - 6*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*c^
2*f^3)*(d*x + c)^2 - 4*(3*I*c^2*d*e*f^2 - I*c^3*f^3)*(d*x + c) - 8*(-3*I*c^
2*d*e*f^2 + I*c^3*f^3)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 8*(I*(d*x
+ c)^3*f^3 + 3*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^2 + 3*(I*d^2*e^2*f - 2*I*c*d
*e*f^2 + I*c^2*f^3)*(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 24
*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*(d*x + c)^2*f^3 + I*c^2*f^3 + 2*(I*d*e*f^
2 - I*c*f^3)*(d*x + c))*dilog(I*e^(I*d*x + I*c)) + 4*(3*c^2*d*e*f^2 + (d*x
+ c)^3*f^3 - c^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c
*d*e*f^2 + c^2*f^3)*(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(
d*x + c) + 1) + 48*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*polylog(3, I*e^(I*d*x
+ I*c)))/(a*d^3))/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(130) = 260$.
time = 0.38, size = 492, normalized size = 3.26

$$\frac{6i f^3 \mathrm{polylog}(4, i\cos(dx+c) - \sin(dx+c)) - 6i f^3 \mathrm{polylog}(4, -i\cos(dx+c) - \sin(dx+c)) - 3(i d^2 f^3 x^2 + 2i d^2 f^2 x e + i d^2 f e^2)\mathrm{Li}_2(i\cos(dx+c) - \sin(dx+c)) - 3(-i d^2 f^3 x^2 - 2i d^2 f^2 x e - i d^2 f e^2)\mathrm{Li}_2(-i\cos(dx+c) - \sin(dx+c)) - (c^3 f^3 - 3c^2 d f^2 e + 3c d^2 f e^2 - d^3 e^3)\log(\cos(dx+c) + i\sin(dx+c) + i) + (d^3 f^3 x^3 + c^3 f^3 + 3(d^3 f x + c d^2 f)e^2 + 3(d^3 f^2 x^2 - c^2 d f^2)e)\log(i\cos(dx+c) + \sin(dx+c) + 1) + (d^3 f^3 x^3 + c^3 f^3 + 3(d^3 f x + c d^2 f)e^2 + 3(d^3 f^2 x^2 - c^2 d f^2)e)\log(-i\cos(dx+c) + \sin(dx+c) + 1) - (c^3 f^3 - 3c^2 d f^2 e + 3c d^2 f e^2 - d^3 e^3)\log(-\cos(dx+c) + i\sin(dx+c) + i) + 6(d f^3 x + d f^2 e)\mathrm{polylog}(3, i\cos(dx+c) - \sin(dx+c)) + 6(d f^3 x + d f^2 e)\mathrm{polylog}(3, -i\cos(dx+c) - \sin(dx+c))}{a d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (6*I*f^3*polylog(4, I*cos(d*x + c) - sin(d*x + c)) - 6*I*f^3*polylog(4, -I*
cos(d*x + c) - sin(d*x + c)) - 3*(I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e + I*d^2*f
*e^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2*f^
2*x*e - I*d^2*f*e^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) - (c^3*f^3 - 3*c
^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3)*log(cos(d*x + c) + I*sin(d*x + c) + I
) + (d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c
^2*d*f^2)*e)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + (d^3*f^3*x^3 + c^3*f^
3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*log(-I*cos(d
*x + c) + sin(d*x + c) + 1) - (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^
3*e^3)*log(-cos(d*x + c) + I*sin(d*x + c) + I) + 6*(d*f^3*x + d*f^2*e)*poly
log(3, I*cos(d*x + c) - sin(d*x + c)) + 6*(d*f^3*x + d*f^2*e)*polylog(3, -I
*cos(d*x + c) - sin(d*x + c)))/(a*d^4)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3 \cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3 x^3 \cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2 \cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx \cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*cos(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*cos
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*cos(c + d*x)/(sin
(c + d*x) + 1), x) + Integral(3*e**2*f*x*cos(c + d*x)/(sin(c + d*x) + 1), x
))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e+f\,x\right)}^3}{a+a\,\sin\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e + f*x)^3)/(a + a*sin(c + d*x)), x)
```

# 3.252 $\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=114

$$-\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{4if(e+fx)\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+\frac{4f^2\mathrm{Li}_3\left(ie^{i(c+dx)}\right)}{ad^3}$$

```
[Out] -1/3*I*(f*x+e)^3/a/f+2*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a/d-4*I*f*(f*x+e)*p
olylog(2,I*exp(I*(d*x+c)))/a/d^2+4*f^2*polylog(3,I*exp(I*(d*x+c)))/a/d^3
```

**Rubi** [A]
time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4613, 2221, 2611, 2320, 6724}

$$\frac{4f^2\mathrm{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}-\frac{4if(e+fx)\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{i(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-1/3*I)*(e + f*x)^3)/(a*f) + (2*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(
a*d) - ((4*I)*f*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) + (4*f^2*P
olyLog[3, I*E^(I*(c + d*x))])/(a*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 4613

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + Dist[2, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - I*b*E^(I*(c + d*x))
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 - b^2,
0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{i(e+fx)^3}{3af}+2\int \frac{e^{i(c+dx)}(e+fx)^2}{a-iae^{i(c+dx)}}\,dx\\
&= -\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{(4f)\int(e+fx)\log\left(1-ie^{i(c-}\right.}{ad}\\
&= -\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{4if(e+fx)\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+\\
&= -\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{4if(e+fx)\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+\\
&= -\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{4if(e+fx)\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+
\end{aligned}$$

**Mathematica** [A]
time = 0.61, size = 198, normalized size = 1.74

$$\frac{\frac{x(3e^2+3efx+f^2x^2)\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)}{\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)}+\frac{2\left(3d^2(e+fx)^2\log(1-i\cos(c+dx)+\sin(c+dx))-6idf(e+fx)\mathrm{Li}_2(i\cos(c+dx)-\sin(c+dx))+6f^2\mathrm{Li}_3(i\cos(c+dx)-\sin(c+dx))+\frac{d^3x\left(3e^2+3efx+f^2x^2\right)(-i\cos(c)+\sin(c))}{\cos(c)+i(1+\sin(c))}\right)}{d^3}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] ((x*(3*e^2 + 3*e*f*x + f^2*x^2)*(Cos[c/2] - Sin[c/2]))/(Cos[c/2] + Sin[c/2]
) + (2*(3*d^2*(e + f*x)^2*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] - (6*I)*d*
```

```
f*(e + f*x)*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]] + 6*f^2*PolyLog[3, I*
Cos[c + d*x] - Sin[c + d*x]] + (d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*((-I)*Cos
[c] + Sin[c]))/(Cos[c] + I*(1 + Sin[c]))))/d^3)/(3*a)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(101) = 202$.
time = 0.13, size = 433, normalized size = 3.80

| method | result |
|---|---|
| risch | $\frac{2ic^2f^2x}{d^2a} - \frac{2ief\,c^2}{d^2a} - \frac{4if^2\,\mathrm{polylog}(2,ie^{i(dx+c)})x}{d^2a} - \frac{if^2x^3}{3a} + \frac{2\ln(e^{i(dx+c)}+i)e^2}{da} + \frac{2f^2c^2\ln(e^{i(dx+c)}+i)}{d^3a} + \frac{4efc\ln(e^{i(dx+c)})}{d^2a}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d^2/a*c^2*f^2*x-2*I/d^2/a*e*f*c^2-1/3*I*f^2/a*x^3+1/3*I/f/a*e^3+2/d/a*l
n(exp(I*(d*x+c))+I)*e^2+2/d^3/a*f^2*c^2*ln(exp(I*(d*x+c))+I)+4/d^2/a*e*f*c*
ln(exp(I*(d*x+c)))-2/d^3/a*f^2*c^2*ln(exp(I*(d*x+c)))-2/d/a*ln(exp(I*(d*x+c
)))*e^2+4/3*I/d^3/a*c^3*f^2-4*I/d^2/a*e*f*polylog(2,I*exp(I*(d*x+c)))-4*I/d
/a*e*f*c*x+4*f^2*polylog(3,I*exp(I*(d*x+c)))/a/d^3-4*I/d^2/a*f^2*polylog(2,
I*exp(I*(d*x+c)))*x+I/a*e^2*x-2/d^3/a*f^2*c^2*ln(1-I*exp(I*(d*x+c)))-I*f/a*
e*x^2+2/d/a*f^2*ln(1-I*exp(I*(d*x+c)))*x^2-4/d^2/a*e*f*c*ln(exp(I*(d*x+c))+
I)+4/d/a*e*f*ln(1-I*exp(I*(d*x+c)))*x+4/d^2/a*e*f*ln(1-I*exp(I*(d*x+c)))*c
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(95) = 190$.
time = 0.34, size = 297, normalized size = 2.61

$$-\frac{\frac{6\,cef\log(ad\sin(dx+c)+ad)}{ad} - \frac{3\,e^2\log(a\sin(dx+c)+a)}{a} - \frac{-i\,(dx+c)^3f^2 - 3i\,(dx+c)c^2f^2 + 6i\,c^2f^2\arctan(\sin(dx+c)+1,\cos(dx+c)) - 3\,(i\,def - i\,cf^2)(dx+c)^2 + 12\,f^2\mathrm{Li}_3(i\,e^{(i\,dx+i\,c)}) - 6\left(i\,(dx+c)^2f^2 + 2\,(i\,def - i\,cf^2)(dx+c)\right)\arctan(\cos(dx+c),\sin(dx+c)+1) - 12\,(i\,def + i\,(dx+c)f^2 - i\,cf^2)\mathrm{Li}_2(i\,e^{(i\,dx+i\,c)}) + 3\left((dx+c)^2f^2 + c^2f^2 + 2\,(def - cf^2)(dx+c)\right)\log\left(\cos(dx+c)^2 + \sin(dx+c)^2 + 2\,\sin(dx+c) + 1\right)}{ad^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(6*c*e*f*log(a*d*sin(d*x + c) + a*d)/(a*d) - 3*e^2*log(a*sin(d*x + c)
+ a)/a - (-I*(d*x + c)^3*f^2 - 3*I*(d*x + c)*c^2*f^2 + 6*I*c^2*f^2*arctan2(
sin(d*x + c) + 1, cos(d*x + c)) - 3*(I*d*e*f - I*c*f^2)*(d*x + c)^2 + 12*f^
2*polylog(3, I*e^(I*d*x + I*c)) - 6*(I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I*c*f
^2)*(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 12*(I*d*e*f + I*(d
*x + c)*f^2 - I*c*f^2)*dilog(I*e^(I*d*x + I*c)) + 3*((d*x + c)^2*f^2 + c^2*
f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*
sin(d*x + c) + 1))/(a*d^2))/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(98) = 196$.
time = 0.35, size = 308, normalized size = 2.70

$$\frac{2\,f^2\mathrm{polylog}(3, i\,\cos(dx+c) - \sin(dx+c)) + 2\,f^2\mathrm{polylog}(3, -i\,\cos(dx+c) - \sin(dx+c)) - 2\,(i\,df^2x + i\,dfe)\mathrm{Li}_2(i\,\cos(dx+c) - \sin(dx+c)) - 2\,(-i\,df^2x - i\,dfe)\mathrm{Li}_2(-i\,\cos(dx+c) - \sin(dx+c)) + (c^2f^2 - 2\,cdfe + d^2e^2)\log(\cos(dx+c) + i\,\sin(dx+c) + i) + (d^2f^2x^2 - c^2f^2 + 2\,(d^2fx + cdf)e)\log(i\,\cos(dx+c) + \sin(dx+c) + 1) + (d^2f^2x^2 - c^2f^2 + 2\,(d^2fx + cdf)e)\log(-i\,\cos(dx+c) + \sin(dx+c) + 1) + (c^2f^2 - 2\,cdfe + d^2e^2)\log(-\cos(dx+c) + i\,\sin(dx+c) + i)}{ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*f^2*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 2*f^2*polylog(3, -I*cos(
d*x + c) - sin(d*x + c)) - 2*(I*d*f^2*x + I*d*f*e)*dilog(I*cos(d*x + c) - s
in(d*x + c)) - 2*(-I*d*f^2*x - I*d*f*e)*dilog(-I*cos(d*x + c) - sin(d*x + c
)) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*log(cos(d*x + c) + I*sin(d*x + c) + I)
 + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*log(I*cos(d*x + c) + sin
(d*x + c) + 1) + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*log(-I*cos
(d*x + c) + sin(d*x + c) + 1) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*log(-cos(d*
x + c) + I*sin(d*x + c) + I))/(a*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2 \cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*cos(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*cos
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*cos(c + d*x)/(sin(c + d
*x) + 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,{\left(e+f\,x\right)}^2}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e + f*x)^2)/(a + a*sin(c + d*x)), x)
```

# 3.253 $\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=79

$$-\frac{i(e+fx)^2}{2af}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{2if\text{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}$$

```
[Out] -1/2*I*(f*x+e)^2/a/f+2*(f*x+e)*ln(1-I*exp(I*(d*x+c)))/a/d-2*I*f*polylog(2,I
*exp(I*(d*x+c)))/a/d^2
```

**Rubi** [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4613, 2221, 2317, 2438}

$$-\frac{2if\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{i(e+fx)^2}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-1/2*I)*(e + f*x)^2)/(a*f) + (2*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*
d) - ((2*I)*f*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4613

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
```

```
))), x] + Dist[2, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - I*b*E^(I*(c + d*x))
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 - b^2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{i(e+fx)^2}{2af}+2\int \frac{e^{i(c+dx)}(e+fx)}{a-iae^{i(c+dx)}}\,dx \\
&= -\frac{i(e+fx)^2}{2af}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{(2f)\int \log\left(1-ie^{i(c+dx)}\right)\,dx}{ad} \\
&= -\frac{i(e+fx)^2}{2af}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}+\frac{(2if)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{i
}{ad^2} \\
&= -\frac{i(e+fx)^2}{2af}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{2if\text{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}
\end{aligned}
$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 246 vs. 2(79) = 158.
time = 0.29, size = 246, normalized size = 3.11

$$
\frac{-ic^2f+icf\pi-2icdfx+idf\pi x-id^2fx^2+4f\pi\log\left(1+e^{-i(c+dx)}\right)+4cf\log\left(1-ie^{i(c+dx)}\right)+2f\pi\log\left(1-ie^{i(c+dx)}\right)+4dfx\log\left(1-ie^{i(c+dx)}\right)-4f\pi\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4de\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-4cf\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2f\pi\log\left(\sin\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)-4if\text{Li}_2\left(ie^{i(c+dx)}\right)}{2ad^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I)*c^2*f + I*c*f*Pi - (2*I)*c*d*f*x + I*d*f*Pi*x - I*d^2*f*x^2 + 4*f*Pi*
Log[1 + E^((-I)*(c + d*x))] + 4*c*f*Log[1 - I*E^(I*(c + d*x))] + 2*f*Pi*Log
[1 - I*E^(I*(c + d*x))] + 4*d*f*x*Log[1 - I*E^(I*(c + d*x))] - 4*f*Pi*Log[C
os[(c + d*x)/2]] + 4*d*e*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 4*c*f*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*f*Pi*Log[Sin[(2*c + Pi + 2*d*x)
/4]] - (4*I)*f*PolyLog[2, I*E^(I*(c + d*x))])/(2*a*d^2)
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(69) = 138.
time = 0.15, size = 203, normalized size = 2.57

| method | result |
|---|---|
| risch | $-\frac{ifx^2}{2a}+\frac{iex}{a}+\frac{2\ln\left(e^{i(dx+c)}+i\right)e}{da}-\frac{2\ln\left(e^{i(dx+c)}\right)e}{da}-\frac{2ifcx}{da}-\frac{ifc^2}{d^2a}+\frac{2f\ln\left(1-ie^{i(dx+c)}\right)x}{da}+\frac{2f\ln\left(1-ie^{i(dx+c)}\right)c}{d^2a}-$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/a*f*x^2+I/a*e*x+2/d/a*ln(exp(I*(d*x+c))+I)*e-2/d/a*ln(exp(I*(d*x+c))
)*e-2*I/d/a*f*c*x-I/d^2/a*f*c^2+2/d/a*f*ln(1-I*exp(I*(d*x+c)))*x+2/d^2/a*f*
ln(1-I*exp(I*(d*x+c)))*c-2*I*f*polylog(2,I*exp(I*(d*x+c)))/a/d^2-2/d^2/a*f*
c*ln(exp(I*(d*x+c))+I)+2/d^2/a*f*c*ln(exp(I*(d*x+c)))
```

**Maxima** [A]
time = 0.34, size = 116, normalized size = 1.47

$$\frac{-i\,d^2fx^2-2i\,d^2ex-4i\,dfx\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)+4i\,de\arctan\left(\sin\left(dx+c\right)+1,\cos\left(dx+c\right)\right)-4i\,f\mathrm{Li}_2\left(i\,e^{\left(i\,dx+i\,c\right)}\right)+2\left(dfx+de\right)\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*d^2*f*x^2 - 2*I*d^2*e*x - 4*I*d*f*x*arctan2(cos(d*x + c), sin(d*x +
 c) + 1) + 4*I*d*e*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 4*I*f*dilog(I*
e^(I*d*x + I*c)) + 2*(d*f*x + d*e)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*
sin(d*x + c) + 1))/(a*d^2)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(66) = 132$.
time = 0.36, size = 160, normalized size = 2.03

$$\frac{-i\,f\mathrm{Li}_2(i\,\cos\left(dx+c\right)-\sin\left(dx+c\right))+i\,f\mathrm{Li}_2(-i\,\cos\left(dx+c\right)-\sin\left(dx+c\right))-(cf-de)\log\left(\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)+i\right)+(dfx+cf)\log\left(i\,\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)+(dfx+cf)\log\left(-i\,\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)-(cf-de)\log\left(-\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)+i\right)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-I*f*dilog(I*cos(d*x + c) - sin(d*x + c)) + I*f*dilog(-I*cos(d*x + c) - si
n(d*x + c)) - (c*f - d*e)*log(cos(d*x + c) + I*sin(d*x + c) + I) + (d*f*x +
 c*f)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + (d*f*x + c*f)*log(-I*cos(d*x
 + c) + sin(d*x + c) + 1) - (c*f - d*e)*log(-cos(d*x + c) + I*sin(d*x + c)
+ I))/(a*d^2)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e\cos\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx+\int \frac{fx\cos\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*cos(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f*x*cos(c + d*x)
/(sin(c + d*x) + 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(e+f\,x\right)}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e + f*x))/(a + a*sin(c + d*x)), x)
```

# 3.254 $\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=16

$$\frac{\log(1+\sin(c+dx))}{ad}$$

```
[Out] ln(1+sin(d*x+c))/a/d
```

**Rubi** [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 31}

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{\log(1+\sin(c+dx))}{ad}\end{aligned}$$

**Mathematica** [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{\log(1+\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

**Maple [A]**
time = 0.05, size = 19, normalized size = 1.19

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(a+a\sin(dx+c))}{da}$ | 19 |
| default | $\frac{\ln(a+a\sin(dx+c))}{da}$ | 19 |
| risch | $-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2\ln(e^{i(dx+c)}+i)}{ad}$ | 40 |
| norman | $\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$ | 44 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*ln(a+a*sin(d*x+c))/a
```

**Maxima [A]**
time = 0.28, size = 18, normalized size = 1.12

$$\frac{\log\left(a\sin\left(dx+c\right)+a\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] log(a*sin(d*x + c) + a)/(a*d)
```

**Fricas [A]**
time = 0.36, size = 16, normalized size = 1.00

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] log(sin(d*x + c) + 1)/(a*d)
```

**Sympy [A]**
time = 0.23, size = 24, normalized size = 1.50

$$\begin{cases} \frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad} & \text{for}\ d\neq 0 \\ \frac{x\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d), Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)
, True))
```

**Giac** [A]
time = 4.07, size = 19, normalized size = 1.19

$$\frac{\log\left(|a\sin\left(dx+c\right)+a|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(a*sin(d*x + c) + a))/(a*d)
```

**Mupad** [B]
time = 0.05, size = 16, normalized size = 1.00

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x)),x)
```

```
[Out] log(sin(c + d*x) + 1)/(a*d)
```

# 3.255 $\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=29

$$\operatorname{Int}\left(\frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 2.03, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(c+dx)}{(e+fx)\,(a+a\,\sin(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/((e + f*x)*(a + a*sin(c + d*x))),x)
[Out] int(cos(c + d*x)/((e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.256 $\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=29

$$\text{Int}\left(\frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 2.61, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(fx+e)^2\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2 + 2*a*f*x
*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos\left(c+dx\right)}{e^2\sin\left(c+dx\right)+e^2+2efx\sin\left(c+dx\right)+2efx+f^2x^2\sin\left(c+dx\right)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2*
e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(c+d\,x)}{(e+f\,x)^2\,(a+a\,\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/((e + f*x)^2*(a + a*sin(c + d*x))),x)
[Out] int(cos(c + d*x)/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.257 $\int \frac{(e+fx)^3 \cos^2(c+dx)}{a+a\sin(c+dx)} \, dx$

**Optimal**. Leaf size=99

$$\frac{(e+fx)^4}{4af}-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}+\frac{(e+fx)^3\cos(c+dx)}{ad}+\frac{6f^3\sin(c+dx)}{ad^4}-\frac{3f(e+fx)^2\sin(c+dx)}{ad^2}$$

```
[Out] 1/4*(f*x+e)^4/a/f-6*f^2*(f*x+e)*cos(d*x+c)/a/d^3+(f*x+e)^3*cos(d*x+c)/a/d+6
*f^3*sin(d*x+c)/a/d^4-3*f*(f*x+e)^2*sin(d*x+c)/a/d^2
```

**Rubi** [A]
time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4619, 32, 3377, 2717}

$$\frac{6f^3\sin(c+dx)}{ad^4}-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}-\frac{3f(e+fx)^2\sin(c+dx)}{ad^2}+\frac{(e+fx)^3\cos(c+dx)}{ad}+\frac{(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*a*f) - (6*f^2*(e + f*x)*Cos[c + d*x])/(a*d^3) + ((e + f*x)^3
*Cos[c + d*x])/(a*d) + (6*f^3*Sin[c + d*x])/(a*d^4) - (3*f*(e + f*x)^2*Sin[
c + d*x])/(a*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
```

```
- b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\,dx}{a} - \frac{\int (e+fx)^3\sin(c+dx)\,dx}{a}$$
$$= \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{(3f)\int (e+fx)^2\cos(c+dx)\,dx}{ad}$$
$$= \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{3f(e+fx)^2\sin(c+dx)}{ad^2} + \frac{(6f^2)\int(e+$$
$$= \frac{(e+fx)^4}{4af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{3f(e+fx)^2}{a}$$
$$= \frac{(e+fx)^4}{4af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c+dx)}{ad} + \frac{6f^3\sin(c+}{ad^4}$$

**Mathematica** [**A**]
time = 0.29, size = 102, normalized size = 1.03

$$\frac{d^4x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)+4d(e+fx)\left(-6f^2+d^2(e+fx)^2\right)\cos(c+dx)-12f\left(-2f^2+d^2(e+fx)^2\right)\sin(c+dx)}{4ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + 4*d*(e + f*x)*(-6*f^2
+ d^2*(e + f*x)^2)*Cos[c + d*x] - 12*f*(-2*f^2 + d^2*(e + f*x)^2)*Sin[c + d
*x])/(4*a*d^4)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(97) = 194$.
time = 0.18, size = 436, normalized size = 4.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4/a*(-c^3*f^3*cos(d*x+c)+3*c^2*d*e*f^2*cos(d*x+c)-3*f^3*c^2*(sin(d*x+c)
-(d*x+c)*cos(d*x+c))-3*c*d^2*e^2*f*cos(d*x+c)+6*c*d*e*f^2*(sin(d*x+c)-(d*x+
c)*cos(d*x+c))+3*f^3*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*
x+c))+d^3*e^3*cos(d*x+c)-3*d^2*e^2*f*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-3*d*e*
f^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-f^3*(-(d*x+c)
^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-c^3
*f^3*(d*x+c)+3*c^2*d*e*f^2*(d*x+c)+3/2*f^3*c^2*(d*x+c)^2-3*c*d^2*e^2*f*(d*x
```

```
+c)-3*c*d*e*f^2*(d*x+c)^2-f^3*c*(d*x+c)^3+d^3*e^3*(d*x+c)+3/2*d^2*e^2*f*(d*
x+c)^2+d*e*f^2*(d*x+c)^3+1/4*f^3*(d*x+c)^4)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(97) = 194$.
time = 0.50, size = 534, normalized size = 5.39

$$-\frac{8c^3f^3\left(\frac{1}{ad^3+\frac{ad^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad^3}\right)-24c^2ef^2\left(\frac{1}{ad^2+\frac{ad^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad^2}\right)+24ce^2f\left(\frac{1}{ad+\frac{ad\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right)-8e^3\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)-\frac{6\left((dx+c)^2+2(dx+c)\cos(dx+c)-2\sin(dx+c)\right)e^2f}{ad}+\frac{12\left((dx+c)^2+2(dx+c)\cos(dx+c)-2\sin(dx+c)\right)cef^2}{ad^2}-\frac{6\left((dx+c)^2+2(dx+c)\cos(dx+c)-2\sin(dx+c)\right)c^2f^3}{ad^3}-\frac{4\left((dx+c)^3+3\left((dx+c)^2-2\right)\cos(dx+c)-6(dx+c)\sin(dx+c)\right)ef^2}{ad^2}+\frac{4\left((dx+c)^3+3\left((dx+c)^2-2\right)\cos(dx+c)-6(dx+c)\sin(dx+c)\right)cf^3}{ad^3}-\frac{\left((dx+c)^4+4\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-12\left((dx+c)^2-2\right)\sin(dx+c)\right)f^3}{ad^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*c^3*f^3*(1/(a*d^3 + a*d^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^3)) - 24*c^2*e*f^2*(1/(a*d^2 + a
*d^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/(a*d^2)) + 24*c*e^2*f*(1/(a*d + a*d*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d)) - 8*e^3*(arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)) - 6*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*e^2*f/(
a*d) + 12*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*c*e*f^2
/(a*d^2) - 6*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*c^2*
f^3/(a*d^3) - 4*((d*x + c)^3 + 3*((d*x + c)^2 - 2)*cos(d*x + c) - 6*(d*x +
c)*sin(d*x + c))*e*f^2/(a*d^2) + 4*((d*x + c)^3 + 3*((d*x + c)^2 - 2)*cos(d
*x + c) - 6*(d*x + c)*sin(d*x + c))*c*f^3/(a*d^3) - ((d*x + c)^4 + 4*((d*x
+ c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 12*((d*x + c)^2 - 2)*sin(d*x + c))*f^3
/(a*d^3))/d
```

**Fricas** [A]
time = 0.35, size = 155, normalized size = 1.57

$$\frac{d^4f^3x^4+4\,d^4f^2x^3e+6\,d^4fx^2e^2+4\,d^4xe^3+4\,(d^3f^3x^3+3\,d^3fxe^2-6\,df^3x+d^3e^3+3\,(d^3f^2x^2-2\,df^2)e)\cos(dx+c)-12\,(d^2f^3x^2+2\,d^2f^2xe+d^2fe^2-2\,f^3)\sin(dx+c)}{4\,ad^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^4*f^3*x^4 + 4*d^4*f^2*x^3*e + 6*d^4*f*x^2*e^2 + 4*d^4*x*e^3 + 4*(d^3
*f^3*x^3 + 3*d^3*f*x*e^2 - 6*d*f^3*x + d^3*e^3 + 3*(d^3*f^2*x^2 - 2*d*f^2)*
e)*cos(d*x + c) - 12*(d^2*f^3*x^2 + 2*d^2*f^2*x*e + d^2*f*e^2 - 2*f^3)*sin(
d*x + c))/(a*d^4)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(88) = 176$.
time = 2.84, size = 984, normalized size = 9.94

$$\begin{cases} \frac{4d^4e^3x\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{4d^4e^3x}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{6d^4e^2fx^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{6d^4e^2fx^2}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{4d^4ef^2x^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{4d^4ef^2x^3}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{d^4f^3x^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{d^4f^3x^4}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{8d^3e^3}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}-\frac{12d^3e^2fx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{12d^3e^2fx}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}-\frac{12d^3ef^2x^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{12d^3ef^2x^2}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}-\frac{4d^3f^3x^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{4d^3f^3x^3}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}-\frac{24d^2e^2f\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}-\frac{48d^2ef^2x\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}-\frac{24d^2f^3x^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}-\frac{48def^2}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{24df^3x\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}-\frac{24df^3x}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4}+\frac{48f^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^4} & \text{for}\: d \neq 0 \\ \frac{\left(e^3x+\frac{3e^2fx^2}{2}+ef^2x^3+\frac{f^3x^4}{4}\right)\cos^2(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((4*d**4*e**3*x*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2
+ 4*a*d**4) + 4*d**4*e**3*x/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 6*d
**4*e**2*f*x**2*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**
4) + 6*d**4*e**2*f*x**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 4*d**4*
e*f**2*x**3*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) +
 4*d**4*e*f**2*x**3/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + d**4*f**3*x
**4*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + d**4*f*
*3*x**4/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 8*d**3*e**3/(4*a*d**4*t
an(c/2 + d*x/2)**2 + 4*a*d**4) - 12*d**3*e**2*f*x*tan(c/2 + d*x/2)**2/(4*a*
d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 12*d**3*e**2*f*x/(4*a*d**4*tan(c/2 +
 d*x/2)**2 + 4*a*d**4) - 12*d**3*e*f**2*x**2*tan(c/2 + d*x/2)**2/(4*a*d**4*
tan(c/2 + d*x/2)**2 + 4*a*d**4) + 12*d**3*e*f**2*x**2/(4*a*d**4*tan(c/2 + d
*x/2)**2 + 4*a*d**4) - 4*d**3*f**3*x**3*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c
/2 + d*x/2)**2 + 4*a*d**4) + 4*d**3*f**3*x**3/(4*a*d**4*tan(c/2 + d*x/2)**2
 + 4*a*d**4) - 24*d**2*e**2*f*tan(c/2 + d*x/2)/(4*a*d**4*tan(c/2 + d*x/2)**
2 + 4*a*d**4) - 48*d**2*e*f**2*x*tan(c/2 + d*x/2)/(4*a*d**4*tan(c/2 + d*x/2
)**2 + 4*a*d**4) - 24*d**2*f**3*x**2*tan(c/2 + d*x/2)/(4*a*d**4*tan(c/2 + d
*x/2)**2 + 4*a*d**4) - 48*d*e*f**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4
) + 24*d*f**3*x*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**
4) - 24*d*f**3*x/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 48*f**3*tan(c/
2 + d*x/2)/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4), Ne(d, 0)), ((e**3*x +
 3*e**2*f*x**2/2 + e*f**2*x**3 + f**3*x**4/4)*cos(c)**2/(a*sin(c) + a), Tru
e))
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 24528 vs. $2(101) = 202$.
time = 7.28, size = 24528, normalized size = 247.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(d^4*f^3*x^4*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - d^4*f^3*x^4*tan(1/2
*d*x)^3*tan(1/2*c)^4*tan(c)^2 - d^4*f^3*x^4*tan(1/2*d*x)^2*tan(1/2*c)^5*tan
(c)^2 - 4*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 + d^4*f^3*x^4*
tan(1/2*d*x)^3*tan(1/2*c)^5 + 2*d^4*f^3*x^4*tan(1/2*d*x)^3*tan(1/2*c)^3*tan
(c)^2 - d^4*f^3*x^4*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 4*d^4*f^2*x^3*e*
tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 + d^4*f^3*x^4*tan(1/2*d*x)*tan(1/2*c)^
5*tan(c)^2 + 4*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 + 4*d^3*f
^3*x^3*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - d^4*f^3*x^4*tan(1/2*d*x)^3*ta
n(1/2*c)^4 - d^4*f^3*x^4*tan(1/2*d*x)^2*tan(1/2*c)^5 + 4*d^4*f^2*x^3*e*tan(
1/2*d*x)^3*tan(1/2*c)^5 - 32*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(
c) - 2*d^4*f^3*x^4*tan(1/2*d*x)^3*tan(1/2*c)^2*tan(c)^2 - 2*d^4*f^3*x^4*tan
```

(1/2*d*x)^2*tan(1/2*c)^3*tan(c)^2 + 24*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - d^4*f^3*x^4*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 + 4*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 - 4*d^3*f^3*x^3*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 - d^4*f^3*x^4*tan(1/2*c)^5*tan(c)^2 - 4*d^4*f^2*x^3*e*tan(1/2*d*x)*tan(1/2*c)^5*tan(c)^2 - 4*d^3*f^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 - 6*d^4*f*x^2*e^2*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - 24*d^3*f^2*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 + 2*d^4*f^3*x^4*tan(1/2*d*x)^3*tan(1/2*c)^3 - d^4*f^3*x^4*tan(1/2*d*x)^2*tan(1/2*c)^4 - 4*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^4 + d^4*f^3*x^4*tan(1/2*d*x)*tan(1/2*c)^5 - 4*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*c)^5 + 4*d^3*f^3*x^3*tan(1/2*d*x)^3*tan(1/2*c)^5 + 32*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 32*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c) + d^4*f^3*x^4*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 2*d^4*f^3*x^4*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 24*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^2*tan(c)^2 + 2*d^4*f^3*x^4*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 24*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(c)^2 - d^4*f^3*x^4*tan(1/2*c)^4*tan(c)^2 + 4*d^4*f^2*x^3*e*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 - 20*d^3*f^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 6*d^4*f*x^2*e^2*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 + 4*d^4*f^2*x^3*e*tan(1/2*c)^5*tan(c)^2 - 4*d^3*f^3*x^3*tan(1/2*d*x)*tan(1/2*c)^5*tan(c)^2 + 6*d^4*f*x^2*e^2*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 - 2*d^4*f^3*x^4*tan(1/2*d*x)^3*tan(1/2*c)^2 - 2*d^4*f^3*x^4*tan(1/2*d*x)^2*tan(1/2*c)^3 - 24*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^3 - d^4*f^3*x^4*tan(1/2*d*x)*tan(1/2*c)^4 - 4*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*c)^4 - 4*d^3*f^3*x^3*tan(1/2*d*x)^3*tan(1/2*c)^4 - d^4*f^3*x^4*tan(1/2*c)^5 + 4*d^4*f^2*x^3*e*tan(1/2*d*x)*tan(1/2*c)^5 - 4*d^3*f^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^5 + 6*d^4*f*x^2*e^2*tan(1/2*d*x)^3*tan(1/2*c)^5 + 12*d^3*f^2*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^5 + 32*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^2*tan(c) + 32*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(c) - 32*d^4*f^2*x^3*e*tan(1/2*d*x)*tan(1/2*c)^4*tan(c) - 48*d^4*f*x^2*e^2*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c) - 96*d^3*f^2*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c) + 48*d^3*f^2*x^2*e*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c) - d^4*f^3*x^4*tan(1/2*d*x)^3*tan(c)^2 - d^4*f^3*x^4*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 - 4*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 2*d^4*f^3*x^4*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 - 24*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 2*d^4*f^3*x^4*tan(1/2*c)^3*tan(c)^2 + 24*d^4*f^2*x^3*e*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 16*d^3*f^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(c)^2 + 36*d^4*f*x^2*e^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 24*d^3*f^2*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 4*d^4*f^2*x^3*e*tan(1/2*c)^4*tan(c)^2 + 20*d^3*f^3*x^3*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 + 6*d^4*f*x^2*e^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 - 24*d^3*f^2*x^2*e*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 24*d^2*f^3*x^2*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 + 4*d^3*f^3*x^3*tan(1/2*c)^5*tan(c)^2 - 6*d^4*f*x^2*e^2*tan(1/2*d*x)*tan(1/2*c)^5*tan(c)^2 + 24*d^2*f^3*x^2*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 - 4*d^4*x*e^3*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - 24*d^3*f*x*e^2*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 + d^4*f^3*x^4*tan(1/2*d*x)^3*tan(1/2*c) - 2*d^4*f^3*x^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + 24*

```
d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^2 + 2*d^4*f^3*x^4*tan(1/2*d*x)*tan(
1/2*c)^3 + 24*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*c)^3 - d^4*f^3*x^4*tan(1
/2*c)^4 - 4*d^4*f^2*x^3*e*tan(1/2*d*x)*tan(1/2*c)^4 - 20*d^3*f^3*x^3*tan(1/
2*d*x)^2*tan(1/2*c)^4 - 6*d^4*f*x^2*e^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 12*d^
3*f^2*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^4 - 4*d^4*f^2*x^3*e*tan(1/2*c)^5 - 4*
d^3*f^3*x^3*tan(1/2*d*x)*tan(1/2*c)^5 - 6*d^4*f*x^2*e^2*tan(1/2*d*x)^2*tan(
1/2*c)^5 - 12*d^3*f^2*x^2*e*tan(1/2*d*x)^2*tan(1/2*c)^5 - 32*d^4*f^2*x^3*e*
tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 32*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(c) + 32*d^4*f^2*x^3*e*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 48*d^4*f*
x^2*e^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 32*d^4*f^2*x^3*e*tan(1/2*c)^4*
tan(c) + 48*d^4*f*x^2*e^2*tan(1/2*d*x)^2*tan(1/...
```

**Mupad** [B]
time = 3.02, size = 184, normalized size = 1.86

$$\frac{e^3\,x+\frac{3\,e^2\,f\,x^2}{2}+e\,f^2\,x^3+\frac{f^3\,x^4}{4}}{a}-\frac{d\,\left(6\,x\,\cos\left(c+d\,x\right)\,f^3+6\,e\,\cos\left(c+d\,x\right)\,f^2\right)+d^2\,\left(3\,f^3\,x^2\,\sin\left(c+d\,x\right)+3\,e^2\,f\,\sin\left(c+d\,x\right)+6\,e\,f^2\,x\,\sin\left(c+d\,x\right)\right)-d^3\,\left(e^3\,\cos\left(c+d\,x\right)+f^3\,x^3\,\cos\left(c+d\,x\right)+3\,e^2\,f\,x\,\cos\left(c+d\,x\right)+3\,e\,f^2\,x^2\,\cos\left(c+d\,x\right)\right)-6\,f^3\,\sin\left(c+d\,x\right)}{a\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] (e^3*x + (f^3*x^4)/4 + (3*e^2*f*x^2)/2 + e*f^2*x^3)/a - (d*(6*e*f^2*cos(c +
 d*x) + 6*f^3*x*cos(c + d*x)) + d^2*(3*f^3*x^2*sin(c + d*x) + 3*e^2*f*sin(c
 + d*x) + 6*e*f^2*x*sin(c + d*x)) - d^3*(e^3*cos(c + d*x) + f^3*x^3*cos(c +
 d*x) + 3*e^2*f*x*cos(c + d*x) + 3*e*f^2*x^2*cos(c + d*x)) - 6*f^3*sin(c +
d*x))/(a*d^4)
```

# 3.258 $\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=75

$$\frac{(e+fx)^3}{3af}-\frac{2f^2\cos(c+dx)}{ad^3}+\frac{(e+fx)^2\cos(c+dx)}{ad}-\frac{2f(e+fx)\sin(c+dx)}{ad^2}$$

```
[Out] 1/3*(f*x+e)^3/a/f-2*f^2*cos(d*x+c)/a/d^3+(f*x+e)^2*cos(d*x+c)/a/d-2*f*(f*x+
e)*sin(d*x+c)/a/d^2
```

**Rubi** [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4619, 32, 3377, 2718}

$$-\frac{2f^2\cos(c+dx)}{ad^3}-\frac{2f(e+fx)\sin(c+dx)}{ad^2}+\frac{(e+fx)^2\cos(c+dx)}{ad}+\frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(3*a*f) - (2*f^2*Cos[c + d*x])/(a*d^3) + ((e + f*x)^2*Cos[c + d
*x])/(a*d) - (2*f*(e + f*x)*Sin[c + d*x])/(a*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
```

- b^2, 0]

Rubi steps

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\,dx}{a} - \frac{\int (e+fx)^2\sin(c+dx)\,dx}{a}$$
$$= \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(2f)\int (e+fx)\cos(c+dx)\,dx}{ad}$$
$$= \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{2f(e+fx)\sin(c+dx)}{ad^2} + \frac{(2f^2)\int \sin(c}{ad^2}$$
$$= \frac{(e+fx)^3}{3af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{2f(e+fx)\sin(c+c}{ad^2}$$

**Mathematica [A]**
time = 0.23, size = 74, normalized size = 0.99

$$\frac{d^3x(3e^2+3efx+f^2x^2)+3(-2f^2+d^2(e+fx)^2)\cos(c+dx)-6df(e+fx)\sin(c+dx)}{3ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] (d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(-2*f^2 + d^2*(e + f*x)^2)*Cos[c + d
*x] - 6*d*f*(e + f*x)*Sin[c + d*x])/(3*a*d^3)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(73) = 146.
time = 0.14, size = 215, normalized size = 2.87

| method | result |
|---|---|
| risch | $\frac{f^2x^3}{3a}+\frac{fe\,x^2}{a}+\frac{e^2x}{a}+\frac{e^3}{3af}+\frac{(d^2x^2f^2+2d^2efx+d^2e^2-2f^2)\cos(dx+c)}{a\,d^3}-\frac{2f(fx+e)\sin(dx+c)}{a\,d^2}$ |
| derivativedivides | $-\frac{-c^2f^2\cos(dx+c)+2cdef\cos(dx+c)-2f^2c(\sin(dx+c)-(dx+c)\cos(dx+c))-d^2e^2\cos(dx+c)+2def(\sin(dx+c)-(dx+c)}{}$ |
| default | $-\frac{-c^2f^2\cos(dx+c)+2cdef\cos(dx+c)-2f^2c(\sin(dx+c)-(dx+c)\cos(dx+c))-d^2e^2\cos(dx+c)+2def(\sin(dx+c)-(dx+c)}{}$ |
| norman | $\frac{\frac{2d^2e^2+4def-4f^2}{a\,d^3}+\frac{4fe\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^2a}+\frac{(2d^2e^2-4f^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\,d^3}+\frac{e(de+2f)x}{da}+\frac{f(de+f)x^2}{da}+\frac{(d^2e^2-2def-4f^2)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{a\,d^2}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3/a*(-c^2*f^2*cos(d*x+c)+2*c*d*e*f*cos(d*x+c)-2*f^2*c*(sin(d*x+c)-(d*x
+c)*cos(d*x+c))-d^2*e^2*cos(d*x+c)+2*d*e*f*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+
f^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-c^2*f^2*(d*x+
c)+2*c*d*e*f*(d*x+c)+f^2*c*(d*x+c)^2-d^2*e^2*(d*x+c)-d*e*f*(d*x+c)^2-1/3*f^
2*(d*x+c)^3)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(73) = 146$.
time = 0.51, size = 309, normalized size = 4.12

$$\frac{6\,c^2f^2\left(\frac{1}{ad^2+\frac{ad^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad^2}\right)-12\,cef\left(\frac{1}{ad+\frac{ad\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right)+6\,e^2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)+\frac{3\left((dx+c)^2+2\,(dx+c)\cos(dx+c)-2\,\sin(dx+c)\right)ef}{ad}-\frac{3\left((dx+c)^2+2\,(dx+c)\cos(dx+c)-2\,\sin(dx+c)\right)cf^2}{ad^2}+\frac{\left((dx+c)^3+3\left((dx+c)^2-2\right)\cos(dx+c)-6\,(dx+c)\sin(dx+c)\right)f^2}{ad^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(6*c^2*f^2*(1/(a*d^2 + a*d^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + arc
tan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^2)) - 12*c*e*f*(1/(a*d + a*d*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/
(a*d)) + 6*e^2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)) + 3*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c)
 - 2*sin(d*x + c))*e*f/(a*d) - 3*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) -
2*sin(d*x + c))*c*f^2/(a*d^2) + ((d*x + c)^3 + 3*((d*x + c)^2 - 2)*cos(d*x
+ c) - 6*(d*x + c)*sin(d*x + c))*f^2/(a*d^2))/d
```

**Fricas** [A]
time = 0.33, size = 97, normalized size = 1.29

$$\frac{d^3f^2x^3+3\,d^3fx^2e+3\,d^3xe^2+3\left(d^2f^2x^2+2\,d^2fxe+d^2e^2-2\,f^2\right)\cos\left(dx+c\right)-6\left(df^2x+dfe\right)\sin\left(dx+c\right)}{3\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(d^3*f^2*x^3 + 3*d^3*f*x^2*e + 3*d^3*x*e^2 + 3*(d^2*f^2*x^2 + 2*d^2*f*x
*e + d^2*e^2 - 2*f^2)*cos(d*x + c) - 6*(d*f^2*x + d*f*e)*sin(d*x + c))/(a*d
^3)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(65) = 130$.
time = 2.93, size = 605, normalized size = 8.07

$$\begin{cases} \frac{3d^3e^2x\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}+\frac{3d^3e^2x}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}+\frac{3d^3efx^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}+\frac{3d^3efx^2}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}+\frac{d^3f^2x^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}+\frac{d^3f^2x^3}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}+\frac{6d^2e^2}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}-\frac{6d^2efx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}+\frac{6d^2efx}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}-\frac{3d^2f^2x^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}+\frac{3d^2f^2x^2}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}-\frac{12def\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}-\frac{12df^2x\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3}-\frac{12f^2}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3} & \text{for}\: d \neq 0 \\ \frac{\left(e^2x+efx^2+\frac{f^2x^3}{3}\right)\cos^2\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*d**3*e**2*x*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2
+ 3*a*d**3) + 3*d**3*e**2*x/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + 3*d
**3*e*f*x**2*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3)
+ 3*d**3*e*f*x**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + d**3*f**2*x**
3*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + d**3*f**2
*x**3/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + 6*d**2*e**2/(3*a*d**3*tan
(c/2 + d*x/2)**2 + 3*a*d**3) - 6*d**2*e*f*x*tan(c/2 + d*x/2)**2/(3*a*d**3*t
an(c/2 + d*x/2)**2 + 3*a*d**3) + 6*d**2*e*f*x/(3*a*d**3*tan(c/2 + d*x/2)**2
 + 3*a*d**3) - 3*d**2*f**2*x**2*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x
/2)**2 + 3*a*d**3) + 3*d**2*f**2*x**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d
**3) - 12*d*e*f*tan(c/2 + d*x/2)/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3)
- 12*d*f**2*x*tan(c/2 + d*x/2)/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) -
12*f**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3), Ne(d, 0)), ((e**2*x + e*
f*x**2 + f**2*x**3/3)*cos(c)**2/(a*sin(c) + a), True))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 33978 vs. $2(76) = 152$.
time = 7.83, size = 33978, normalized size = 453.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*(d^3*f^2*x^3*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - d^3*f^2*x^3*tan(1/2
*d*x)^3*tan(1/2*c)^4*tan(c)^2 - d^3*f^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^5*tan
(c)^2 - 3*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 + d^3*f^2*x^3*ta
n(1/2*d*x)^3*tan(1/2*c)^5 + 2*d^3*f^2*x^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c
)^2 - d^3*f^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 3*d^3*f*x^2*e*tan(
1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 + d^3*f^2*x^3*tan(1/2*d*x)*tan(1/2*c)^5*ta
n(c)^2 + 3*d^3*f*x^2*e*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 + 3*d^2*f^2*x^2
*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - d^3*f^2*x^3*tan(1/2*d*x)^3*tan(1/2*
c)^4 - d^3*f^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^5 + 3*d^3*f*x^2*e*tan(1/2*d*x)
^3*tan(1/2*c)^5 - 24*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c) - 2*d^3
*f^2*x^3*tan(1/2*d*x)^3*tan(1/2*c)^2*tan(c)^2 - 2*d^3*f^2*x^3*tan(1/2*d*x)^
2*tan(1/2*c)^3*tan(c)^2 + 18*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)
^2 - d^3*f^2*x^3*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 + 3*d^3*f*x^2*e*tan(1/2
*d*x)^2*tan(1/2*c)^4*tan(c)^2 - 3*d^2*f^2*x^2*tan(1/2*d*x)^3*tan(1/2*c)^4*t
an(c)^2 - d^3*f^2*x^3*tan(1/2*c)^5*tan(c)^2 - 3*d^3*f*x^2*e*tan(1/2*d*x)*ta
n(1/2*c)^5*tan(c)^2 - 3*d^2*f^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 -
3*d^3*x*e^2*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - 12*d^2*f*x*e*tan(1/2*d*x
)^3*tan(1/2*c)^5*tan(c)^2 + 2*d^3*f^2*x^3*tan(1/2*d*x)^3*tan(1/2*c)^3 - d^3
*f^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^4 - 3*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(1/2
*c)^4 + d^3*f^2*x^3*tan(1/2*d*x)*tan(1/2*c)^5 - 3*d^3*f*x^2*e*tan(1/2*d*x)^
2*tan(1/2*c)^5 + 3*d^2*f^2*x^2*tan(1/2*d*x)^3*tan(1/2*c)^5 + 24*d^3*f*x^2*e
```

*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 24*d^3*f*x^2*e*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c) - 12*d^2*f*x*e*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c) - 6*pi*d*f*e*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c) + d^3*f^2*x^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 2*d^3*f^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 18*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^2*tan(c)^2 + 2*d^3*f^2*x^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 18*d^3*f*x^2*e*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(c)^2 - d^3*f^2*x^3*tan(1/2*c)^4*tan(c)^2 + 3*d^3*f*x^2*e*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 - 15*d^2*f^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 3*d^3*x*e^2*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 + 24*d^2*f*x*e*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 + 12*pi*d*f*e*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 + 3*d^3*f*x^2*e*tan(1/2*c)^5*tan(c)^2 - 3*d^2*f^2*x^2*tan(1/2*d*x)*tan(1/2*c)^5*tan(c)^2 + 3*d^3*x*e^2*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 + 6*d*f*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - 2*d^3*f^2*x^3*tan(1/2*d*x)^3*tan(1/2*c)^2 - 2*d^3*f^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^3 - 18*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^3 - d^3*f^2*x^3*tan(1/2*d*x)*tan(1/2*c)^4 - 3*d^3*f*x^2*e*tan(1/2*d*x)^2*tan(1/2*c)^4 - 3*d^2*f^2*x^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - d^3*f^2*x^3*tan(1/2*c)^5 + 3*d^3*f*x^2*e*tan(1/2*d*x)*tan(1/2*c)^5 - 3*d^2*f^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)^5 + 3*d^3*x*e^2*tan(1/2*d*x)^3*tan(1/2*c)^5 + 6*d^2*f*x*e*tan(1/2*d*x)^3*tan(1/2*c)^5 + 24*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^2*tan(c) + 24*d^3*f*x^2*e*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(c) - 24*d^3*f*x^2*e*tan(1/2*d*x)*tan(1/2*c)^4*tan(c) - 24*d^3*x*e^2*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c) - 36*d^2*f*x*e*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c) + 6*pi*d*f*e*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c) + 36*d^2*f*x*e*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c) + 6*pi*d*f*e*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c) + 6*pi*d*f*e*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c) + 12*d*f*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*e*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c) + 6*d^2*e^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c) - d^3*f^2*x^3*tan(1/2*d*x)^3*tan(c)^2 - d^3*f^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 - 3*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 2*d^3*f^2*x^3*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 - 18*d^3*f*x^2*e*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 2*d^3*f^2*x^3*tan(1/2*c)^3*tan(c)^2 + 18*d^3*f*x^2*e*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 12*d^2*f^2*x^2*tan(1/2*d*x)^

```
2*tan(1/2*c)^3*tan(c)^2 + 18*d^3*x*e^2*tan(1/2*...
```

**Mupad [B]**
time = 2.96, size = 110, normalized size = 1.47

$$\frac{e^2\,x+e\,f\,x^2+\frac{f^2\,x^3}{3}}{a}-\frac{2\,f^2\,\cos\left(c+d\,x\right)-d^2\,\left(e^2\,\cos\left(c+d\,x\right)+f^2\,x^2\,\cos\left(c+d\,x\right)+2\,e\,f\,x\,\cos\left(c+d\,x\right)\right)+d\,\left(2\,x\,\sin\left(c+d\,x\right)\,f^2+2\,e\,\sin\left(c+d\,x\right)\,f\right)}{a\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
[Out] (e^2*x + (f^2*x^3)/3 + e*f*x^2)/a - (2*f^2*cos(c + d*x) - d^2*(e^2*cos(c +
d*x) + f^2*x^2*cos(c + d*x) + 2*e*f*x*cos(c + d*x)) + d*(2*f^2*x*sin(c + d*
x) + 2*e*f*sin(c + d*x)))/(a*d^3)
```

# 3.259 $\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=51

$$\frac{ex}{a}+\frac{fx^2}{2a}+\frac{(e+fx)\cos(c+dx)}{ad}-\frac{f\sin(c+dx)}{ad^2}$$

```
[Out] e*x/a+1/2*f*x^2/a+(f*x+e)*cos(d*x+c)/a/d-f*sin(d*x+c)/a/d^2
```

**Rubi** [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {4619, 3377, 2717}

$$-\frac{f\sin(c+dx)}{ad^2}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e*x)/a + (f*x^2)/(2*a) + ((e + f*x)*Cos[c + d*x])/(a*d) - (f*Sin[c + d*x])
/(a*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\,dx}{a}-\frac{\int (e+fx)\sin(c+dx)\,dx}{a} \\
&= \frac{ex}{a}+\frac{fx^2}{2a}+\frac{(e+fx)\cos(c+dx)}{ad}-\frac{f\int \cos(c+dx)\,dx}{ad} \\
&= \frac{ex}{a}+\frac{fx^2}{2a}+\frac{(e+fx)\cos(c+dx)}{ad}-\frac{f\sin(c+dx)}{ad^2}
\end{aligned}
$$

**Mathematica [A]**
time = 0.29, size = 53, normalized size = 1.04

$$
-\frac{(c+dx)(-2de+cf-dfx)-2d(e+fx)\cos(c+dx)+2f\sin(c+dx)}{2ad^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*((c + d*x)*(-2*d*e + c*f - d*f*x) - 2*d*(e + f*x)*Cos[c + d*x] + 2*f*S
in[c + d*x])/(a*d^2)
```

**Maple [A]**
time = 0.11, size = 78, normalized size = 1.53

| method | result |
|---|---|
| risch | $\frac{ex}{a}+\frac{f\,x^2}{2a}+\frac{(fx+e)\cos(dx+c)}{ad}-\frac{f\sin(dx+c)}{a\,d^2}$ |
| derivativedivides | $\frac{-fc\cos(dx+c)+ed\cos(dx+c)-f(\sin(dx+c)-(dx+c)\cos(dx+c))-fc(dx+c)+ed(dx+c)+\frac{f(dx+c)^2}{2}}{d^2a}$ |
| default | $\frac{-fc\cos(dx+c)+ed\cos(dx+c)-f(\sin(dx+c)-(dx+c)\cos(dx+c))-fc(dx+c)+ed(dx+c)+\frac{f(dx+c)^2}{2}}{d^2a}$ |
| norman | $\frac{\frac{2e}{da}+\frac{f\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{f\,x^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(de+f)x}{da}-\frac{2f\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,d^2}+\frac{(2de-2f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\,d^2}+\frac{(de-f)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{da}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2/a*(-f*c*cos(d*x+c)+e*d*cos(d*x+c)-f*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-f
*c*(d*x+c)+e*d*(d*x+c)+1/2*f*(d*x+c)^2)
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(49) = 98.
time = 0.49, size = 151, normalized size = 2.96

$$
-\frac{4\,cf\left(\frac{1}{ad+\frac{ad\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right)-4\,e\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)-\frac{\left((dx+c)^2+2\,(dx+c)\cos(dx+c)-2\,\sin(dx+c)\right)f}{ad}}{2\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(4*c*f*(1/(a*d + a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/(a*d)) - 4*e*(arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)) - ((d*x + c)^2
+ 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*f/(a*d))/d
```

**Fricas** [A]
time = 0.36, size = 51, normalized size = 1.00

$$\frac{d^2 f x^2 + 2\,d^2 x e + 2\,(dfx + de)\cos{(dx + c)} - 2\,f\sin{(dx + c)}}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*f*x^2 + 2*d^2*x*e + 2*(d*f*x + d*e)*cos(d*x + c) - 2*f*sin(d*x + c
))/(a*d^2)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(41) = 82$.
time = 1.62, size = 326, normalized size = 6.39

$$\begin{cases} \frac{2d^2ex\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{2ad^2\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}+2ad^2} + \frac{2d^2ex}{2ad^2\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}+2ad^2} + \frac{d^2fx^2\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{2ad^2\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}+2ad^2} + \frac{d^2fx^2}{2ad^2\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}+2ad^2} + \frac{4de}{2ad^2\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}+2ad^2} - \frac{2dfx\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{2ad^2\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}+2ad^2} + \frac{2dfx}{2ad^2\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}+2ad^2} - \frac{4f\tan{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{2ad^2\tan^2{\left(\frac{c}{2}+\frac{dx}{2}\right)}+2ad^2} & \text{for}\: d \neq 0 \\ \frac{\left(ex+\frac{fx^2}{2}\right)\cos^2{(c)}}{a\sin{(c)}+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*d**2*e*x*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2
*a*d**2) + 2*d**2*e*x/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + d**2*f*x*
*2*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + d**2*f*x
**2/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + 4*d*e/(2*a*d**2*tan(c/2 + d
*x/2)**2 + 2*a*d**2) - 2*d*f*x*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/
2)**2 + 2*a*d**2) + 2*d*f*x/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) - 4*f
*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2), Ne(d, 0)), ((e
*x + f*x**2/2)*cos(c)**2/(a*sin(c) + a), True))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 13408 vs. $2(51) = 102$.
time = 4.41, size = 13408, normalized size = 262.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*(d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 - d^2*f*x^2*tan(1/2*d*x
)^3*tan(1/2*c)^4*tan(c)^2 - d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2
- 2*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c)^2 + d^2*f*x^2*tan(1/2*d*x)^3
*tan(1/2*c)^5 + 2*d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - d^2*f*x^
2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 2*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*c
)^4*tan(c)^2 + d^2*f*x^2*tan(1/2*d*x)*tan(1/2*c)^5*tan(c)^2 + 2*d^2*x*e*tan
(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 + 2*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^5*tan
(c)^2 - d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - d^2*f*x^2*tan(1/2*d*x)^2*ta
n(1/2*c)^5 + 2*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*c)^5 - 16*d^2*x*e*tan(1/2*d*x
)^3*tan(1/2*c)^4*tan(c) + 4*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^5*tan(c) - 2*d^2*f*x^2*tan(1/2*
d*x)^3*tan(1/2*c)^2*tan(c)^2 - 2*d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(
c)^2 + 12*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - d^2*f*x^2*tan(1/2*
d*x)*tan(1/2*c)^4*tan(c)^2 + 2*d^2*x*e*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2
 - 2*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 - 8*d*e*log(2*(tan(1/2*d*x)
^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)
^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c
)^2 - d^2*f*x^2*tan(1/2*c)^5*tan(c)^2 - 2*d^2*x*e*tan(1/2*d*x)*tan(1/2*c)^5
*tan(c)^2 - 2*d*f*x*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 - 4*d*e*tan(1/2*d*
x)^3*tan(1/2*c)^5*tan(c)^2 + 2*d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - d^2*
f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^4 - 2*d^2*x*e*tan(1/2*d*x)^3*tan(1/2*c)^4 +
 d^2*f*x^2*tan(1/2*d*x)*tan(1/2*c)^5 - 2*d^2*x*e*tan(1/2*d*x)^2*tan(1/2*c)^
5 + 2*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^5 + 16*d^2*x*e*tan(1/2*d*x)^3*tan(1/2
*c)^3*tan(c) + 16*d^2*x*e*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c) - 4*d*e*log(2*
(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)
^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1
/2*c)^4*tan(c) - 4*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c) + d^2*f*x^2*tan(1/2*d*x)^3*tan(
1/2*c)*tan(c)^2 - 2*d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 12*d^2
*x*e*tan(1/2*d*x)^3*tan(1/2*c)^2*tan(c)^2 + 2*d^2*f*x^2*tan(1/2*d*x)*tan(1/
2*c)^3*tan(c)^2 - 12*d^2*x*e*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(c)^2 + 8*d*e*l
og(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2
*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
```

```
an(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*
tan(1/2*c)^3*tan(c)^2 - d^2*f*x^2*tan(1/2*c)^4*tan(c)^2 + 2*d^2*x*e*tan(1/2
*d*x)*tan(1/2*c)^4*tan(c)^2 - 10*d*f*x*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2
 + 8*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 2*d^2*x*e*tan(1/2*c)^5*tan(c)^2 - 2*d*f*x
*tan(1/2*d*x)*tan(1/2*c)^5*tan(c)^2 - 2*d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)
^2 - 2*d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^3 - 12*d^2*x*e*tan(1/2*d*x)^3*ta
n(1/2*c)^3 - d^2*f*x^2*tan(1/2*d*x)*tan(1/2*c)^4 - 2*d^2*x*e*tan(1/2*d*x)^2
*tan(1/2*c)^4 - 2*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^4 + 8*d*e*log(2*(tan(1/2*
d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
- 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^4 -
 d^2*f*x^2*tan(1/2*c)^5 + 2*d^2*x*e*tan(1/2*d*x)*tan(1/2*c)^5 - 2*d*f*x*tan
(1/2*d*x)^2*tan(1/2*c)^5 + 2*d*e*tan(1/2*d*x)^3*tan(1/2*c)^5 + 16*d^2*x*e*t
an(1/2*d*x)^3*tan(1/2*c)^2*tan(c) + 16*d^2*x*e*tan(1/2*d*x)^2*tan(1/2*c)^3*
tan(c) - 24*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1
/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/...
```

**Mupad [B]**
time = 2.94, size = 53, normalized size = 1.04

$$\frac{\frac{f\,x^2}{2}+e\,x}{a}-\frac{f\,\sin\left(c+d\,x\right)-d\,\left(e\,\cos\left(c+d\,x\right)+f\,x\,\cos\left(c+d\,x\right)\right)}{a\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(e + f*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] (e*x + (f*x^2)/2)/a - (f*sin(c + d*x) - d*(e*cos(c + d*x) + f*x*cos(c + d*x
)))/(a*d^2)
```

# 3.260 $\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=19

$$\frac{x}{a}+\frac{\cos(c+dx)}{ad}$$

```
[Out] x/a+cos(d*x+c)/a/d
```

**Rubi** [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2761, 8}

$$\frac{\cos(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cos(c+dx)}{ad}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}+\frac{\cos(c+dx)}{ad}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(19) = 38.
time = 0.10, size = 97, normalized size = 5.11

$$-\frac{\cos^3(c+dx)\left(2\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+(-1+\sin(c+dx))\sqrt{1+\sin(c+dx)}\right)}{ad(-1+\sin(c+dx))^2(1+\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^3*(2*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + (-1 + Sin[c + d*x])*Sqrt[1 + Sin[c + d*x]]))/(a*d*(-1 + Sin[c + d
*x])^2*(1 + Sin[c + d*x])^(3/2)))
```

**Maple** [A]
time = 0.10, size = 35, normalized size = 1.84

| method | result |
|---|---|
| risch | $\frac{x}{a}+\frac{\cos(dx+c)}{ad}$ |
| derivativedivides | $\frac{\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ |
| default | $\frac{\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$ |
| norman | $\frac{\frac{x}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2(\mathrm{ta}}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/(1+tan(1/2*d*x+1/2*c)^2)+arctan(tan(1/2*d*x+1/2*c)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.
time = 0.48, size = 52, normalized size = 2.74

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2))/d
```

**Fricas** [A]
time = 0.38, size = 17, normalized size = 0.89

$$\frac{dx+\cos(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x + cos(d*x + c))/(a*d)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 1.17, size = 88, normalized size = 4.63

$$\begin{cases} \frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + d*x/(a
*d*tan(c/2 + d*x/2)**2 + a*d) + 2/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)
), (x*cos(c)**2/(a*sin(c) + a), True))
```

**Giac** [A]
time = 5.11, size = 34, normalized size = 1.79

$$\frac{\frac{dx+c}{a} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a + 2/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

**Mupad** [B]
time = 2.77, size = 29, normalized size = 1.53

$$\frac{x}{a} + \frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

```
[Out] x/a + 2/(a*d*(tan(c/2 + (d*x)/2)^2 + 1))
```

# 3.261 $\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))} \, dx$

**Optimal**. Leaf size=72

$$\frac{\log(e+fx)}{af} - \frac{\text{Ci}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af} - \frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}$$

```
[Out] ln(f*x+e)/a/f-cos(c-d*e/f)*Si(d*e/f+d*x)/a/f-Ci(d*e/f+d*x)*sin(c-d*e/f)/a/f
```

**Rubi** [A]
time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4619, 31, 3384, 3380, 3383}

$$-\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af} - \frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af} + \frac{\log(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Log[e + f*x]/(a*f) - (CosIntegral[(d*e)/f + d*x]*Sin[c - (d*e)/f])/(a*f) -
(Cos[c - (d*e)/f]*SinIntegral[(d*e)/f + d*x])/(a*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx &= \frac{\int \frac{1}{e+fx}\,dx}{a} - \frac{\int \frac{\sin(c+dx)}{e+fx}\,dx}{a} \\
&= \frac{\log(e+fx)}{af} - \frac{\cos\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a} - \frac{\sin\left(c-\frac{de}{f}\right)\int \frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}\,d}{a} \\
&= \frac{\log(e+fx)}{af} - \frac{\operatorname{Ci}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af} - \frac{\cos\left(c-\frac{de}{f}\right)\operatorname{Si}\left(\frac{de}{f}+dx\right)}{af}
\end{aligned}$$

**Mathematica [A]**
time = 0.16, size = 58, normalized size = 0.81

$$\frac{\log(e+fx)-\operatorname{Ci}\left(d\left(\frac{e}{f}+x\right)\right)\sin\left(c-\frac{de}{f}\right)-\cos\left(c-\frac{de}{f}\right)\operatorname{Si}\left(d\left(\frac{e}{f}+x\right)\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Log[e + f*x] - CosIntegral[d*(e/f + x)]*Sin[c - (d*e)/f] - Cos[c - (d*e)/f
]*SinIntegral[d*(e/f + x)])/(a*f)
```

**Maple [A]**
time = 0.14, size = 104, normalized size = 1.44

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{-\frac{\mathrm{sinIntegral}\left(-dx-c-\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)}{f}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}-\frac{\ln(-cf+de+f(dx+c))}{f}}{a}$ | 104 |
| default | $-\frac{-\frac{\mathrm{sinIntegral}\left(-dx-c-\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)}{f}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}-\frac{\ln(-cf+de+f(dx+c))}{f}}{a}$ | 104 |
| risch | $\frac{\ln(fx+e)}{af}-\frac{i\,e^{\frac{i(cf-de)}{f}}\,\mathrm{expIntegral}\left(1,-idx-ic-\frac{-icf+ide}{f}\right)}{2af}+\frac{i\,e^{-\frac{i(cf-de)}{f}}\,\mathrm{expIntegral}\left(1,idx+ic-\frac{i(cf-de)}{f}\right)}{2af}$ | 117 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(-Si(-d*x-c-(-c*f+d*e)/f)*cos((-c*f+d*e)/f)/f-Ci(d*x+c+(-c*f+d*e)/f)*s
in((-c*f+d*e)/f)/f-ln(-c*f+d*e+f*(d*x+c))/f)
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.33, size = 163, normalized size = 2.26

$$\frac{d\left(i\,E_1\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)-i\,E_1\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)+d\left(E_1\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_1\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\sin\left(-\frac{de-cf}{f}\right)+2\,d\log\left(de+(dx+c)f-cf\right)}{2\,adf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(d*(I*exp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) - I*exp_inte
gral_e(1, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) + d*(exp
_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_integral_e(1, -(I*d
*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) + 2*d*log(d*e + (d*x +
c)*f - c*f))/(a*d*f)
```

**Fricas** [A]
time = 0.33, size = 95, normalized size = 1.32

$$\frac{\left(\mathrm{Ci}\left(\frac{dfx+de}{f}\right)+\mathrm{Ci}\left(-\frac{dfx+de}{f}\right)\right)\sin\left(-\frac{cf-de}{f}\right)-2\,\cos\left(-\frac{cf-de}{f}\right)\mathrm{Si}\left(\frac{dfx+de}{f}\right)+2\,\log\left(fx+e\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral((d*f*x + d*e)/f) + cos_integral(-(d*f*x + d*e)/f))*sin(-
(c*f - d*e)/f) - 2*cos(-(c*f - d*e)/f)*sin_integral((d*f*x + d*e)/f) + 2*lo
g(f*x + e))/(a*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2\left(c+dx\right)}{e\sin\left(c+dx\right)+e+fx\sin\left(c+dx\right)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.52, size = 716, normalized size = 9.94

$$-\frac{\Im\left(\mathrm{Ci}\left(dx+\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{de}{2f}\right)^2-\Im\left(\mathrm{Ci}\left(-dx-\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{de}{2f}\right)^2-2\log\left(|fx+e|\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{de}{2f}\right)^2+2\,\mathrm{Si}\left(\frac{dfx+de}{f}\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{de}{2f}\right)^2+2\Re\left(\mathrm{Ci}\left(dx+\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{de}{2f}\right)+2\Re\left(\mathrm{Ci}\left(-dx-\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{de}{2f}\right)-2\Re\left(\mathrm{Ci}\left(dx+\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{de}{2f}\right)^2-2\Re\left(\mathrm{Ci}\left(-dx-\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{de}{2f}\right)^2-\Im\left(\mathrm{Ci}\left(dx+\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)^2+\Im\left(\mathrm{Ci}\left(-dx-\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)^2-2\log\left(|fx+e|\right)\tan\left(\frac{1}{2}c\right)^2-2\,\mathrm{Si}\left(\frac{dfx+de}{f}\right)\tan\left(\frac{1}{2}c\right)^2+4\Im\left(\mathrm{Ci}\left(dx+\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{de}{2f}\right)-4\Im\left(\mathrm{Ci}\left(-dx-\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{de}{2f}\right)+8\,\mathrm{Si}\left(\frac{dfx+de}{f}\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{de}{2f}\right)-\Im\left(\mathrm{Ci}\left(dx+\frac{de}{f}\right)\right)\tan\left(\frac{de}{2f}\right)^2+\Im\left(\mathrm{Ci}\left(-dx-\frac{de}{f}\right)\right)\tan\left(\frac{de}{2f}\right)^2-2\log\left(|fx+e|\right)\tan\left(\frac{de}{2f}\right)^2-2\,\mathrm{Si}\left(\frac{dfx+de}{f}\right)\tan\left(\frac{de}{2f}\right)^2+2\Re\left(\mathrm{Ci}\left(dx+\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)+2\Re\left(\mathrm{Ci}\left(-dx-\frac{de}{f}\right)\right)\tan\left(\frac{1}{2}c\right)-2\Re\left(\mathrm{Ci}\left(dx+\frac{de}{f}\right)\right)\tan\left(\frac{de}{2f}\right)-2\Re\left(\mathrm{Ci}\left(-dx-\frac{de}{f}\right)\right)\tan\left(\frac{de}{2f}\right)+\Im\left(\mathrm{Ci}\left(dx+\frac{de}{f}\right)\right)-\Im\left(\mathrm{Ci}\left(-dx-\frac{de}{f}\right)\right)-2\log\left(|fx+e|\right)+2\,\mathrm{Si}\left(\frac{dfx+de}{f}\right)}{2\left(af\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{de}{2f}\right)^2+af\tan\left(\frac{1}{2}c\right)^2+af\tan\left(\frac{de}{2f}\right)^2+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 -
imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 - 2*log
(abs(f*x + e))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + 2*sin_integral((d*f*x + d*e)
/f)*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + 2*real_part(cos_integral(d*x + d*e/f))*
tan(1/2*c)^2*tan(1/2*d*e/f) + 2*real_part(cos_integral(-d*x - d*e/f))*tan(1
/2*c)^2*tan(1/2*d*e/f) - 2*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)*
tan(1/2*d*e/f)^2 - 2*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)*tan(1
/2*d*e/f)^2 - imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2 + imag_part
(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2 - 2*log(abs(f*x + e))*tan(1/2*c)^
2 - 2*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)^2 + 4*imag_part(cos_integral
(d*x + d*e/f))*tan(1/2*c)*tan(1/2*d*e/f) - 4*imag_part(cos_integral(-d*x -
d*e/f))*tan(1/2*c)*tan(1/2*d*e/f) + 8*sin_integral((d*f*x + d*e)/f)*tan(1/2
*c)*tan(1/2*d*e/f) - imag_part(cos_integral(d*x + d*e/f))*tan(1/2*d*e/f)^2
+ imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*e/f)^2 - 2*log(abs(f*x +
e))*tan(1/2*d*e/f)^2 - 2*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*e/f)^2 + 2
*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c) + 2*real_part(cos_integral
(-d*x - d*e/f))*tan(1/2*c) - 2*real_part(cos_integral(d*x + d*e/f))*tan(1/2
*d*e/f) - 2*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*e/f) + imag_par
t(cos_integral(d*x + d*e/f)) - imag_part(cos_integral(-d*x - d*e/f)) - 2*lo
g(abs(f*x + e)) + 2*sin_integral((d*f*x + d*e)/f))/(a*f*tan(1/2*c)^2*tan(1/
2*d*e/f)^2 + a*f*tan(1/2*c)^2 + a*f*tan(1/2*d*e/f)^2 + a*f)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/((e + f*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)^2/((e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.262 $\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=95

$$-\frac{1}{af(e+fx)}-\frac{d\cos\left(c-\frac{de}{f}\right)\operatorname{Ci}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{\sin(c+dx)}{af(e+fx)}+\frac{d\sin\left(c-\frac{de}{f}\right)\operatorname{Si}\left(\frac{de}{f}+dx\right)}{af^2}$$

```
[Out] -1/a/f/(f*x+e)-d*Ci(d*e/f+d*x)*cos(c-d*e/f)/a/f^2+d*Si(d*e/f+d*x)*sin(c-d*e
/f)/a/f^2+sin(d*x+c)/a/f/(f*x+e)
```

**Rubi** [A]
time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4619, 32, 3378, 3384, 3380, 3383}

$$-\frac{d\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{d\sin\left(c-\frac{de}{f}\right)\operatorname{Si}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{\sin(c+dx)}{af(e+fx)}-\frac{1}{af(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] -(1/(a*f*(e + f*x))) - (d*Cos[c - (d*e)/f]*CosIntegral[(d*e)/f + d*x])/(a*f
^2) + Sin[c + d*x]/(a*f*(e + f*x)) + (d*Sin[c - (d*e)/f]*SinIntegral[(d*e)/
f + d*x])/(a*f^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx &= \frac{\int \frac{1}{(e+fx)^2}\,dx}{a}-\frac{\int \frac{\sin(c+dx)}{(e+fx)^2}\,dx}{a}\\ &=-\frac{1}{af(e+fx)}+\frac{\sin(c+dx)}{af(e+fx)}-\frac{d\int \frac{\cos(c+dx)}{e+fx}\,dx}{af}\\ &=-\frac{1}{af(e+fx)}+\frac{\sin(c+dx)}{af(e+fx)}-\frac{\left(d\cos\left(c-\frac{de}{f}\right)\right)\int \frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{af}+\frac{\left(d\,\mathrm{s}\right.}{}\\ &=-\frac{1}{af(e+fx)}-\frac{d\cos\left(c-\frac{de}{f}\right)\mathrm{Ci}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{\sin(c+dx)}{af(e+fx)}+\frac{d\sin\left(c-\right.}{}\end{aligned}$$

**Mathematica [A]**
time = 0.23, size = 80, normalized size = 0.84

$$\frac{-d(e+fx)\cos\left(c-\frac{de}{f}\right)\mathrm{Ci}\left(d\left(\frac{e}{f}+x\right)\right)+f(-1+\sin(c+dx))+d(e+fx)\sin\left(c-\frac{de}{f}\right)\mathrm{Si}\left(d\left(\frac{e}{f}+x\right)\right)}{af^2(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] (-(d*(e + f*x)*Cos[c - (d*e)/f]*CosIntegral[d*(e/f + x)]) + f*(-1 + Sin[c +
 d*x]) + d*(e + f*x)*Sin[c - (d*e)/f]*SinIntegral[d*(e/f + x)])/(a*f^2*(e +
 f*x))
```

**Maple [A]**
time = 0.20, size = 137, normalized size = 1.44

| method | result |
|---|---|
| derivativedivides | $\frac{d\left(\frac{\sin(dx+c)}{(-cf+de+f(dx+c))f}-\frac{-\frac{\text{sinIntegral}\left(-dx-c-\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}+\frac{\text{cosineIntegral}\left(dx+c+\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)}{f}}{f}-\frac{1}{(-c\ldots}\right)}{a}$ |
| default | $\frac{d\left(\frac{\sin(dx+c)}{(-cf+de+f(dx+c))f}-\frac{-\frac{\text{sinIntegral}\left(-dx-c-\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}+\frac{\text{cosineIntegral}\left(dx+c+\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)}{f}}{f}-\frac{1}{(-c\ldots}\right)}{a}$ |
| risch | $-\frac{1}{af(fx+e)}+\frac{d\,e^{\frac{i(cf-de)}{f}}\text{expIntegral}\left(1,-idx-ic-\frac{-icf+ide}{f}\right)}{2a\,f^2}+\frac{d\,e^{-\frac{i(cf-de)}{f}}\text{expIntegral}\left(1,idx+ic-\frac{i(cf-de)}{f}\right)}{2a\,f^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] d/a*(sin(d*x+c)/(-c*f+d*e+f*(d*x+c))/f-(-Si(-d*x-c-(-c*f+d*e)/f)*sin((-c*f+
d*e)/f)/f+Ci(d*x+c+(-c*f+d*e)/f)*cos((-c*f+d*e)/f)/f)/f-1/(-c*f+d*e+f*(d*x+
c))/f)
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.36, size = 172, normalized size = 1.81

$$\frac{d^2\left(i\,E_2\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)-i\,E_2\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)+d^2\left(E_2\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_2\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\sin\left(-\frac{de-cf}{f}\right)-2\,d^2}{2\,(adef+(dx+c)af^2-acf^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(d^2*(I*exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) - I*exp_in
tegral_e(2, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) + d^2*
(exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_integral_e(2, -
(I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) - 2*d^2)/((a*d*e*f
+ (d*x + c)*a*f^2 - a*c*f^2)*d)
```

**Fricas [A]**
time = 0.35, size = 137, normalized size = 1.44

$$-\frac{2\,(dfx+de)\sin\left(-\frac{cf-de}{f}\right)\text{Si}\left(\frac{dfx+de}{f}\right)+\left((dfx+de)\,\text{Ci}\left(\frac{dfx+de}{f}\right)+(dfx+de)\,\text{Ci}\left(-\frac{dfx+de}{f}\right)\right)\cos\left(-\frac{cf-de}{f}\right)-2\,f\sin\left(dx+c\right)+2\,f}{2\,(af^3x+af^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(d*f*x + d*e)*sin(-(c*f - d*e)/f)*sin_integral((d*f*x + d*e)/f) + (
(d*f*x + d*e)*cos_integral((d*f*x + d*e)/f) + (d*f*x + d*e)*cos_integral(-(
```

```
d*f*x + d*e)/f))*cos(-(c*f - d*e)/f) - 2*f*sin(d*x + c) + 2*f)/(a*f^3*x + a
*f^2*e)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.33, size = 3408, normalized size = 35.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(d*f*x*real_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*d*e/f)^2 + d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(1/2*d*e/f)^2 - 2*d*f*x*imag_part(cos_integral(d*x + d
*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) + 2*d*f*x*imag_part(cos_i
ntegral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) - 4*d*f*x
*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) +
 2*d*f*x*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)*tan
(1/2*d*e/f)^2 - 2*d*f*x*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^
2*tan(1/2*c)*tan(1/2*d*e/f)^2 + 4*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f)^2 + d*e*real_part(cos_integral(d*x + d*
e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + d*e*real_part(cos_inte
gral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f)^2 - d*f*x*re
al_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2 - d*f*x*real
_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*d*f*x*rea
l_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f)
+ 4*d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*d*e/f) - 2*d*e*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*d*e/f) + 2*d*e*imag_part(cos_integral(-d*x - d*e/f))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) - 4*d*e*sin_integral((d*f*x + d*e)
/f)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) - d*f*x*real_part(cos_integr
al(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*d*e/f)^2 - d*f*x*real_part(cos_inte
gral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*d*e/f)^2 + 2*d*e*imag_part(cos_i
ntegral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f)^2 - 2*d*e*im
```

ag_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f)^2 + 4*d*e*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f)^2 + d*f*x*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 - 2*d*f*x*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c) + 2*d*f*x*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c) - 4*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*x)^2*tan(1/2*c) - d*e*real_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2 - d*e*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*d*f*x*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*d*e/f) - 2*d*f*x*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*d*e/f) + 4*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*x)^2*tan(1/2*d*e/f) + 4*d*e*real_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f) + 4*d*e*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f) - 2*d*f*x*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f) + 2*d*f*x*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f) - 4*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)^2*tan(1/2*d*e/f) - d*e*real_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*d*e/f)^2 - d*e*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*d*e/f)^2 + 2*d*f*x*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)*tan(1/2*d*e/f)^2 - 2*d*f*x*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)*tan(1/2*d*e/f)^2 + 4*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)*tan(1/2*d*e/f)^2 + d*e*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + d*e*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + 2*f*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + d*f*x*real_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2 + d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2 - 2*d*e*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c) + 2*d*e*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c) - 4*d*e*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*x)^2*tan(1/2*c) - d*f*x*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2 - d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2 + 2*d*e*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*d*e/f) - 2*d*e*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*d*e/f) + 4*d*e*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*x)^2*tan(1/2*d*e/f) + 4*d*f*x*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)*tan(1/2*d*e/f) + 4*d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)*tan(1/2*d*e/f) - 2*d*e*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f) + 2*d*e*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f) - 4*d*e*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)^2*tan(1/2*d*e/f) - d*f*x*real_part(cos_integral(d*x + d*e/f))*tan(1/2*d*e/f)^2 - d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*e/f)^2 + 2*d*e*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)*tan(1/2*d*e/f)^2 - 2*d*e*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)*tan(1/2*d*e/f)^2 + 4*d*e*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)*tan(1/2*d*e/f)^2 + 4*f*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f)^2 + 4*f*tan...

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{(e+f\,x)^2\,\left(a+a\,\sin(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/((e + f*x)^2*(a + a*sin(c + d*x))),x)
[Out] int(cos(c + d*x)^2/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.263 $\int \frac{(e+fx)^3 \cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=219

$$-\frac{3f^3x}{8ad^3}+\frac{(e+fx)^3}{4ad}-\frac{6f^3\cos(c+dx)}{ad^4}+\frac{3f(e+fx)^2\cos(c+dx)}{ad^2}-\frac{6f^2(e+fx)\sin(c+dx)}{ad^3}+\frac{(e+fx)^3\sin(c+}{ad}$$

```
[Out] -3/8*f^3*x/a/d^3+1/4*(f*x+e)^3/a/d-6*f^3*cos(d*x+c)/a/d^4+3*f*(f*x+e)^2*cos
(d*x+c)/a/d^2-6*f^2*(f*x+e)*sin(d*x+c)/a/d^3+(f*x+e)^3*sin(d*x+c)/a/d+3/8*f
^3*cos(d*x+c)*sin(d*x+c)/a/d^4-3/4*f*(f*x+e)^2*cos(d*x+c)*sin(d*x+c)/a/d^2+
3/4*f^2*(f*x+e)*sin(d*x+c)^2/a/d^3-1/2*(f*x+e)^3*sin(d*x+c)^2/a/d
```

**Rubi** [A]

time = 0.15, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4619, 3377, 2718, 4489, 3392, 32, 2715, 8}

$$-\frac{6f^3\cos(c+dx)}{ad^4}+\frac{3f^3\sin(c+dx)\cos(c+dx)}{8ad^4}+\frac{3f^2(e+fx)\sin^2(c+dx)}{4ad^3}-\frac{6f^2(e+fx)\sin(c+dx)}{ad^3}+\frac{3f(e+fx)^2\cos(c+dx)}{ad^2}-\frac{3f(e+fx)^2\sin(c+dx)\cos(c+dx)}{4ad^2}-\frac{(e+fx)^3\sin^2(c+dx)}{2ad}+\frac{(e+fx)^3\sin(c+dx)}{ad}-\frac{3f^3x}{8ad^3}+\frac{(e+fx)^3}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*f^3*x)/(8*a*d^3) + (e + f*x)^3/(4*a*d) - (6*f^3*Cos[c + d*x])/(a*d^4) +
 (3*f*(e + f*x)^2*Cos[c + d*x])/(a*d^2) - (6*f^2*(e + f*x)*Sin[c + d*x])/(a
*d^3) + ((e + f*x)^3*Sin[c + d*x])/(a*d) + (3*f^3*Cos[c + d*x]*Sin[c + d*x]
)/(8*a*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^2) + (3*f^
2*(e + f*x)*Sin[c + d*x]^2)/(4*a*d^3) - ((e + f*x)^3*Sin[c + d*x]^2)/(2*a*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^3\cos(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)\,dx}{a} \\
&= \frac{(e+fx)^3\sin(c+dx)}{ad} - \frac{(e+fx)^3\sin^2(c+dx)}{2ad} + \frac{(3f)\int (e+fx)^2\sin^2(c+ \cdots}{2ad} \\
&= \frac{3f(e+fx)^2\cos(c+dx)}{ad^2} + \frac{(e+fx)^3\sin(c+dx)}{ad} - \frac{3f(e+fx)^2\cos(c+dx \cdots}{4ad^2} \\
&= \frac{(e+fx)^3}{4ad} + \frac{3f(e+fx)^2\cos(c+dx)}{ad^2} - \frac{6f^2(e+fx)\sin(c+dx)}{ad^3} + \frac{(e+fx \cdots}{} \\
&= -\frac{3f^3x}{8ad^3} + \frac{(e+fx)^3}{4ad} - \frac{6f^3\cos(c+dx)}{ad^4} + \frac{3f(e+fx)^2\cos(c+dx)}{ad^2} - \frac{6f^2(e \cdots}{}
\end{aligned}
$$

**Mathematica** [**A**]
time = 0.61, size = 132, normalized size = 0.60

$$
\frac{96f(-2f^2+d^2(e+fx)^2)\cos(c+dx)+4d(e+fx)\left(-3f^2+2d^2(e+fx)^2\right)\cos(2(c+dx))+4(8d(e+fx)\left(-6f^2+d^2(e+fx)^2\right)-3f(-f^2+2d^2(e+fx)^2)\cos(c+dx))\sin(c+dx)}{32ad^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (96*f*(-2*f^2 + d^2*(e + f*x)^2)*Cos[c + d*x] + 4*d*(e + f*x)*(-3*f^2 + 2*d
^2*(e + f*x)^2)*Cos[2*(c + d*x)] + 4*(8*d*(e + f*x)*(-6*f^2 + d^2*(e + f*x)
^2) - 3*f*(-f^2 + 2*d^2*(e + f*x)^2)*Cos[c + d*x])*Sin[c + d*x])/(32*a*d^4)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(207) = 414$.
time = 0.13, size = 736, normalized size = 3.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4/a*(-1/2*c^3*f^3*cos(d*x+c)^2+3/2*c^2*d*e*f^2*cos(d*x+c)^2-3*f^3*c^2*(
-1/2*(d*x+c)*cos(d*x+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*x+1/4*c)-3/2*c*d^
2*e^2*f*cos(d*x+c)^2+6*c*d*e*f^2*(-1/2*(d*x+c)*cos(d*x+c)^2+1/4*cos(d*x+c)*
sin(d*x+c)+1/4*d*x+1/4*c)+3*f^3*c*(-1/2*(d*x+c)^2*cos(d*x+c)^2+(d*x+c)*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/4*(d*x+c)^2-1/4*sin(d*x+c)^2)+1/2*d
^3*e^3*cos(d*x+c)^2-3*d^2*e^2*f*(-1/2*(d*x+c)*cos(d*x+c)^2+1/4*cos(d*x+c)*s
in(d*x+c)+1/4*d*x+1/4*c)-3*d*e*f^2*(-1/2*(d*x+c)^2*cos(d*x+c)^2+(d*x+c)*(1/
2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/4*(d*x+c)^2-1/4*sin(d*x+c)^2)-f^3*
(-1/2*(d*x+c)^3*cos(d*x+c)^2+3/2*(d*x+c)^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d
*x+1/2*c)+3/4*(d*x+c)*cos(d*x+c)^2-3/8*cos(d*x+c)*sin(d*x+c)-3/8*d*x-3/8*c-
1/2*(d*x+c)^3)-sin(d*x+c)*c^3*f^3+3*sin(d*x+c)*c^2*d*e*f^2+3*f^3*c^2*(cos(d
*x+c)+(d*x+c)*sin(d*x+c))-3*sin(d*x+c)*c*d^2*e^2*f-6*c*d*e*f^2*(cos(d*x+c)+
```

```
(d*x+c)*sin(d*x+c))-3*f^3*c*((d*x+c)^2*sin(d*x+c)-2*sin(d*x+c)+2*(d*x+c)*co
s(d*x+c))+sin(d*x+c)*d^3*e^3+3*d^2*e^2*f*(cos(d*x+c)+(d*x+c)*sin(d*x+c))+3*
d*e*f^2*((d*x+c)^2*sin(d*x+c)-2*sin(d*x+c)+2*(d*x+c)*cos(d*x+c))+f^3*((d*x+
c)^3*sin(d*x+c)+3*(d*x+c)^2*cos(d*x+c)-6*cos(d*x+c)-6*(d*x+c)*sin(d*x+c)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(207) = 414$.
time = 0.32, size = 572, normalized size = 2.61

$$-\frac{\frac{8\left(\sin(dx+c)^2-2\sin(dx+c)\right)e^3}{a}-\frac{24\left(\sin(dx+c)^2-2\sin(dx+c)\right)ce^2f}{ad}+\frac{24\left(\sin(dx+c)^2-2\sin(dx+c)\right)c^2ef^2}{ad^2}-\frac{8\left(\sin(dx+c)^2-2\sin(dx+c)\right)c^3f^3}{ad^3}-\frac{6(2(dx+c)\cos(2dx+2c)+8(dx+c)\sin(dx+c)+8\cos(dx+c)-\sin(2dx+2c))e^2f}{ad}+\frac{12(2(dx+c)\cos(2dx+2c)+8(dx+c)\sin(dx+c)+8\cos(dx+c)-\sin(2dx+2c))cef^2}{ad^2}-\frac{6(2(dx+c)\cos(2dx+2c)+8(dx+c)\sin(dx+c)+8\cos(dx+c)-\sin(2dx+2c))c^2f^3}{ad^3}-\frac{6\left(\left(2(dx+c)^2-1\right)\cos(2dx+2c)+16(dx+c)\cos(dx+c)-2(dx+c)\sin(2dx+2c)+8\left((dx+c)^2-2\right)\sin(dx+c)\right)ef^2}{ad^2}+\frac{6\left(\left(2(dx+c)^2-1\right)\cos(2dx+2c)+16(dx+c)\cos(dx+c)-2(dx+c)\sin(2dx+2c)+8\left((dx+c)^2-2\right)\sin(dx+c)\right)cf^3}{ad^3}-\frac{\left(2\left(2(dx+c)^3-3dx-3c\right)\cos(2dx+2c)+48\left((dx+c)^2-2\right)\cos(dx+c)-3\left(2(dx+c)^2-1\right)\sin(2dx+2c)+16\left((dx+c)^3-6dx-6c\right)\sin(dx+c)\right)f^3}{ad^3}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(sin(d*x + c)^2 - 2*sin(d*x + c))*e^3/a - 24*(sin(d*x + c)^2 - 2*s
in(d*x + c))*c*e^2*f/(a*d) + 24*(sin(d*x + c)^2 - 2*sin(d*x + c))*c^2*e*f^2
/(a*d^2) - 8*(sin(d*x + c)^2 - 2*sin(d*x + c))*c^3*f^3/(a*d^3) - 6*(2*(d*x
+ c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x + c) + 8*cos(d*x + c) - sin(2*d
*x + 2*c))*e^2*f/(a*d) + 12*(2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin
(d*x + c) + 8*cos(d*x + c) - sin(2*d*x + 2*c))*c*e*f^2/(a*d^2) - 6*(2*(d*x
+ c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x + c) + 8*cos(d*x + c) - sin(2*d
*x + 2*c))*c^2*f^3/(a*d^3) - 6*((2*(d*x + c)^2 - 1)*cos(2*d*x + 2*c) + 16*(
d*x + c)*cos(d*x + c) - 2*(d*x + c)*sin(2*d*x + 2*c) + 8*((d*x + c)^2 - 2)*
sin(d*x + c))*e*f^2/(a*d^2) + 6*((2*(d*x + c)^2 - 1)*cos(2*d*x + 2*c) + 16*
(d*x + c)*cos(d*x + c) - 2*(d*x + c)*sin(2*d*x + 2*c) + 8*((d*x + c)^2 - 2)
*sin(d*x + c))*c*f^3/(a*d^3) - (2*(2*(d*x + c)^3 - 3*d*x - 3*c)*cos(2*d*x +
 2*c) + 48*((d*x + c)^2 - 2)*cos(d*x + c) - 3*(2*(d*x + c)^2 - 1)*sin(2*d*x
 + 2*c) + 16*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*f^3/(a*d^3))/d
```

**Fricas** [A]
time = 0.36, size = 266, normalized size = 1.21

$$-\frac{2d^3f^3x^3+6d^3f^2x^2e+6d^3fxe^2-3df^3x-2\left(2d^3f^3x^3+6d^3fxe^2-3df^3x+2d^3e^3+3\left(2d^3f^2x^2-df^2\right)e\right)\cos(dx+c)^2-24\left(d^2f^3x^2+2d^2f^2xe+d^2fe^2-2f^3\right)\cos(dx+c)-\left(8d^3f^3x^3+24d^3fxe^2-48df^3x+8d^3e^3-3\left(2d^2f^3x^2+4d^2f^2xe+2d^2fe^2-f^3\right)\cos(dx+c)+24\left(d^3f^2x^2-2df^2\right)e\right)\sin(dx+c)}{8ad^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*d^3*f^3*x^3 + 6*d^3*f^2*x^2*e + 6*d^3*f*x*e^2 - 3*d*f^3*x - 2*(2*d^
3*f^3*x^3 + 6*d^3*f*x*e^2 - 3*d*f^3*x + 2*d^3*e^3 + 3*(2*d^3*f^2*x^2 - d*f^
2)*e)*cos(d*x + c)^2 - 24*(d^2*f^3*x^2 + 2*d^2*f^2*x*e + d^2*f*e^2 - 2*f^3)
*cos(d*x + c) - (8*d^3*f^3*x^3 + 24*d^3*f*x*e^2 - 48*d*f^3*x + 8*d^3*e^3 -
3*(2*d^2*f^3*x^2 + 4*d^2*f^2*x*e + 2*d^2*f*e^2 - f^3)*cos(d*x + c) + 24*(d^
3*f^2*x^2 - 2*d*f^2)*e)*sin(d*x + c))/(a*d^4)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 2725 vs. $2(204) = 408$.

time = 5.32, size = 2725, normalized size = 12.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*d**3*e**3*tan(c/2 + d*x/2)**3/(8*a*d**4*tan(c/2 + d*x/2)**4 +
 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 16*d**3*e**3*tan(c/2 + d*x/2)*
*2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4
) + 16*d**3*e**3*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4
*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 6*d**3*e**2*f*x*tan(c/2 + d*x/2)**4/(8*a
*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 48*
d**3*e**2*f*x*tan(c/2 + d*x/2)**3/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4
*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 36*d**3*e**2*f*x*tan(c/2 + d*x/2)**2/(8*
a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 48
*d**3*e**2*f*x*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*t
an(c/2 + d*x/2)**2 + 8*a*d**4) + 6*d**3*e**2*f*x/(8*a*d**4*tan(c/2 + d*x/2)
**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 6*d**3*e*f**2*x**2*tan(c/
2 + d*x/2)**4/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2
 + 8*a*d**4) + 48*d**3*e*f**2*x**2*tan(c/2 + d*x/2)**3/(8*a*d**4*tan(c/2 +
d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 36*d**3*e*f**2*x**2
*tan(c/2 + d*x/2)**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*
x/2)**2 + 8*a*d**4) + 48*d**3*e*f**2*x**2*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/
2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 6*d**3*e*f**2*x
**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**
4) + 2*d**3*f**3*x**3*tan(c/2 + d*x/2)**4/(8*a*d**4*tan(c/2 + d*x/2)**4 + 1
6*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 16*d**3*f**3*x**3*tan(c/2 + d*x/
2)**3/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d
**4) - 12*d**3*f**3*x**3*tan(c/2 + d*x/2)**2/(8*a*d**4*tan(c/2 + d*x/2)**4
+ 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 16*d**3*f**3*x**3*tan(c/2 + d
*x/2)/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d
**4) + 2*d**3*f**3*x**3/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 +
 d*x/2)**2 + 8*a*d**4) + 12*d**2*e**2*f*tan(c/2 + d*x/2)**3/(8*a*d**4*tan(c
/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 48*d**2*e**2*f
*tan(c/2 + d*x/2)**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*
x/2)**2 + 8*a*d**4) - 12*d**2*e**2*f*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d
*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 48*d**2*e**2*f/(8*a*
d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 48*d
**2*e*f**2*x*tan(c/2 + d*x/2)**4/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*
tan(c/2 + d*x/2)**2 + 8*a*d**4) + 24*d**2*e*f**2*x*tan(c/2 + d*x/2)**3/(8*a
*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 24*
d**2*e*f**2*x*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*ta
n(c/2 + d*x/2)**2 + 8*a*d**4) + 48*d**2*e*f**2*x/(8*a*d**4*tan(c/2 + d*x/2)
**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 24*d**2*f**3*x**2*tan(c/2
```

```
 + d*x/2)**4/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2
+ 8*a*d**4) + 12*d**2*f**3*x**2*tan(c/2 + d*x/2)**3/(8*a*d**4*tan(c/2 + d*x
/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 12*d**2*f**3*x**2*tan(
c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2
+ 8*a*d**4) + 24*d**2*f**3*x**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*t
an(c/2 + d*x/2)**2 + 8*a*d**4) - 96*d*e*f**2*tan(c/2 + d*x/2)**3/(8*a*d**4*
tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 24*d*e*f*
*2*tan(c/2 + d*x/2)**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 +
d*x/2)**2 + 8*a*d**4) - 96*d*e*f**2*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d*
x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 3*d*f**3*x*tan(c/2 +
d*x/2)**4/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8
*a*d**4) - 96*d*f**3*x*tan(c/2 + d*x/2)**3/(8*a*d**4*tan(c/2 + d*x/2)**4 +
16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 18*d*f**3*x*tan(c/2 + d*x/2)**2
/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4)
- 96*d*f**3*x*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*ta
n(c/2 + d*x/2)**2 + 8*a*d**4) - 3*d*f**3*x/(8*a*d**4*tan(c/2 + d*x/2)**4 +
16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 6*f**3*tan(c/2 + d*x/2)**3/(8*a
*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 96*
f**3*tan(c/2 + d*x/2)**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2
+ d*x/2)**2 + 8*a*d**4) + 6*f**3*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d*x/2
)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 96*f**3/(8*a*d**4*tan(c/
2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4), Ne(d, 0)), ((e**
3*x + 3*e**2*f*x**2/2 + e*f**2*x**3 + f**3*x**4/4)*cos(c)**3/(a*sin(c) + a)
, True))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 55344 vs. $2(214) = 428$.
time = 9.11, size = 55344, normalized size = 252.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(36*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 - 12*d^4*f^2
*x^3*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^7 + 2*d^3*f^3*x^3*tan(1/2*d*x)^
5*tan(3/2*c)^2*tan(1/2*c)^7 - 36*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(3/2*c)^2*
tan(1/2*c)^5 + 12*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^6 - 36
*d^4*f^2*x^3*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 - 18*d^3*f^3*x^3*ta
n(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 + 12*d^4*f^2*x^3*e*tan(1/2*d*x)^4*ta
n(3/2*c)*tan(1/2*c)^7 - 18*d^3*f^3*x^3*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*
c)^7 + 6*d^3*f^2*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 - 120*d^4*f
^2*x^3*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 + 180*d^4*f^2*x^3*e*tan(1
/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^5 - 36*d^4*f^2*x^3*e*tan(1/2*d*x)^4*tan(3/2
*c)^2*tan(1/2*c)^5 + 6*d^3*f^3*x^3*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5
```

- 36*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(1/2*c)^6 + 12*d^4*f^2*x^3*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^6 + 72*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^6 - 2*d^3*f^3*x^3*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 + 54*d^4*f*x^2*e^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 + 126*d^3*f^2*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 + 2*d^3*f^3*x^3*tan(1/2*d*x)^5*tan(1/2*c)^7 - 24*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(3/2*c)*tan(1/2*c)^7 - 18*d^4*f*x^2*e^2*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^7 - 60*d^3*f^2*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^7 + 4*d^3*f^3*x^3*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^7 - 54*d^3*f^2*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^7 + 24*d^2*f^3*x^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 + 120*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^3 - 180*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^4 + 120*d^4*f^2*x^3*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^4 - 22*d^3*f^3*x^3*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 + 36*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(1/2*c)^5 - 180*d^4*f^2*x^3*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^5 - 72*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^5 + 42*d^3*f^3*x^3*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^5 - 54*d^4*f*x^2*e^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 - 54*d^3*f^2*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 + 36*d^4*f^2*x^3*e*tan(1/2*d*x)^4*tan(1/2*c)^6 - 18*d^3*f^3*x^3*tan(1/2*d*x)^5*tan(1/2*c)^6 + 24*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(3/2*c)*tan(1/2*c)^6 + 18*d^4*f*x^2*e^2*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^6 + 60*d^3*f^2*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^6 - 72*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(3/2*c)^2*tan(1/2*c)^6 + 60*d^3*f^3*x^3*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^6 - 54*d^4*f*x^2*e^2*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 + 30*d^3*f^2*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 - 12*d^2*f^3*x^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 - 18*d^3*f^3*x^3*tan(1/2*d*x)^4*tan(1/2*c)^7 - 6*d^3*f^2*x^2*e*tan(1/2*d*x)^5*tan(1/2*c)^7 + 24*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(3/2*c)*tan(1/2*c)^7 + 18*d^4*f*x^2*e^2*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^7 - 12*d^3*f^2*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^7 - 4*d^3*f^3*x^3*tan(1/2*d*x)^2*tan(3/2*c)^2*tan(1/2*c)^7 + 12*d^3*f^2*x^2*e*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^7 - 12*d^2*f^3*x^2*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^7 + 6*d^3*f*x*e^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 + 48*d^2*f^2*x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 + 36*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^2 - 180*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^3 + 120*d^4*f^2*x^3*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^3 + 22*d^3*f^3*x^3*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^3 + 120*d^4*f^2*x^3*e*tan(1/2*d*x)^5*tan(1/2*c)^4 - 180*d^4*f^2*x^3*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^4 - 240*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^4 + 58*d^3*f^3*x^3*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^4 - 180*d^4*f*x^2*e^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 - 126*d^3*f^2*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 + 36*d^4*f^2*x^3*e*tan(1/2*d*x)^4*tan(1/2*c)^5 + 6*d^3*f^3*x^3*tan(1/2*d*x)^5*tan(1/2*c)^5 + 360*d^4*f^2*x^3*e*tan(1/2*d*x)^3*tan(3/2*c)*tan(1/2*c)^5 + 270*d^4*f*x^2*e^2*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^5 + 396*d^3*f^2*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^5 - 72*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(3/2*c)^2*tan(1/2*c)^5 + 108*d^3*f^3*x^3*tan(1/

```
2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^5 - 54*d^4*f*x^2*e^2*tan(1/2*d*x)^4*tan(3/
2*c)^2*tan(1/2*c)^5 + 54*d^3*f^2*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*
c)^5 + 12*d^2*f^3*x^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 - 72*d^4*f^2
*x^3*e*tan(1/2*d*x)^3*tan(1/2*c)^6 - 2*d^3*f^3*x^3*tan(1/2*d*x)^4*tan(1/2*c
)^6 - 54*d^4*f*x^2*e^2*tan(1/2*d*x)^5*tan(1/2*c)^6 - 126*d^3*f^2*x^2*e*tan(
1/2*d*x)^5*tan(1/2*c)^6 + 24*d^4*f^2*x^3*e*tan(1/2*d*x)^2*tan(3/2*c)*tan(1/
2*c)^6 + 18*d^4*f*x^2*e^2*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^6 - 300*d^3*
f^2*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^6 + 36*d^4*f^2*x^3*e*tan(1/2
*d*x)*tan(3/2*c)^2*tan(1/2*c)^6 + 60*d^3*f^3*x^3*tan(1/2*d*x)^2*tan(3/2*c)^
2*tan(1/2*c)^6 + 108*d^4*f*x^2*e^2*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^6
 + 108*d^3*f^2*x^2*e*tan(1/2*d*x)^3*tan(3/2*c)^...
```

**Mupad** [B]
time = 3.49, size = 339, normalized size = 1.55

$$\frac{\frac{3 f^3 \sin(2c+2dx)}{2} - 48 f^3 \cos(c+dx) + 8 d^3 e^3 \sin(c+dx) + 2 d^3 e^3 \cos(2c+2dx) - 3 d^2 e^2 f \sin(2c+2dx) + 24 d^2 f^3 x^2 \cos(c+dx) + 8 d^3 f^3 x^3 \sin(c+dx) - 48 d e f^2 \sin(c+dx) - 48 d f^3 x \sin(c+dx) + 2 d^3 f^3 x^3 \cos(2c+2dx) - 3 d^2 f^3 x^2 \sin(2c+2dx) - 3 d e f^2 \cos(2c+2dx) + 24 d^2 e^2 f \cos(c+dx) - 3 d f^3 x \cos(2c+2dx) + 48 d^2 e f^2 x \cos(c+dx) + 24 d^3 e^2 f x \sin(c+dx) + 6 d^3 e^2 f x \cos(2c+2dx) - 6 d^2 e f^2 x \sin(2c+2dx) + 24 d^3 e f^2 x^2 \sin(c+dx) + 6 d^3 e f^2 x^2 \cos(2c+2dx)}{8 a d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] ((3*f^3*sin(2*c + 2*d*x))/2 - 48*f^3*cos(c + d*x) + 8*d^3*e^3*sin(c + d*x)
+ 2*d^3*e^3*cos(2*c + 2*d*x) - 3*d^2*e^2*f*sin(2*c + 2*d*x) + 24*d^2*f^3*x^
2*cos(c + d*x) + 8*d^3*f^3*x^3*sin(c + d*x) - 48*d*e*f^2*sin(c + d*x) - 48*
d*f^3*x*sin(c + d*x) + 2*d^3*f^3*x^3*cos(2*c + 2*d*x) - 3*d^2*f^3*x^2*sin(2
*c + 2*d*x) - 3*d*e*f^2*cos(2*c + 2*d*x) + 24*d^2*e^2*f*cos(c + d*x) - 3*d*
f^3*x*cos(2*c + 2*d*x) + 48*d^2*e*f^2*x*cos(c + d*x) + 24*d^3*e^2*f*x*sin(c
 + d*x) + 6*d^3*e^2*f*x*cos(2*c + 2*d*x) - 6*d^2*e*f^2*x*sin(2*c + 2*d*x) +
 24*d^3*e*f^2*x^2*sin(c + d*x) + 6*d^3*e*f^2*x^2*cos(2*c + 2*d*x))/(8*a*d^4
)
```

# 3.264 $\int \frac{(e+fx)^2 \cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=161

$$\frac{efx}{2ad}+\frac{f^2x^2}{4ad}+\frac{2f(e+fx)\cos(c+dx)}{ad^2}-\frac{2f^2\sin(c+dx)}{ad^3}+\frac{(e+fx)^2\sin(c+dx)}{ad}-\frac{f(e+fx)\cos(c+dx)\sin(c}{2ad^2}$$

```
[Out] 1/2*e*f*x/a/d+1/4*f^2*x^2/a/d+2*f*(f*x+e)*cos(d*x+c)/a/d^2-2*f^2*sin(d*x+c)
/a/d^3+(f*x+e)^2*sin(d*x+c)/a/d-1/2*f*(f*x+e)*cos(d*x+c)*sin(d*x+c)/a/d^2+1
/4*f^2*sin(d*x+c)^2/a/d^3-1/2*(f*x+e)^2*sin(d*x+c)^2/a/d
```

**Rubi** [A]
time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4619, 3377, 2717, 4489, 3391}

$$\frac{f^2\sin^2(c+dx)}{4ad^3}-\frac{2f^2\sin(c+dx)}{ad^3}+\frac{2f(e+fx)\cos(c+dx)}{ad^2}-\frac{f(e+fx)\sin(c+dx)\cos(c+dx)}{2ad^2}-\frac{(e+fx)^2\sin^2(c+dx)}{2ad}+\frac{(e+fx)^2\sin(c+dx)}{ad}+\frac{efx}{2ad}+\frac{f^2x^2}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e*f*x)/(2*a*d) + (f^2*x^2)/(4*a*d) + (2*f*(e + f*x)*Cos[c + d*x])/(a*d^2)
- (2*f^2*Sin[c + d*x])/(a*d^3) + ((e + f*x)^2*Sin[c + d*x])/(a*d) - (f*(e +
 f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d^2) + (f^2*Sin[c + d*x]^2)/(4*a*d^3)
 - ((e + f*x)^2*Sin[c + d*x]^2)/(2*a*d)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^2\cos(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)\,dx}{a} \\
&= \frac{(e+fx)^2\sin(c+dx)}{ad} - \frac{(e+fx)^2\sin^2(c+dx)}{2ad} + \frac{f\int (e+fx)\sin^2(c+dx)\,dx}{ad} \\
&= \frac{2f(e+fx)\cos(c+dx)}{ad^2} + \frac{(e+fx)^2\sin(c+dx)}{ad} - \frac{f(e+fx)\cos(c+dx)\sin(}{2ad^2} \\
&= \frac{efx}{2ad} + \frac{f^2x^2}{4ad} + \frac{2f(e+fx)\cos(c+dx)}{ad^2} - \frac{2f^2\sin(c+dx)}{ad^3} + \frac{(e+fx)^2\sin(c+}{ad}
\end{aligned}
$$

**Mathematica [A]**
time = 0.68, size = 95, normalized size = 0.59

$$\frac{16df(e+fx)\cos(c+dx)+(-f^2+2d^2(e+fx)^2)\cos(2(c+dx))-4(-2(-2f^2+d^2(e+fx)^2)+df(e+fx)\cos(c+dx))\sin(c+dx)}{8ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (16*d*f*(e + f*x)*Cos[c + d*x] + (-f^2 + 2*d^2*(e + f*x)^2)*Cos[2*(c + d*x)
] - 4*(-2*(-2*f^2 + d^2*(e + f*x)^2) + d*f*(e + f*x)*Cos[c + d*x])*Sin[c +
d*x])/(8*a*d^3)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(151) = 302$.
time = 0.20, size = 339, normalized size = 2.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3/a*(-1/2*c^2*f^2*cos(d*x+c)^2+c*d*e*f*cos(d*x+c)^2-2*f^2*c*(-1/2*(d*x
+c)*cos(d*x+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*x+1/4*c)-1/2*d^2*e^2*cos(d
*x+c)^2+2*d*e*f*(-1/2*(d*x+c)*cos(d*x+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*
x+1/4*c)+f^2*(-1/2*(d*x+c)^2*cos(d*x+c)^2+(d*x+c)*(1/2*cos(d*x+c)*sin(d*x+c
)+1/2*d*x+1/2*c)-1/4*(d*x+c)^2-1/4*sin(d*x+c)^2)-sin(d*x+c)*c^2*f^2+2*sin(d
*x+c)*c*d*e*f+2*f^2*c*(cos(d*x+c)+(d*x+c)*sin(d*x+c))-sin(d*x+c)*d^2*e^2-2*
d*e*f*(cos(d*x+c)+(d*x+c)*sin(d*x+c))-f^2*((d*x+c)^2*sin(d*x+c)-2*sin(d*x+c
)+2*(d*x+c)*cos(d*x+c)))
```

**Maxima** [A]
time = 0.29, size = 289, normalized size = 1.80

$$-\frac{\frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)e^2}{a}-\frac{8\left(\sin(dx+c)^2-2\sin(dx+c)\right)cef}{ad}+\frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)c^2f^2}{ad^2}-\frac{2\left(2\left(dx+c\right)\cos(2dx+2c)+8\left(dx+c\right)\sin(dx+c)+8\cos(dx+c)-\sin(2dx+2c)\right)ef}{ad}+\frac{2\left(2\left(dx+c\right)\cos(2dx+2c)+8\left(dx+c\right)\sin(dx+c)+8\cos(dx+c)-\sin(2dx+2c)\right)cf^2}{ad^2}-\frac{\left(\left(2\left(dx+c\right)^2-1\right)\cos(2dx+2c)+16\left(dx+c\right)\cos(dx+c)-2\left(dx+c\right)\sin(2dx+2c)+8\left(\left(dx+c\right)^2-2\right)\sin(dx+c)\right)f^2}{ad^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(sin(d*x + c)^2 - 2*sin(d*x + c))*e^2/a - 8*(sin(d*x + c)^2 - 2*sin
(d*x + c))*c*e*f/(a*d) + 4*(sin(d*x + c)^2 - 2*sin(d*x + c))*c^2*f^2/(a*d^2
) - 2*(2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x + c) + 8*cos(d*x
+ c) - sin(2*d*x + 2*c))*e*f/(a*d) + 2*(2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d
*x + c)*sin(d*x + c) + 8*cos(d*x + c) - sin(2*d*x + 2*c))*c*f^2/(a*d^2) - (
(2*(d*x + c)^2 - 1)*cos(2*d*x + 2*c) + 16*(d*x + c)*cos(d*x + c) - 2*(d*x +
 c)*sin(2*d*x + 2*c) + 8*((d*x + c)^2 - 2)*sin(d*x + c))*f^2/(a*d^2))/d
```

**Fricas** [A]
time = 0.34, size = 152, normalized size = 0.94

$$-\frac{d^2f^2x^2+2\,d^2fxe-\left(2\,d^2f^2x^2+4\,d^2fxe+2\,d^2e^2-f^2\right)\cos\left(dx+c\right)^2-8\left(df^2x+dfe\right)\cos\left(dx+c\right)-2\left(2\,d^2f^2x^2+4\,d^2fxe+2\,d^2e^2-4\,f^2-\left(df^2x+dfe\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^2*f^2*x^2 + 2*d^2*f*x*e - (2*d^2*f^2*x^2 + 4*d^2*f*x*e + 2*d^2*e^2
- f^2)*cos(d*x + c)^2 - 8*(d*f^2*x + d*f*e)*cos(d*x + c) - 2*(2*d^2*f^2*x^2
 + 4*d^2*f*x*e + 2*d^2*e^2 - 4*f^2 - (d*f^2*x + d*f*e)*cos(d*x + c))*sin(d*
x + c))/(a*d^3)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1528 vs. $2(141) = 282$.
time = 3.96, size = 1528, normalized size = 9.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*d**2*e**2*tan(c/2 + d*x/2)**3/(4*a*d**3*tan(c/2 + d*x/2)**4 +
8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) - 8*d**2*e**2*tan(c/2 + d*x/2)**2/
(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) +
8*d**2*e**2*tan(c/2 + d*x/2)/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c
/2 + d*x/2)**2 + 4*a*d**3) + 2*d**2*e*f*x*tan(c/2 + d*x/2)**4/(4*a*d**3*tan
(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 16*d**2*e*f*x
*tan(c/2 + d*x/2)**3/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x
/2)**2 + 4*a*d**3) - 12*d**2*e*f*x*tan(c/2 + d*x/2)**2/(4*a*d**3*tan(c/2 +
d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 16*d**2*e*f*x*tan(c/
2 + d*x/2)/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4
*a*d**3) + 2*d**2*e*f*x/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 +
d*x/2)**2 + 4*a*d**3) + d**2*f**2*x**2*tan(c/2 + d*x/2)**4/(4*a*d**3*tan(c/
2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 8*d**2*f**2*x**2
*tan(c/2 + d*x/2)**3/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x
/2)**2 + 4*a*d**3) - 6*d**2*f**2*x**2*tan(c/2 + d*x/2)**2/(4*a*d**3*tan(c/2
 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 8*d**2*f**2*x**2*
tan(c/2 + d*x/2)/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)*
*2 + 4*a*d**3) + d**2*f**2*x**2/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*ta
n(c/2 + d*x/2)**2 + 4*a*d**3) + 4*d*e*f*tan(c/2 + d*x/2)**3/(4*a*d**3*tan(c
/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 16*d*e*f*tan(c/
2 + d*x/2)**2/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2
+ 4*a*d**3) - 4*d*e*f*tan(c/2 + d*x/2)/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*
d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 16*d*e*f/(4*a*d**3*tan(c/2 + d*x/2)*
*4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) - 8*d*f**2*x*tan(c/2 + d*x/2)
**4/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3
) + 4*d*f**2*x*tan(c/2 + d*x/2)**3/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3
*tan(c/2 + d*x/2)**2 + 4*a*d**3) - 4*d*f**2*x*tan(c/2 + d*x/2)/(4*a*d**3*ta
n(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 8*d*f**2*x/(
4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) - 1
6*f**2*tan(c/2 + d*x/2)**3/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2
 + d*x/2)**2 + 4*a*d**3) + 4*f**2*tan(c/2 + d*x/2)**2/(4*a*d**3*tan(c/2 + d
*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) - 16*f**2*tan(c/2 + d*x
/2)/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3
), Ne(d, 0)), ((e**2*x + e*f*x**2 + f**2*x**3/3)*cos(c)**3/(a*sin(c) + a),
True))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 38869 vs. $2(156) = 312$.
time = 9.38, size = 38869, normalized size = 241.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

[Out] 1/8*(36*d^3*f*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 - 12*d^3*f*x^2
*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^7 + 2*d^2*f^2*x^2*tan(1/2*d*x)^5*ta
n(3/2*c)^2*tan(1/2*c)^7 - 36*d^3*f*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/
2*c)^5 + 12*d^3*f*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^6 - 36*d^3*f*x
^2*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 - 18*d^2*f^2*x^2*tan(1/2*d*x)
^5*tan(3/2*c)^2*tan(1/2*c)^6 + 12*d^3*f*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)*tan
(1/2*c)^7 - 18*d^2*f^2*x^2*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^7 + 4*d^2
*f*x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 - 120*d^3*f*x^2*e*tan(1/2*d
*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 + 180*d^3*f*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)
*tan(1/2*c)^5 - 36*d^3*f*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^5 + 6
*d^2*f^2*x^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 - 36*d^3*f*x^2*e*tan(
1/2*d*x)^5*tan(1/2*c)^6 + 12*d^3*f*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*
c)^6 + 72*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^6 - 2*d^2*f^2*
x^2*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 + 36*d^3*x*e^2*tan(1/2*d*x)^5*
tan(3/2*c)^2*tan(1/2*c)^6 + 84*d^2*f*x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/
2*c)^6 + 2*d^2*f^2*x^2*tan(1/2*d*x)^5*tan(1/2*c)^7 - 24*d^3*f*x^2*e*tan(1/2
*d*x)^3*tan(3/2*c)*tan(1/2*c)^7 - 12*d^3*x*e^2*tan(1/2*d*x)^5*tan(3/2*c)*ta
n(1/2*c)^7 - 40*d^2*f*x*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^7 + 4*d^2*f^
2*x^2*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^7 - 36*d^2*f*x*e*tan(1/2*d*x)^
4*tan(3/2*c)^2*tan(1/2*c)^7 + 16*d*f^2*x*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/
2*c)^7 + 120*d^3*f*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^3 - 180*d^3
*f*x^2*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^4 + 120*d^3*f*x^2*e*tan(1/2*d
*x)^4*tan(3/2*c)^2*tan(1/2*c)^4 - 22*d^2*f^2*x^2*tan(1/2*d*x)^5*tan(3/2*c)^
2*tan(1/2*c)^4 + 36*d^3*f*x^2*e*tan(1/2*d*x)^5*tan(1/2*c)^5 - 180*d^3*f*x^2
*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^5 - 72*d^3*f*x^2*e*tan(1/2*d*x)^3*t
an(3/2*c)^2*tan(1/2*c)^5 + 42*d^2*f^2*x^2*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1
/2*c)^5 - 36*d^3*x*e^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 - 36*d^2*f*
x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 + 36*d^3*f*x^2*e*tan(1/2*d*x)^
4*tan(1/2*c)^6 - 18*d^2*f^2*x^2*tan(1/2*d*x)^5*tan(1/2*c)^6 + 24*d^3*f*x^2*
e*tan(1/2*d*x)^3*tan(3/2*c)*tan(1/2*c)^6 + 12*d^3*x*e^2*tan(1/2*d*x)^5*tan(
3/2*c)*tan(1/2*c)^6 + 40*d^2*f*x*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^6 -
72*d^3*f*x^2*e*tan(1/2*d*x)^2*tan(3/2*c)^2*tan(1/2*c)^6 + 60*d^2*f^2*x^2*t
an(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^6 - 36*d^3*x*e^2*tan(1/2*d*x)^4*tan(3
/2*c)^2*tan(1/2*c)^6 + 20*d^2*f*x*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^
6 - 8*d*f^2*x*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 - 18*d^2*f^2*x^2*tan
(1/2*d*x)^4*tan(1/2*c)^7 - 4*d^2*f*x*e*tan(1/2*d*x)^5*tan(1/2*c)^7 + 24*d^3
*f*x^2*e*tan(1/2*d*x)^2*tan(3/2*c)*tan(1/2*c)^7 + 12*d^3*x*e^2*tan(1/2*d*x)
^4*tan(3/2*c)*tan(1/2*c)^7 - 8*d^2*f*x*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*
c)^7 - 4*d^2*f^2*x^2*tan(1/2*d*x)^2*tan(3/2*c)^2*tan(1/2*c)^7 + 8*d^2*f*x*e
*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^7 - 8*d*f^2*x*tan(1/2*d*x)^4*tan(3/
2*c)^2*tan(1/2*c)^7 + 2*d^2*e^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 +
16*d*f*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 + 36*d^3*f*x^2*e*tan(1/2*
d*x)^5*tan(3/2*c)^2*tan(1/2*c)^2 - 180*d^3*f*x^2*e*tan(1/2*d*x)^5*tan(3/2*c
)*tan(1/2*c)^3 + 120*d^3*f*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^3 +
22*d^2*f^2*x^2*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^3 + 120*d^3*f*x^2*e*

```
tan(1/2*d*x)^5*tan(1/2*c)^4 - 180*d^3*f*x^2*e*tan(1/2*d*x)^4*tan(3/2*c)*tan
(1/2*c)^4 - 240*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^4 + 58*d
^2*f^2*x^2*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^4 - 120*d^3*x*e^2*tan(1/2
*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 - 84*d^2*f*x*e*tan(1/2*d*x)^5*tan(3/2*c)^
2*tan(1/2*c)^4 + 36*d^3*f*x^2*e*tan(1/2*d*x)^4*tan(1/2*c)^5 + 6*d^2*f^2*x^2
*tan(1/2*d*x)^5*tan(1/2*c)^5 + 360*d^3*f*x^2*e*tan(1/2*d*x)^3*tan(3/2*c)*ta
n(1/2*c)^5 + 180*d^3*x*e^2*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^5 + 264*d^2
*f*x*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^5 - 72*d^3*f*x^2*e*tan(1/2*d*x)
^2*tan(3/2*c)^2*tan(1/2*c)^5 + 108*d^2*f^2*x^2*tan(1/2*d*x)^3*tan(3/2*c)^2*
tan(1/2*c)^5 - 36*d^3*x*e^2*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^5 + 36*d
^2*f*x*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^5 + 8*d*f^2*x*tan(1/2*d*x)^
5*tan(3/2*c)^2*tan(1/2*c)^5 + 108*d^2*e^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d
*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 - 72*d
^3*f*x^2*e*tan(1/2*d*x)^3*tan(1/2*c)^6 - 2*d^2*f^2*x^2*tan(1/2*d*x)^4*tan(1
/2*c)^6 - 36*d^3*x*e^2*tan(1/2*d*x)^5*tan(1/2*c)^6 - 84*d^2*f*x*e*tan(1/2*d
*x)^5*tan(1/2*c)^6 + 24*d^3*f*x^2*e*tan(1/2*d*x)^2*tan(3/2*c)*tan(1/2*c)^6
+ 12*d^3*x*e^2*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^6 - 200*d^2*f*x*e*tan(1
/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^6 - 72*d^2*e^2*log(2*(tan(1/2*d*x)^4*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/...
```

**Mupad** [B]
time = 3.20, size = 187, normalized size = 1.16

$$\frac{8d^2e^2\sin(c+dx) - f^2\cos(2c+2dx) - 16f^2\sin(c+dx) + 2d^2e^2\cos(2c+2dx) + 8d^2f^2x^2\sin(c+dx) - 2def\sin(2c+2dx) + 16df^2x\cos(c+dx) + 2d^2f^2x^2\cos(2c+2dx) - 2df^2x\sin(2c+2dx) + 16def\cos(c+dx) + 4d^2efx\cos(2c+2dx) + 16d^2efx\sin(c+dx)}{8ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] (8*d^2*e^2*sin(c + d*x) - f^2*cos(2*c + 2*d*x) - 16*f^2*sin(c + d*x) + 2*d^
2*e^2*cos(2*c + 2*d*x) + 8*d^2*f^2*x^2*sin(c + d*x) - 2*d*e*f*sin(2*c + 2*d
*x) + 16*d*f^2*x*cos(c + d*x) + 2*d^2*f^2*x^2*cos(2*c + 2*d*x) - 2*d*f^2*x*
sin(2*c + 2*d*x) + 16*d*e*f*cos(c + d*x) + 4*d^2*e*f*x*cos(2*c + 2*d*x) + 1
6*d^2*e*f*x*sin(c + d*x))/(8*a*d^3)
```

# 3.265 $\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=91

$$\frac{fx}{4ad}+\frac{f\cos(c+dx)}{ad^2}+\frac{(e+fx)\sin(c+dx)}{ad}-\frac{f\cos(c+dx)\sin(c+dx)}{4ad^2}-\frac{(e+fx)\sin^2(c+dx)}{2ad}$$

```
[Out] 1/4*f*x/a/d+f*cos(d*x+c)/a/d^2+(f*x+e)*sin(d*x+c)/a/d-1/4*f*cos(d*x+c)*sin(
d*x+c)/a/d^2-1/2*(f*x+e)*sin(d*x+c)^2/a/d
```

**Rubi** [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4619, 3377, 2718, 4489, 2715, 8}

$$\frac{f\cos(c+dx)}{ad^2}-\frac{f\sin(c+dx)\cos(c+dx)}{4ad^2}-\frac{(e+fx)\sin^2(c+dx)}{2ad}+\frac{(e+fx)\sin(c+dx)}{ad}+\frac{fx}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (f*x)/(4*a*d) + (f*Cos[c + d*x])/(a*d^2) + ((e + f*x)*Sin[c + d*x])/(a*d) -
 (f*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^2) - ((e + f*x)*Sin[c + d*x]^2)/(2*a*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\cos(c+dx)\,dx}{a} - \frac{\int (e+fx)\cos(c+dx)\sin(c+dx)\,dx}{a}\\ &= \frac{(e+fx)\sin(c+dx)}{ad} - \frac{(e+fx)\sin^2(c+dx)}{2ad} + \frac{f\int \sin^2(c+dx)\,dx}{2ad} - \frac{f\int \sin(\\ &= \frac{f\cos(c+dx)}{ad^2} + \frac{(e+fx)\sin(c+dx)}{ad} - \frac{f\cos(c+dx)\sin(c+dx)}{4ad^2} - \frac{(e+fx)}{2\\ &= \frac{fx}{4ad} + \frac{f\cos(c+dx)}{ad^2} + \frac{(e+fx)\sin(c+dx)}{ad} - \frac{f\cos(c+dx)\sin(c+dx)}{4ad^2} - \frac{(e\end{aligned}$$

**Mathematica** [**A**]
time = 0.74, size = 52, normalized size = 0.57

$$\frac{-f\cos(c+dx)(-4+\sin(c+dx))+d(e+fx)(\cos(2(c+dx))+4\sin(c+dx))}{4ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-(f*Cos[c + d*x]*(-4 + Sin[c + d*x])) + d*(e + f*x)*(Cos[2*(c + d*x)] + 4*
Sin[c + d*x]))/(4*a*d^2)
```

**Maple** [**A**]
time = 0.14, size = 113, normalized size = 1.24

| method | result |
|---|---|

| | |
|---|---|
| risch | $\frac{f\cos(dx+c)}{a\,d^2}+\frac{(fx+e)\sin(dx+c)}{ad}+\frac{(fx+e)\cos(2dx+2c)}{4ad}-\frac{f\sin(2dx+2c)}{8a\,d^2}$ |
| derivativedivides | $\frac{-\frac{fc\left(\cos^2(dx+c)\right)}{2}+\frac{\left(\cos^2(dx+c)\right)de}{2}-f\left(-\frac{(dx+c)\left(\cos^2(dx+c)\right)}{2}+\frac{\cos(dx+c)\sin(dx+c)}{4}+\frac{dx}{4}+\frac{c}{4}\right)-\sin(dx+c)cf+\sin(dx+c)d\ldots}{d^2a}$ |
| default | $\frac{-\frac{fc\left(\cos^2(dx+c)\right)}{2}+\frac{\left(\cos^2(dx+c)\right)de}{2}-f\left(-\frac{(dx+c)\left(\cos^2(dx+c)\right)}{2}+\frac{\cos(dx+c)\sin(dx+c)}{4}+\frac{dx}{4}+\frac{c}{4}\right)-\sin(dx+c)cf+\sin(dx+c)d\ldots}{d^2a}$ |
| norman | $\frac{2f}{a\,d^2}+\frac{(2de+2f)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,d^2}+\frac{(2de+4f)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,d^2}+\frac{fx}{4ad}+\frac{5f\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\,d^2}+\frac{7f\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\,d^2}+\frac{(4de+f)\left(\tan^6\left(\frac{dx}{2}\ldots\right.\right.}{2a\,d^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2/a*(-1/2*f*c*cos(d*x+c)^2+1/2*cos(d*x+c)^2*d*e-f*(-1/2*(d*x+c)*cos(d*x
+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*x+1/4*c)-sin(d*x+c)*c*f+sin(d*x+c)*d*
e+f*(cos(d*x+c)+(d*x+c)*sin(d*x+c)))
```

**Maxima [A]**
time = 0.28, size = 114, normalized size = 1.25

$$-\frac{\frac{4\left(\sin(dx+c)^2-2\,\sin(dx+c)\right)e}{a}-\frac{4\left(\sin(dx+c)^2-2\,\sin(dx+c)\right)cf}{ad}-\frac{(2\,(dx+c)\cos(2\,dx+2\,c)+8\,(dx+c)\sin(dx+c)+8\,\cos(dx+c)-\sin(2\,dx+2\,c))f}{ad}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(sin(d*x + c)^2 - 2*sin(d*x + c))*e/a - 4*(sin(d*x + c)^2 - 2*sin(d
*x + c))*c*f/(a*d) - (2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x +
c) + 8*cos(d*x + c) - sin(2*d*x + 2*c))*f/(a*d))/d
```

**Fricas [A]**
time = 0.35, size = 69, normalized size = 0.76

$$-\frac{dfx-2\,(dfx+de)\cos\left(dx+c\right)^2-4\,f\cos\left(dx+c\right)-\left(4\,dfx-f\cos\left(dx+c\right)+4\,de\right)\sin\left(dx+c\right)}{4\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d*f*x - 2*(d*f*x + d*e)*cos(d*x + c)^2 - 4*f*cos(d*x + c) - (4*d*f*x
- f*cos(d*x + c) + 4*d*e)*sin(d*x + c))/(a*d^2)
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 724 vs. 2(78) = 156.
time = 2.94, size = 724, normalized size = 7.96

$$\begin{cases} \frac{8de\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}-\frac{8de\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}+\frac{8de\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}+\frac{dfx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}+\frac{8dfx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}-\frac{6dfx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}+\frac{8dfx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}+\frac{dfx}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}+\frac{2f\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}+\frac{8f\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}-\frac{2f\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2}+\frac{8f}{4ad^2\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad^2} & \text{for}\: d \neq 0 \\ \frac{\left(ex+\frac{fx^2}{2}\right)\cos^3(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)

[Out] Piecewise((8*d*e*tan(c/2 + d*x/2)**3/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d*
*2*tan(c/2 + d*x/2)**2 + 4*a*d**2) - 8*d*e*tan(c/2 + d*x/2)**2/(4*a*d**2*ta
n(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + 8*d*e*tan(c/
2 + d*x/2)/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4
*a*d**2) + d*f*x*tan(c/2 + d*x/2)**4/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d*
*2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + 8*d*f*x*tan(c/2 + d*x/2)**3/(4*a*d**2*
tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) - 6*d*f*x*ta
n(c/2 + d*x/2)**2/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)
**2 + 4*a*d**2) + 8*d*f*x*tan(c/2 + d*x/2)/(4*a*d**2*tan(c/2 + d*x/2)**4 +
8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + d*f*x/(4*a*d**2*tan(c/2 + d*x/2)
**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + 2*f*tan(c/2 + d*x/2)**3/(4
*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + 8*
f*tan(c/2 + d*x/2)**2/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*
x/2)**2 + 4*a*d**2) - 2*f*tan(c/2 + d*x/2)/(4*a*d**2*tan(c/2 + d*x/2)**4 +
8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + 8*f/(4*a*d**2*tan(c/2 + d*x/2)**
4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2), Ne(d, 0)), ((e*x + f*x**2/2)*
cos(c)**3/(a*sin(c) + a), True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 26480 vs. $2(87) = 174$.
time = 10.74, size = 26480, normalized size = 290.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/4*(18*d^2*x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 - 6*d^2*x*e*tan(1/
2*d*x)^5*tan(3/2*c)*tan(1/2*c)^7 + d*f*x*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/
2*c)^7 - 18*d^2*x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 + 6*d^2*x*e*ta
n(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^6 - 18*d^2*x*e*tan(1/2*d*x)^4*tan(3/2*c)
^2*tan(1/2*c)^6 - 9*d*f*x*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 + 6*d^2*
x*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^7 - 9*d*f*x*tan(1/2*d*x)^4*tan(3/2
*c)^2*tan(1/2*c)^7 + d*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 - 60*d^2*
x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 + 90*d^2*x*e*tan(1/2*d*x)^5*ta
n(3/2*c)*tan(1/2*c)^5 - 18*d^2*x*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^5
 + 3*d*f*x*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 + 54*d*e*log(2*(tan(1/2
*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*c)^2*
tan(1/2*c)^5 - 18*d^2*x*e*tan(1/2*d*x)^5*tan(1/2*c)^6 + 6*d^2*x*e*tan(1/2*d

*x)^4*tan(3/2*c)*tan(1/2*c)^6 - 36*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^6 + 36*d^2*x*e*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^6 - d*f*x*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 + 21*d*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^6 + d*f*x*tan(1/2*d*x)^5*tan(1/2*c)^7 + 6*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^7 - 12*d^2*x*e*tan(1/2*d*x)^3*tan(3/2*c)*tan(1/2*c)^7 - 10*d*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^7 + 2*d*f*x*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^7 - 9*d*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^7 + 4*f*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^7 + 60*d^2*x*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^3 - 90*d^2*x*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^4 + 60*d^2*x*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^4 - 11*d*f*x*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 - 54*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^4 + 18*d^2*x*e*tan(1/2*d*x)^5*tan(1/2*c)^5 - 90*d^2*x*e*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^5 + 36*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^5 - 36*d^2*x*e*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c)^5 + 21*d*f*x*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^5 - 54*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^5 - 9*d*e*tan(1/2*d*x)^5*tan(3/2*c)^2*tan(1/2*c)^5 + 18*d^2*x*e*tan(1/2*d*x)^4*tan(1/2*c)^6 - 9*d*f*x*tan(1/2*d*x)^5*tan(1/2*c)^6 - 6*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^6 + 12*d^2*x*e*tan(1/2*d*x)^3*tan(3/2*c)*tan(1/2*c)^6 + 36*d*e*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(3/2*c)*tan(1/2*c)^6

```
+ 10*d*e*tan(1/2*d*x)^5*tan(3/2*c)*tan(1/2*c)^6 - 36*d^2*x*e*tan(1/2*d*x)^2
*tan(3/2*c)^2*tan(1/2*c)^6 + 30*d*f*x*tan(1/2*d*x)^3*tan(3/2*c)^2*tan(1/2*c
)^6 + 5*d*e*tan(1/2*d*x)^4*tan(3/2*c)^2*tan(1/2*c)^6 - 2*f*tan(1/2*d*x)^5*t
an(3/2*c)^2*tan(1/2*c)^6 - 9*d*f*x*tan(1/2*d*x)^4*tan(1/2*c)^7 - 6*d*e*log(
2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*
x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta...
```

**Mupad [B]**
time = 3.04, size = 84, normalized size = 0.92

$$-\frac{\frac{f\,\sin\left(2\,c+2\,d\,x\right)}{2}+8\,f\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,d\,e\,\sin\left(c+d\,x\right)+2\,d\,e\,\sin\left(c+d\,x\right)^2-4\,d\,f\,x\,\sin\left(c+d\,x\right)+d\,f\,x\,\left(2\,\sin\left(c+d\,x\right)^2-1\right)}{4\,a\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(e + f*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] -((f*sin(2*c + 2*d*x))/2 + 8*f*sin(c/2 + (d*x)/2)^2 - 4*d*e*sin(c + d*x) +
2*d*e*sin(c + d*x)^2 - 4*d*f*x*sin(c + d*x) + d*f*x*(2*sin(c + d*x)^2 - 1))
/(4*a*d^2)
```

# 3.266 $\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=32

$$\frac{\sin(c+dx)}{ad} - \frac{\sin^2(c+dx)}{2ad}$$

```
[Out] sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d
```

**Rubi** [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2746\}$

$$\frac{\sin(c+dx)}{ad} - \frac{\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)\,dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\sin(c+dx)}{ad} - \frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

**Mathematica** [A]
time = 0.04, size = 24, normalized size = 0.75

$$-\frac{(-2+\sin(c+dx))\sin(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
[Out] -1/2*((-2 + Sin[c + d*x])*Sin[c + d*x])/(a*d)
```

**Maple** [A]
time = 0.08, size = 28, normalized size = 0.88

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)}{ad}$ | 28 |
| default | $-\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)}{ad}$ | 28 |
| risch | $\frac{\sin(dx+c)}{ad}+\frac{\cos(2dx+2c)}{4ad}$ | 32 |
| norman | $\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$ | 105 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/a/d*(1/2*sin(d*x+c)^2-sin(d*x+c))
```

**Maxima** [A]
time = 0.28, size = 25, normalized size = 0.78

$$-\frac{\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

**Fricas** [A]
time = 0.37, size = 25, normalized size = 0.78

$$\frac{\cos\left(dx+c\right)^2+2\,\sin\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(cos(d*x + c)^2 + 2*sin(d*x + c))/(a*d)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(22) = 44$.

time = 2.19, size = 158, normalized size = 4.94

$$\begin{cases} \frac{2\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^3(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2 +
 d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*
tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**4 +
2*a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a), T
rue))
```

**Giac** [A]
time = 4.62, size = 25, normalized size = 0.78

$$-\frac{\sin(dx+c)^2 - 2\sin(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

**Mupad** [B]
time = 2.64, size = 22, normalized size = 0.69

$$-\frac{\sin(c+d\,x)\,(\sin(c+d\,x)-2)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)*(sin(c + d*x) - 2))/(2*a*d)
```

# 3.267 $\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=128

$$\frac{\cos\left(c-\frac{de}{f}\right)\text{Ci}\left(\frac{de}{f}+dx\right)}{af}-\frac{\text{Ci}\left(\frac{2de}{f}+2dx\right)\sin\left(2c-\frac{2de}{f}\right)}{2af}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}-\frac{\cos\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}\right.}{2af}$$

```
[Out] Ci(d*e/f+d*x)*cos(c-d*e/f)/a/f-1/2*cos(2*c-2*d*e/f)*Si(2*d*e/f+2*d*x)/a/f-1
/2*Ci(2*d*e/f+2*d*x)*sin(2*c-2*d*e/f)/a/f-Si(d*e/f+d*x)*sin(c-d*e/f)/a/f
```

**Rubi** [A]
time = 0.20, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4619, 3384, 3380, 3383, 4491, 12}

$$-\frac{\sin\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)}{2af}+\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}-\frac{\cos\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Cos[c - (d*e)/f]*CosIntegral[(d*e)/f + d*x])/(a*f) - (CosIntegral[(2*d*e)/
f + 2*d*x]*Sin[2*c - (2*d*e)/f])/(2*a*f) - (Sin[c - (d*e)/f]*SinIntegral[(d
*e)/f + d*x])/(a*f) - (Cos[2*c - (2*d*e)/f]*SinIntegral[(2*d*e)/f + 2*d*x])
/(2*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx &= \frac{\int \frac{\cos(c+dx)}{e+fx}\,dx}{a} - \frac{\int \frac{\cos(c+dx)\sin(c+dx)}{e+fx}\,dx}{a}\\ &= -\frac{\int \frac{\sin(2c+2dx)}{2(e+fx)}\,dx}{a} + \frac{\cos\left(c-\frac{de}{f}\right)\int \frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a} - \frac{\sin\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{d}{f}\right.}{e+}}{a}\\ &= \frac{\cos\left(c-\frac{de}{f}\right)\mathrm{Ci}\left(\frac{de}{f}+dx\right)}{af} - \frac{\sin\left(c-\frac{de}{f}\right)\mathrm{Si}\left(\frac{de}{f}+dx\right)}{af} - \frac{\int \frac{\sin(2c+2dx)}{e+fx}}{2a}\\ &= \frac{\cos\left(c-\frac{de}{f}\right)\mathrm{Ci}\left(\frac{de}{f}+dx\right)}{af} - \frac{\sin\left(c-\frac{de}{f}\right)\mathrm{Si}\left(\frac{de}{f}+dx\right)}{af} - \frac{\cos\left(2c-\frac{2d}{f}\right.}{}\\ &= \frac{\cos\left(c-\frac{de}{f}\right)\mathrm{Ci}\left(\frac{de}{f}+dx\right)}{af} - \frac{\mathrm{Ci}\left(\frac{2de}{f}+2dx\right)\sin\left(2c-\frac{2de}{f}\right)}{2af} - \frac{\sin\left(c-\right.}{}\end{aligned}$$

**Mathematica [A]**
time = 0.31, size = 105, normalized size = 0.82

$$-\frac{-2\cos\left(c-\frac{de}{f}\right)\mathrm{Ci}\left(d\left(\frac{e}{f}+x\right)\right)+\mathrm{Ci}\left(\frac{2d(e+fx)}{f}\right)\sin\left(2c-\frac{2de}{f}\right)+2\sin\left(c-\frac{de}{f}\right)\mathrm{Si}\left(d\left(\frac{e}{f}+x\right)\right)+\cos\left(2c-\frac{2de}{f}\right)\mathrm{Si}\left(\frac{2d(e+fx)}{f}\right)}{2af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] -1/2*(-2*Cos[c - (d*e)/f]*CosIntegral[d*(e/f + x)] + CosIntegral[(2*d*(e +
f*x))/f]*Sin[2*c - (2*d*e)/f] + 2*Sin[c - (d*e)/f]*SinIntegral[d*(e/f + x)]
 + Cos[2*c - (2*d*e)/f]*SinIntegral[(2*d*(e + f*x))/f])/(a*f)
```

**Maple** [A]
time = 0.18, size = 163, normalized size = 1.27

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\mathrm{sinIntegral}\left(-2dx-2c-\frac{2(-cf+de)}{f}\right)\cos\left(\frac{-2cf+2de}{f}\right)}{2f}+\frac{\mathrm{cosineIntegral}\left(2dx+2c+\frac{-2cf+2de}{f}\right)\sin\left(\frac{-2cf+2de}{f}\right)}{2f}-\frac{\mathrm{sinIntegral}\left(-dx-c-\right.}{}}{a}$ |
| default | $\frac{\frac{\mathrm{sinIntegral}\left(-2dx-2c-\frac{2(-cf+de)}{f}\right)\cos\left(\frac{-2cf+2de}{f}\right)}{2f}+\frac{\mathrm{cosineIntegral}\left(2dx+2c+\frac{-2cf+2de}{f}\right)\sin\left(\frac{-2cf+2de}{f}\right)}{2f}-\frac{\mathrm{sinIntegral}\left(-dx-c-\right.}{}}{a}$ |
| risch | $-\frac{e^{-\frac{i(cf-de)}{f}}\mathrm{expIntegral}\left(1,idx+ic-\frac{i(cf-de)}{f}\right)}{2af}-\frac{e^{\frac{i(cf-de)}{f}}\mathrm{expIntegral}\left(1,-idx-ic-\frac{-icf+ide}{f}\right)}{2af}-\frac{ie^{\frac{2i(cf-de)}{f}}\mathrm{expI}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*Si(-2*d*x-2*c-2*(-c*f+d*e)/f)*cos(2*(-c*f+d*e)/f)/f+1/2*Ci(2*d*x+2
*c+2*(-c*f+d*e)/f)*sin(2*(-c*f+d*e)/f)/f-Si(-d*x-c-(-c*f+d*e)/f)*sin((-c*f+
d*e)/f)/f+Ci(d*x+c+(-c*f+d*e)/f)*cos((-c*f+d*e)/f)/f)
```

**Maxima** [C] Result contains complex when optimal does not.
time = 0.38, size = 282, normalized size = 2.20

$$-\frac{2\,d\left(E_1\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_1\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)-d\left(-i\,E_1\left(\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)+i\,E_1\left(-\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)\right)\cos\left(-\frac{2\,(de-cf)}{f}\right)+2\,d\left(-i\,E_1\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+i\,E_1\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\sin\left(-\frac{de-cf}{f}\right)-d\left(E_1\left(\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)+E_1\left(-\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)\right)\sin\left(-\frac{2\,(de-cf)}{f}\right)}{4\,adf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*(exp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_integ
ral_e(1, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) - d*(-I*e
xp_integral_e(1, 2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f) + I*exp_integral_e(1
, -2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f))*cos(-2*(d*e - c*f)/f) + 2*d*(-I*e
xp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + I*exp_integral_e(1, -
(I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) - d*(exp_integral_e
(1, 2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f) + exp_integral_e(1, -2*(-I*d*e -
I*(d*x + c)*f + I*c*f)/f))*sin(-2*(d*e - c*f)/f))/(a*d*f)
```

**Fricas** [A]
time = 0.36, size = 166, normalized size = 1.30

$$\frac{2\left(\mathrm{Ci}\left(\frac{dfx+de}{f}\right)+\mathrm{Ci}\left(-\frac{dfx+de}{f}\right)\right)\cos\left(-\frac{cf-de}{f}\right)+\left(\mathrm{Ci}\left(\frac{2\,(dfx+de)}{f}\right)+\mathrm{Ci}\left(-\frac{2\,(dfx+de)}{f}\right)\right)\sin\left(-\frac{2\,(cf-de)}{f}\right)-2\,\cos\left(-\frac{2\,(cf-de)}{f}\right)\mathrm{Si}\left(\frac{2\,(dfx+de)}{f}\right)+4\,\sin\left(-\frac{cf-de}{f}\right)\mathrm{Si}\left(\frac{dfx+de}{f}\right)}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(cos_integral((d*f*x + d*e)/f) + cos_integral(-(d*f*x + d*e)/f))*cos
(-(c*f - d*e)/f) + (cos_integral(2*(d*f*x + d*e)/f) + cos_integral(-2*(d*f*
x + d*e)/f))*sin(-2*(c*f - d*e)/f) - 2*cos(-2*(c*f - d*e)/f)*sin_integral(2
*(d*f*x + d*e)/f) + 4*sin(-(c*f - d*e)/f)*sin_integral((d*f*x + d*e)/f))/(a
*f)
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

**Giac** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.61, size = 4828, normalized size = 37.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(3*pi + 3*pi*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 2*imag_part(
cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 +
 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*tan(
1/2*d*e/f)^2 - 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^4*tan(d*e/
f)^2*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^
4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 4*sin_integral(2*(d*f*x + d*e)/f)*tan(1/2
*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 8*imag_part(cos_integral(d*x + d*e/f)
)*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) - 8*imag_part(cos_integral(-d*x
- d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) + 16*sin_integral((d*f*x
 + d*e)/f)*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) - 4*real_part(cos_integ
ral(2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)*tan(1/2*d*e/f)^2 - 4*real_par
t(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)*tan(1/2*d*e/f)^2
- 8*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*
d*e/f)^2 + 8*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^3*tan(d*e/f)^
2*tan(1/2*d*e/f)^2 + 8*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^
3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 8*real_part(cos_integral(-2*d*x - 2*d*e/f
))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 16*sin_integral((d*f*x + d*
e)/f)*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 3*pi*tan(1/2*c)^4*tan(d*
```

e/f)^2 - 2*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)
^2 + 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2
+ 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2 + 4*real
_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2 - 4*sin_integra
l(2*(d*f*x + d*e)/f)*tan(1/2*c)^4*tan(d*e/f)^2 - 16*real_part(cos_integral(
d*x + d*e/f))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f) - 16*real_part(cos_i
ntegral(-d*x - d*e/f))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f) + 3*pi*tan(
1/2*c)^4*tan(1/2*d*e/f)^2 + 2*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(
1/2*c)^4*tan(1/2*d*e/f)^2 - 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan
(1/2*c)^4*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2
*c)^4*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)
^4*tan(1/2*d*e/f)^2 + 4*sin_integral(2*(d*f*x + d*e)/f)*tan(1/2*c)^4*tan(1/
2*d*e/f)^2 - 16*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^3*tan(d
*e/f)*tan(1/2*d*e/f)^2 + 16*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1
/2*c)^3*tan(d*e/f)*tan(1/2*d*e/f)^2 - 32*sin_integral(2*(d*f*x + d*e)/f)*ta
n(1/2*c)^3*tan(d*e/f)*tan(1/2*d*e/f)^2 + 6*pi*tan(1/2*c)^2*tan(d*e/f)^2*tan
(1/2*d*e/f)^2 + 12*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^2*ta
n(d*e/f)^2*tan(1/2*d*e/f)^2 - 12*imag_part(cos_integral(-2*d*x - 2*d*e/f))*
tan(1/2*c)^2*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 24*sin_integral(2*(d*f*x + d*e
)/f)*tan(1/2*c)^2*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(
2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f) - 4*real_part(cos_integral(-2*d*x
 - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f) + 8*imag_part(cos_integral(d*x + d*e/f
))*tan(1/2*c)^3*tan(d*e/f)^2 - 8*imag_part(cos_integral(-d*x - d*e/f))*tan(
1/2*c)^3*tan(d*e/f)^2 + 8*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*
c)^3*tan(d*e/f)^2 + 8*real_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^
3*tan(d*e/f)^2 + 16*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)^3*tan(d*e/f)^2
 + 8*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^4*tan(1/2*d*e/f) - 8*i
mag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^4*tan(1/2*d*e/f) + 16*sin_i
ntegral((d*f*x + d*e)/f)*tan(1/2*c)^4*tan(1/2*d*e/f) - 8*imag_part(cos_inte
gral(d*x + d*e/f))*tan(1/2*c)^3*tan(1/2*d*e/f)^2 + 8*imag_part(cos_integral
(-d*x - d*e/f))*tan(1/2*c)^3*tan(1/2*d*e/f)^2 - 8*real_part(cos_integral(2*
d*x + 2*d*e/f))*tan(1/2*c)^3*tan(1/2*d*e/f)^2 - 8*real_part(cos_integral(-2
*d*x - 2*d*e/f))*tan(1/2*c)^3*tan(1/2*d*e/f)^2 - 16*sin_integral((d*f*x + d
*e)/f)*tan(1/2*c)^3*tan(1/2*d*e/f)^2 + 24*real_part(cos_integral(2*d*x + 2*
d*e/f))*tan(1/2*c)^2*tan(d*e/f)*tan(1/2*d*e/f)^2 + 24*real_part(cos_integra
l(-2*d*x - 2*d*e/f))*tan(1/2*c)^2*tan(d*e/f)*tan(1/2*d*e/f)^2 - 8*imag_part
(cos_integral(d*x + d*e/f))*tan(1/2*c)*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 8*im
ag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)*tan(d*e/f)^2*tan(1/2*d*e/f)^
2 - 8*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)*tan(d*e/f)^2*tan(
1/2*d*e/f)^2 - 8*real_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)*tan(d
*e/f)^2*tan(1/2*d*e/f)^2 - 16*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)*tan(
d*e/f)^2*tan(1/2*d*e/f)^2 + 3*pi*tan(1/2*c)^4 + 2*imag_part(cos_integral(2*
d*x + 2*d*e/f))*tan(1/2*c)^4 - 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*
tan(1/2*c)^4 + 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^4 + 4*real
_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^4 + 4*sin_integral(2*(d*f*x +

```
d*e)/f)*tan(1/2*c)^4 - 16*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*
c)^3*tan(d*e/f) + 16*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^3
*tan(d*e/f) - 32*sin_integral(2*(d*f*x + d*e)/f)*tan(1/2*c)^3*tan(d*e/f) +
6*pi*tan(1/2*c)^2*tan(d*e/f)^2 + 12*imag_part(c...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)^3}{(e+f\,x)\ (a+a\ \sin (c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/((e + f*x)*(a + a*sin(c + d*x))),x)
[Out] int(cos(c + d*x)^3/((e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.268 $\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=175

$$-\frac{\cos(c+dx)}{af(e+fx)}-\frac{d\cos\left(2c-\frac{2de}{f}\right)\mathrm{Ci}\left(\frac{2de}{f}+2dx\right)}{af^2}-\frac{d\mathrm{Ci}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af^2}+\frac{\sin(2c+2dx)}{2af(e+fx)}-\frac{d\cos\left(c-\frac{de}{f}\right)}{af}$$

```
[Out] -d*Ci(2*d*e/f+2*d*x)*cos(2*c-2*d*e/f)/a/f^2-cos(d*x+c)/a/f/(f*x+e)-d*cos(c-
d*e/f)*Si(d*e/f+d*x)/a/f^2+d*Si(2*d*e/f+2*d*x)*sin(2*c-2*d*e/f)/a/f^2-d*Ci(
d*e/f+d*x)*sin(c-d*e/f)/a/f^2+1/2*sin(2*d*x+2*c)/a/f/(f*x+e)
```

**Rubi** [A]

time = 0.22, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4619, 3378, 3384, 3380, 3383, 4491, 12}

$$-\frac{d\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af^2}-\frac{d\cos\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)}{af^2}+\frac{d\sin\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{af^2}-\frac{d\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{\sin(2c+2dx)}{2af(e+fx)}-\frac{\cos(c+dx)}{af(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] -(Cos[c + d*x]/(a*f*(e + f*x))) - (d*Cos[2*c - (2*d*e)/f]*CosIntegral[(2*d*
e)/f + 2*d*x])/(a*f^2) - (d*CosIntegral[(d*e)/f + d*x]*Sin[c - (d*e)/f])/(a
*f^2) + Sin[2*c + 2*d*x]/(2*a*f*(e + f*x)) - (d*Cos[c - (d*e)/f]*SinIntegra
l[(d*e)/f + d*x])/(a*f^2) + (d*Sin[2*c - (2*d*e)/f]*SinIntegral[(2*d*e)/f +
 2*d*x])/(a*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\cos(c+dx)}{(e+fx)^2}\,dx}{a} - \frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}\,dx}{a}$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{\int \frac{\sin(2c+2dx)}{2(e+fx)^2}\,dx}{a} - \frac{d\int \frac{\sin(c+dx)}{e+fx}\,dx}{af}$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{\int \frac{\sin(2c+2dx)}{(e+fx)^2}\,dx}{2a} - \frac{\left(d\cos\left(c-\frac{de}{f}\right)\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{af} -$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{d\text{Ci}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af^2} + \frac{\sin(2c+2dx)}{2af(e+fx)} - \frac{d\cos\left(\right.}{}$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{d\text{Ci}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af^2} + \frac{\sin(2c+2dx)}{2af(e+fx)} - \frac{d\cos\left(\right.}{}$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{d\cos\left(2c-\frac{2de}{f}\right)\text{Ci}\left(\frac{2de}{f}+2dx\right)}{af^2} - \frac{d\text{Ci}\left(\frac{de}{f}+dx\right)\sin\left(\right.}{af^2}$$

## Mathematica [A]

time = 0.50, size = 203, normalized size = 1.16

$$\frac{-2f\cos(c+dx) - 2d(e+fx)\cos\left(2c-\frac{2de}{f}\right)\text{Ci}\left(\frac{2d(e+fx)}{f}\right) - 2d(e+fx)\text{Ci}\left(d\left(\frac{e}{f}+x\right)\right)\sin\left(c-\frac{de}{f}\right) + f\sin(2(c+dx)) - 2de\cos\left(c-\frac{de}{f}\right)\text{Si}\left(d\left(\frac{e}{f}+x\right)\right) - 2dfx\cos\left(c-\frac{de}{f}\right)\text{Si}\left(d\left(\frac{e}{f}+x\right)\right) + 2de\sin\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2d(e+fx)}{f}\right) + 2dfx\sin\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2d(e+fx)}{f}\right)}{2af^2(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] (-2*f*Cos[c + d*x] - 2*d*(e + f*x)*Cos[2*c - (2*d*e)/f]*CosIntegral[(2*d*(e
 + f*x))/f] - 2*d*(e + f*x)*CosIntegral[d*(e/f + x)]*Sin[c - (d*e)/f] + f*S
in[2*(c + d*x)] - 2*d*e*Cos[c - (d*e)/f]*SinIntegral[d*(e/f + x)] - 2*d*f*x
*Cos[c - (d*e)/f]*SinIntegral[d*(e/f + x)] + 2*d*e*Sin[2*c - (2*d*e)/f]*Sin
Integral[(2*d*(e + f*x))/f] + 2*d*f*x*Sin[2*c - (2*d*e)/f]*SinIntegral[(2*d
*(e + f*x))/f])/(2*a*f^2*(e + f*x))
```

## Maple [A]

time = 0.26, size = 235, normalized size = 1.34

| method | result |
|---|---|
| derivativedivides | $-\frac{d\left(-\frac{\sin(2dx+2c)}{2(-cf+de+f(dx+c))f}+\frac{-\frac{2\,\text{sinIntegral}\left(-2dx-2c-\frac{2(-cf+de)}{f}\right)\sin\left(\frac{-2cf+2de}{f}\right)}{f}+\frac{2\,\text{cosineIntegral}\left(2dx+2c+\frac{-2cf+2de}{f}\right)\text{co}}{f}}{2f}\right.}{}$ |

| | |
|---|---|
| default | $-\frac{d\left(-\frac{\sin(2dx+2c)}{2(-cf+de+f(dx+c))f}+\frac{-\frac{2\,\mathrm{sinIntegral}\left(-2dx-2c-\frac{2(-cf+de)}{f}\right)\sin\left(\frac{-2cf+2de}{f}\right)}{f}+\frac{2\,\mathrm{cosineIntegral}\left(2dx+2c+\frac{-2cf+2de}{f}\right)}{f}}{2f}\right.}{}$ |
| risch | $\frac{id\,\mathrm{e}^{-\frac{i(cf-de)}{f}}\,\mathrm{expIntegral}\left(1,idx+ic-\frac{i(cf-de)}{f}\right)}{2a\,f^{2}}-\frac{id\,\mathrm{e}^{\frac{i(cf-de)}{f}}\,\mathrm{expIntegral}\left(1,-idx-ic-\frac{-icf+ide}{f}\right)}{2a\,f^{2}}+\frac{d\,\mathrm{e}^{\frac{2i(cf-de)}{f}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -d/a*(-1/2*sin(2*d*x+2*c)/(-c*f+d*e+f*(d*x+c))/f+1/2*(-2*Si(-2*d*x-2*c-2*(-
c*f+d*e)/f)*sin(2*(-c*f+d*e)/f)/f+2*Ci(2*d*x+2*c+2*(-c*f+d*e)/f)*cos(2*(-c*
f+d*e)/f)/f)/f+cos(d*x+c)/(-c*f+d*e+f*(d*x+c))/f+(-Si(-d*x-c-(-c*f+d*e)/f)*
cos((-c*f+d*e)/f)/f-Ci(d*x+c+(-c*f+d*e)/f)*sin((-c*f+d*e)/f)/f)/f)
```

**Maxima [C]** Result contains complex when optimal does not.
time = 0.40, size = 309, normalized size = 1.77

$$-\frac{2\,d^{2}\left(E_{2}\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_{2}\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)-d^{2}\left(-i\,E_{2}\left(\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)+i\,E_{2}\left(-\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)\right)\cos\left(-\frac{2\,(de-cf)}{f}\right)+2\,d^{2}\left(-i\,E_{2}\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+i\,E_{2}\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\sin\left(-\frac{de-cf}{f}\right)-d^{2}\left(E_{2}\left(\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)+E_{2}\left(-\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)\right)\sin\left(-\frac{2\,(de-cf)}{f}\right)}{4\,(adef+(dx+c)af^{2}-acf^{2})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*d^2*(exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_int
egral_e(2, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) - d^2*(
-I*exp_integral_e(2, 2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f) + I*exp_integral
_e(2, -2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f))*cos(-2*(d*e - c*f)/f) + 2*d^2
*(-I*exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + I*exp_integral_
e(2, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) - d^2*(exp_in
tegral_e(2, 2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f) + exp_integral_e(2, -2*(-
I*d*e - I*(d*x + c)*f + I*c*f)/f))*sin(-2*(d*e - c*f)/f))/((a*d*e*f + (d*x
+ c)*a*f^2 - a*c*f^2)*d)
```

**Fricas [A]**
time = 0.37, size = 258, normalized size = 1.47

$$\frac{2\,f\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,(dfx+de)\sin\left(-\frac{2\,(cf-de)}{f}\right)\mathrm{Si}\left(\frac{2\,(dfx+de)}{f}\right)-2\,(dfx+de)\cos\left(-\frac{cf-de}{f}\right)\mathrm{Si}\left(\frac{dfx+de}{f}\right)-2\,f\cos\left(dx+c\right)-\left((dfx+de)\,\mathrm{Ci}\left(\frac{2\,(dfx+de)}{f}\right)+(dfx+de)\,\mathrm{Ci}\left(-\frac{2\,(dfx+de)}{f}\right)\right)\cos\left(-\frac{2\,(cf-de)}{f}\right)+\left((dfx+de)\,\mathrm{Ci}\left(\frac{dfx+de}{f}\right)+(dfx+de)\,\mathrm{Ci}\left(-\frac{dfx+de}{f}\right)\right)\sin\left(-\frac{cf-de}{f}\right)}{2\,(af^{3}x+af^{2}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*f*cos(d*x + c)*sin(d*x + c) - 2*(d*f*x + d*e)*sin(-2*(c*f - d*e)/f)*
sin_integral(2*(d*f*x + d*e)/f) - 2*(d*f*x + d*e)*cos(-(c*f - d*e)/f)*sin_i
ntegral((d*f*x + d*e)/f) - 2*f*cos(d*x + c) - ((d*f*x + d*e)*cos_integral(2
*(d*f*x + d*e)/f) + (d*f*x + d*e)*cos_integral(-2*(d*f*x + d*e)/f))*cos(-2*
```

```
(c*f - d*e)/f) + ((d*f*x + d*e)*cos_integral((d*f*x + d*e)/f) + (d*f*x + d*
e)*cos_integral(-(d*f*x + d*e)/f))*sin(-(c*f - d*e)/f))/(a*f^3*x + a*f^2*e)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.47, size = 49990, normalized size = 285.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(d*f*x*imag_part(cos_integral(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*
tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - d*f*x*imag_part(cos_integral(-
d*x - d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d
*e/f)^2 - d*f*x*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(d*x)^2*tan(1/2
*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - d*f*x*real_part(cos_in
tegral(-2*d*x - 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)
^2*tan(1/2*d*e/f)^2 + 2*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(d*x)^2*tan(
1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 2*d*f*x*real_part(c
os_integral(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)
^2*tan(1/2*d*e/f) + 2*d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(d*x)^
2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) + 2*d*f*x*imag_pa
rt(cos_integral(2*d*x + 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*ta
n(d*e/f)*tan(1/2*d*e/f)^2 - 2*d*f*x*imag_part(cos_integral(-2*d*x - 2*d*e/f
))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)*tan(1/2*d*e/f)^2 + 4*d
*f*x*sin_integral(2*(d*f*x + d*e)/f)*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4
*tan(d*e/f)*tan(1/2*d*e/f)^2 - 4*d*f*x*imag_part(cos_integral(2*d*x + 2*d*e
/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 +
 4*d*f*x*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^
2*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 2*d*f*x*real_part(cos_integr
al(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/
2*d*e/f)^2 - 2*d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(d*x)^2*tan(1
/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 8*d*f*x*sin_integral
(2*(d*f*x + d*e)/f)*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)^2*tan
(1/2*d*e/f)^2 + d*e*imag_part(cos_integral(d*x + d*e/f))*tan(d*x)^2*tan(1/2
*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - d*e*imag_part(cos_inte
```

```
gral(-d*x - d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan
(1/2*d*e/f)^2 - d*e*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(d*x)^2*tan
(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - d*e*real_part(cos_
integral(-2*d*x - 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/
f)^2*tan(1/2*d*e/f)^2 + 2*d*e*sin_integral((d*f*x + d*e)/f)*tan(d*x)^2*tan(
1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - d*f*x*imag_part(cos
_integral(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2
 + d*f*x*imag_part(cos_integral(-d*x - d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*ta
n(1/2*c)^4*tan(d*e/f)^2 - d*f*x*real_part(cos_integral(2*d*x + 2*d*e/f))*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2 - d*f*x*real_part(cos_int
egral(-2*d*x - 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^
2 - 2*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2
*c)^4*tan(d*e/f)^2 + 4*d*f*x*imag_part(cos_integral(d*x + d*e/f))*tan(d*x)^
2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f) - 4*d*f*x*imag_pa
rt(cos_integral(-d*x - d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(d
*e/f)^2*tan(1/2*d*e/f) + 8*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(d*x)^2*t
an(1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f) + 2*d*e*real_part(co
s_integral(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^
2*tan(1/2*d*e/f) + 2*d*e*real_part(cos_integral(-d*x - d*e/f))*tan(d*x)^2*t
an(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) + d*f*x*imag_part(co
s_integral(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(1/2*d*e
/f)^2 - d*f*x*imag_part(cos_integral(-d*x - d*e/f))*tan(d*x)^2*tan(1/2*d*x)
^2*tan(1/2*c)^4*tan(1/2*d*e/f)^2 + d*f*x*real_part(cos_integral(2*d*x + 2*d
*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(1/2*d*e/f)^2 + d*f*x*real
_part(cos_integral(-2*d*x - 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^
4*tan(1/2*d*e/f)^2 + 2*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(d*x)^2*tan(1
/2*d*x)^2*tan(1/2*c)^4*tan(1/2*d*e/f)^2 - 8*d*f*x*real_part(cos_integral(2*
d*x + 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)*tan(1/2*d
*e/f)^2 - 8*d*f*x*real_part(cos_integral(-2*d*x - 2*d*e/f))*tan(d*x)^2*tan(
1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)*tan(1/2*d*e/f)^2 + 2*d*e*imag_part(cos_i
ntegral(2*d*x + 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)
*tan(1/2*d*e/f)^2 - 2*d*e*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(d*x
)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)*tan(1/2*d*e/f)^2 + 4*d*e*sin_int
egral(2*(d*f*x + d*e)/f)*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)*
tan(1/2*d*e/f)^2 + 6*d*f*x*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(d*x
)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 6*d*f*x*rea
l_part(cos_integral(-2*d*x - 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 4*d*e*imag_part(cos_integral(2*d*x + 2*d
*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2
 + 4*d*e*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^
2*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 2*d*e*real_part(cos_integral
(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/...
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/((e + f*x)^2*(a + a*sin(c + d*x))),x)
[Out] int(cos(c + d*x)^3/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.269 $\int \frac{(e+fx)^3 \sec(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=502

$$-\frac{3if(e+fx)^2}{2ad^2}-\frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3}-\frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{3f^2(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{ad^3}+$$

```
[Out] -3/2*I*f*(f*x+e)^2/a/d^2-I*(f*x+e)^3*arctan(exp(I*(d*x+c)))/a/d+3*I*f^3*pol
ylog(2,-I*exp(I*(d*x+c)))/a/d^4+3*f^2*(f*x+e)*ln(1+exp(2*I*(d*x+c)))/a/d^3-
6*I*f^2*(f*x+e)*arctan(exp(I*(d*x+c)))/a/d^3-3*I*f^3*polylog(2,I*exp(I*(d*x
+c)))/a/d^4-3/2*I*f^3*polylog(2,-exp(2*I*(d*x+c)))/a/d^4-3*I*f^3*polylog(4,
-I*exp(I*(d*x+c)))/a/d^4-3/2*I*f*(f*x+e)^2*polylog(2,I*exp(I*(d*x+c)))/a/d^
2-3*f^2*(f*x+e)*polylog(3,-I*exp(I*(d*x+c)))/a/d^3+3*f^2*(f*x+e)*polylog(3,
I*exp(I*(d*x+c)))/a/d^3+3/2*I*f*(f*x+e)^2*polylog(2,-I*exp(I*(d*x+c)))/a/d^
2+3*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4-3/2*f*(f*x+e)^2*sec(d*x+c)/a/d^
2-1/2*(f*x+e)^3*sec(d*x+c)^2/a/d+3/2*f*(f*x+e)^2*tan(d*x+c)/a/d^2+1/2*(f*x+
e)^3*sec(d*x+c)*tan(d*x+c)/a/d
```

**Rubi** [A]

time = 0.32, antiderivative size = 502, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4627, 4271, 4266, 2317, 2438, 2611, 6744, 2320, 6724, 4494, 4269, 3800, 2221}

$$\frac{3if^3\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^4}-\frac{3if^3\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^4}-\frac{3if^3\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2ad^4}-\frac{3if^3\text{PolyLog}\left(4,-ie^{i(c+dx)}\right)}{ad^4}+\frac{3if^3\text{PolyLog}\left(4,ie^{i(c+dx)}\right)}{ad^4}-\frac{3f^2(e+fx)\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^3}+\frac{3f^2(e+fx)\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}+\frac{3if(e+fx)^2\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{2ad^2}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{2ad^2}-\frac{6if^2(e+fx)\text{ArcTan}\left(e^{i(c+dx)}\right)}{ad^3}-\frac{i(e+fx)^3\text{ArcTan}\left(e^{i(c+dx)}\right)}{ad}+\frac{3f^2(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{ad^3}+\frac{3f(e+fx)^2\tan(c+dx)}{2ad^2}-\frac{3f(e+fx)^2\sec(c+dx)}{2ad^2}-\frac{(e+fx)^3\sec^2(c+dx)}{2ad}+\frac{(e+fx)^3\tan(c+dx)\sec(c+dx)}{2ad}-\frac{3if(e+fx)^2}{2ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (((-3*I)/2)*f*(e + f*x)^2)/(a*d^2) - ((6*I)*f^2*(e + f*x)*ArcTan[E^(I*(c +
d*x))])/(a*d^3) - (I*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/(a*d) + (3*f^2*(e
 + f*x)*Log[1 + E^((2*I)*(c + d*x))])/(a*d^3) + ((3*I)*f^3*PolyLog[2, (-I)*
E^(I*(c + d*x))])/(a*d^4) + (((3*I)/2)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(I*(
c + d*x))])/(a*d^2) - ((3*I)*f^3*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^4) - (
((3*I)/2)*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) - (((3*I)/2)
*f^3*PolyLog[2, -E^((2*I)*(c + d*x))])/(a*d^4) - (3*f^2*(e + f*x)*PolyLog[3
, (-I)*E^(I*(c + d*x))])/(a*d^3) + (3*f^2*(e + f*x)*PolyLog[3, I*E^(I*(c +
d*x))])/(a*d^3) - ((3*I)*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))])/(a*d^4) + ((
3*I)*f^3*PolyLog[4, I*E^(I*(c + d*x))])/(a*d^4) - (3*f*(e + f*x)^2*Sec[c +
d*x])/(2*a*d^2) - ((e + f*x)^3*Sec[c + d*x]^2)/(2*a*d) + (3*f*(e + f*x)^2*T
an[c + d*x])/(2*a*d^2) + ((e + f*x)^3*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4627

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sec^3(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\sec^2(c+dx)\tan(c+dx)\,dx}{a}$$
$$= -\frac{3f(e+fx)^2\sec(c+dx)}{2ad^2} - \frac{(e+fx)^3\sec^2(c+dx)}{2ad} + \frac{(e+fx)^3\sec(c+dx)\,\mathrm{ta}}{2ad}$$
$$= -\frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{3f(e+fx)^2\,\mathrm{se}}{2ad^2}$$
$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad}$$
$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad}$$
$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad}$$
$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1438 vs. $2(502) = 1004$.
time = 8.89, size = 1438, normalized size = 2.86

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(8*a*(Cos[c/2] - Sin[c/2])*
(Cos[c/2] + Sin[c/2])) - ((Cos[c] + I*Sin[c])*((-I)*e^3*x - ((3*I)/2)*e^2*f
*x^2 - I*e*f^2*x^3 - (I/4)*f^3*x^4 + (e^3*Log[1 + I*Cos[c + d*x] - Sin[c +
d*x]])/d + (3*e^2*f*x*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]])/d + (3*e*f^2*
x^2*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]])/d + (f^3*x^3*Log[1 + I*Cos[c +
d*x] - Sin[c + d*x]])/d + ((6*I)*f^3*PolyLog[4, (-I)*Cos[c + d*x] + Sin[c +
 d*x]])/d^4 - (I*e^3*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin
[c]))/d - ((3*I)*e^2*f*x*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I
*Sin[c]))/d - ((3*I)*e*f^2*x^2*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[
c] - I*Sin[c]))/d - (I*f^3*x^3*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[
c] - I*Sin[c]))/d + (6*f^3*PolyLog[4, (-I)*Cos[c + d*x] + Sin[c + d*x]]*(Co
s[c] - I*Sin[c]))/d^4 + (6*f^2*(e + f*x)*PolyLog[3, (-I)*Cos[c + d*x] + Sin
[c + d*x]]*(Cos[c] + I*(-1 + Sin[c]))*(Cos[c] - I*Sin[c]))/d^3 + (3*f*(e +
f*x)^2*PolyLog[2, (-I)*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(-1
 - I*Cos[c] + Sin[c]))/d^2))/(2*a*(Cos[c] + I*(-1 + Sin[c]))) - ((Cos[c] +
I*Sin[c])*(I*d^2*e^3*x + (12*I)*e*f^2*x + ((3*I)/2)*d^2*e^2*f*x^2 + (6*I)*f
```

```
^3*x^2 + I*d^2*e*f^2*x^3 + (I/4)*d^2*f^3*x^4 - 3*d*e^2*f*x*Log[1 - I*Cos[c
+ d*x] + Sin[c + d*x]] - (12*f^3*x*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]])/
d - 3*d*e*f^2*x^2*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] - d*f^3*x^3*Log[1
- I*Cos[c + d*x] + Sin[c + d*x]] - d*e^3*Log[Cos[c + d*x] + I*(1 + Sin[c +
d*x])] - (12*e*f^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])])/d - ((6*I)*f^3
*PolyLog[4, I*Cos[c + d*x] - Sin[c + d*x]])/d^2 - (3*I)*d*e^2*f*x*Log[1 - I
*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - ((12*I)*f^3*x*Log[1 - I
*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]))/d - (3*I)*d*e*f^2*x^2*Lo
g[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - I*d*f^3*x^3*Log[
1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - I*d*e^3*Log[Cos[c
+ d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]) - ((12*I)*e*f^2*Log[Cos[
c + d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]))/d + (6*f^3*PolyLog[4,
 I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin[c]))/d^2 - (6*f^2*(e + f*x)
*PolyLog[3, I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*
(1 + Sin[c])))/d + (3*f*(4*f^2 + d^2*(e + f*x)^2)*PolyLog[2, I*Cos[c + d*x]
 - Sin[c + d*x]]*(I*Cos[c] + Sin[c])*(Cos[c] + I*(1 + Sin[c])))/d^2))/(2*a*
d^2*(Cos[c] + I*(1 + Sin[c]))) - (e + f*x)^3/(2*a*d*(Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2])^2) + (3*(e^2*f*Sin[(d*x)/2] + 2*e*f^2*x*Sin[(d*x)/2] + f
^3*x^2*Sin[(d*x)/2]))/(a*d^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Si
n[c/2 + (d*x)/2]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(444) = 888$.
time = 0.30, size = 1265, normalized size = 2.52

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1265 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 3*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4+6/a/d^3*f^3*ln(1-I*exp(I*(d*x+c))
)*x+6/a/d^4*f^3*ln(1-I*exp(I*(d*x+c)))*c-1/2/a/d^4*f^3*c^3*ln(1+I*exp(I*(d*
x+c)))+6/a/d^4*f^3*c*ln(exp(I*(d*x+c)))-6/a/d^4*f^3*c*ln(exp(I*(d*x+c))+I)+
1/2/a/d^4*f^3*c^3*ln(exp(I*(d*x+c))-I)-6/a/d^3*e*f^2*ln(exp(I*(d*x+c)))+6/a
/d^3*e*f^2*ln(exp(I*(d*x+c))+I)-3/a/d^3*e*f^2*polylog(3,-I*exp(I*(d*x+c)))-
3/a/d^3*f^3*polylog(3,-I*exp(I*(d*x+c)))*x-6*I/a/d^4*f^3*polylog(2,I*exp(I*
(d*x+c)))-3*I/a/d^2*f^3*x^2-3*I/a/d^4*c^2*f^3+3/2/a/d^3*ln(1+I*exp(I*(d*x+c
)))*c^2*e*f^2-3/2/a/d*ln(1+I*exp(I*(d*x+c)))*e*f^2*x^2-1/2/a/d*f^3*ln(1+I*e
xp(I*(d*x+c)))*x^3-I*(d*f^3*x^3*exp(I*(d*x+c))+3*d*e*f^2*x^2*exp(I*(d*x+c))
+3*d*e^2*f*x*exp(I*(d*x+c))+d*e^3*exp(I*(d*x+c))+3*f^3*x^2-3*I*f^3*x^2*exp(
I*(d*x+c))+6*e*f^2*x-6*I*e*f^2*x*exp(I*(d*x+c))+3*e^2*f-3*I*e^2*f*exp(I*(d*
x+c)))/d^2/(exp(I*(d*x+c))+I)^2/a+1/2/d/a*ln(exp(I*(d*x+c))+I)*e^3-1/2/a/d*
e^3*ln(exp(I*(d*x+c))-I)+3*I/a/d^2*polylog(2,-I*exp(I*(d*x+c)))*e*f^2*x-3*I
/a/d^2*polylog(2,I*exp(I*(d*x+c)))*e*f^2*x+3/2/d/a*e*f^2*ln(1-I*exp(I*(d*x+
```

```
c)))*x^2-3/2/d^3/a*e*f^2*ln(1-I*exp(I*(d*x+c)))*c^2+1/2/d/a*f^3*ln(1-I*exp(
I*(d*x+c)))*x^3+1/2/d^4/a*f^3*ln(1-I*exp(I*(d*x+c)))*c^3+3/2/d/a*e^2*f*ln(1
-I*exp(I*(d*x+c)))*x+3/2/d^2/a*e^2*f*ln(1-I*exp(I*(d*x+c)))*c-3/2/d^2/a*e^2
*f*c*ln(exp(I*(d*x+c))+I)+3/2/d^3/a*e*f^2*c^2*ln(exp(I*(d*x+c))+I)+3/2/a/d^
2*e^2*f*c*ln(exp(I*(d*x+c))-I)-3/2/a/d*ln(1+I*exp(I*(d*x+c)))*e^2*f*x-3/2/a
/d^2*ln(1+I*exp(I*(d*x+c)))*c*e^2*f-3/2/a/d^3*e*f^2*c^2*ln(exp(I*(d*x+c))-I
)-3/2*I/a/d^2*f^3*polylog(2,I*exp(I*(d*x+c)))*x^2+3/2*I/a/d^2*e^2*f*polylog
(2,-I*exp(I*(d*x+c)))-3/2*I/a/d^2*e^2*f*polylog(2,I*exp(I*(d*x+c)))-6*I/a/d
^3*f^3*c*x+3/2*I/a/d^2*f^3*polylog(2,-I*exp(I*(d*x+c)))*x^2+3/d^3/a*e*f^2*p
olylog(3,I*exp(I*(d*x+c)))-1/2/d^4/a*f^3*c^3*ln(exp(I*(d*x+c))+I)+3/d^3/a*f
^3*polylog(3,I*exp(I*(d*x+c)))*x-3*I*f^3*polylog(4,-I*exp(I*(d*x+c)))/a/d^4
```

**Maxima** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3854 vs. $2(421) = 842$.
time = 1.19, size = 3854, normalized size = 7.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(3*c*e^2*f*(2/(a*d*sin(d*x + c) + a*d) - log(sin(d*x + c) + 1)/(a*d) +
log(sin(d*x + c) - 1)/(a*d)) + e^3*(log(sin(d*x + c) + 1)/a - log(sin(d*x +
 c) - 1)/a - 2/(a*sin(d*x + c) + a)) - 4*(12*d^2*e^2*f - 24*c*d*e*f^2 + 12*
c^2*f^3 + 2*(3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3 - (3*(c^2 + 4)*d*e*f^2
- (c^3 + 12*c)*f^3)*cos(2*d*x + 2*c) + 2*(3*(-I*c^2 - 4*I)*d*e*f^2 + (I*c^3
 + 12*I*c)*f^3)*cos(d*x + c) + (3*(-I*c^2 - 4*I)*d*e*f^2 + (I*c^3 + 12*I*c)
*f^3)*sin(2*d*x + 2*c) + 2*(3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3)*sin(d*x
 + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 2*(3*c^2*d*e*f^2 - c^3*f^3
 - (3*c^2*d*e*f^2 - c^3*f^3)*cos(2*d*x + 2*c) - 2*(3*I*c^2*d*e*f^2 - I*c^3*
f^3)*cos(d*x + c) - (3*I*c^2*d*e*f^2 - I*c^3*f^3)*sin(2*d*x + 2*c) + 2*(3*c
^2*d*e*f^2 - c^3*f^3)*sin(d*x + c))*arctan2(sin(d*x + c) - 1, cos(d*x + c))
 - 2*((d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*
c*d*e*f^2 + (c^2 + 4)*f^3)*(d*x + c) - ((d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f^
3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)*(d*x + c))*cos
(2*d*x + 2*c) - 2*(I*(d*x + c)^3*f^3 + 3*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^2
+ 3*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + (I*c^2 + 4*I)*f^3)*(d*x + c))*cos(d*x +
c) - (I*(d*x + c)^3*f^3 + 3*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^2 + 3*(I*d^2*e^
2*f - 2*I*c*d*e*f^2 + (I*c^2 + 4*I)*f^3)*(d*x + c))*sin(2*d*x + 2*c) + 2*((
d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f
^2 + (c^2 + 4)*f^3)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*x
+ c) + 1) - 2*((d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e
^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c) - ((d*x + c)^3*f^3 + 3*(d*e*f^2 - c
*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*cos(2*
d*x + 2*c) - 2*(I*(d*x + c)^3*f^3 + 3*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^2 + 3
```

```
*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*c^2*f^3)*(d*x + c))*cos(d*x + c) - (I*(d*
x + c)^3*f^3 + 3*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^2 + 3*(I*d^2*e^2*f - 2*I*c
*d*e*f^2 + I*c^2*f^3)*(d*x + c))*sin(2*d*x + 2*c) + 2*((d*x + c)^3*f^3 + 3*
(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x
+ c))*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x + c) + 1) + 12*((d*x + c
)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(2*d*x + 2*c) + 4*((d*x + c)^3*
f^3 - 3*I*d^2*e^2*f + 3*(c^2 + 2*I*c)*d*e*f^2 - (c^3 + 3*I*c^2)*f^3 + 3*(d*
e*f^2 - (c - I)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c - I)*d*e*f^2 + (c^2
- 2*I*c)*f^3)*(d*x + c))*cos(d*x + c) - 6*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x +
 c)^2*f^3 + (c^2 + 4)*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c) - (d^2*e^2*f - 2*
c*d*e*f^2 + (d*x + c)^2*f^3 + (c^2 + 4)*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c)
)*cos(2*d*x + 2*c) - 2*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*(d*x + c)^2*f^3 + (
I*c^2 + 4*I)*f^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c))*cos(d*x + c) - (I*d^2
*e^2*f - 2*I*c*d*e*f^2 + I*(d*x + c)^2*f^3 + (I*c^2 + 4*I)*f^3 + 2*(I*d*e*f
^2 - I*c*f^3)*(d*x + c))*sin(2*d*x + 2*c) + 2*(d^2*e^2*f - 2*c*d*e*f^2 + (d
*x + c)^2*f^3 + (c^2 + 4)*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*sin(d*x + c)
)*dilog(I*e^(I*d*x + I*c)) + 6*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 +
 c^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c) - (d^2*e^2*f - 2*c*d*e*f^2 + (d*x
+ c)^2*f^3 + c^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(2*d*x + 2*c) + 2*
(-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*(d*x + c)^2*f^3 - I*c^2*f^3 + 2*(-I*d*e*f
^2 + I*c*f^3)*(d*x + c))*cos(d*x + c) + (-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*(
d*x + c)^2*f^3 - I*c^2*f^3 + 2*(-I*d*e*f^2 + I*c*f^3)*(d*x + c))*sin(2*d*x
+ 2*c) + 2*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 + c^2*f^3 + 2*(d*e*f^
2 - c*f^3)*(d*x + c))*sin(d*x + c))*dilog(-I*e^(I*d*x + I*c)) - (I*(d*x + c
)^3*f^3 - 3*(-I*c^2 - 4*I)*d*e*f^2 + (-I*c^3 - 12*I*c)*f^3 - 3*(-I*d*e*f^2
+ I*c*f^3)*(d*x + c)^2 - 3*(-I*d^2*e^2*f + 2*I*c*d*e*f^2 + (-I*c^2 - 4*I)*f
^3)*(d*x + c) + (-I*(d*x + c)^3*f^3 - 3*(I*c^2 + 4*I)*d*e*f^2 + (I*c^3 + 12
*I*c)*f^3 - 3*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^2 - 3*(I*d^2*e^2*f - 2*I*c*d*
e*f^2 + (I*c^2 + 4*I)*f^3)*(d*x + c))*cos(2*d*x + 2*c) + 2*((d*x + c)^3*f^3
 + 3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2
 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)*(d*x + c))*cos(d*x + c) + ((
d*x + c)^3*f^3 + 3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3 + 3*(d*e*f^2 - c*f^
3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)*(d*x + c))*sin
(2*d*x + 2*c) - 2*(-I*(d*x + c)^3*f^3 + 3*(-I*c^2 - 4*I)*d*e*f^2 + (I*c^3 +
 12*I*c)*f^3 + 3*(-I*d*e*f^2 + I*c*f^3)*(d*x + c)^2 + 3*(-I*d^2*e^2*f + 2*I
*c*d*e*f^2 + (-I*c^2 - 4*I)*f^3)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^
2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - (-3*I*c^2*d*e*f^2 - I*(d*x + c)^
3*f^3 + I*c^3*f^3 - 3*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^2 - 3*(I*d^2*e^2*f -
2*I*c*d*e*f^2 + I*c^2*f^3)*(d*x + c) + (3*I*c^2*d*e*f^2 + I*(d*x + c)^3*f^3
 - I*c^3*f^3 - 3*(-I*d*e*f^2 + I*c*f^3)*(d*x + c)^2 - 3*(-I*d^2*e^2*f + 2*I
*c*d*e*f^2 - I*c^2*f^3)*(d*x + c))*cos(2*d*x + 2*c) - 2*(3*c^2*d*e*f^2 + (d
*x + c)^3*f^3 - c^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f -
2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*cos(d*x + c) ...
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than

twice the leaf count of optimal. 1871 vs. 2(433) = 866.
time = 0.48, size = 1871, normalized size = 3.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^3*f^3*x^3 + 6*d^3*f^2*x^2*e + 6*d^3*f*x*e^2 + 2*d^3*e^3 + 6*(d^2*
f^3*x^2 + 2*d^2*f^2*x*e + d^2*f*e^2)*cos(d*x + c) + 3*(I*d^2*f^3*x^2 + 2*I*
d^2*f^2*x*e + I*d^2*f*e^2 + (I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e + I*d^2*f*e^2)
*sin(d*x + c))*dilog(I*cos(d*x + c) + sin(d*x + c)) + 3*(I*d^2*f^3*x^2 + 2*
I*d^2*f^2*x*e + I*d^2*f*e^2 + 4*I*f^3 + (I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e +
I*d^2*f*e^2 + 4*I*f^3)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) +
 3*(-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2 + (-I*d^2*f^3*x^2 - 2*I*
d^2*f^2*x*e - I*d^2*f*e^2)*sin(d*x + c))*dilog(-I*cos(d*x + c) + sin(d*x +
c)) + 3*(-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2 - 4*I*f^3 + (-I*d^2
*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2 - 4*I*f^3)*sin(d*x + c))*dilog(-I*
cos(d*x + c) - sin(d*x + c)) + (3*c*d^2*f*e^2 - 3*(c^2 + 4)*d*f^2*e + (c^3
+ 12*c)*f^3 - d^3*e^3 + (3*c*d^2*f*e^2 - 3*(c^2 + 4)*d*f^2*e + (c^3 + 12*c)
*f^3 - d^3*e^3)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - (c^3
*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3 + (c^3*f^3 - 3*c^2*d*f^2*e +
 3*c*d^2*f*e^2 - d^3*e^3)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x + c) +
 I) - (d^3*f^3*x^3 + 12*d*f^3*x + (c^3 + 12*c)*f^3 + 3*(d^3*f*x + c*d^2*f)*
e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e + (d^3*f^3*x^3 + 12*d*f^3*x + (c^3 + 12
*c)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*sin(d*
x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) + (d^3*f^3*x^3 + c^3*f^3 + 3
*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e + (d^3*f^3*x^3 + c
^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*sin(d*x
 + c))*log(I*cos(d*x + c) - sin(d*x + c) + 1) - (d^3*f^3*x^3 + 12*d*f^3*x +
 (c^3 + 12*c)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)
*e + (d^3*f^3*x^3 + 12*d*f^3*x + (c^3 + 12*c)*f^3 + 3*(d^3*f*x + c*d^2*f)*e
^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*sin(d*x + c))*log(-I*cos(d*x + c) + sin
(d*x + c) + 1) + (d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^
3*f^2*x^2 - c^2*d*f^2)*e + (d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e
^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*sin(d*x + c))*log(-I*cos(d*x + c) - sin
(d*x + c) + 1) + (3*c*d^2*f*e^2 - 3*(c^2 + 4)*d*f^2*e + (c^3 + 12*c)*f^3 -
d^3*e^3 + (3*c*d^2*f*e^2 - 3*(c^2 + 4)*d*f^2*e + (c^3 + 12*c)*f^3 - d^3*e^3
)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) - (c^3*f^3 - 3*c^2*
d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3 + (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^
2 - d^3*e^3)*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + I) + 6*(-I*
f^3*sin(d*x + c) - I*f^3)*polylog(4, I*cos(d*x + c) + sin(d*x + c)) + 6*(-I
*f^3*sin(d*x + c) - I*f^3)*polylog(4, I*cos(d*x + c) - sin(d*x + c)) + 6*(I
*f^3*sin(d*x + c) + I*f^3)*polylog(4, -I*cos(d*x + c) + sin(d*x + c)) + 6*(
I*f^3*sin(d*x + c) + I*f^3)*polylog(4, -I*cos(d*x + c) - sin(d*x + c)) + 6*
```

```
(d*f^3*x + d*f^2*e + (d*f^3*x + d*f^2*e)*sin(d*x + c))*polylog(3, I*cos(d*x
 + c) + sin(d*x + c)) - 6*(d*f^3*x + d*f^2*e + (d*f^3*x + d*f^2*e)*sin(d*x
+ c))*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 6*(d*f^3*x + d*f^2*e + (d
*f^3*x + d*f^2*e)*sin(d*x + c))*polylog(3, -I*cos(d*x + c) + sin(d*x + c))
- 6*(d*f^3*x + d*f^2*e + (d*f^3*x + d*f^2*e)*sin(d*x + c))*polylog(3, -I*co
s(d*x + c) - sin(d*x + c)))/(a*d^4*sin(d*x + c) + a*d^4)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3 \sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3 \sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2 \sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx \sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*sec
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sec(c + d*x)/(sin
(c + d*x) + 1), x) + Integral(3*e**2*f*x*sec(c + d*x)/(sin(c + d*x) + 1), x
))/a
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.270 $\int \frac{(e+fx)^2\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=278

$$-\frac{i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3}+\frac{f^2\log(\cos(c+dx))}{ad^3}+\frac{if(e+fx)\mathrm{Li}_2\left(-ie^{i(c+dx)}\right)}{ad^2}-\frac{if(e$$

```
[Out] -I*(f*x+e)^2*arctan(exp(I*(d*x+c)))/a/d+f^2*arctanh(sin(d*x+c))/a/d^3+f^2*l
n(cos(d*x+c))/a/d^3+I*f*(f*x+e)*polylog(2,-I*exp(I*(d*x+c)))/a/d^2-I*f*(f*x
+e)*polylog(2,I*exp(I*(d*x+c)))/a/d^2-f^2*polylog(3,-I*exp(I*(d*x+c)))/a/d^
3+f^2*polylog(3,I*exp(I*(d*x+c)))/a/d^3-f*(f*x+e)*sec(d*x+c)/a/d^2-1/2*(f*x
+e)^2*sec(d*x+c)^2/a/d+f*(f*x+e)*tan(d*x+c)/a/d^2+1/2*(f*x+e)^2*sec(d*x+c)*
tan(d*x+c)/a/d
```

**Rubi** [A]
time = 0.17, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4627, 4271, 3855, 4266, 2611, 2320, 6724, 4494, 4269, 3556}

$$-\frac{f^2\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^3}+\frac{f^2\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}+\frac{if(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^2}-\frac{if(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}-\frac{i(e+fx)^2\text{ArcTan}\left(e^{i(c+dx)}\right)}{ad}+\frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3}+\frac{f^2\log(\cos(c+dx))}{ad^3}+\frac{f(e+fx)\tan(c+dx)}{ad^2}-\frac{f(e+fx)\sec(c+dx)}{ad^2}-\frac{(e+fx)^2\sec^2(c+dx)}{2ad}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/(a*d) + (f^2*ArcTanh[Sin[c + d*x
]])/(a*d^3) + (f^2*Log[Cos[c + d*x]])/(a*d^3) + (I*f*(e + f*x)*PolyLog[2, (
-I)*E^(I*(c + d*x))])/(a*d^2) - (I*f*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))
])/(a*d^2) - (f^2*PolyLog[3, (-I)*E^(I*(c + d*x))])/(a*d^3) + (f^2*PolyLog[
3, I*E^(I*(c + d*x))])/(a*d^3) - (f*(e + f*x)*Sec[c + d*x])/(a*d^2) - ((e +
 f*x)^2*Sec[c + d*x]^2)/(2*a*d) + (f*(e + f*x)*Tan[c + d*x])/(a*d^2) + ((e
+ f*x)^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4627

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
```

```
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sec^3(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)\sec(c+dx)}{ad^2} - \frac{(e+fx)^2\sec^2(c+dx)}{2ad} + \frac{(e+fx)^2\sec(c+dx)\tan}{2ad}$$

$$= -\frac{i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3} - \frac{f(e+fx)\sec(c+dx}{ad^2}$$

$$= -\frac{i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3} + \frac{f^2\log(\cos(c+dx))}{ad^3}$$

$$= -\frac{i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3} + \frac{f^2\log(\cos(c+dx))}{ad^3}$$

$$= -\frac{i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3} + \frac{f^2\log(\cos(c+dx))}{ad^3}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 748 vs. 2(278) = 556.
time = 8.66, size = 748, normalized size = 2.69

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(6*a*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[
c/2])) + (-3*d^2*(e + f*x)^2*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]] + (6*I)
*d*f*(e + f*x)*PolyLog[2, (-I)*Cos[c + d*x] + Sin[c + d*x]] - 6*f^2*PolyLog
[3, (-I)*Cos[c + d*x] + Sin[c + d*x]] + (I*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2
)*(Cos[c] + I*Sin[c]))/(Cos[c] + I*(-1 + Sin[c])))/(6*a*d^3) - ((I/6)*(Cos[
c] + I*Sin[c])*(3*d^3*e^2*x + 12*d*f^2*x + 3*d^3*e*f*x^2 + d^3*f^2*x^3 + (6
*I)*d^2*e*f*x*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] + (3*I)*d^2*f^2*x^2*Lo
```

```
g[1 - I*Cos[c + d*x] + Sin[c + d*x]] + (3*I)*d^2*e^2*Log[Cos[c + d*x] + I*(
1 + Sin[c + d*x])] + (12*I)*f^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])] -
6*d^2*e*f*x*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - 3*
d^2*f^2*x^2*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - 3*
d^2*e^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]) - 12*f
^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]) - 6*f^2*Pol
yLog[3, I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])) + 6*d*f*(e
 + f*x)*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[
c] + I*(1 + Sin[c]))))/(a*d^3*(Cos[c] + I*(1 + Sin[c]))) - (e + f*x)^2/(2*a
*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (2*(e*f*Sin[(d*x)/2] + f^
2*x*Sin[(d*x)/2]))/(a*d^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c
/2 + (d*x)/2]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(257) = 514$.
time = 0.19, size = 677, normalized size = 2.44

| method | result |
|---|---|
| risch | $\frac{ief\,\mathrm{polylog}\left(2,-ie^{i(dx+c)}\right)}{d^2a}+\frac{f^2c^2\ln\left(e^{i(dx+c)}+i\right)}{2d^3a}-\frac{f^2c^2\ln\left(1-ie^{i(dx+c)}\right)}{2d^3a}-\frac{i\left(d f^2x^2e^{i(dx+c)}+2defx\,e^{i(dx+c)}+d\,e^2e^{i(dx+c)}+2\right.}{d^2\left(e^{i(dx+c)}+\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(d*f^2*x^2*exp(I*(d*x+c))+2*d*e*f*x*exp(I*(d*x+c))+d*e^2*exp(I*(d*x+c))+
2*f^2*x-2*I*f^2*x*exp(I*(d*x+c))+2*e*f-2*I*e*f*exp(I*(d*x+c)))/d^2/(exp(I*(
d*x+c))+I)^2/a+1/2/d^3/a*f^2*c^2*ln(exp(I*(d*x+c))+I)-1/2/d^3/a*f^2*c^2*ln(
1-I*exp(I*(d*x+c)))-I/d^2/a*e*f*polylog(2,I*exp(I*(d*x+c)))-1/2/d/a*ln(1+I*
exp(I*(d*x+c)))*f^2*x^2+I/d^2/a*e*f*polylog(2,-I*exp(I*(d*x+c)))+I/d^2/a*po
lylog(2,-I*exp(I*(d*x+c)))*f^2*x-I/d^2/a*polylog(2,I*exp(I*(d*x+c)))*f^2*x+
1/2/d/a*f^2*ln(1-I*exp(I*(d*x+c)))*x^2-1/2/d/a*e^2*ln(exp(I*(d*x+c))-I)+2/d
^3/a*f^2*ln(exp(I*(d*x+c))+I)-2/d^3/a*f^2*ln(exp(I*(d*x+c)))+f^2*polylog(3,
I*exp(I*(d*x+c)))/a/d^3+1/2/d/a*ln(exp(I*(d*x+c))+I)*e^2+1/2/d^3/a*ln(1+I*e
xp(I*(d*x+c)))*c^2*f^2-1/2/d^3/a*f^2*c^2*ln(exp(I*(d*x+c))-I)-1/d^2/a*e*f*c
*ln(exp(I*(d*x+c))+I)+1/d/a*e*f*ln(1-I*exp(I*(d*x+c)))*x+1/d^2/a*e*f*ln(1-I
*exp(I*(d*x+c)))*c-f^2*polylog(3,-I*exp(I*(d*x+c)))/a/d^3+1/d^2/a*e*f*c*ln(
exp(I*(d*x+c))-I)-1/d/a*ln(1+I*exp(I*(d*x+c)))*e*f*x-1/d^2/a*ln(1+I*exp(I*(
d*x+c)))*c*e*f
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1932 vs. $2(248) = 496$.
time = 0.52, size = 1932, normalized size = 6.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/4*(2*c*e*f*(2/(a*d*sin(d*x + c) + a*d) - log(sin(d*x + c) + 1)/(a*d) + lo
g(sin(d*x + c) - 1)/(a*d)) + e^2*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c
) - 1)/a - 2/(a*sin(d*x + c) + a)) - 4*(8*(d*x + c)*f^2*cos(2*d*x + 2*c) +
8*I*(d*x + c)*f^2*sin(2*d*x + 2*c) + 8*d*e*f - 8*c*f^2 - 2*((c^2 + 4)*f^2*c
os(2*d*x + 2*c) - 2*(-I*c^2 - 4*I)*f^2*cos(d*x + c) - (-I*c^2 - 4*I)*f^2*si
n(2*d*x + 2*c) - 2*(c^2 + 4)*f^2*sin(d*x + c) - (c^2 + 4)*f^2)*arctan2(sin(
d*x + c) + 1, cos(d*x + c)) + 2*(c^2*f^2*cos(2*d*x + 2*c) + 2*I*c^2*f^2*cos
(d*x + c) + I*c^2*f^2*sin(2*d*x + 2*c) - 2*c^2*f^2*sin(d*x + c) - c^2*f^2)*
arctan2(sin(d*x + c) - 1, cos(d*x + c)) - 2*((d*x + c)^2*f^2 + 2*(d*e*f - c
*f^2)*(d*x + c) - ((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(2*d*x
 + 2*c) - 2*(I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I*c*f^2)*(d*x + c))*cos(d*x +
 c) - (I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I*c*f^2)*(d*x + c))*sin(2*d*x + 2*c
) + 2*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(d*x + c))*arctan2
(cos(d*x + c), sin(d*x + c) + 1) - 2*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(
d*x + c) - ((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(2*d*x + 2*c)
 - 2*(I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I*c*f^2)*(d*x + c))*cos(d*x + c) - (
I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I*c*f^2)*(d*x + c))*sin(2*d*x + 2*c) + 2*(
(d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*
x + c), -sin(d*x + c) + 1) + 4*((d*x + c)^2*f^2 - 2*I*d*e*f + (c^2 + 2*I*c)
*f^2 + 2*(d*e*f - (c - I)*f^2)*(d*x + c))*cos(d*x + c) - 4*(d*e*f + (d*x +
c)*f^2 - c*f^2 - (d*e*f + (d*x + c)*f^2 - c*f^2)*cos(2*d*x + 2*c) - 2*(I*d*
e*f + I*(d*x + c)*f^2 - I*c*f^2)*cos(d*x + c) - (I*d*e*f + I*(d*x + c)*f^2
- I*c*f^2)*sin(2*d*x + 2*c) + 2*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(d*x + c
))*dilog(I*e^(I*d*x + I*c)) + 4*(d*e*f + (d*x + c)*f^2 - c*f^2 - (d*e*f + (
d*x + c)*f^2 - c*f^2)*cos(2*d*x + 2*c) + 2*(-I*d*e*f - I*(d*x + c)*f^2 + I*
c*f^2)*cos(d*x + c) + (-I*d*e*f - I*(d*x + c)*f^2 + I*c*f^2)*sin(2*d*x + 2*
c) + 2*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(d*x + c))*dilog(-I*e^(I*d*x + I*
c)) - (I*(d*x + c)^2*f^2 + (I*c^2 + 4*I)*f^2 - 2*(-I*d*e*f + I*c*f^2)*(d*x
+ c) + (-I*(d*x + c)^2*f^2 + (-I*c^2 - 4*I)*f^2 - 2*(I*d*e*f - I*c*f^2)*(d*
x + c))*cos(2*d*x + 2*c) + 2*((d*x + c)^2*f^2 + (c^2 + 4)*f^2 + 2*(d*e*f -
c*f^2)*(d*x + c))*cos(d*x + c) + ((d*x + c)^2*f^2 + (c^2 + 4)*f^2 + 2*(d*e*
f - c*f^2)*(d*x + c))*sin(2*d*x + 2*c) - 2*(-I*(d*x + c)^2*f^2 + (-I*c^2 -
4*I)*f^2 + 2*(-I*d*e*f + I*c*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)
^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - (-I*(d*x + c)^2*f^2 - I*c^2*f^2
 - 2*(I*d*e*f - I*c*f^2)*(d*x + c) + (I*(d*x + c)^2*f^2 + I*c^2*f^2 - 2*(-I
*d*e*f + I*c*f^2)*(d*x + c))*cos(2*d*x + 2*c) - 2*((d*x + c)^2*f^2 + c^2*f^
2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(d*x + c) - ((d*x + c)^2*f^2 + c^2*f^2
+ 2*(d*e*f - c*f^2)*(d*x + c))*sin(2*d*x + 2*c) - 2*(I*(d*x + c)^2*f^2 + I*
c^2*f^2 + 2*(I*d*e*f - I*c*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2
 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*(I*f^2*cos(2*d*x + 2*c) - 2*f^2
*cos(d*x + c) - f^2*sin(2*d*x + 2*c) - 2*I*f^2*sin(d*x + c) - I*f^2)*polylo
g(3, I*e^(I*d*x + I*c)) + 4*(-I*f^2*cos(2*d*x + 2*c) + 2*f^2*cos(d*x + c) +
 f^2*sin(2*d*x + 2*c) + 2*I*f^2*sin(d*x + c) + I*f^2)*polylog(3, -I*e^(I*d*
```

```
x + I*c)) + 4*(I*(d*x + c)^2*f^2 + 2*d*e*f + (I*c^2 - 2*c)*f^2 + 2*(I*d*e*f
 + (-I*c - 1)*f^2)*(d*x + c))*sin(d*x + c))/(-4*I*a*d^2*cos(2*d*x + 2*c) +
8*a*d^2*cos(d*x + c) + 4*a*d^2*sin(2*d*x + 2*c) + 8*I*a*d^2*sin(d*x + c) +
4*I*a*d^2))/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(255) = 510$.
time = 0.43, size = 1087, normalized size = 3.91

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*f^2*x^2 + 4*d^2*f*x*e + 2*d^2*e^2 + 4*(d*f^2*x + d*f*e)*cos(d*x
 + c) + 2*(I*d*f^2*x + I*d*f*e + (I*d*f^2*x + I*d*f*e)*sin(d*x + c))*dilog(
I*cos(d*x + c) + sin(d*x + c)) + 2*(I*d*f^2*x + I*d*f*e + (I*d*f^2*x + I*d*
f*e)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) + 2*(-I*d*f^2*x - I
*d*f*e + (-I*d*f^2*x - I*d*f*e)*sin(d*x + c))*dilog(-I*cos(d*x + c) + sin(d
*x + c)) + 2*(-I*d*f^2*x - I*d*f*e + (-I*d*f^2*x - I*d*f*e)*sin(d*x + c))*d
ilog(-I*cos(d*x + c) - sin(d*x + c)) + (2*c*d*f*e - (c^2 + 4)*f^2 - d^2*e^2
 + (2*c*d*f*e - (c^2 + 4)*f^2 - d^2*e^2)*sin(d*x + c))*log(cos(d*x + c) + I
*sin(d*x + c) + I) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2 + (c^2*f^2 - 2*c*d*f*e
+ d^2*e^2)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x + c) + I) - (d^2*f^2*
x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x
 + c*d*f)*e)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) + (d^2*f^
2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f
*x + c*d*f)*e)*sin(d*x + c))*log(I*cos(d*x + c) - sin(d*x + c) + 1) - (d^2*
f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2
*f*x + c*d*f)*e)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d
^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e + (d^2*f^2*x^2 - c^2*f^2 + 2*(
d^2*f*x + c*d*f)*e)*sin(d*x + c))*log(-I*cos(d*x + c) - sin(d*x + c) + 1) +
 (2*c*d*f*e - (c^2 + 4)*f^2 - d^2*e^2 + (2*c*d*f*e - (c^2 + 4)*f^2 - d^2*e^
2)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (c^2*f^2 - 2*c*d
*f*e + d^2*e^2 + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*sin(d*x + c))*log(-cos(d*x
 + c) - I*sin(d*x + c) + I) + 2*(f^2*sin(d*x + c) + f^2)*polylog(3, I*cos(d
*x + c) + sin(d*x + c)) - 2*(f^2*sin(d*x + c) + f^2)*polylog(3, I*cos(d*x +
 c) - sin(d*x + c)) + 2*(f^2*sin(d*x + c) + f^2)*polylog(3, -I*cos(d*x + c)
 + sin(d*x + c)) - 2*(f^2*sin(d*x + c) + f^2)*polylog(3, -I*cos(d*x + c) -
sin(d*x + c)))/(a*d^3*sin(d*x + c) + a*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2 \sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*sec
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sec(c + d*x)/(sin(c + d
*x) + 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.271 $\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=172

$$-\frac{i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{if\text{Li}_2\left(-ie^{i(c+dx)}\right)}{2ad^2}-\frac{if\text{Li}_2\left(ie^{i(c+dx)}\right)}{2ad^2}-\frac{f\sec(c+dx)}{2ad^2}-\frac{(e+fx)\sec^2(c+dx)}{2ad}+\frac{f}{}$$

```
[Out] -I*(f*x+e)*arctan(exp(I*(d*x+c)))/a/d+1/2*I*f*polylog(2,-I*exp(I*(d*x+c)))/
a/d^2-1/2*I*f*polylog(2,I*exp(I*(d*x+c)))/a/d^2-1/2*f*sec(d*x+c)/a/d^2-1/2*
(f*x+e)*sec(d*x+c)^2/a/d+1/2*f*tan(d*x+c)/a/d^2+1/2*(f*x+e)*sec(d*x+c)*tan(
d*x+c)/a/d
```

**Rubi** [A]
time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4627, 4270, 4266, 2317, 2438, 4494, 3852, 8}

$$\frac{if\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{2ad^2}-\frac{if\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{2ad^2}-\frac{i(e+fx)\text{ArcTan}\left(e^{i(c+dx)}\right)}{ad}+\frac{f\tan(c+dx)}{2ad^2}-\frac{f\sec(c+dx)}{2ad^2}-\frac{(e+fx)\sec^2(c+dx)}{2ad}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)*ArcTan[E^(I*(c + d*x))])/(a*d) + ((I/2)*f*PolyLog[2, (-I)*E
^(I*(c + d*x))])/(a*d^2) - ((I/2)*f*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2)
- (f*Sec[c + d*x])/(2*a*d^2) - ((e + f*x)*Sec[c + d*x]^2)/(2*a*d) + (f*Tan[
c + d*x])/(2*a*d^2) + ((e + f*x)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4627

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\sec^3(c+dx)\,dx}{a} - \frac{\int (e+fx)\sec^2(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f\sec(c+dx)}{2ad^2} - \frac{(e+fx)\sec^2(c+dx)}{2ad} + \frac{(e+fx)\sec(c+dx)\tan(c+dx)}{2ad} +$$

$$= -\frac{i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f\sec(c+dx)}{2ad^2} - \frac{(e+fx)\sec^2(c+dx)}{2ad} + \frac{(e+}{}$$

$$= -\frac{i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f\sec(c+dx)}{2ad^2} - \frac{(e+fx)\sec^2(c+dx)}{2ad} + \frac{f\tan(}{2}$$

$$= -\frac{i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{if\text{Li}_2\left(-ie^{i(c+dx)}\right)}{2ad^2} - \frac{if\text{Li}_2\left(ie^{i(c+dx)}\right)}{2ad^2} - \frac{f\sec(c}{2ad}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 655 vs. $2(172) = 344$.
time = 2.96, size = 655, normalized size = 3.81

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(2*d*(e + f*x) - 4*f*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]) + (c + d*x)*(c*f - d*(2*e + f*x))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^2 + d*e*(c + d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^2 - c*f*(c + d*x + 2*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + d*e*(c + d*x - 2
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2 - c*f*(c + d*x - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^2 - (f*((-1)^(3/4)*(c + d*x)^2 + ((-3*I)*Pi
*(c + d*x) - 4*Pi*Log[1 + E^((-I)*(c + d*x))] + 2*(-2*c + Pi - 2*d*x)*Log[1
 + I*E^(I*(c + d*x))] + 4*Pi*Log[Cos[(c + d*x)/2]] - 2*Pi*Log[Sin[(2*c - Pi
 + 2*d*x)/4]] + (4*I)*PolyLog[2, (-I)*E^(I*(c + d*x))])/Sqrt[2])*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^2)/Sqrt[2] + (f*((-1)^(1/4)*(c + d*x)^2 + ((-I)
*Pi*(c + d*x) - 4*Pi*Log[1 + E^((-I)*(c + d*x))] - 2*(2*c + Pi + 2*d*x)*Log
[1 - I*E^(I*(c + d*x))] + 4*Pi*Log[Cos[(c + d*x)/2]] + 2*Pi*Log[Sin[(2*c +
Pi + 2*d*x)/4]] + (4*I)*PolyLog[2, I*E^(I*(c + d*x))])/Sqrt[2])*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^2)/Sqrt[2])/(a*d^2*(1 + Sin[c + d*x]))
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(149) = 298$.
time = 0.20, size = 303, normalized size = 1.76

| method | result |
|---|---|
| risch | $-\frac{i\left(dfx\,\mathrm{e}^{i(dx+c)}+de\,\mathrm{e}^{i(dx+c)}+f-if\,\mathrm{e}^{i(dx+c)}\right)}{d^2\left(\mathrm{e}^{i(dx+c)}+i\right)^2a}-\frac{e\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)e}{2da}-\frac{f\ln\left(1+i\mathrm{e}^{i(dx+c)}\right)x}{2ad}-\frac{f\ln\left(1+i\mathrm{e}^{i(dx+c)}\right)}{2a\,d^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(d*f*x*exp(I*(d*x+c))+d*e*exp(I*(d*x+c))+f-I*f*exp(I*(d*x+c)))/d^2/(exp(
I*(d*x+c))+I)^2/a-1/2/a/d*e*ln(exp(I*(d*x+c))-I)+1/2/d/a*ln(exp(I*(d*x+c))+
I)*e-1/2/a/d*f*ln(1+I*exp(I*(d*x+c)))*x-1/2/a/d^2*f*ln(1+I*exp(I*(d*x+c)))*
c+1/2*I*f*polylog(2,-I*exp(I*(d*x+c)))/a/d^2+1/2/d/a*f*ln(1-I*exp(I*(d*x+c)
))*x+1/2/d^2/a*f*ln(1-I*exp(I*(d*x+c)))*c-1/2*I*f*polylog(2,I*exp(I*(d*x+c)
))/a/d^2+1/2/a/d^2*f*c*ln(exp(I*(d*x+c))-I)-1/2/d^2/a*f*c*ln(exp(I*(d*x+c))
+I)
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 725 vs. 2(144) = 288.
time = 0.41, size = 725, normalized size = 4.22

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*(d*e*cos(2*d*x + 2*c) + 2*I*d*e*cos(d*x + c) + I*d*e*sin(2*d*x + 2*c) -
2*d*e*sin(d*x + c) - d*e)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 2*(d*e*
cos(2*d*x + 2*c) + 2*I*d*e*cos(d*x + c) + I*d*e*sin(2*d*x + 2*c) - 2*d*e*si
n(d*x + c) - d*e)*arctan2(sin(d*x + c) - 1, cos(d*x + c)) - 2*(d*f*x*cos(2*
d*x + 2*c) + 2*I*d*f*x*cos(d*x + c) + I*d*f*x*sin(2*d*x + 2*c) - 2*d*f*x*si
n(d*x + c) - d*f*x)*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*(d*f*x*cos(
2*d*x + 2*c) + 2*I*d*f*x*cos(d*x + c) + I*d*f*x*sin(2*d*x + 2*c) - 2*d*f*x*
sin(d*x + c) - d*f*x)*arctan2(cos(d*x + c), -sin(d*x + c) + 1) - 4*(d*f*x +
 d*e - I*f)*cos(d*x + c) - 2*(f*cos(2*d*x + 2*c) + 2*I*f*cos(d*x + c) + I*f
*sin(2*d*x + 2*c) - 2*f*sin(d*x + c) - f)*dilog(I*e^(I*d*x + I*c)) + 2*(f*c
os(2*d*x + 2*c) + 2*I*f*cos(d*x + c) + I*f*sin(2*d*x + 2*c) - 2*f*sin(d*x +
 c) - f)*dilog(-I*e^(I*d*x + I*c)) + (I*d*f*x + I*d*e + (-I*d*f*x - I*d*e)*
cos(2*d*x + 2*c) + 2*(d*f*x + d*e)*cos(d*x + c) + (d*f*x + d*e)*sin(2*d*x +
 2*c) - 2*(-I*d*f*x - I*d*e)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c
)^2 + 2*sin(d*x + c) + 1) + (-I*d*f*x - I*d*e + (I*d*f*x + I*d*e)*cos(2*d*x
 + 2*c) - 2*(d*f*x + d*e)*cos(d*x + c) - (d*f*x + d*e)*sin(2*d*x + 2*c) - 2
*(I*d*f*x + I*d*e)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*si
n(d*x + c) + 1) - 4*(I*d*f*x + I*d*e + f)*sin(d*x + c) - 4*f)/(-4*I*a*d^2*c
os(2*d*x + 2*c) + 8*a*d^2*cos(d*x + c) + 4*a*d^2*sin(2*d*x + 2*c) + 8*I*a*d
^2*sin(d*x + c) + 4*I*a*d^2)
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(147) = 294$.
time = 0.41, size = 517, normalized size = 3.01

$$-\frac{2\,dfx + 2\,f\cos\left(dx+c\right) - \left(-i\,f\sin\left(dx+c\right) - i\,f\right){\rm Li}_2\left(i\,\cos\left(dx+c\right) + \sin\left(dx+c\right)\right) - \left(-i\,f\sin\left(dx+c\right) - i\,f\right){\rm Li}_2\left(i\,\cos\left(dx+c\right) - \sin\left(dx+c\right)\right) - \left(i\,f\sin\left(dx+c\right) + i\,f\right){\rm Li}_2\left(-i\,\cos\left(dx+c\right) + \sin\left(dx+c\right)\right) - \left(i\,f\sin\left(dx+c\right) + i\,f\right){\rm Li}_2\left(-i\,\cos\left(dx+c\right) - \sin\left(dx+c\right)\right) + 2\,de + \left(cf - de + \left(cf - de\right)\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right) + i\,\sin\left(dx+c\right) + i\right) - \left(cf - de + \left(cf - de\right)\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right) - i\,\sin\left(dx+c\right) + i\right) - \left(dfx + cf + \left(dfx + cf\right)\sin\left(dx+c\right)\right)\log\left(i\,\cos\left(dx+c\right) + \sin\left(dx+c\right) + 1\right) + \left(dfx + cf + \left(dfx + cf\right)\sin\left(dx+c\right)\right)\log\left(i\,\cos\left(dx+c\right) - \sin\left(dx+c\right) + 1\right) - \left(dfx + cf + \left(dfx + cf\right)\sin\left(dx+c\right)\right)\log\left(-i\,\cos\left(dx+c\right) + \sin\left(dx+c\right) + 1\right) + \left(dfx + cf + \left(dfx + cf\right)\sin\left(dx+c\right)\right)\log\left(-i\,\cos\left(dx+c\right) - \sin\left(dx+c\right) + 1\right) + \left(cf - de + \left(cf - de\right)\sin\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right) + i\,\sin\left(dx+c\right) + i\right) - \left(cf - de + \left(cf - de\right)\sin\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right) - i\,\sin\left(dx+c\right) + i\right)}{4\left(ad^2\sin\left(dx+c\right) + ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*f*x + 2*f*cos(d*x + c) - (-I*f*sin(d*x + c) - I*f)*dilog(I*cos(d*
x + c) + sin(d*x + c)) - (-I*f*sin(d*x + c) - I*f)*dilog(I*cos(d*x + c) - s
in(d*x + c)) - (I*f*sin(d*x + c) + I*f)*dilog(-I*cos(d*x + c) + sin(d*x + c
)) - (I*f*sin(d*x + c) + I*f)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + 2*d*e
 + (c*f - d*e + (c*f - d*e)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c)
 + I) - (c*f - d*e + (c*f - d*e)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x
 + c) + I) - (d*f*x + c*f + (d*f*x + c*f)*sin(d*x + c))*log(I*cos(d*x + c)
+ sin(d*x + c) + 1) + (d*f*x + c*f + (d*f*x + c*f)*sin(d*x + c))*log(I*cos(
d*x + c) - sin(d*x + c) + 1) - (d*f*x + c*f + (d*f*x + c*f)*sin(d*x + c))*l
og(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d*f*x + c*f + (d*f*x + c*f)*sin(d
*x + c))*log(-I*cos(d*x + c) - sin(d*x + c) + 1) + (c*f - d*e + (c*f - d*e)
*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) - (c*f - d*e + (c*f
- d*e)*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + I))/(a*d^2*sin(d*
x + c) + a*d^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e\sec\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx + \int \frac{fx\sec\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f*x*sec(c + d*x)
/(sin(c + d*x) + 1), x))/a
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.272 $\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=37

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a+a\sin(c+dx))}$$

```
[Out] 1/2*arctanh(sin(d*x+c))/a/d-1/2/d/(a+a*sin(d*x+c))
```

**Rubi** [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2746, 46, 212}

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2} + \frac{1}{2a(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{1}{2d(a+a\sin(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{2d}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{1}{2d(a+a\sin(c+dx))}$$

**Mathematica** [**A**]
time = 0.04, size = 30, normalized size = 0.81

$$\frac{\tanh^{-1}(\sin(c+dx)) - \frac{1}{1+\sin(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] (ArcTanh[Sin[c + d*x]] - (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

**Maple** [**A**]
time = 0.09, size = 43, normalized size = 1.16

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{1}{2(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4}-\frac{\ln(\sin(dx+c)-1)}{4}}{da}$ | 43 |
| default | $\frac{-\frac{1}{2(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4}-\frac{\ln(\sin(dx+c)-1)}{4}}{da}$ | 43 |
| norman | $\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$ | 71 |
| risch | $-\frac{i e^{i(dx+c)}}{da\left(e^{i(dx+c)}+i\right)^2}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{2ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{2ad}$ | 76 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/2/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c))-1/4*ln(sin(d*x+c)-1))
```

**Maxima** [**A**]
time = 0.28, size = 47, normalized size = 1.27

$$\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} - \frac{2}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a - 2/(a*sin(d*x + c)
+ a))/d
```

**Fricas** [A]
time = 0.41, size = 58, normalized size = 1.57

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)-(\sin(dx+c)+1)\log(-\sin(dx+c)+1)-2}{4(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2)/(a*d*sin(d*x + c) + a*d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x) + 1), x)/a
```

**Giac** [A]
time = 6.41, size = 58, normalized size = 1.57

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a}-\frac{\log(|\sin(dx+c)-1|)}{a}-\frac{\sin(dx+c)+3}{a(\sin(dx+c)+1)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c) - 1))/a - (sin(d*x
 + c) + 3)/(a*(sin(d*x + c) + 1)))/d
```

**Mupad** [B]
time = 0.08, size = 33, normalized size = 0.89

$$\frac{\operatorname{atanh}(\sin(c+d\,x))}{2\,a\,d}-\frac{1}{2\,d\,(a+a\,\sin(c+d\,x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] atanh(sin(c + d*x))/(2*a*d) - 1/(2*d*(a + a*sin(c + d*x)))
```

# 3.273 $\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=29

$$\text{Int}\left(\frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 13.93, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*(d*f*x + d*e)*cos(d*x + c)^2 + 2*(d*f*x + d*e)*sin(d*x + c)^2 - (f*cos(
d*x + c) + (d*f*x + d*e)*sin(d*x + c))*cos(2*d*x + 2*c) - f*cos(d*x + c) -
(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x
 + a*d^2*e^2)*cos(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2
*e^2)*cos(d*x + c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*
x + c)*sin(2*d*x + 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2
*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c)^
2 - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + 2*(a*d^2*f^2*x^2 + 2*a*d
^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*cos(2*d*x + 2*c) + 4*(a*d^2*f^2*x^2 + 2
*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*integrate(1/2*(d^2*f^2*x^2 + 2*d^2*
e*f*x + d^2*e^2 + 4*f^2)*cos(d*x + c)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*
e^2*f*x + a*d^2*e^3)*cos(d*x + c)^2 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)^2 + 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*
f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)), x) - (a*d^2*f^2*x^2 +
 2*a*d^2*e*f*x + a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*co
s(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x +
c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)*sin(2*d*x
 + 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2*d*x + 2*c)^2 +
4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c)^2 - 2*(a*d^2*f^2
*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2
*e^2)*sin(d*x + c))*cos(2*d*x + 2*c) + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a
*d^2*e^2)*sin(d*x + c))*integrate(1/2*cos(d*x + c)/(a*f*x + (a*f*x + a*e)*c
os(d*x + c)^2 + (a*f*x + a*e)*sin(d*x + c)^2 + a*e - 2*(a*f*x + a*e)*sin(d*
x + c)), x) + ((d*f*x + d*e)*cos(d*x + c) - f*sin(d*x + c) - f)*sin(2*d*x +
 2*c) + (d*f*x + d*e)*sin(d*x + c))/(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*
e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(2*d*x + 2*c)^2 + 4*(a
*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)^2 + 4*(a*d^2*f^2*x^2
 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)*sin(2*d*x + 2*c) + (a*d^2*f^2*x^
2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*
d^2*e*f*x + a*d^2*e^2)*sin(d*x + c)^2 - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x +
a*d^2*e^2 + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*cos
(2*d*x + 2*c) + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(sec(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec (c+dx)}{e\sin (c+dx)+e+fx\sin (c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
[Out] Integral(sec(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos (c+d\,x)\,\left(e+f\,x\right)\,\left(a+a\,\sin (c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(e + f*x)*(a + a*sin(c + d*x))),x)
[Out] int(1/(cos(c + d*x)*(e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.274 $\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=29

$$\operatorname{Int}\left(\frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 22.48, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(fx+e)^2\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*(d*f*x + d*e)*cos(d*x + c)^2 + 2*(d*f*x + d*e)*sin(d*x + c)^2 - (2*f*co
s(d*x + c) + (d*f*x + d*e)*sin(d*x + c))*cos(2*d*x + 2*c) - 2*f*cos(d*x + c
) - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d
^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2
*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)
*cos(d*x + c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x +
a*d^2*e^3)*cos(d*x + c)*sin(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x
^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^3*x^3 + 3
*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)^2 - 2*(a*d^2*f
^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + 2*(a*d^2*f^3*x^3
 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*cos(2*d*x
 + 2*c) + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^
3)*sin(d*x + c))*integrate(1/2*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 + 12*f^
2)*cos(d*x + c)/(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 +
4*a*d^2*e^3*f*x + a*d^2*e^4 + (a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*
e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*cos(d*x + c)^2 + (a*d^2*f^4*x^4
+ 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*si
n(d*x + c)^2 + 2*(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 +
 4*a*d^2*e^3*f*x + a*d^2*e^4)*sin(d*x + c)), x) - (a*d^2*f^3*x^3 + 3*a*d^2*
e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^
2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2*c)^2 + 4*(a*d^2*f^3*x^3 + 3*
a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(d*x + c)^2 + 4*(a*d^2*f^
3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(d*x + c)*sin(2
*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*
e^3)*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^
2*f*x + a*d^2*e^3)*sin(d*x + c)^2 - 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
3*a*d^2*e^2*f*x + a*d^2*e^3 + 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^
2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*cos(2*d*x + 2*c) + 4*(a*d^2*f^3*x^3 +
3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*integrate(1/
2*cos(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*f*x + a*
e^2)*cos(d*x + c)^2 + (a*f^2*x^2 + 2*a*e*f*x + a*e^2)*sin(d*x + c)^2 - 2*(a
*f^2*x^2 + 2*a*e*f*x + a*e^2)*sin(d*x + c)), x) + ((d*f*x + d*e)*cos(d*x +
```

```
c) - 2*f*sin(d*x + c) - 2*f)*sin(2*d*x + 2*c) + (d*f*x + d*e)*sin(d*x + c))
/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*
f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2*c)
^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*co
s(d*x + c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d
^2*e^3)*cos(d*x + c)*sin(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2
+ 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*
d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)^2 - 2*(a*d^2*f^3*
x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + 2*(a*d^2*f^3*x^3 +
3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*cos(2*d*x +
2*c) + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*
sin(d*x + c))
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2 + 2*a*f*x
*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec (c+dx)}{e^2 \sin (c+dx)+e^2+2efx \sin (c+dx)+2efx+f^2x^2 \sin (c+dx)+f^2x^2}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2*
e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(e + f*x)^2*(a + a*sin(c + d*x))),x)
[Out] int(1/(cos(c + d*x)*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.275 $\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+a\sin(c+dx)}\, dx$

**Optimal**. Leaf size=475

$$-\frac{2i(e+fx)^3}{3ad}-\frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2}+\frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4}+\frac{2f(e+fx)^2\log\left(1+e^{2i(c+dx)}\right)}{ad^2}+\frac{f^3\,\mathrm{lo}}{}$$

```
[Out] -2*I*f^2*(f*x+e)*polylog(2,-exp(2*I*(d*x+c)))/a/d^3+I*f^2*(f*x+e)*polylog(2
,-I*exp(I*(d*x+c)))/a/d^3+f^3*arctanh(sin(d*x+c))/a/d^4+2*f*(f*x+e)^2*ln(1+
exp(2*I*(d*x+c)))/a/d^2+f^3*ln(cos(d*x+c))/a/d^4-I*f*(f*x+e)^2*arctan(exp(I
*(d*x+c)))/a/d^2-I*f^2*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/a/d^3-2/3*I*(f*x
+e)^3/a/d-f^3*polylog(3,-I*exp(I*(d*x+c)))/a/d^4+f^3*polylog(3,I*exp(I*(d*x
+c)))/a/d^4+f^3*polylog(3,-exp(2*I*(d*x+c)))/a/d^4-f^2*(f*x+e)*sec(d*x+c)/a
/d^3-1/2*f*(f*x+e)^2*sec(d*x+c)^2/a/d^2-1/3*(f*x+e)^3*sec(d*x+c)^3/a/d+f^2*
(f*x+e)*tan(d*x+c)/a/d^3+2/3*(f*x+e)^3*tan(d*x+c)/a/d+1/2*f*(f*x+e)^2*sec(d
*x+c)*tan(d*x+c)/a/d^2+1/3*(f*x+e)^3*sec(d*x+c)^2*tan(d*x+c)/a/d
```

**Rubi** [A]
time = 0.38, antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4627, 4271, 4269, 3556, 3800, 2221, 2611, 2320, 6724, 4494, 3855, 4266}

$$-\frac{f^3\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^4}+\frac{f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}+\frac{f^3\text{PolyLog}\left(3,-e^{2i(c+dx)}\right)}{ad^4}+\frac{if^2(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^3}-\frac{if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{2if^2(e+fx)\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{ad^3}-\frac{if(e+fx)^2\text{ArcTan}\left(e^{i(c+dx)}\right)}{ad^2}+\frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4}+\frac{f^3\log(\cos(c+dx))}{ad^4}+\frac{f^2(e+fx)\tan(c+dx)}{ad^3}-\frac{f^2(e+fx)\sec(c+dx)}{ad^3}+\frac{2f(e+fx)^2\log\left(1+e^{2i(c+dx)}\right)}{ad^2}-\frac{f(e+fx)^2\sec^2(c+dx)}{2ad^2}+\frac{f(e+fx)^2\tan(c+dx)\sec(c+dx)}{2ad^2}+\frac{2(e+fx)^3\tan(c+dx)}{3ad}-\frac{(e+fx)^3\sec^3(c+dx)}{3ad}+\frac{(e+fx)^3\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{2i(e+fx)^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (((-2*I)/3)*(e + f*x)^3)/(a*d) - (I*f*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/
(a*d^2) + (f^3*ArcTanh[Sin[c + d*x]])/(a*d^4) + (2*f*(e + f*x)^2*Log[1 + E^
((2*I)*(c + d*x))])/(a*d^2) + (f^3*Log[Cos[c + d*x]])/(a*d^4) + (I*f^2*(e +
 f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/(a*d^3) - (I*f^2*(e + f*x)*PolyLog[
2, I*E^(I*(c + d*x))])/(a*d^3) - ((2*I)*f^2*(e + f*x)*PolyLog[2, -E^((2*I)*
(c + d*x))])/(a*d^3) - (f^3*PolyLog[3, (-I)*E^(I*(c + d*x))])/(a*d^4) + (f^
3*PolyLog[3, I*E^(I*(c + d*x))])/(a*d^4) + (f^3*PolyLog[3, -E^((2*I)*(c + d
*x))])/(a*d^4) - (f^2*(e + f*x)*Sec[c + d*x])/(a*d^3) - (f*(e + f*x)^2*Sec[
c + d*x]^2)/(2*a*d^2) - ((e + f*x)^3*Sec[c + d*x]^3)/(3*a*d) + (f^2*(e + f*
x)*Tan[c + d*x])/(a*d^3) + (2*(e + f*x)^3*Tan[c + d*x])/(3*a*d) + (f*(e + f
*x)^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*d^2) + ((e + f*x)^3*Sec[c + d*x]^2*Ta
n[c + d*x])/(3*a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4627

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sec^4(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\sec^3(c+dx)\tan(c+dx)\,dx}{a}$$
$$= -\frac{f(e+fx)^2\sec^2(c+dx)}{2ad^2} - \frac{(e+fx)^3\sec^3(c+dx)}{3ad} + \frac{(e+fx)^3\sec^2(c+dx)}{3ad}$$
$$= -\frac{f^2(e+fx)\sec(c+dx)}{ad^3} - \frac{f(e+fx)^2\sec^2(c+dx)}{2ad^2} - \frac{(e+fx)^3\sec^3(c+dx)}{3ad}$$
$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{f^3}{}$$
$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{2f}{}$$
$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{2f}{}$$
$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{2f}{}$$
$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{2f}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1200 vs. $2(475) = 950$.
time = 9.45, size = 1200, normalized size = 2.53

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (f*(3*d^2*(e + f*x)^2*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]] - (6*I)*d*f*(e
 + f*x)*PolyLog[2, (-I)*Cos[c + d*x] + Sin[c + d*x]] + 6*f^2*PolyLog[3, (-I
)*Cos[c + d*x] + Sin[c + d*x]] + (d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*((-I)*C
os[c] + Sin[c]))/(Cos[c] + I*(-1 + Sin[c]))))/(2*a*d^4) - (f*(Cos[c] + I*Si
n[c])*((5*I)*d^2*e^2*x + (4*I)*f^2*x + (5*I)*d^2*e*f*x^2 + ((5*I)/3)*d^2*f^
2*x^3 - 10*d*e*f*x*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] - 5*d*f^2*x^2*Log
[1 - I*Cos[c + d*x] + Sin[c + d*x]] - 5*d*e^2*Log[Cos[c + d*x] + I*(1 + Sin
[c + d*x])] - (4*f^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])])/d - (10*I)*d
*e*f*x*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - (5*I)*d
*f^2*x^2*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - (5*I)
*d*e^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]) - ((4*I
)*f^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]))/d - (10
*f^2*PolyLog[3, I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c]
```

```
+ I*(1 + Sin[c])))/d + 10*f*(e + f*x)*PolyLog[2, I*Cos[c + d*x] - Sin[c + d
*x]]*(I*Cos[c] + Sin[c])*(Cos[c] + I*(1 + Sin[c]))))/(2*a*d^3*(Cos[c] + I*(
1 + Sin[c]))) + (e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Si
n[(d*x)/2] + f^3*x^3*Sin[(d*x)/2])/(2*a*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
(d*x)/2] - Sin[c/2 + (d*x)/2])) + (e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2
] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2])/(3*a*d*(Cos[c/2] + Sin
[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (-(d*e^3*Cos[c/2]) -
3*e^2*f*Cos[c/2] - 3*d*e^2*f*x*Cos[c/2] - 6*e*f^2*x*Cos[c/2] - 3*d*e*f^2*x^
2*Cos[c/2] - 3*f^3*x^2*Cos[c/2] - d*f^3*x^3*Cos[c/2] + d*e^3*Sin[c/2] - 3*e
^2*f*Sin[c/2] + 3*d*e^2*f*x*Sin[c/2] - 6*e*f^2*x*Sin[c/2] + 3*d*e*f^2*x^2*S
in[c/2] - 3*f^3*x^2*Sin[c/2] + d*f^3*x^3*Sin[c/2])/(6*a*d^2*(Cos[c/2] + Sin
[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (5*d^2*e^3*Sin[(d*x)/
2] + 12*e*f^2*Sin[(d*x)/2] + 15*d^2*e^2*f*x*Sin[(d*x)/2] + 12*f^3*x*Sin[(d*
x)/2] + 15*d^2*e*f^2*x^2*Sin[(d*x)/2] + 5*d^2*f^3*x^3*Sin[(d*x)/2])/(6*a*d^
3*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1123 vs. 2(438) = 876.
time = 0.35, size = 1124, normalized size = 2.37

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1124 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 3*f^3*polylog(3,-I*exp(I*(d*x+c)))/a/d^4+5/2/a/d^2*f^3*ln(1-I*exp(I*(d*x+c)
))*x^2-5/2/a/d^4*f^3*ln(1-I*exp(I*(d*x+c)))*c^2-8*I/a/d^2*c*e*f^2*x-4/a/d^2
*f*ln(exp(I*(d*x+c)))*e^2-4/a/d^4*f^3*c^2*ln(exp(I*(d*x+c)))+5/2/a/d^4*f^3*
c^2*ln(exp(I*(d*x+c))+I)+5/2/a/d^2*f*ln(exp(I*(d*x+c))+I)*e^2+8/a/d^3*f^2*e
*c*ln(exp(I*(d*x+c)))-3/a/d^3*e*f^2*c*ln(exp(I*(d*x+c))-I)+3/a/d^3*ln(1+I*e
xp(I*(d*x+c)))*c*e*f^2+3/a/d^2*ln(1+I*exp(I*(d*x+c)))*e*f^2*x+3/2/a/d^2*ln(
1+I*exp(I*(d*x+c)))*f^3*x^2-1/3*(8*d^2*e^3*exp(I*(d*x+c))+3*I*d*e^2*f*exp(I
*(d*x+c))+24*d^2*e*f^2*x^2*exp(I*(d*x+c))+24*d^2*e^2*f*x*exp(I*(d*x+c))+3*I
*d*f^3*x^2*exp(3*I*(d*x+c))+4*I*d^2*f^3*x^3+6*I*d*e*f^2*x*exp(I*(d*x+c))+3*
I*d*e^2*f*exp(3*I*(d*x+c))+6*I*f^3*x*exp(2*I*(d*x+c))+4*I*d^2*e^3+6*f^3*x*e
xp(I*(d*x+c))+6*f^2*e*exp(I*(d*x+c))+6*I*e*f^2+3*I*d*f^3*x^2*exp(I*(d*x+c))
+6*I*e*f^2*exp(2*I*(d*x+c))+12*I*d^2*e^2*f*x+8*d^2*f^3*x^3*exp(I*(d*x+c))+1
2*I*d^2*e*f^2*x^2+6*I*f^3*x+6*f^3*x*exp(3*I*(d*x+c))+6*e*f^2*exp(3*I*(d*x+c
))+6*I*d*e*f^2*x*exp(3*I*(d*x+c)))/(exp(I*(d*x+c))-I)/(exp(I*(d*x+c))+I)^3/
d^3/a+5*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4-3/2/a/d^4*ln(1+I*exp(I*(d*x+c
)))*c^2*f^3-3*I/a/d^3*polylog(2,-I*exp(I*(d*x+c)))*f^3*x-5*I/a/d^3*polylog(
2,I*exp(I*(d*x+c)))*f^3*x-4*I/a/d^3*c^2*e*f^2-4*I/a/d*e*f^2*x^2+4*I/a/d^3*c
^2*f^3*x-3*I/a/d^3*e*f^2*polylog(2,-I*exp(I*(d*x+c)))-5*I/a/d^3*e*f^2*polyl
og(2,I*exp(I*(d*x+c)))-5/a/d^3*f^2*e*c*ln(exp(I*(d*x+c))+I)+8/3*I/a/d^4*c^3
```

```
*f^3-4/3*I/a/d*x^3*f^3+3/2/a/d^2*e^2*f*ln(exp(I*(d*x+c))-I)+3/2/a/d^4*f^3*c
^2*ln(exp(I*(d*x+c))-I)+2/a/d^4*f^3*ln(exp(I*(d*x+c))+I)-2/a/d^4*f^3*ln(exp
(I*(d*x+c)))+5/a/d^2*f^2*e*ln(1-I*exp(I*(d*x+c)))*x+5/a/d^3*f^2*e*ln(1-I*ex
p(I*(d*x+c)))*c
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5130 vs. 2(426) = 852.
time = 1.30, size = 5130, normalized size = 10.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(24*c^2*e*f^2*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d^2 + 2*a*
d^2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*d^2*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 - a*d^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + 6*(4*(8*(d*x + c)*cos
(d*x + c) - sin(3*d*x + 3*c) - sin(d*x + c))*cos(4*d*x + 4*c) + 16*(2*d*x +
 4*(d*x + c)*sin(d*x + c) + 2*c + cos(d*x + c))*cos(3*d*x + 3*c) + 8*cos(3*
d*x + 3*c)^2 + 8*cos(d*x + c)^2 + 5*(2*(2*sin(3*d*x + 3*c) + 2*sin(d*x + c)
 + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c)^2 - 8*cos(
3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3*c) + cos(d*
x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x + c) + 1)*sin(
3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(d*x + c) - 1
)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(2*(2*sin(3
*d*x + 3*c) + 2*sin(d*x + c) + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4
*cos(3*d*x + 3*c)^2 - 8*cos(3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 -
4*(cos(3*d*x + 3*c) + cos(d*x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 -
 4*(2*sin(d*x + c) + 1)*sin(3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x
 + c)^2 - 4*sin(d*x + c) - 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
*x + c) + 1) + 4*(4*d*x + 8*(d*x + c)*sin(d*x + c) + 4*c + cos(3*d*x + 3*c)
 + cos(d*x + c))*sin(4*d*x + 4*c) - 4*(16*(d*x + c)*cos(d*x + c) - 4*sin(d*
x + c) - 1)*sin(3*d*x + 3*c) + 8*sin(3*d*x + 3*c)^2 + 8*sin(d*x + c)^2 + 4*
sin(d*x + c))*c*e*f^2/(a*d^2*cos(4*d*x + 4*c)^2 + 4*a*d^2*cos(3*d*x + 3*c)^
2 + 8*a*d^2*cos(3*d*x + 3*c)*cos(d*x + c) + 4*a*d^2*cos(d*x + c)^2 + a*d^2*
sin(4*d*x + 4*c)^2 + 4*a*d^2*sin(3*d*x + 3*c)^2 + 4*a*d^2*sin(d*x + c)^2 +
4*a*d^2*sin(d*x + c) + a*d^2 - 2*(2*a*d^2*sin(3*d*x + 3*c) + 2*a*d^2*sin(d*
x + c) + a*d^2)*cos(4*d*x + 4*c) + 4*(a*d^2*cos(3*d*x + 3*c) + a*d^2*cos(d*
x + c))*sin(4*d*x + 4*c) + 4*(2*a*d^2*sin(d*x + c) + a*d^2)*sin(3*d*x + 3*c
)) - 24*c*e^2*f*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d + 2*a*d*sin
(d*x + c)/(cos(d*x + c) + 1) - 2*a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 -
a*d*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*(4*(8*(d*x + c)*cos(d*x + c) -
 sin(3*d*x + 3*c) - sin(d*x + c))*cos(4*d*x + 4*c) + 16*(2*d*x + 4*(d*x + c
```

```
)*sin(d*x + c) + 2*c + cos(d*x + c))*cos(3*d*x + 3*c) + 8*cos(3*d*x + 3*c)^
2 + 8*cos(d*x + c)^2 + 5*(2*(2*sin(3*d*x + 3*c) + 2*sin(d*x + c) + 1)*cos(4
*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c)^2 - 8*cos(3*d*x + 3*c
)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3*c) + cos(d*x + c))*sin
(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x + c) + 1)*sin(3*d*x + 3*c
) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(d*x + c) - 1)*log(cos(d
*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(2*(2*sin(3*d*x + 3*c)
 + 2*sin(d*x + c) + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x
+ 3*c)^2 - 8*cos(3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*
x + 3*c) + cos(d*x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d
*x + c) + 1)*sin(3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4
*sin(d*x + c) - 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1
) + 4*(4*d*x + 8*(d*x + c)*sin(d*x + c) + 4*c + cos(3*d*x + 3*c) + cos(d*x
+ c))*sin(4*d*x + 4*c) - 4*(16*(d*x + c)*cos(d*x + c) - 4*sin(d*x + c) - 1)
*sin(3*d*x + 3*c) + 8*sin(3*d*x + 3*c)^2 + 8*sin(d*x + c)^2 + 4*sin(d*x + c
))*e^2*f/(a*d*cos(4*d*x + 4*c)^2 + 4*a*d*cos(3*d*x + 3*c)^2 + 8*a*d*cos(3*d
*x + 3*c)*cos(d*x + c) + 4*a*d*cos(d*x + c)^2 + a*d*sin(4*d*x + 4*c)^2 + 4*
a*d*sin(3*d*x + 3*c)^2 + 4*a*d*sin(d*x + c)^2 + 4*a*d*sin(d*x + c) + a*d -
2*(2*a*d*sin(3*d*x + 3*c) + 2*a*d*sin(d*x + c) + a*d)*cos(4*d*x + 4*c) + 4*
(a*d*cos(3*d*x + 3*c) + a*d*cos(d*x + c))*sin(4*d*x + 4*c) + 4*(2*a*d*sin(d
*x + c) + a*d)*sin(3*d*x + 3*c)) + 8*e^3*(sin(d*x + c)/(cos(d*x + c) + 1) +
 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 - 1)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 12*(24*d*e*f^2
- 8*(2*c^3 + 3*c)*f^3 - 6*((5*c^2 + 4)*f^3*cos(4*d*x + 4*c) - 2*(-5*I*c^2 -
 4*I)*f^3*cos(3*d*x + 3*c) - 2*(-5*I*c^2 - 4*I)*f^3*cos(d*x + c) - (-5*I*c^
2 - 4*I)*f^3*sin(4*d*x + 4*c) - 2*(5*c^2 + 4)*f^3*sin(3*d*x + 3*c) - 2*(5*c
^2 + 4)*f^3*sin(d*x + c) - (5*c^2 + 4)*f^3)*arctan2(sin(d*x + c) + 1, cos(d
*x + c)) - 18*(c^2*f^3*cos(4*d*x + 4*c) + 2*I*c^2*f^3*cos(3*d*x + 3*c) + 2*
I*c^2*f^3*cos(d*x + c) + I*c^2*f^3*sin(4*d*x + 4*c) - 2*c^2*f^3*sin(3*d*x +
 3*c) - 2*c^2*f^3*sin(d*x + c) - c^2*f^3)*arctan2(sin(d*x + c) - 1, cos(d*x
 + c)) - 30*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c) - ((d*x + c)^2
*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(4*d*x + 4*c) - 2*(I*(d*x + c)^2*f
^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c))*cos(3*d*x + 3*c) - 2*(I*(d*x + c)^2
*f^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c))*cos(d*x + c) - (I*(d*x + c)^2*f^3
 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c))*sin(4*d*x...
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1548 vs. $2(439) = 878$.
time = 0.48, size = 1548, normalized size = 3.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*d^3*f^3*x^3 + 12*d^3*f^2*x^2*e + 12*d^3*f*x*e^2 + 4*d^3*e^3 - 4*(2*
d^3*f^3*x^3 + 6*d^3*f*x*e^2 + 3*d*f^3*x + 2*d^3*e^3 + 3*(2*d^3*f^2*x^2 + d*
f^2)*e)*cos(d*x + c)^2 - 6*(d^2*f^3*x^2 + 2*d^2*f^2*x*e + d^2*f*e^2)*cos(d*
x + c) - 18*((-I*d*f^3*x - I*d*f^2*e)*cos(d*x + c)*sin(d*x + c) + (-I*d*f^3
*x - I*d*f^2*e)*cos(d*x + c))*dilog(I*cos(d*x + c) + sin(d*x + c)) - 30*((I
*d*f^3*x + I*d*f^2*e)*cos(d*x + c)*sin(d*x + c) + (I*d*f^3*x + I*d*f^2*e)*c
os(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - 18*((I*d*f^3*x + I*d*f^
2*e)*cos(d*x + c)*sin(d*x + c) + (I*d*f^3*x + I*d*f^2*e)*cos(d*x + c))*dilo
g(-I*cos(d*x + c) + sin(d*x + c)) - 30*((-I*d*f^3*x - I*d*f^2*e)*cos(d*x +
c)*sin(d*x + c) + (-I*d*f^3*x - I*d*f^2*e)*cos(d*x + c))*dilog(-I*cos(d*x +
 c) - sin(d*x + c)) - 3*((10*c*d*f^2*e - (5*c^2 + 4)*f^3 - 5*d^2*f*e^2)*cos
(d*x + c)*sin(d*x + c) + (10*c*d*f^2*e - (5*c^2 + 4)*f^3 - 5*d^2*f*e^2)*cos
(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) + 9*((c^2*f^3 - 2*c*d*f^2
*e + d^2*f*e^2)*cos(d*x + c)*sin(d*x + c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*
e^2)*cos(d*x + c))*log(cos(d*x + c) - I*sin(d*x + c) + I) + 15*((d^2*f^3*x^
2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*x + c)*sin(d*x + c) + (d^2*f
^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*x + c))*log(I*cos(d*x +
 c) + sin(d*x + c) + 1) + 9*((d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^
2)*e)*cos(d*x + c)*sin(d*x + c) + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c
*d*f^2)*e)*cos(d*x + c))*log(I*cos(d*x + c) - sin(d*x + c) + 1) + 15*((d^2*
f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*x + c)*sin(d*x + c) +
(d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*cos(d*x + c))*log(-I*co
s(d*x + c) + sin(d*x + c) + 1) + 9*((d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x +
 c*d*f^2)*e)*cos(d*x + c)*sin(d*x + c) + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^
2*x + c*d*f^2)*e)*cos(d*x + c))*log(-I*cos(d*x + c) - sin(d*x + c) + 1) - 3
*((10*c*d*f^2*e - (5*c^2 + 4)*f^3 - 5*d^2*f*e^2)*cos(d*x + c)*sin(d*x + c)
+ (10*c*d*f^2*e - (5*c^2 + 4)*f^3 - 5*d^2*f*e^2)*cos(d*x + c))*log(-cos(d*x
 + c) + I*sin(d*x + c) + I) + 9*((c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(d*
x + c)*sin(d*x + c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*cos(d*x + c))*log
(-cos(d*x + c) - I*sin(d*x + c) + I) + 18*(f^3*cos(d*x + c)*sin(d*x + c) +
f^3*cos(d*x + c))*polylog(3, I*cos(d*x + c) + sin(d*x + c)) + 30*(f^3*cos(d
*x + c)*sin(d*x + c) + f^3*cos(d*x + c))*polylog(3, I*cos(d*x + c) - sin(d*
x + c)) + 18*(f^3*cos(d*x + c)*sin(d*x + c) + f^3*cos(d*x + c))*polylog(3,
-I*cos(d*x + c) + sin(d*x + c)) + 30*(f^3*cos(d*x + c)*sin(d*x + c) + f^3*c
os(d*x + c))*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) + 8*(d^3*f^3*x^3 +
3*d^3*f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*sin(d*x + c))/(a*d^4*cos(d*x + c
)*sin(d*x + c) + a*d^4*cos(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3\sec^2(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{f^3x^3\sec^2(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{3ef^2x^2\sec^2(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{3e^2fx\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*
sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sec(c + d*x
)**2/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sec(c + d*x)**2/(sin(c +
d*x) + 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.276 $\int \frac{(e+fx)^2 \sec^2(c+dx)}{a+a\sin(c+dx)}\, dx$

**Optimal**. Leaf size=343

$$-\frac{2i(e+fx)^2}{3ad}-\frac{2if(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{3ad^2}+\frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2}+\frac{if^2\text{Li}_2\left(-ie^{i(c+dx)}\right)}{3ad^3}-\frac{if^2\text{Li}_2\left(ie^{i(c+dx)}\right)}{3ad^3}$$

```
[Out] -2/3*I*(f*x+e)^2/a/d-2/3*I*f*(f*x+e)*arctan(exp(I*(d*x+c)))/a/d^2+4/3*f*(f*
x+e)*ln(1+exp(2*I*(d*x+c)))/a/d^2+1/3*I*f^2*polylog(2,-I*exp(I*(d*x+c)))/a/
d^3-1/3*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3-2/3*I*f^2*polylog(2,-exp(2*
I*(d*x+c)))/a/d^3-1/3*f^2*sec(d*x+c)/a/d^3-1/3*f*(f*x+e)*sec(d*x+c)^2/a/d^2
-1/3*(f*x+e)^2*sec(d*x+c)^3/a/d+1/3*f^2*tan(d*x+c)/a/d^3+2/3*(f*x+e)^2*tan(
d*x+c)/a/d+1/3*f*(f*x+e)*sec(d*x+c)*tan(d*x+c)/a/d^2+1/3*(f*x+e)^2*sec(d*x+
c)^2*tan(d*x+c)/a/d
```

**Rubi** [A]
time = 0.25, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4627, 4271, 3852, 8, 4269, 3800, 2221, 2317, 2438, 4494, 4270, 4266}

$$\frac{if^2\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{3ad^3}-\frac{if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{3ad^3}-\frac{2if^2\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{3ad^3}-\frac{2if(e+fx)\text{ArcTan}\left(e^{i(c+dx)}\right)}{3ad^2}+\frac{f^2\tan(c+dx)}{3ad^3}-\frac{f^2\sec(c+dx)}{3ad^3}+\frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2}-\frac{f(e+fx)\sec^2(c+dx)}{3ad^2}+\frac{f(e+fx)\tan(c+dx)\sec(c+dx)}{3ad^2}+\frac{2(e+fx)^2\tan(c+dx)}{3ad}-\frac{(e+fx)^2\sec^3(c+dx)}{3ad}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{2i(e+fx)^2}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (((-2*I)/3)*(e + f*x)^2)/(a*d) - (((2*I)/3)*f*(e + f*x)*ArcTan[E^(I*(c + d*
x))])/(a*d^2) + (4*f*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/(3*a*d^2) + ((
I/3)*f^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/(a*d^3) - ((I/3)*f^2*PolyLog[2,
I*E^(I*(c + d*x))])/(a*d^3) - (((2*I)/3)*f^2*PolyLog[2, -E^((2*I)*(c + d*x)
)])/(a*d^3) - (f^2*Sec[c + d*x])/(3*a*d^3) - (f*(e + f*x)*Sec[c + d*x]^2)/(
3*a*d^2) - ((e + f*x)^2*Sec[c + d*x]^3)/(3*a*d) + (f^2*Tan[c + d*x])/(3*a*d
^3) + (2*(e + f*x)^2*Tan[c + d*x])/(3*a*d) + (f*(e + f*x)*Sec[c + d*x]*Tan[
c + d*x])/(3*a*d^2) + ((e + f*x)^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4627

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sec^4(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)\sec^2(c+dx)}{3ad^2} - \frac{(e+fx)^2\sec^3(c+dx)}{3ad} + \frac{(e+fx)^2\sec^2(c+dx)\,\mathrm{t}}{3ad}$$

$$= -\frac{f^2\sec(c+dx)}{3ad^3} - \frac{f(e+fx)\sec^2(c+dx)}{3ad^2} - \frac{(e+fx)^2\sec^3(c+dx)}{3ad} + \frac{2(e+}{}$$

$$= -\frac{2i(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{3ad^2} - \frac{f^2\sec(c+dx)}{3ad^3} - \frac{f(e+fx)\,\mathrm{s}}{3a}$$

$$= -\frac{2i(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{3ad^2} + \frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2}$$

$$= -\frac{2i(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{3ad^2} + \frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2}$$

$$= -\frac{2i(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{3ad^2} + \frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2}$$

**Mathematica** [A]
time = 5.16, size = 564, normalized size = 1.64

$$\frac{12df(e+fx)\log(1+i\cos(c+dx)-\sin(c+dx))+20df(e+fx)\log(1-i\cos(c+dx)+\sin(c+dx))-20if^2\text{Li}_2(i\cos(c+dx)-\sin(c+dx))-12if^2\text{Li}_2(-i\cos(c+dx)+\sin(c+dx))+\frac{6d^2fx(2e+fx)(-i\cos(c)+\sin(c))}{\cos(c)+i(-1+\sin(c))}+\frac{10d^2fx(2e+fx)(-i\cos(c)+\sin(c))}{\cos(c)+i(1+\sin(c))}+\frac{-2f^2\cos(c)-2df(e+fx)\cos(dx)+2d^2e^2\cos(c+dx)+4f^2\cos(c+dx)+4d^2efx\cos(c+dx)+2d^2f^2x^2\cos(c+dx)-2def\cos(2c+dx)-2df^2x\cos(2c+dx)-4d^2e^2\cos(c+2dx)-2f^2\cos(c+2dx)-8d^2efx\cos(c+2dx)-4d^2f^2x^2\cos(c+2dx)+8d^2e^2\sin(dx)+2f^2\sin(dx)+16d^2efx\sin(dx)+8d^2f^2x^2\sin(dx)+d^2e^2\sin(2(c+dx))+2f^2\sin(2(c+dx))+2d^2efx\sin(2(c+dx))+d^2f^2x^2\sin(2(c+dx))-2f^2\sin(2c+dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}}{12ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (12*d*f*(e + f*x)*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]] + 20*d*f*(e + f*x)
*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] - (20*I)*f^2*PolyLog[2, I*Cos[c + d
*x] - Sin[c + d*x]] - (12*I)*f^2*PolyLog[2, (-I)*Cos[c + d*x] + Sin[c + d*x
]] + (6*d^2*f*x*(2*e + f*x)*((-I)*Cos[c] + Sin[c]))/(Cos[c] + I*(-1 + Sin[c
])) + (10*d^2*f*x*(2*e + f*x)*((-I)*Cos[c] + Sin[c]))/(Cos[c] + I*(1 + Sin[
c])) + (-2*f^2*Cos[c] - 2*d*f*(e + f*x)*Cos[d*x] + 2*d^2*e^2*Cos[c + d*x] +
 4*f^2*Cos[c + d*x] + 4*d^2*e*f*x*Cos[c + d*x] + 2*d^2*f^2*x^2*Cos[c + d*x]
 - 2*d*e*f*Cos[2*c + d*x] - 2*d*f^2*x*Cos[2*c + d*x] - 4*d^2*e^2*Cos[c + 2*
d*x] - 2*f^2*Cos[c + 2*d*x] - 8*d^2*e*f*x*Cos[c + 2*d*x] - 4*d^2*f^2*x^2*Co
s[c + 2*d*x] + 8*d^2*e^2*Sin[d*x] + 2*f^2*Sin[d*x] + 16*d^2*e*f*x*Sin[d*x]
+ 8*d^2*f^2*x^2*Sin[d*x] + d^2*e^2*Sin[2*(c + d*x)] + 2*f^2*Sin[2*(c + d*x)
] + 2*d^2*e*f*x*Sin[2*(c + d*x)] + d^2*f^2*x^2*Sin[2*(c + d*x)] - 2*f^2*Sin
[2*c + d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/(12*a*d^3)
```

**Maple** [A]
time = 0.24, size = 573, normalized size = 1.67

| method | result |
|---|---|
| risch | $-\frac{2\left(if^2+f^2 e^{3i(dx+c)}+4d^2f^2x^2 e^{i(dx+c)}+2id^2x^2f^2+if^2 e^{2i(dx+c)}+8d^2efx\, e^{i(dx+c)}+id\,f^2x\, e^{i(dx+c)}+idef\, e^{i(dx+c)}+id\,f^2x\, e^{3i(dx+c)}\right.}{3\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)^3 d^3a}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(I*f^2+f^2*exp(3*I*(d*x+c))+4*d^2*f^2*x^2*exp(I*(d*x+c))+2*I*d^2*x^2*f
^2+I*f^2*exp(2*I*(d*x+c))+8*d^2*e*f*x*exp(I*(d*x+c))+I*d*f^2*x*exp(I*(d*x+c
))+I*d*e*f*exp(I*(d*x+c))+I*d*f^2*x*exp(3*I*(d*x+c))+I*d*e*f*exp(3*I*(d*x+c
))+4*I*d^2*e*f*x+f^2*exp(I*(d*x+c))+4*d^2*e^2*exp(I*(d*x+c))+2*I*d^2*e^2)/(
exp(I*(d*x+c))-I)/(exp(I*(d*x+c))+I)^3/d^3/a+1/d^2/a*e*f*ln(exp(I*(d*x+c))-
I)+5/3/a/d^2*f*ln(exp(I*(d*x+c))+I)*e-8/3/a/d^2*f*ln(exp(I*(d*x+c)))*e-1/d^
3/a*f^2*c*ln(exp(I*(d*x+c))-I)-5/3/a/d^3*f^2*c*ln(exp(I*(d*x+c))+I)+8/3/a/d
^3*f^2*c*ln(exp(I*(d*x+c)))-4/3*I/d^3/a*f^2*c^2-I/d^3/a*f^2*polylog(2,-I*ex
p(I*(d*x+c)))-8/3*I/d^2/a*f^2*c*x+1/d^2/a*f^2*ln(1+I*exp(I*(d*x+c)))*x+1/d^
3/a*f^2*ln(1+I*exp(I*(d*x+c)))*c-4/3*I/d/a*f^2*x^2+5/3/a/d^2*f^2*ln(1-I*exp
(I*(d*x+c)))*x+5/3/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c-5/3*I/d^3/a*f^2*polyl
og(2,I*exp(I*(d*x+c)))
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1328 vs. $2(290) = 580$.
time = 0.64, size = 1328, normalized size = 3.87

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(8*d^2*e^2 + 4*f^2*cos(2*d*x + 2*c) + 4*I*f^2*sin(2*d*x + 2*c) + 4*f^2 - 1
0*(d*e*f*cos(4*d*x + 4*c) + 2*I*d*e*f*cos(3*d*x + 3*c) + 2*I*d*e*f*cos(d*x
+ c) + I*d*e*f*sin(4*d*x + 4*c) - 2*d*e*f*sin(3*d*x + 3*c) - 2*d*e*f*sin(d*
x + c) - d*e*f)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 6*(d*e*f*cos(4*d*
x + 4*c) + 2*I*d*e*f*cos(3*d*x + 3*c) + 2*I*d*e*f*cos(d*x + c) + I*d*e*f*si
n(4*d*x + 4*c) - 2*d*e*f*sin(3*d*x + 3*c) - 2*d*e*f*sin(d*x + c) - d*e*f)*a
rctan2(sin(d*x + c) - 1, cos(d*x + c)) + 10*(d*f^2*x*cos(4*d*x + 4*c) + 2*I
*d*f^2*x*cos(3*d*x + 3*c) + 2*I*d*f^2*x*cos(d*x + c) + I*d*f^2*x*sin(4*d*x
+ 4*c) - 2*d*f^2*x*sin(3*d*x + 3*c) - 2*d*f^2*x*sin(d*x + c) - d*f^2*x)*arc
tan2(cos(d*x + c), sin(d*x + c) + 1) - 6*(d*f^2*x*cos(4*d*x + 4*c) + 2*I*d*
f^2*x*cos(3*d*x + 3*c) + 2*I*d*f^2*x*cos(d*x + c) + I*d*f^2*x*sin(4*d*x + 4
*c) - 2*d*f^2*x*sin(3*d*x + 3*c) - 2*d*f^2*x*sin(d*x + c) - d*f^2*x)*arctan
2(cos(d*x + c), -sin(d*x + c) + 1) + 8*(d^2*f^2*x^2 + 2*d^2*e*f*x)*cos(4*d*
x + 4*c) + 4*(4*I*d^2*f^2*x^2 + d*e*f - I*f^2 + (8*I*d^2*e*f + d*f^2)*x)*co
s(3*d*x + 3*c) + 4*(-4*I*d^2*e^2 + d*f^2*x + d*e*f - I*f^2)*cos(d*x + c) +
10*(f^2*cos(4*d*x + 4*c) + 2*I*f^2*cos(3*d*x + 3*c) + 2*I*f^2*cos(d*x + c)
+ I*f^2*sin(4*d*x + 4*c) - 2*f^2*sin(3*d*x + 3*c) - 2*f^2*sin(d*x + c) - f^
2)*dilog(I*e^(I*d*x + I*c)) + 6*(f^2*cos(4*d*x + 4*c) + 2*I*f^2*cos(3*d*x +
 3*c) + 2*I*f^2*cos(d*x + c) + I*f^2*sin(4*d*x + 4*c) - 2*f^2*sin(3*d*x + 3
*c) - 2*f^2*sin(d*x + c) - f^2)*dilog(-I*e^(I*d*x + I*c)) + 5*(-I*d*f^2*x -
 I*d*e*f + (I*d*f^2*x + I*d*e*f)*cos(4*d*x + 4*c) - 2*(d*f^2*x + d*e*f)*cos
(3*d*x + 3*c) - 2*(d*f^2*x + d*e*f)*cos(d*x + c) - (d*f^2*x + d*e*f)*sin(4*
d*x + 4*c) + 2*(-I*d*f^2*x - I*d*e*f)*sin(3*d*x + 3*c) + 2*(-I*d*f^2*x - I*
d*e*f)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) +
 1) + 3*(-I*d*f^2*x - I*d*e*f + (I*d*f^2*x + I*d*e*f)*cos(4*d*x + 4*c) - 2*
(d*f^2*x + d*e*f)*cos(3*d*x + 3*c) - 2*(d*f^2*x + d*e*f)*cos(d*x + c) - (d*
f^2*x + d*e*f)*sin(4*d*x + 4*c) + 2*(-I*d*f^2*x - I*d*e*f)*sin(3*d*x + 3*c)
 + 2*(-I*d*f^2*x - I*d*e*f)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)
^2 - 2*sin(d*x + c) + 1) + 8*(I*d^2*f^2*x^2 + 2*I*d^2*e*f*x)*sin(4*d*x + 4*
c) - 4*(4*d^2*f^2*x^2 - I*d*e*f - f^2 + (8*d^2*e*f - I*d*f^2)*x)*sin(3*d*x
+ 3*c) + 4*(4*d^2*e^2 + I*d*f^2*x + I*d*e*f + f^2)*sin(d*x + c))/(-6*I*a*d^
3*cos(4*d*x + 4*c) + 12*a*d^3*cos(3*d*x + 3*c) + 12*a*d^3*cos(d*x + c) + 6*
a*d^3*sin(4*d*x + 4*c) + 12*I*a*d^3*sin(3*d*x + 3*c) + 12*I*a*d^3*sin(d*x +
 c) + 6*I*a*d^3)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(298) = 596$.

time = 0.44, size = 868, normalized size = 2.53

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^2*f^2*x^2 + 4*d^2*f*x*e - 2*(2*d^2*f^2*x^2 + 4*d^2*f*x*e + 2*d^2*e
^2 + f^2)*cos(d*x + c)^2 + 2*d^2*e^2 - 2*(d*f^2*x + d*f*e)*cos(d*x + c) - 3
*(-I*f^2*cos(d*x + c)*sin(d*x + c) - I*f^2*cos(d*x + c))*dilog(I*cos(d*x +
c) + sin(d*x + c)) - 5*(I*f^2*cos(d*x + c)*sin(d*x + c) + I*f^2*cos(d*x + c
))*dilog(I*cos(d*x + c) - sin(d*x + c)) - 3*(I*f^2*cos(d*x + c)*sin(d*x + c
) + I*f^2*cos(d*x + c))*dilog(-I*cos(d*x + c) + sin(d*x + c)) - 5*(-I*f^2*c
os(d*x + c)*sin(d*x + c) - I*f^2*cos(d*x + c))*dilog(-I*cos(d*x + c) - sin(
d*x + c)) - 5*((c*f^2 - d*f*e)*cos(d*x + c)*sin(d*x + c) + (c*f^2 - d*f*e)*
cos(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 3*((c*f^2 - d*f*e)*c
os(d*x + c)*sin(d*x + c) + (c*f^2 - d*f*e)*cos(d*x + c))*log(cos(d*x + c) -
 I*sin(d*x + c) + I) + 5*((d*f^2*x + c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*
f^2*x + c*f^2)*cos(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) + 3*((d
*f^2*x + c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*f^2*x + c*f^2)*cos(d*x + c))
*log(I*cos(d*x + c) - sin(d*x + c) + 1) + 5*((d*f^2*x + c*f^2)*cos(d*x + c)
*sin(d*x + c) + (d*f^2*x + c*f^2)*cos(d*x + c))*log(-I*cos(d*x + c) + sin(d
*x + c) + 1) + 3*((d*f^2*x + c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*f^2*x +
c*f^2)*cos(d*x + c))*log(-I*cos(d*x + c) - sin(d*x + c) + 1) - 5*((c*f^2 -
d*f*e)*cos(d*x + c)*sin(d*x + c) + (c*f^2 - d*f*e)*cos(d*x + c))*log(-cos(d
*x + c) + I*sin(d*x + c) + I) - 3*((c*f^2 - d*f*e)*cos(d*x + c)*sin(d*x + c
) + (c*f^2 - d*f*e)*cos(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + I) +
 4*(d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2)*sin(d*x + c))/(a*d^3*cos(d*x + c)*
sin(d*x + c) + a*d^3*cos(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2\sec^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\sec^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sec(c + d*x)**2/(
sin(c + d*x) + 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.277 $\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=152

$$\frac{f\tanh^{-1}(\sin(c+dx))}{6ad^2}+\frac{2f\log(\cos(c+dx))}{3ad^2}-\frac{f\sec^2(c+dx)}{6ad^2}-\frac{(e+fx)\sec^3(c+dx)}{3ad}+\frac{2(e+fx)\tan(c+dx}{3ad}$$

```
[Out] 1/6*f*arctanh(sin(d*x+c))/a/d^2+2/3*f*ln(cos(d*x+c))/a/d^2-1/6*f*sec(d*x+c)
^2/a/d^2-1/3*(f*x+e)*sec(d*x+c)^3/a/d+2/3*(f*x+e)*tan(d*x+c)/a/d+1/6*f*sec(
d*x+c)*tan(d*x+c)/a/d^2+1/3*(f*x+e)*sec(d*x+c)^2*tan(d*x+c)/a/d
```

**Rubi** [A]
time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4627, 4270, 4269, 3556, 4494, 3853, 3855}

$$-\frac{f\sec^2(c+dx)}{6ad^2}+\frac{f\tanh^{-1}(\sin(c+dx))}{6ad^2}+\frac{2f\log(\cos(c+dx))}{3ad^2}+\frac{f\tan(c+dx)\sec(c+dx)}{6ad^2}+\frac{2(e+fx)\tan(c+dx)}{3ad}-\frac{(e+fx)\sec^3(c+dx)}{3ad}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (f*ArcTanh[Sin[c + d*x]])/(6*a*d^2) + (2*f*Log[Cos[c + d*x]])/(3*a*d^2) - (
f*Sec[c + d*x]^2)/(6*a*d^2) - ((e + f*x)*Sec[c + d*x]^3)/(3*a*d) + (2*(e +
f*x)*Tan[c + d*x])/(3*a*d) + (f*Sec[c + d*x]*Tan[c + d*x])/(6*a*d^2) + ((e
+ f*x)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4627

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\sec^4(c+dx)\,dx}{a} - \frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)\,dx}{a} \\
&= -\frac{f\sec^2(c+dx)}{6ad^2} - \frac{(e+fx)\sec^3(c+dx)}{3ad} + \frac{(e+fx)\sec^2(c+dx)\tan(c+dx)}{3ad} \\
&= -\frac{f\sec^2(c+dx)}{6ad^2} - \frac{(e+fx)\sec^3(c+dx)}{3ad} + \frac{2(e+fx)\tan(c+dx)}{3ad} + \frac{f\sec(c+}{} \\
&= \frac{f\tanh^{-1}(\sin(c+dx))}{6ad^2} + \frac{2f\log(\cos(c+dx))}{3ad^2} - \frac{f\sec^2(c+dx)}{6ad^2} - \frac{(e+fx)\sec}{3ad}
\end{aligned}
$$

**Mathematica [A]**
time = 0.99, size = 231, normalized size = 1.52

$$
\frac{-2d(e+fx)(\cos(2(c+dx))-2\sin(c+dx))+\cos(c+dx)\left(de-f-cf+3f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+5f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\left(de-cf+3f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+5f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)}{6ad^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*d*(e + f*x)*(Cos[2*(c + d*x)] - 2*Sin[c + d*x]) + Cos[c + d*x]*(d*e - f
 - c*f + 3*f*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 5*f*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + (d*e - c*f + 3*f*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 5*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[c + d*x]))/(
6*a*d^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])*(1 + Sin[c + d*x]))
```

**Maple [A]**
time = 0.18, size = 181, normalized size = 1.19

| method | result |
|---|---|
| risch | $-\frac{4ifx}{3ad}-\frac{4ifc}{3a\,d^{2}}-\frac{i\left(f\,{e}^{3i(dx+c)}+4dxf-8idfx\,{e}^{i(dx+c)}+4de+{e}^{i(dx+c)}f-8ide\,{e}^{i(dx+c)}\right)}{3\left({e}^{i(dx+c)}+i\right)^{3}d^{2}\left({e}^{i(dx+c)}-i\right)a}+\frac{5f\ln\left({e}^{i(dx+c)}+i\right)}{6a\,d^{2}}+$ |
| derivativedivides | $\frac{\frac{fc}{3\cos(dx+c)^{3}}-\frac{ed}{3\cos(dx+c)^{3}}-f\left(\frac{dx+c}{3\cos(dx+c)^{3}}-\frac{\sec(dx+c)\tan(dx+c)}{6}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{6}\right)+fc\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)}{d^{2}}$ |
| default | $\frac{\frac{fc}{3\cos(dx+c)^{3}}-\frac{ed}{3\cos(dx+c)^{3}}-f\left(\frac{dx+c}{3\cos(dx+c)^{3}}-\frac{\sec(dx+c)\tan(dx+c)}{6}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{6}\right)+fc\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)}{d^{2}}$ |
| norman | $\frac{\frac{2e}{3da}-\frac{2e\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{fx}{3ad}+\frac{(-6de+f)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a\,d^{2}}-\frac{(2de+f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a\,d^{2}}-\frac{4fx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{2fx\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2/a*(1/3*f*c/cos(d*x+c)^3-1/3*e*d/cos(d*x+c)^3-f*(1/3*(d*x+c)/cos(d*x+c
)^3-1/6*sec(d*x+c)*tan(d*x+c)-1/6*ln(sec(d*x+c)+tan(d*x+c)))+f*c*(-2/3-1/3*
sec(d*x+c)^2)*tan(d*x+c)-e*d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+f*(1/3*(d*x
+c)*sin(d*x+c)/cos(d*x+c)^3-1/6/cos(d*x+c)^2+2/3*(d*x+c)*tan(d*x+c)+2/3*ln(
cos(d*x+c))))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. 2(138) = 276.
time = 0.31, size = 1115, normalized size = 7.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(8*c*f*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d + 2*a*d*sin(d*
x + c)/(cos(d*x + c) + 1) - 2*a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*d
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (4*(8*(d*x + c)*cos(d*x + c) - sin(
```

```
3*d*x + 3*c) - sin(d*x + c))*cos(4*d*x + 4*c) + 16*(2*d*x + 4*(d*x + c)*sin
(d*x + c) + 2*c + cos(d*x + c))*cos(3*d*x + 3*c) + 8*cos(3*d*x + 3*c)^2 + 8
*cos(d*x + c)^2 + 5*(2*(2*sin(3*d*x + 3*c) + 2*sin(d*x + c) + 1)*cos(4*d*x
+ 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c)^2 - 8*cos(3*d*x + 3*c)*cos
(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3*c) + cos(d*x + c))*sin(4*d*
x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x + c) + 1)*sin(3*d*x + 3*c) - 4
*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(d*x + c) - 1)*log(cos(d*x +
c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(2*(2*sin(3*d*x + 3*c) + 2*
sin(d*x + c) + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c
)^2 - 8*cos(3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3
*c) + cos(d*x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x +
c) + 1)*sin(3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(
d*x + c) - 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4
*(4*d*x + 8*(d*x + c)*sin(d*x + c) + 4*c + cos(3*d*x + 3*c) + cos(d*x + c))
*sin(4*d*x + 4*c) - 4*(16*(d*x + c)*cos(d*x + c) - 4*sin(d*x + c) - 1)*sin(
3*d*x + 3*c) + 8*sin(3*d*x + 3*c)^2 + 8*sin(d*x + c)^2 + 4*sin(d*x + c))*f/
(a*d*cos(4*d*x + 4*c)^2 + 4*a*d*cos(3*d*x + 3*c)^2 + 8*a*d*cos(3*d*x + 3*c)
*cos(d*x + c) + 4*a*d*cos(d*x + c)^2 + a*d*sin(4*d*x + 4*c)^2 + 4*a*d*sin(3
*d*x + 3*c)^2 + 4*a*d*sin(d*x + c)^2 + 4*a*d*sin(d*x + c) + a*d - 2*(2*a*d*
sin(3*d*x + 3*c) + 2*a*d*sin(d*x + c) + a*d)*cos(4*d*x + 4*c) + 4*(a*d*cos(
3*d*x + 3*c) + a*d*cos(d*x + c))*sin(4*d*x + 4*c) + 4*(2*a*d*sin(d*x + c) +
 a*d)*sin(3*d*x + 3*c)) - 8*e*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a
 + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

**Fricas** [A]
time = 0.38, size = 159, normalized size = 1.05

$$\frac{4\,dfx - 8\,(dfx + de)\cos\left(dx + c\right)^{2} - 2\,f\cos\left(dx + c\right) + 4\,de + 5\,\left(f\cos\left(dx + c\right)\sin\left(dx + c\right) + f\cos\left(dx + c\right)\right)\log\left(\sin\left(dx + c\right) + 1\right) + 3\,\left(f\cos\left(dx + c\right)\sin\left(dx + c\right) + f\cos\left(dx + c\right)\right)\log\left(-\sin\left(dx + c\right) + 1\right) + 8\,(dfx + de)\sin\left(dx + c\right)}{12\,\left(ad^{2}\cos\left(dx + c\right)\sin\left(dx + c\right) + ad^{2}\cos\left(dx + c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*d*f*x - 8*(d*f*x + d*e)*cos(d*x + c)^2 - 2*f*cos(d*x + c) + 4*d*e +
 5*(f*cos(d*x + c)*sin(d*x + c) + f*cos(d*x + c))*log(sin(d*x + c) + 1) + 3
*(f*cos(d*x + c)*sin(d*x + c) + f*cos(d*x + c))*log(-sin(d*x + c) + 1) + 8*
(d*f*x + d*e)*sin(d*x + c))/(a*d^2*cos(d*x + c)*sin(d*x + c) + a*d^2*cos(d*
x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e\sec^{2}\left(c + dx\right)}{\sin\left(c + dx\right) + 1}\, dx + \int \frac{fx\sec^{2}\left(c + dx\right)}{\sin\left(c + dx\right) + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f*x*sec(c + d
*x)**2/(sin(c + d*x) + 1), x))/a
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 6656 vs. $2(141) = 282$.
time = 6.41, size = 6656, normalized size = 43.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(4*d*f*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 16*d*f*x*tan(1/2*d*x)^4*tan(1/
2*c)^3 + 16*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^4 + 4*d*e*tan(1/2*d*x)^4*tan(1/
2*c)^4 - 3*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*
c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*t
an(1/2*d*x)^4*tan(1/2*c)^4 - 5*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 24*d*f*x*tan(1/2*d*x)^4*ta
n(1/2*c)^2 - 64*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^3 + 16*d*e*tan(1/2*d*x)^4*t
an(1/2*c)^3 + 6*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan
(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 +
1))*tan(1/2*d*x)^4*tan(1/2*c)^3 + 10*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)
^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
+ 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^3 - 24*d*f*x*tan(1/2*d*x
)^2*tan(1/2*c)^4 + 16*d*e*tan(1/2*d*x)^3*tan(1/2*c)^4 + 6*f*log(2*(tan(1/2*
d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^4 +
 10*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*
tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*
```

```
d*x)^3*tan(1/2*c)^4 + 2*f*tan(1/2*d*x)^4*tan(1/2*c)^4 + 16*d*f*x*tan(1/2*d*
x)^4*tan(1/2*c) - 24*d*e*tan(1/2*d*x)^4*tan(1/2*c)^2 - 64*d*e*tan(1/2*d*x)^
3*tan(1/2*c)^3 + 12*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 20*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*
d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2
*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 16*d*f*x*tan(1/2
*d*x)*tan(1/2*c)^4 - 24*d*e*tan(1/2*d*x)^2*tan(1/2*c)^4 + 4*d*f*x*tan(1/2*d
*x)^4 + 64*d*f*x*tan(1/2*d*x)^3*tan(1/2*c) + 16*d*e*tan(1/2*d*x)^4*tan(1/2*
c) - 6*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^4*tan(1/2*c) - 10*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c) + 144*d*f*x*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 36*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^3*tan(1/2*c)^2 - 60*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^2 + 64*d*f*x*tan(1/2*d*x)*tan(
1/2*c)^3 - 36*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1
/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)
)*tan(1/2*d*x)^2*tan(1/2*c)^3 - 60*f*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*t...
```

**Mupad [B]**
time = 7.67, size = 240, normalized size = 1.58

$$\frac{2\,\left(d\,e+d\,f\,x\right)}{3\,a\,d^2\,\left(3\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}-{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,3{}\mathrm{i}-{\mathrm{e}}^{c\,3{}\mathrm{i}+d\,x\,3{}\mathrm{i}}+1{}\mathrm{i}\right)}-\frac{3\,d\,e+3\,d\,f\,x+f\,2{}\mathrm{i}}{6\,a\,d^2\,\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1{}\mathrm{i}\right)}+\frac{e+f\,x}{2\,a\,d\,\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}-\mathrm{i}\right)}-\frac{\left(24\,d\,e+24\,d\,f\,x-f\,8{}\mathrm{i}\right)\,1{}\mathrm{i}}{24\,a\,d^2\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}-1+{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,2{}\mathrm{i}\right)}-\frac{f\,x\,4{}\mathrm{i}}{3\,a\,d}+\frac{f\,\ln\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}-\mathrm{i}\right)}{2\,a\,d^2}+\frac{5\,f\,\ln\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1{}\mathrm{i}\right)}{6\,a\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (2*(d*e + d*f*x))/(3*a*d^2*(3*exp(c*1i + d*x*1i) - exp(c*2i + d*x*2i)*3i -
exp(c*3i + d*x*3i) + 1i)) - (f*2i + 3*d*e + 3*d*f*x)/(6*a*d^2*(exp(c*1i + d
*x*1i) + 1i)) + (e + f*x)/(2*a*d*(exp(c*1i + d*x*1i) - 1i)) - ((24*d*e - f*
8i + 24*d*f*x)*1i)/(24*a*d^2*(exp(c*1i + d*x*1i)*2i + exp(c*2i + d*x*2i) -
1)) - (f*x*4i)/(3*a*d) + (f*log(exp(c*1i + d*x*1i) - 1i))/(2*a*d^2) + (5*f*
log(exp(c*1i + d*x*1i) + 1i))/(6*a*d^2)
```

# 3.278 $\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=42

$$-\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}+\frac{2\tan(c+dx)}{3ad}$$

```
[Out] -1/3*sec(d*x+c)/d/(a+a*sin(d*x+c))+2/3*tan(d*x+c)/a/d
```

**Rubi** [A]

time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2751, 3852, 8}

$$\frac{2\tan(c+dx)}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*Sec[c + d*x]/(d*(a + a*Sin[c + d*x])) + (2*Tan[c + d*x])/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}+\frac{2\int \sec^2(c+dx)\,dx}{3a}$$
$$= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}-\frac{2\text{Subst}(\int 1\,dx,x,-\tan(c+dx))}{3ad}$$
$$= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}+\frac{2\tan(c+dx)}{3ad}$$

**Mathematica** [A]
time = 0.07, size = 45, normalized size = 1.07

$$\frac{-\cos(2(c+dx))\sec(c+dx)+2\tan(c+dx)}{3ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-(Cos[2*(c + d*x)]*Sec[c + d*x]) + 2*Tan[c + d*x])/(3*a*d*(1 + Sin[c + d*x
]))
```

**Maple** [A]
time = 0.12, size = 70, normalized size = 1.67

| method | result | size |
|---|---|---|
| risch | $-\frac{4\left(2\,\mathrm{e}^{i(dx+c)}+i\right)}{3\left(\mathrm{e}^{i(dx+c)}+i\right)^{3}\left(\mathrm{e}^{i(dx+c)}-i\right)da}$ | 51 |
| derivativedivides | $\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{da}$ | 70 |
| default | $\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{da}$ | 70 |
| norman | $\frac{-\frac{2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2}{3ad}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3da}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$ | 92 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(-1/4/(tan(1/2*d*x+1/2*c)-1)-1/3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(tan(1/
2*d*x+1/2*c)+1)^2-3/4/(tan(1/2*d*x+1/2*c)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(38) = 76$.

time = 0.28, size = 129, normalized size = 3.07

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/((a + 2*a*sin(d*x + c)/(cos(
d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4)*d)
```

**Fricas** [A]
time = 0.35, size = 49, normalized size = 1.17

$$-\frac{2\cos(dx+c)^2-2\sin(dx+c)-1}{3\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(d*x + c)^2 - 2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)*sin(d*x + c)
 + a*d*cos(d*x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

**Giac** [A]
time = 3.30, size = 67, normalized size = 1.60

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) + (9*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

**Mupad [B]**
time = 2.80, size = 71, normalized size = 1.69

$$-\frac{2\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*(tan(c/2 + (d*x)/2) + 3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^3 -
 1))/(3*a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^3)
```

# 3.279 $\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 20.01, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(4*f^2*cos(2*d*x + 2*c)*cos(d*x + c) - 2*(d*f^2*x + d*e*f)*cos(3*d*x +
 3*c)^2 + 2*f^2*cos(d*x + c) - 2*(d*f^2*x + d*e*f)*cos(d*x + c)^2 - 2*(d*f^
2*x + d*e*f)*sin(3*d*x + 3*c)^2 - 2*(d*f^2*x + d*e*f)*sin(d*x + c)^2 + (2*f
^2*cos(3*d*x + 3*c) - 2*f^2*sin(2*d*x + 2*c) + 2*(4*d^2*f^2*x^2 + 8*d^2*e*f
*x + 4*d^2*e^2 + f^2)*cos(d*x + c) + (d*f^2*x + d*e*f)*sin(3*d*x + 3*c) + (
d*f^2*x + d*e*f)*sin(d*x + c))*cos(4*d*x + 4*c) + 2*(4*d^2*f^2*x^2 + 8*d^2*
e*f*x + 4*d^2*e^2 + 2*f^2*cos(2*d*x + 2*c) + f^2 - 2*(d*f^2*x + d*e*f)*cos(
d*x + c) + 8*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*sin(d*x + c))*cos(3*d*x
+ 3*c) + 3*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3
 + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(4*
d*x + 4*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d
^3*e^3)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3
*e^2*f*x + a*d^3*e^3)*cos(3*d*x + 3*c)*cos(d*x + c) + 4*(a*d^3*f^3*x^3 + 3*
a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(d*x + c)^2 + (a*d^3*f^3*
x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(4*d*x + 4*c)^2 +
 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(3*
d*x + 3*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d
^3*e^3)*sin(d*x + c)^2 - 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2
*f*x + a*d^3*e^3 + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x +
 a*d^3*e^3)*sin(3*d*x + 3*c) + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d
^3*e^2*f*x + a*d^3*e^3)*sin(d*x + c))*cos(4*d*x + 4*c) + 4*((a*d^3*f^3*x^3
+ 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(3*d*x + 3*c) + (a*d^
3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(d*x + c))*
sin(4*d*x + 4*c) + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x +
 a*d^3*e^3 + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3
*e^3)*sin(d*x + c))*sin(3*d*x + 3*c) + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2
 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(d*x + c))*integrate(1/6*(5*d^2*f^3*x^2
+ 10*d^2*e*f^2*x + 5*d^2*e^2*f + 12*f^3)*cos(d*x + c)/(a*d^3*f^4*x^4 + 4*a*
d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 + (a*d^3*
f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3
*e^4)*cos(d*x + c)^2 + (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2
*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(d*x + c)^2 + 2*(a*d^3*f^4*x^4 + 4*a
```

```
*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(d*x
 + c)), x) - 3*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d
^3*e^3*f + (a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e
^3*f)*cos(4*d*x + 4*c)^2 + 4*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e
^2*f^2*x + a*d^3*e^3*f)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^4*x^3 + 3*a*d^3*e*f
^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*cos(3*d*x + 3*c)*cos(d*x + c) + 4
*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*cos(
d*x + c)^2 + (a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3
*e^3*f)*sin(4*d*x + 4*c)^2 + 4*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3
*e^2*f^2*x + a*d^3*e^3*f)*sin(3*d*x + 3*c)^2 + 4*(a*d^3*f^4*x^3 + 3*a*d^3*e
*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*sin(d*x + c)^2 - 2*(a*d^3*f^4*x
^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f + 2*(a*d^3*f^4*x^3
 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*sin(3*d*x + 3*c) +
2*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*sin
(d*x + c))*cos(4*d*x + 4*c) + 4*((a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d
^3*e^2*f^2*x + a*d^3*e^3*f)*cos(3*d*x + 3*c) + (a*d^3*f^4*x^3 + 3*a*d^3*e*f
^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*cos(d*x + c))*sin(4*d*x + 4*c) +
4*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f + 2*
(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*sin(d
*x + c))*sin(3*d*x + 3*c) + 4*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*
e^2*f^2*x + a*d^3*e^3*f)*sin(d*x + c))*integrate(1/2*cos(d*x + c)/(a*d*f^2*
x^2 + 2*a*d*e*f*x + a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*cos(d*x
 + c)^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*sin(d*x + c)^2 - 2*(a*d*f^2
*x^2 + 2*a*d*e*f*x + a*d*e^2)*sin(d*x + c)), x) + (4*d^2*f^2*x^2 + 8*d^2*e*
f*x + 4*d^2*e^2 + 2*f^2*cos(2*d*x + 2*c) + 2*f^2*sin(3*d*x + 3*c) + 2*f^2 -
 (d*f^2*x + d*e*f)*cos(3*d*x + 3*c) - (d*f^2*x + d*e*f)*cos(d*x + c) + 2*(4
*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 + f^2)*sin(d*x + c))*sin(4*d*x + 4*c
) - (d*f^2*x + d*e*f - 4*f^2*sin(2*d*x + 2*c) + 16*(d^2*f^2*x^2 + 2*d^2*e*f
*x + d^2*e^2)*cos(d*x + c) + 4*(d*f^2*x + d*e*f)*sin(d*x + c))*sin(3*d*x +
3*c) + 2*(2*f^2*sin(d*x + c) + f^2)*sin(2*d*x + 2*c) - (d*f^2*x + d*e*f)*si
n(d*x + c))/(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^
3 + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(4
*d*x + 4*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*
d^3*e^3)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^
3*e^2*f*x + a*d^3*e^3)*cos(3*d*x + 3*c)*cos(d*x...
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos(c+d\,x)^2\,(e+f\,x)\,(a+a\,\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(e + f*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.280 $\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 25.66, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(fx+e)^2\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(12*f^2*cos(2*d*x + 2*c)*cos(d*x + c) - 4*(d*f^2*x + d*e*f)*cos(3*d*x
+ 3*c)^2 + 6*f^2*cos(d*x + c) - 4*(d*f^2*x + d*e*f)*cos(d*x + c)^2 - 4*(d*f
^2*x + d*e*f)*sin(3*d*x + 3*c)^2 - 4*(d*f^2*x + d*e*f)*sin(d*x + c)^2 + 2*(
3*f^2*cos(3*d*x + 3*c) - 3*f^2*sin(2*d*x + 2*c) + (4*d^2*f^2*x^2 + 8*d^2*e*
f*x + 4*d^2*e^2 + 3*f^2)*cos(d*x + c) + (d*f^2*x + d*e*f)*sin(3*d*x + 3*c)
+ (d*f^2*x + d*e*f)*sin(d*x + c))*cos(4*d*x + 4*c) + 2*(4*d^2*f^2*x^2 + 8*d
^2*e*f*x + 4*d^2*e^2 + 6*f^2*cos(2*d*x + 2*c) + 3*f^2 - 4*(d*f^2*x + d*e*f)
*cos(d*x + c) + 8*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*sin(d*x + c))*cos(3
*d*x + 3*c) + 3*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 +
4*a*d^3*e^3*f*x + a*d^3*e^4 + (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*
e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*cos(4*d*x + 4*c)^2 + 4*(a*d^3*f^
4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e
^4)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2
*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*cos(3*d*x + 3*c)*cos(d*x + c) + 4*(
a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x +
 a*d^3*e^4)*cos(d*x + c)^2 + (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e
^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(4*d*x + 4*c)^2 + 4*(a*d^3*f^4
*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^
4)*sin(3*d*x + 3*c)^2 + 4*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*
f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(d*x + c)^2 - 2*(a*d^3*f^4*x^4 +
4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 + 2*(
a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x +
 a*d^3*e^4)*sin(3*d*x + 3*c) + 2*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d
^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(d*x + c))*cos(4*d*x + 4*c
) + 4*((a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e
^3*f*x + a*d^3*e^4)*cos(3*d*x + 3*c) + (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 +
 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*cos(d*x + c))*sin(4*d*x
 + 4*c) + 4*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*
d^3*e^3*f*x + a*d^3*e^4 + 2*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^
2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(d*x + c))*sin(3*d*x + 3*c) + 4
*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x
 + a*d^3*e^4)*sin(d*x + c))*integrate(1/3*(5*d^2*f^3*x^2 + 10*d^2*e*f^2*x +
```

```
5*d^2*e^2*f + 24*f^3)*cos(d*x + c)/(a*d^3*f^5*x^5 + 5*a*d^3*e*f^4*x^4 + 10
*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*x + a*d^3*e^5 + (
a*d^3*f^5*x^5 + 5*a*d^3*e*f^4*x^4 + 10*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2
*x^2 + 5*a*d^3*e^4*f*x + a*d^3*e^5)*cos(d*x + c)^2 + (a*d^3*f^5*x^5 + 5*a*d
^3*e*f^4*x^4 + 10*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*
x + a*d^3*e^5)*sin(d*x + c)^2 + 2*(a*d^3*f^5*x^5 + 5*a*d^3*e*f^4*x^4 + 10*a
*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*x + a*d^3*e^5)*sin(
d*x + c)), x) - 3*(a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2
+ 4*a*d^3*e^3*f^2*x + a*d^3*e^4*f + (a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*
a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2*x + a*d^3*e^4*f)*cos(4*d*x + 4*c)^2 + 4
*(a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2
*x + a*d^3*e^4*f)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3
 + 6*a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2*x + a*d^3*e^4*f)*cos(3*d*x + 3*c)*
cos(d*x + c) + 4*(a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 +
 4*a*d^3*e^3*f^2*x + a*d^3*e^4*f)*cos(d*x + c)^2 + (a*d^3*f^5*x^4 + 4*a*d^3
*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2*x + a*d^3*e^4*f)*sin(4*d
*x + 4*c)^2 + 4*(a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 +
4*a*d^3*e^3*f^2*x + a*d^3*e^4*f)*sin(3*d*x + 3*c)^2 + 4*(a*d^3*f^5*x^4 + 4*
a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2*x + a*d^3*e^4*f)*si
n(d*x + c)^2 - 2*(a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 +
 4*a*d^3*e^3*f^2*x + a*d^3*e^4*f + 2*(a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6
*a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2*x + a*d^3*e^4*f)*sin(3*d*x + 3*c) + 2*
(a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2*
x + a*d^3*e^4*f)*sin(d*x + c))*cos(4*d*x + 4*c) + 4*((a*d^3*f^5*x^4 + 4*a*d
^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2*x + a*d^3*e^4*f)*cos(3
*d*x + 3*c) + (a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 + 4*
a*d^3*e^3*f^2*x + a*d^3*e^4*f)*cos(d*x + c))*sin(4*d*x + 4*c) + 4*(a*d^3*f^
5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2*x + a*d^3
*e^4*f + 2*(a*d^3*f^5*x^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 + 4*a*d
^3*e^3*f^2*x + a*d^3*e^4*f)*sin(d*x + c))*sin(3*d*x + 3*c) + 4*(a*d^3*f^5*x
^4 + 4*a*d^3*e*f^4*x^3 + 6*a*d^3*e^2*f^3*x^2 + 4*a*d^3*e^3*f^2*x + a*d^3*e^
4*f)*sin(d*x + c))*integrate(cos(d*x + c)/(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 +
3*a*d*e^2*f*x + a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x +
a*d*e^3)*cos(d*x + c)^2 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x +
a*d*e^3)*sin(d*x + c)^2 - 2*(a*d*f^3*x^3 + 3*a*...
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2 + 2*a*f
*x*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
[Out] Integral(sec(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(e + f*x)^2*(a + a*sin(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^2*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.281 $\int \frac{(e+fx)^3 \sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=698

$$-\frac{if(e+fx)^2}{2ad^2}-\frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3}-\frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad}+\frac{f^2(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{ad^3}+$$

```
[Out] -1/2*I*f*(f*x+e)^2/a/d^2-3/4*I*(f*x+e)^3*arctan(exp(I*(d*x+c)))/a/d+5/2*I*f
^3*polylog(2,-I*exp(I*(d*x+c)))/a/d^4+f^2*(f*x+e)*ln(1+exp(2*I*(d*x+c)))/a/
d^3-5*I*f^2*(f*x+e)*arctan(exp(I*(d*x+c)))/a/d^3-5/2*I*f^3*polylog(2,I*exp(
I*(d*x+c)))/a/d^4-1/2*I*f^3*polylog(2,-exp(2*I*(d*x+c)))/a/d^4-9/4*I*f^3*po
lylog(4,-I*exp(I*(d*x+c)))/a/d^4-9/8*I*f*(f*x+e)^2*polylog(2,I*exp(I*(d*x+c
)))/a/d^2-9/4*f^2*(f*x+e)*polylog(3,-I*exp(I*(d*x+c)))/a/d^3+9/4*f^2*(f*x+e
)*polylog(3,I*exp(I*(d*x+c)))/a/d^3+9/8*I*f*(f*x+e)^2*polylog(2,-I*exp(I*(d
*x+c)))/a/d^2+9/4*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4-1/4*f^3*sec(d*x+c
)/a/d^4-9/8*f*(f*x+e)^2*sec(d*x+c)/a/d^2-1/4*f^2*(f*x+e)*sec(d*x+c)^2/a/d^3
-1/4*f*(f*x+e)^2*sec(d*x+c)^3/a/d^2-1/4*(f*x+e)^3*sec(d*x+c)^4/a/d+1/4*f^3*
tan(d*x+c)/a/d^4+1/2*f*(f*x+e)^2*tan(d*x+c)/a/d^2+1/4*f^2*(f*x+e)*sec(d*x+c
)*tan(d*x+c)/a/d^3+3/8*(f*x+e)^3*sec(d*x+c)*tan(d*x+c)/a/d+1/4*f*(f*x+e)^2*
sec(d*x+c)^2*tan(d*x+c)/a/d^2+1/4*(f*x+e)^3*sec(d*x+c)^3*tan(d*x+c)/a/d
```

**Rubi** [A]
time = 0.48, antiderivative size = 698, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 16, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4627, 4271, 4270, 4266, 2317, 2438, 2611, 6744, 2320, 6724, 4494, 3852, 8, 4269, 3800, 2221}

$$\frac{5if^3\text{PolyLog}(2,-ie^{i(c+dx)})}{2ad^4}-\frac{5if^3\text{PolyLog}(2,ie^{i(c+dx)})}{2ad^4}-\frac{if^3\text{PolyLog}(2,-e^{2i(c+dx)})}{2ad^4}-\frac{9if^3\text{PolyLog}(4,-ie^{i(c+dx)})}{4ad^4}+\frac{9if^3\text{PolyLog}(4,ie^{i(c+dx)})}{4ad^4}-\frac{9f^2(e+fx)\text{PolyLog}(3,-ie^{i(c+dx)})}{4ad^3}+\frac{9f^2(e+fx)\text{PolyLog}(3,ie^{i(c+dx)})}{4ad^3}+\frac{9if(e+fx)^2\text{PolyLog}(2,-ie^{i(c+dx)})}{8ad^2}-\frac{9if(e+fx)^2\text{PolyLog}(2,ie^{i(c+dx)})}{8ad^2}-\frac{5if^2(e+fx)\text{ArcTan}(e^{i(c+dx)})}{ad^3}-\frac{3i(e+fx)^3\text{ArcTan}(e^{i(c+dx)})}{4ad}+\frac{f^3\tan(c+dx)}{4ad^4}-\frac{f^3\sec(c+dx)}{4ad^4}+\frac{f^2(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{ad^3}-\frac{f^2(e+fx)\sec^2(c+dx)}{4ad^3}+\frac{f^2(e+fx)\tan(c+dx)\sec(c+dx)}{4ad^3}+\frac{f(e+fx)^2\tan(c+dx)}{2ad^2}-\frac{f(e+fx)^2\sec^3(c+dx)}{4ad^2}-\frac{9f(e+fx)^2\sec(c+dx)}{8ad^2}+\frac{f(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{4ad^2}-\frac{(e+fx)^3\sec^4(c+dx)}{4ad}+\frac{(e+fx)^3\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3(e+fx)^3\tan(c+dx)\sec(c+dx)}{8ad}-\frac{if(e+fx)^2}{2ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-1/2*I)*f*(e + f*x)^2)/(a*d^2) - ((5*I)*f^2*(e + f*x)*ArcTan[E^(I*(c + d*
x))])/(a*d^3) - (((3*I)/4)*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/(a*d) + (f^
2*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/(a*d^3) + (((5*I)/2)*f^3*PolyLog[
2, (-I)*E^(I*(c + d*x))])/(a*d^4) + (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, (-I
)*E^(I*(c + d*x))])/(a*d^2) - (((5*I)/2)*f^3*PolyLog[2, I*E^(I*(c + d*x))])
/(a*d^4) - (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2)
- ((I/2)*f^3*PolyLog[2, -E^((2*I)*(c + d*x))])/(a*d^4) - (9*f^2*(e + f*x)*P
olyLog[3, (-I)*E^(I*(c + d*x))])/(4*a*d^3) + (9*f^2*(e + f*x)*PolyLog[3, I*
E^(I*(c + d*x))])/(4*a*d^3) - (((9*I)/4)*f^3*PolyLog[4, (-I)*E^(I*(c + d*x)
)])/(a*d^4) + (((9*I)/4)*f^3*PolyLog[4, I*E^(I*(c + d*x))])/(a*d^4) - (f^3*
Sec[c + d*x])/(4*a*d^4) - (9*f*(e + f*x)^2*Sec[c + d*x])/(8*a*d^2) - (f^2*(
e + f*x)*Sec[c + d*x]^2)/(4*a*d^3) - (f*(e + f*x)^2*Sec[c + d*x]^3)/(4*a*d^
2) - ((e + f*x)^3*Sec[c + d*x]^4)/(4*a*d) + (f^3*Tan[c + d*x])/(4*a*d^4) +
```

```
(f*(e + f*x)^2*Tan[c + d*x])/(2*a*d^2) + (f^2*(e + f*x)*Sec[c + d*x]*Tan[c
+ d*x])/(4*a*d^3) + (3*(e + f*x)^3*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + (f*
(e + f*x)^2*Sec[c + d*x]^2*Tan[c + d*x])/(4*a*d^2) + ((e + f*x)^3*Sec[c + d
*x]^3*Tan[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
```

```
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4627

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sec^5(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\sec^4(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)^2\sec^3(c+dx)}{4ad^2} - \frac{(e+fx)^3\sec^4(c+dx)}{4ad} + \frac{(e+fx)^3\sec^3(c+dx}{4ad}$$

$$= -\frac{f^3\sec(c+dx)}{4ad^4} - \frac{9f(e+fx)^2\sec(c+dx)}{8ad^2} - \frac{f^2(e+fx)\sec^2(c+dx)}{4ad^3} - f$$

$$= -\frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} - \frac{f^3\sec(c+}{4ad^4}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right.}{4ad}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right.}{4ad}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right.}{4ad}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right.}{4ad}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2368 vs. 2(698) = 1396.
time = 8.48, size = 2368, normalized size = 3.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*(Cos[c] + I*Sin[c])*((-I)*d^2*e^3*x - (4*I)*e*f^2*x - ((3*I)/2)*d^2*e^2
*f*x^2 - (2*I)*f^3*x^2 - I*d^2*e*f^2*x^3 - (I/4)*d^2*f^3*x^4 + d*e^3*Log[-C
os[c + d*x] - I*(-1 + Sin[c + d*x])] + (4*e*f^2*Log[-Cos[c + d*x] - I*(-1 +
 Sin[c + d*x])])/d + 3*d*e^2*f*x*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]] + (
4*f^3*x*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]])/d + 3*d*e*f^2*x^2*Log[1 + I
*Cos[c + d*x] - Sin[c + d*x]] + d*f^3*x^3*Log[1 + I*Cos[c + d*x] - Sin[c +
d*x]] + ((6*I)*f^3*PolyLog[4, (-I)*Cos[c + d*x] + Sin[c + d*x]])/d^2 - I*d*
e^3*Log[-Cos[c + d*x] - I*(-1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]) - ((4*I)
*e*f^2*Log[-Cos[c + d*x] - I*(-1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]))/d -
(3*I)*d*e^2*f*x*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin[c])
- ((4*I)*f^3*x*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin[c]))/
d - (3*I)*d*e*f^2*x^2*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Si
n[c]) - I*d*f^3*x^3*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin[
c]) + (6*f^3*PolyLog[4, (-I)*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c
]))/d^2 + (6*f^2*(e + f*x)*PolyLog[3, (-I)*Cos[c + d*x] + Sin[c + d*x]]*(Co
s[c] + I*(-1 + Sin[c]))*(Cos[c] - I*Sin[c]))/d + (f*(4*f^2 + 3*d^2*(e + f*x
)^2)*PolyLog[2, (-I)*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(-1 -
 I*Cos[c] + Sin[c]))/d^2))/(8*a*d^2*(Cos[c] + I*(-1 + Sin[c]))) - ((Cos[c]
+ I*Sin[c])*((3*I)*d^2*e^3*x + (28*I)*e*f^2*x + ((9*I)/2)*d^2*e^2*f*x^2 + (
14*I)*f^3*x^2 + (3*I)*d^2*e*f^2*x^3 + ((3*I)/4)*d^2*f^3*x^4 - 9*d*e^2*f*x*L
og[1 - I*Cos[c + d*x] + Sin[c + d*x]] - (28*f^3*x*Log[1 - I*Cos[c + d*x] +
Sin[c + d*x]])/d - 9*d*e*f^2*x^2*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] - 3
*d*f^3*x^3*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]] - 3*d*e^3*Log[Cos[c + d*x
] + I*(1 + Sin[c + d*x])] - (28*e*f^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x
])])/d - ((18*I)*f^3*PolyLog[4, I*Cos[c + d*x] - Sin[c + d*x]])/d^2 - (9*I)
*d*e^2*f*x*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - ((2
8*I)*f^3*x*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]))/d -
(9*I)*d*e*f^2*x^2*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]
) - (3*I)*d*f^3*x^3*Log[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[
c]) - (3*I)*d*e^3*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[
c]) - ((28*I)*e*f^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Si
n[c]))/d + (18*f^3*PolyLog[4, I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Si
n[c]))/d^2 - (18*f^2*(e + f*x)*PolyLog[3, I*Cos[c + d*x] - Sin[c + d*x]]*(C
os[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])))/d + (f*(28*f^2 + 9*d^2*(e + f*
x)^2)*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]]*(I*Cos[c] + Sin[c])*(Cos[c]
 + I*(1 + Sin[c])))/d^2))/(8*a*d^2*(Cos[c] + I*(1 + Sin[c]))) + ((3*e^3*x*C
```

```
os[c])/(4*a) + (((3*I)/4)*e^3*x*Sin[c])/a)/(1 + Cos[2*c] + I*Sin[2*c]) + ((
9*e^2*f*x^2*Cos[c])/(8*a) + (((9*I)/8)*e^2*f*x^2*Sin[c])/a)/(1 + Cos[2*c] +
 I*Sin[2*c]) + ((3*e*f^2*x^3*Cos[c])/(4*a) + (((3*I)/4)*e*f^2*x^3*Sin[c])/a
)/(1 + Cos[2*c] + I*Sin[2*c]) + ((3*f^3*x^4*Cos[c])/(16*a) + (((3*I)/16)*f^
3*x^4*Sin[c])/a)/(1 + Cos[2*c] + I*Sin[2*c]) + (e^3 + 3*e^2*f*x + 3*e*f^2*x
^2 + f^3*x^3)/(8*a*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) - (3*(e^2
*f*Sin[(d*x)/2] + 2*e*f^2*x*Sin[(d*x)/2] + f^3*x^2*Sin[(d*x)/2]))/(4*a*d^2*
(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (-e^3 -
3*e^2*f*x - 3*e*f^2*x^2 - f^3*x^3)/(8*a*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (
d*x)/2])^4) + (e^2*f*Sin[(d*x)/2] + 2*e*f^2*x*Sin[(d*x)/2] + f^3*x^2*Sin[(d
*x)/2])/(4*a*d^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x
)/2])^3) + (-2*d^2*e^3*Cos[c/2] - d*e^2*f*Cos[c/2] - 2*e*f^2*Cos[c/2] - 6*d
^2*e^2*f*x*Cos[c/2] - 2*d*e*f^2*x*Cos[c/2] - 2*f^3*x*Cos[c/2] - 6*d^2*e*f^2
*x^2*Cos[c/2] - d*f^3*x^2*Cos[c/2] - 2*d^2*f^3*x^3*Cos[c/2] - 2*d^2*e^3*Sin
[c/2] + d*e^2*f*Sin[c/2] - 2*e*f^2*Sin[c/2] - 6*d^2*e^2*f*x*Sin[c/2] + 2*d*
e*f^2*x*Sin[c/2] - 2*f^3*x*Sin[c/2] - 6*d^2*e*f^2*x^2*Sin[c/2] + d*f^3*x^2*
Sin[c/2] - 2*d^2*f^3*x^3*Sin[c/2])/(8*a*d^3*(Cos[c/2] + Sin[c/2])*(Cos[c/2
+ (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (7*d^2*e^2*f*Sin[(d*x)/2] + 2*f^3*Sin
[(d*x)/2] + 14*d^2*e*f^2*x*Sin[(d*x)/2] + 7*d^2*f^3*x^2*Sin[(d*x)/2])/(4*a*
d^4*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2160 vs. $2(612) = 1224$.
time = 0.37, size = 2161, normalized size = 3.10

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2161 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 9/4*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4-9/8*I/a/d^2*f^3*polylog(2,I*exp
(I*(d*x+c)))*x^2-2*I/a/d^3*f^3*c*x+9/8*I/a/d^2*e^2*f*polylog(2,-I*exp(I*(d*
x+c)))-9/8*I/a/d^2*e^2*f*polylog(2,I*exp(I*(d*x+c)))+9/8*I/a/d^2*f^3*polylo
g(2,-I*exp(I*(d*x+c)))*x^2+7/2/a/d^3*f^3*ln(1-I*exp(I*(d*x+c)))*x+7/2/a/d^4
*f^3*ln(1-I*exp(I*(d*x+c)))*c-3/8/a/d^4*f^3*c^3*ln(1+I*exp(I*(d*x+c)))+2/a/
d^4*f^3*c*ln(exp(I*(d*x+c)))-7/2/a/d^4*f^3*c*ln(exp(I*(d*x+c))+I)+3/8/a/d^4
*f^3*c^3*ln(exp(I*(d*x+c))-I)-2/a/d^3*e*f^2*ln(exp(I*(d*x+c)))+7/2/a/d^3*e*
f^2*ln(exp(I*(d*x+c))+I)-9/4/a/d^3*e*f^2*polylog(3,-I*exp(I*(d*x+c)))-9/4/a
/d^3*f^3*polylog(3,-I*exp(I*(d*x+c)))*x+3/2/a/d^4*f^3*c*ln(exp(I*(d*x+c))-I
)-3/2/a/d^3*e*f^2*ln(exp(I*(d*x+c))-I)-3/2/a/d^3*f^3*ln(1+I*exp(I*(d*x+c)))
*x-3/2/a/d^4*f^3*ln(1+I*exp(I*(d*x+c)))*c+9/8/a/d^3*ln(1+I*exp(I*(d*x+c)))*
c^2*e*f^2-9/8/a/d*ln(1+I*exp(I*(d*x+c)))*e*f^2*x^2-3/8/a/d*f^3*ln(1+I*exp(I
*(d*x+c)))*x^3-9/4*I*f^3*polylog(4,-I*exp(I*(d*x+c)))/a/d^4+3/8/d/a*ln(exp(
I*(d*x+c))+I)*e^3-3/8/a/d*e^3*ln(exp(I*(d*x+c))-I)+9/8/d/a*e*f^2*ln(1-I*exp
```

```
(I*(d*x+c)))*x^2-9/8/d^3/a*e*f^2*ln(1-I*exp(I*(d*x+c)))*c^2+3/8/d/a*f^3*ln(
1-I*exp(I*(d*x+c)))*x^3+3/8/d^4/a*f^3*ln(1-I*exp(I*(d*x+c)))*c^3+9/8/d/a*e^
2*f*ln(1-I*exp(I*(d*x+c)))*x+9/8/d^2/a*e^2*f*ln(1-I*exp(I*(d*x+c)))*c-9/8/d
^2/a*e^2*f*c*ln(exp(I*(d*x+c))+I)+9/8/d^3/a*e*f^2*c^2*ln(exp(I*(d*x+c))+I)+
9/8/a/d^2*e^2*f*c*ln(exp(I*(d*x+c))-I)-9/8/a/d*ln(1+I*exp(I*(d*x+c)))*e^2*f
*x-9/8/a/d^2*ln(1+I*exp(I*(d*x+c)))*c*e^2*f-9/8/a/d^3*e*f^2*c^2*ln(exp(I*(d
*x+c))-I)+9/4/d^3/a*e*f^2*polylog(3,I*exp(I*(d*x+c)))-3/8/d^4/a*f^3*c^3*ln(
exp(I*(d*x+c))+I)+9/4/d^3/a*f^3*polylog(3,I*exp(I*(d*x+c)))*x+9/4*I/a/d^2*p
olylog(2,-I*exp(I*(d*x+c)))*e*f^2*x-9/4*I/a/d^2*polylog(2,I*exp(I*(d*x+c)))
*e*f^2*x-7/2*I/a/d^4*f^3*polylog(2,I*exp(I*(d*x+c)))-I/a/d^4*c^2*f^3+3/2*I/
a/d^4*f^3*polylog(2,-I*exp(I*(d*x+c)))-I/a/d^2*f^3*x^2-1/4*I*(3*d^3*e^3*exp
(5*I*(d*x+c))-4*I*f^3*exp(3*I*(d*x+c))+2*d^3*e^3*exp(3*I*(d*x+c))-2*I*f^3*e
xp(5*I*(d*x+c))-2*I*f^3*exp(I*(d*x+c))+3*d^3*e^3*exp(I*(d*x+c))+9*d^3*e*f^2
*x^2*exp(I*(d*x+c))+9*d^3*e^2*f*x*exp(I*(d*x+c))+8*d^2*e*f^2*x+9*d^3*e^2*f*
x*exp(5*I*(d*x+c))+4*d^2*e^2*f+4*f^3*exp(2*I*(d*x+c))+2*f^3*exp(4*I*(d*x+c)
)+2*f^3-9*I*d^2*e^2*f*exp(5*I*(d*x+c))+36*d^2*e*f^2*x*exp(4*I*(d*x+c))+9*d^
3*e*f^2*x^2*exp(5*I*(d*x+c))-18*I*d^3*e^2*f*x*exp(2*I*(d*x+c))-16*I*d^2*e*f
^2*x*exp(3*I*(d*x+c))+18*I*d^3*e^2*f*x*exp(4*I*(d*x+c))-18*I*d^2*e*f^2*x*ex
p(5*I*(d*x+c))+6*I*d^3*f^3*x^3*exp(4*I*(d*x+c))-6*I*d^3*f^3*x^3*exp(2*I*(d*
x+c))-8*I*d^2*f^3*x^2*exp(3*I*(d*x+c))-8*I*d^2*e^2*f*exp(3*I*(d*x+c))-9*I*d
^2*f^3*x^2*exp(5*I*(d*x+c))+18*I*d^3*e*f^2*x^2*exp(4*I*(d*x+c))-18*I*d^3*e*
f^2*x^2*exp(2*I*(d*x+c))+4*d^2*f^3*x^2+18*d^2*f^3*x^2*exp(4*I*(d*x+c))+18*d
^2*e^2*f*exp(4*I*(d*x+c))+4*d*e*f^2*exp(3*I*(d*x+c))+2*d*e*f^2*exp(5*I*(d*x
+c))+3*d^3*f^3*x^3*exp(5*I*(d*x+c))+2*d*f^3*x*exp(5*I*(d*x+c))+2*d^3*f^3*x^
3*exp(3*I*(d*x+c))+22*d^2*e^2*f*exp(2*I*(d*x+c))-6*I*d^3*e^3*exp(2*I*(d*x+c
))+6*I*d^3*e^3*exp(4*I*(d*x+c))+22*d^2*f^3*x^2*exp(2*I*(d*x+c))+4*d*f^3*x*e
xp(3*I*(d*x+c))+2*d*f^3*x*exp(I*(d*x+c))+2*d*e*f^2*exp(I*(d*x+c))+3*d^3*f^3
*x^3*exp(I*(d*x+c))+I*d^2*f^3*x^2*exp(I*(d*x+c))+I*d^2*e^2*f*exp(I*(d*x+c))
+6*d^3*e*f^2*x^2*exp(3*I*(d*x+c))+6*d^3*e^2*f*x*exp(3*I*(d*x+c))+44*d^2*e*f
^2*x*exp(2*I*(d*x+c))+2*I*d^2*e*f^2*x*exp(I*(d*x+c)))/(exp(I*(d*x+c))+I)^4/
d^4/(exp(I*(d*x+c))-I)^2/a
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2559 vs. 2(606) = 1212.

time = 0.62, size = 2559, normalized size = 3.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*d^3*f^3*x^3 + 6*d^3*f^2*x^2*e + 6*d^3*f*x*e^2 - 4*(2*d^2*f^3*x^2 +
4*d^2*f^2*x*e + 2*d^2*f*e^2 + f^3)*cos(d*x + c)^3 + 2*d^3*e^3 - 2*(3*d^3*f^
3*x^3 + 9*d^3*f*x*e^2 + 2*d*f^3*x + 3*d^3*e^3 + (9*d^3*f^2*x^2 + 2*d*f^2)*e
)*cos(d*x + c)^2 - 14*(d^2*f^3*x^2 + 2*d^2*f^2*x*e + d^2*f*e^2)*cos(d*x + c
) - 3*((3*I*d^2*f^3*x^2 + 6*I*d^2*f^2*x*e + 3*I*d^2*f*e^2 + 4*I*f^3)*cos(d*
x + c)^2*sin(d*x + c) + (3*I*d^2*f^3*x^2 + 6*I*d^2*f^2*x*e + 3*I*d^2*f*e^2
+ 4*I*f^3)*cos(d*x + c)^2)*dilog(I*cos(d*x + c) + sin(d*x + c)) + ((-9*I*d^
2*f^3*x^2 - 18*I*d^2*f^2*x*e - 9*I*d^2*f*e^2 - 28*I*f^3)*cos(d*x + c)^2*sin
(d*x + c) + (-9*I*d^2*f^3*x^2 - 18*I*d^2*f^2*x*e - 9*I*d^2*f*e^2 - 28*I*f^3
)*cos(d*x + c)^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 3*((-3*I*d^2*f^3*x
^2 - 6*I*d^2*f^2*x*e - 3*I*d^2*f*e^2 - 4*I*f^3)*cos(d*x + c)^2*sin(d*x + c)
 + (-3*I*d^2*f^3*x^2 - 6*I*d^2*f^2*x*e - 3*I*d^2*f*e^2 - 4*I*f^3)*cos(d*x +
 c)^2)*dilog(-I*cos(d*x + c) + sin(d*x + c)) + ((9*I*d^2*f^3*x^2 + 18*I*d^2
*f^2*x*e + 9*I*d^2*f*e^2 + 28*I*f^3)*cos(d*x + c)^2*sin(d*x + c) + (9*I*d^2
*f^3*x^2 + 18*I*d^2*f^2*x*e + 9*I*d^2*f*e^2 + 28*I*f^3)*cos(d*x + c)^2)*dil
og(-I*cos(d*x + c) - sin(d*x + c)) - ((9*c*d^2*f*e^2 - (9*c^2 + 28)*d*f^2*e
 + (3*c^3 + 28*c)*f^3 - 3*d^3*e^3)*cos(d*x + c)^2*sin(d*x + c) + (9*c*d^2*f
*e^2 - (9*c^2 + 28)*d*f^2*e + (3*c^3 + 28*c)*f^3 - 3*d^3*e^3)*cos(d*x + c)^
2)*log(cos(d*x + c) + I*sin(d*x + c) + I) + 3*((3*c*d^2*f*e^2 - (3*c^2 + 4)
*d*f^2*e + (c^3 + 4*c)*f^3 - d^3*e^3)*cos(d*x + c)^2*sin(d*x + c) + (3*c*d^
2*f*e^2 - (3*c^2 + 4)*d*f^2*e + (c^3 + 4*c)*f^3 - d^3*e^3)*cos(d*x + c)^2)*
log(cos(d*x + c) - I*sin(d*x + c) + I) + ((3*d^3*f^3*x^3 + 28*d*f^3*x + (3*
c^3 + 28*c)*f^3 + 9*(d^3*f*x + c*d^2*f)*e^2 + 9*(d^3*f^2*x^2 - c^2*d*f^2)*e
)*cos(d*x + c)^2*sin(d*x + c) + (3*d^3*f^3*x^3 + 28*d*f^3*x + (3*c^3 + 28*c
)*f^3 + 9*(d^3*f*x + c*d^2*f)*e^2 + 9*(d^3*f^2*x^2 - c^2*d*f^2)*e)*cos(d*x
+ c)^2)*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 3*((d^3*f^3*x^3 + 4*d*f^3*
x + (c^3 + 4*c)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^
2)*e)*cos(d*x + c)^2*sin(d*x + c) + (d^3*f^3*x^3 + 4*d*f^3*x + (c^3 + 4*c)*
f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*cos(d*x +
c)^2)*log(I*cos(d*x + c) - sin(d*x + c) + 1) + ((3*d^3*f^3*x^3 + 28*d*f^3*x
 + (3*c^3 + 28*c)*f^3 + 9*(d^3*f*x + c*d^2*f)*e^2 + 9*(d^3*f^2*x^2 - c^2*d*
f^2)*e)*cos(d*x + c)^2*sin(d*x + c) + (3*d^3*f^3*x^3 + 28*d*f^3*x + (3*c^3
+ 28*c)*f^3 + 9*(d^3*f*x + c*d^2*f)*e^2 + 9*(d^3*f^2*x^2 - c^2*d*f^2)*e)*co
s(d*x + c)^2)*log(-I*cos(d*x + c) + sin(d*x + c) + 1) - 3*((d^3*f^3*x^3 + 4
*d*f^3*x + (c^3 + 4*c)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c
^2*d*f^2)*e)*cos(d*x + c)^2*sin(d*x + c) + (d^3*f^3*x^3 + 4*d*f^3*x + (c^3
+ 4*c)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*cos
(d*x + c)^2)*log(-I*cos(d*x + c) - sin(d*x + c) + 1) - ((9*c*d^2*f*e^2 - (9
```

```
*c^2 + 28)*d*f^2*e + (3*c^3 + 28*c)*f^3 - 3*d^3*e^3)*cos(d*x + c)^2*sin(d*x
 + c) + (9*c*d^2*f*e^2 - (9*c^2 + 28)*d*f^2*e + (3*c^3 + 28*c)*f^3 - 3*d^3*
e^3)*cos(d*x + c)^2)*log(-cos(d*x + c) + I*sin(d*x + c) + I) + 3*((3*c*d^2*
f*e^2 - (3*c^2 + 4)*d*f^2*e + (c^3 + 4*c)*f^3 - d^3*e^3)*cos(d*x + c)^2*sin
(d*x + c) + (3*c*d^2*f*e^2 - (3*c^2 + 4)*d*f^2*e + (c^3 + 4*c)*f^3 - d^3*e^
3)*cos(d*x + c)^2)*log(-cos(d*x + c) - I*sin(d*x + c) + I) - 18*(-I*f^3*cos
(d*x + c)^2*sin(d*x + c) - I*f^3*cos(d*x + c)^2)*polylog(4, I*cos(d*x + c)
+ sin(d*x + c)) - 18*(-I*f^3*cos(d*x + c)^2*sin(d*x + c) - I*f^3*cos(d*x +
c)^2)*polylog(4, I*cos(d*x + c) - sin(d*x + c)) - 18*(I*f^3*cos(d*x + c)^2*
sin(d*x + c) + I*f^3*cos(d*x + c)^2)*polylog(4, -I*cos(d*x + c) + sin(d*x +
 c)) - 18*(I*f^3*cos(d*x + c)^2*sin(d*x + c) + I*f^3*cos(d*x + c)^2)*polylo
g(4, -I*cos(d*x + c) - sin(d*x + c)) - 18*((d*f^3*x + d*f^2*e)*cos(d*x + c)
^2*sin(d*x + c) + (d*f^3*x + d*f^2*e)*cos(d*x + c)^2)*polylog(3, I*cos(d*x
+ c) + sin(d*x + c)) + 18*((d*f^3*x + d*f^2*e)*cos(d*x + c)^2*sin(d*x + c)
+ (d*f^3*x + d*f^2*e)*cos(d*x + c)^2)*polylog(3, I*cos(d*x + c) - sin(d*x +
 c)) - 18*((d*f^3*x + d*f^2*e)*cos(d*x + c)^2*sin(d*x + c) + (d*f^3*x + d*f
^2*e)*cos(d*x + c)^2)*polylog(3, -I*cos(d*x + c) + sin(d*x + c)) + 18*((d*f
^3*x + d*f^2*e)*cos(d*x + c)^2*sin(d*x + c) + (d*f^3*x + d*f^2*e)*cos(d*x +
 c)^2)*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) + 2*(3*d^3*f^3*x^3 + 9*d^
3*f^2*x^2*e + 9*d^3*f*x*e^2 + 3*d^3*e^3 - 5*(d^2*f^3*x^2 + 2*d^2*f^2*x*e +
d^2*f*e^2)*cos(d*x + c))*sin(d*x + c))/(a*d^4*cos(d*x + c)^2*sin(d*x + c) +
 a*d^4*cos(d*x + c)^2)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^3 \sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3 \sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2 \sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx \sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
[Out] (Integral(e**3*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*
sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sec(c + d*x
)**3/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sec(c + d*x)**3/(sin(c +
d*x) + 1), x))/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*sec(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.282 $\int \frac{(e+fx)^2 \sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=431

$$-\frac{3i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad}+\frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3}+\frac{f^2\log(\cos(c+dx))}{3ad^3}+\frac{3if(e+fx)\text{Li}_2\left(-ie^{i(c+dx)}\right)}{4ad^2}-$$

```
[Out] 3/4*I*f*(f*x+e)*polylog(2,-I*exp(I*(d*x+c)))/a/d^2+5/6*f^2*arctanh(sin(d*x+
c))/a/d^3+1/3*f^2*ln(cos(d*x+c))/a/d^3-3/4*I*f*(f*x+e)*polylog(2,I*exp(I*(d
*x+c)))/a/d^2-3/4*I*(f*x+e)^2*arctan(exp(I*(d*x+c)))/a/d-3/4*f^2*polylog(3,
-I*exp(I*(d*x+c)))/a/d^3+3/4*f^2*polylog(3,I*exp(I*(d*x+c)))/a/d^3-3/4*f*(f
*x+e)*sec(d*x+c)/a/d^2-1/12*f^2*sec(d*x+c)^2/a/d^3-1/6*f*(f*x+e)*sec(d*x+c)
^3/a/d^2-1/4*(f*x+e)^2*sec(d*x+c)^4/a/d+1/3*f*(f*x+e)*tan(d*x+c)/a/d^2+1/12
*f^2*sec(d*x+c)*tan(d*x+c)/a/d^3+3/8*(f*x+e)^2*sec(d*x+c)*tan(d*x+c)/a/d+1/
6*f*(f*x+e)*sec(d*x+c)^2*tan(d*x+c)/a/d^2+1/4*(f*x+e)^2*sec(d*x+c)^3*tan(d*
x+c)/a/d
```

**Rubi** [A]
time = 0.26, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4627, 4271, 3853, 3855, 4266, 2611, 2320, 6724, 4494, 4270, 4269, 3556}

$$-\frac{3f^2\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{4ad^3}+\frac{3f^2\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{4ad^3}+\frac{3if(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{4ad^2}-\frac{3if(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{4ad^2}-\frac{3i(e+fx)^2\text{ArcTan}\left(e^{i(c+dx)}\right)}{4ad}-\frac{f^2\sec^2(c+dx)}{12ad^3}+\frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3}+\frac{f^2\log(\cos(c+dx))}{3ad^3}+\frac{f^2\tan(c+dx)\sec(c+dx)}{12ad^3}+\frac{f(e+fx)\tan(c+dx)}{3ad^2}-\frac{f(e+fx)\sec^3(c+dx)}{6ad^2}-\frac{3f(e+fx)\sec(c+dx)}{4ad^2}+\frac{f(e+fx)\tan(c+dx)\sec^2(c+dx)}{6ad^2}-\frac{(e+fx)^2\sec^4(c+dx)}{4ad}+\frac{(e+fx)^2\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3(e+fx)^2\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (((-3*I)/4)*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/(a*d) + (5*f^2*ArcTanh[Sin
[c + d*x]])/(6*a*d^3) + (f^2*Log[Cos[c + d*x]])/(3*a*d^3) + (((3*I)/4)*f*(e
 + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/(a*d^2) - (((3*I)/4)*f*(e + f*x)*
PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) - (3*f^2*PolyLog[3, (-I)*E^(I*(c + d
*x))])/(4*a*d^3) + (3*f^2*PolyLog[3, I*E^(I*(c + d*x))])/(4*a*d^3) - (3*f*(
e + f*x)*Sec[c + d*x])/(4*a*d^2) - (f^2*Sec[c + d*x]^2)/(12*a*d^3) - (f*(e
+ f*x)*Sec[c + d*x]^3)/(6*a*d^2) - ((e + f*x)^2*Sec[c + d*x]^4)/(4*a*d) + (
f*(e + f*x)*Tan[c + d*x])/(3*a*d^2) + (f^2*Sec[c + d*x]*Tan[c + d*x])/(12*a
*d^3) + (3*(e + f*x)^2*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + (f*(e + f*x)*Se
c[c + d*x]^2*Tan[c + d*x])/(6*a*d^2) + ((e + f*x)^2*Sec[c + d*x]^3*Tan[c +
d*x])/(4*a*d)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
```

```
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4627

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sec^5(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)\,dx}{a}$$
$$= -\frac{f(e+fx)\sec^3(c+dx)}{6ad^2} - \frac{(e+fx)^2\sec^4(c+dx)}{4ad} + \frac{(e+fx)^2\sec^3(c+dx)}{4ad}$$
$$= -\frac{3f(e+fx)\sec(c+dx)}{4ad^2} - \frac{f^2\sec^2(c+dx)}{12ad^3} - \frac{f(e+fx)\sec^3(c+dx)}{6ad^2} - (e$$
$$= -\frac{3i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3} - \frac{3f(e+fx)\sec(}{4ad^2}$$
$$= -\frac{3i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3} + \frac{f^2\log(\cos(c+a}{3ad^3}$$
$$= -\frac{3i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3} + \frac{f^2\log(\cos(c+a}{3ad^3}$$
$$= -\frac{3i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3} + \frac{f^2\log(\cos(c+a}{3ad^3}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1577 vs. $2(431) = 862$.
time = 8.49, size = 1577, normalized size = 3.66

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*((Cos[c] + I*Sin[c])*((-3*I)*d^2*e^2*x - (4*I)*f^2*x - (3*I)*d^2*e*f*x
^2 - I*d^2*f^2*x^3 + 3*d*e^2*Log[-Cos[c + d*x] - I*(-1 + Sin[c + d*x])] + (
4*f^2*Log[-Cos[c + d*x] - I*(-1 + Sin[c + d*x])])/d + 6*d*e*f*x*Log[1 + I*C
os[c + d*x] - Sin[c + d*x]] + 3*d*f^2*x^2*Log[1 + I*Cos[c + d*x] - Sin[c +
d*x]] - (3*I)*d*e^2*Log[-Cos[c + d*x] - I*(-1 + Sin[c + d*x])]*(Cos[c] - I*
Sin[c]) - ((4*I)*f^2*Log[-Cos[c + d*x] - I*(-1 + Sin[c + d*x])]*(Cos[c] - I
*Sin[c]))/d - (6*I)*d*e*f*x*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c]
- I*Sin[c]) - (3*I)*d*f^2*x^2*Log[1 + I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c
] - I*Sin[c]) + (6*f^2*PolyLog[3, (-I)*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c]
 + I*(-1 + Sin[c]))*(Cos[c] - I*Sin[c]))/d + 6*f*(e + f*x)*PolyLog[2, (-I)*
Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(-1 - I*Cos[c] + Sin[c])))
/(a*d^2*(Cos[c] + I*(-1 + Sin[c]))) - ((I/24)*(Cos[c] + I*Sin[c])*(9*d^3*e^
2*x + 28*d*f^2*x + 9*d^3*e*f*x^2 + 3*d^3*f^2*x^3 + (18*I)*d^2*e*f*x*Log[1 -
 I*Cos[c + d*x] + Sin[c + d*x]] + (9*I)*d^2*f^2*x^2*Log[1 - I*Cos[c + d*x]
+ Sin[c + d*x]] + (9*I)*d^2*e^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])] +
(28*I)*f^2*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])] - 18*d^2*e*f*x*Log[1 -
```

```
I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - 9*d^2*f^2*x^2*Log[1 -
I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c]) - 9*d^2*e^2*Log[Cos[c +
d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]) - 28*f^2*Log[Cos[c + d*x]
+ I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c]) - 18*f^2*PolyLog[3, I*Cos[c + d
*x] - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])) + 18*d*f*(e + f*x)*PolyLog[2,
 I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c]
))))/(a*d^3*(Cos[c] + I*(1 + Sin[c]))) + ((3*e^2*x*Cos[c])/(4*a) + (((3*I)/
4)*e^2*x*Sin[c])/a)/(1 + Cos[2*c] + I*Sin[2*c]) + ((3*e*f*x^2*Cos[c])/(4*a)
 + (((3*I)/4)*e*f*x^2*Sin[c])/a)/(1 + Cos[2*c] + I*Sin[2*c]) + ((f^2*x^3*Co
s[c])/(4*a) + ((I/4)*f^2*x^3*Sin[c])/a)/(1 + Cos[2*c] + I*Sin[2*c]) + (e^2
+ 2*e*f*x + f^2*x^2)/(8*a*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) +
(-(e*f*Sin[(d*x)/2]) - f^2*x*Sin[(d*x)/2])/(2*a*d^2*(Cos[c/2] - Sin[c/2])*(
Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (-e^2 - 2*e*f*x - f^2*x^2)/(8*a
*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + (e*f*Sin[(d*x)/2] + f^2*x
*Sin[(d*x)/2])/(6*a*d^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
 + (d*x)/2])^3) + (-3*d^2*e^2*Cos[c/2] - d*e*f*Cos[c/2] - f^2*Cos[c/2] - 6*
d^2*e*f*x*Cos[c/2] - d*f^2*x*Cos[c/2] - 3*d^2*f^2*x^2*Cos[c/2] - 3*d^2*e^2*
Sin[c/2] + d*e*f*Sin[c/2] - f^2*Sin[c/2] - 6*d^2*e*f*x*Sin[c/2] + d*f^2*x*S
in[c/2] - 3*d^2*f^2*x^2*Sin[c/2])/(12*a*d^3*(Cos[c/2] + Sin[c/2])*(Cos[c/2
+ (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (7*(e*f*Sin[(d*x)/2] + f^2*x*Sin[(d*x
)/2]))/(6*a*d^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)
/2]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1118 vs. $2(382) = 764$.
time = 0.36, size = 1119, normalized size = 2.60

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1119 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*I/d^2/a*e*f*polylog(2,I*exp(I*(d*x+c)))+3/8/d^3/a*f^2*c^2*ln(exp(I*(d*
x+c))+I)-3/8/d^3/a*f^2*c^2*ln(1-I*exp(I*(d*x+c)))-3/8/d/a*ln(1+I*exp(I*(d*x
+c)))*f^2*x^2+3/8/d/a*f^2*ln(1-I*exp(I*(d*x+c)))*x^2-3/8/d/a*e^2*ln(exp(I*(
d*x+c))-I)+7/6/d^3/a*f^2*ln(exp(I*(d*x+c))+I)-2/3/d^3/a*f^2*ln(exp(I*(d*x+c
)))+3/4*f^2*polylog(3,I*exp(I*(d*x+c)))/a/d^3+3/8/d/a*ln(exp(I*(d*x+c))+I)*
e^2-1/12*I*(2*I*d*f^2*x*exp(I*(d*x+c))+2*f^2*exp(5*I*(d*x+c))+4*f^2*exp(3*I
*(d*x+c))+8*d*e*f+6*d^2*f^2*x^2*exp(3*I*(d*x+c))+44*d*f^2*x*exp(2*I*(d*x+c)
)+44*d*e*f*exp(2*I*(d*x+c))+9*d^2*f^2*x^2*exp(5*I*(d*x+c))+36*d*f^2*x*exp(4
*I*(d*x+c))+36*d*e*f*exp(4*I*(d*x+c))-18*I*d^2*e^2*exp(2*I*(d*x+c))+18*I*d^
2*e^2*exp(4*I*(d*x+c))+8*d*f^2*x+36*I*d^2*e*f*x*exp(4*I*(d*x+c))-36*I*d^2*e
*f*x*exp(2*I*(d*x+c))+18*d^2*e*f*x*exp(I*(d*x+c))+9*d^2*e^2*exp(5*I*(d*x+c)
)+6*d^2*e^2*exp(3*I*(d*x+c))+9*d^2*e^2*exp(I*(d*x+c))+2*I*d*e*f*exp(I*(d*x+
```

```
c))+12*d^2*e*f*x*exp(3*I*(d*x+c))+18*d^2*e*f*x*exp(5*I*(d*x+c))-16*I*d*f^2*
x*exp(3*I*(d*x+c))-16*I*d*e*f*exp(3*I*(d*x+c))-18*I*d*f^2*x*exp(5*I*(d*x+c)
)-18*I*d*e*f*exp(5*I*(d*x+c))+18*I*d^2*f^2*x^2*exp(4*I*(d*x+c))-18*I*d^2*f^
2*x^2*exp(2*I*(d*x+c))+2*f^2*exp(I*(d*x+c))+9*d^2*f^2*x^2*exp(I*(d*x+c)))/(
exp(I*(d*x+c))+I)^4/d^3/(exp(I*(d*x+c))-I)^2/a+3/4*I/d^2/a*e*f*polylog(2,-I
*exp(I*(d*x+c)))+3/4*I/d^2/a*polylog(2,-I*exp(I*(d*x+c)))*f^2*x-3/4*I/d^2/a
*polylog(2,I*exp(I*(d*x+c)))*f^2*x+3/8/d^3/a*ln(1+I*exp(I*(d*x+c)))*c^2*f^2
-3/8/d^3/a*f^2*c^2*ln(exp(I*(d*x+c))-I)-3/4/d^2/a*e*f*c*ln(exp(I*(d*x+c))+I
)+3/4/d/a*e*f*ln(1-I*exp(I*(d*x+c)))*x+3/4/d^2/a*e*f*ln(1-I*exp(I*(d*x+c)))
*c-3/4*f^2*polylog(3,-I*exp(I*(d*x+c)))/a/d^3+3/4/d^2/a*e*f*c*ln(exp(I*(d*x
+c))-I)-3/4/d/a*ln(1+I*exp(I*(d*x+c)))*e*f*x-3/4/d^2/a*ln(1+I*exp(I*(d*x+c)
))*c*e*f-1/2/d^3/a*f^2*ln(exp(I*(d*x+c))-I)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1546 vs. 2(383) = 766.
time = 0.48, size = 1546, normalized size = 3.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*d^2*f^2*x^2 + 12*d^2*f*x*e - 16*(d*f^2*x + d*f*e)*cos(d*x + c)^3 -
2*(9*d^2*f^2*x^2 + 18*d^2*f*x*e + 9*d^2*e^2 + 2*f^2)*cos(d*x + c)^2 + 6*d^2
*e^2 - 28*(d*f^2*x + d*f*e)*cos(d*x + c) - 18*((I*d*f^2*x + I*d*f*e)*cos(d*
x + c)^2*sin(d*x + c) + (I*d*f^2*x + I*d*f*e)*cos(d*x + c)^2)*dilog(I*cos(d
*x + c) + sin(d*x + c)) - 18*((I*d*f^2*x + I*d*f*e)*cos(d*x + c)^2*sin(d*x
+ c) + (I*d*f^2*x + I*d*f*e)*cos(d*x + c)^2)*dilog(I*cos(d*x + c) - sin(d*x
 + c)) - 18*((-I*d*f^2*x - I*d*f*e)*cos(d*x + c)^2*sin(d*x + c) + (-I*d*f^2
*x - I*d*f*e)*cos(d*x + c)^2)*dilog(-I*cos(d*x + c) + sin(d*x + c)) - 18*((
-I*d*f^2*x - I*d*f*e)*cos(d*x + c)^2*sin(d*x + c) + (-I*d*f^2*x - I*d*f*e)*
cos(d*x + c)^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) - ((18*c*d*f*e - (9*c
^2 + 28)*f^2 - 9*d^2*e^2)*cos(d*x + c)^2*sin(d*x + c) + (18*c*d*f*e - (9*c^
2 + 28)*f^2 - 9*d^2*e^2)*cos(d*x + c)^2)*log(cos(d*x + c) + I*sin(d*x + c)
+ I) + 3*((6*c*d*f*e - (3*c^2 + 4)*f^2 - 3*d^2*e^2)*cos(d*x + c)^2*sin(d*x
```

```
+ c) + (6*c*d*f*e - (3*c^2 + 4)*f^2 - 3*d^2*e^2)*cos(d*x + c)^2)*log(cos(d*
x + c) - I*sin(d*x + c) + I) + 9*((d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d
*f)*e)*cos(d*x + c)^2*sin(d*x + c) + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x +
c*d*f)*e)*cos(d*x + c)^2)*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 9*((d^2*
f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x + c)^2*sin(d*x + c) + (d
^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x + c)^2)*log(I*cos(d*x
 + c) - sin(d*x + c) + 1) + 9*((d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)
*e)*cos(d*x + c)^2*sin(d*x + c) + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d
*f)*e)*cos(d*x + c)^2)*log(-I*cos(d*x + c) + sin(d*x + c) + 1) - 9*((d^2*f^
2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x + c)^2*sin(d*x + c) + (d^2
*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*cos(d*x + c)^2)*log(-I*cos(d*x
+ c) - sin(d*x + c) + 1) - ((18*c*d*f*e - (9*c^2 + 28)*f^2 - 9*d^2*e^2)*cos
(d*x + c)^2*sin(d*x + c) + (18*c*d*f*e - (9*c^2 + 28)*f^2 - 9*d^2*e^2)*cos(
d*x + c)^2)*log(-cos(d*x + c) + I*sin(d*x + c) + I) + 3*((6*c*d*f*e - (3*c^
2 + 4)*f^2 - 3*d^2*e^2)*cos(d*x + c)^2*sin(d*x + c) + (6*c*d*f*e - (3*c^2 +
 4)*f^2 - 3*d^2*e^2)*cos(d*x + c)^2)*log(-cos(d*x + c) - I*sin(d*x + c) + I
) - 18*(f^2*cos(d*x + c)^2*sin(d*x + c) + f^2*cos(d*x + c)^2)*polylog(3, I*
cos(d*x + c) + sin(d*x + c)) + 18*(f^2*cos(d*x + c)^2*sin(d*x + c) + f^2*co
s(d*x + c)^2)*polylog(3, I*cos(d*x + c) - sin(d*x + c)) - 18*(f^2*cos(d*x +
 c)^2*sin(d*x + c) + f^2*cos(d*x + c)^2)*polylog(3, -I*cos(d*x + c) + sin(d
*x + c)) + 18*(f^2*cos(d*x + c)^2*sin(d*x + c) + f^2*cos(d*x + c)^2)*polylo
g(3, -I*cos(d*x + c) - sin(d*x + c)) + 2*(9*d^2*f^2*x^2 + 18*d^2*f*x*e + 9*
d^2*e^2 - 10*(d*f^2*x + d*f*e)*cos(d*x + c))*sin(d*x + c))/(a*d^3*cos(d*x +
 c)^2*sin(d*x + c) + a*d^3*cos(d*x + c)^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^2\sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sec(c + d*x)**3/(
sin(c + d*x) + 1), x))/a
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sec(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.283 $\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=241

$$-\frac{3i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad}+\frac{3if\text{Li}_2\left(-ie^{i(c+dx)}\right)}{8ad^2}-\frac{3if\text{Li}_2\left(ie^{i(c+dx)}\right)}{8ad^2}-\frac{3f\sec(c+dx)}{8ad^2}-\frac{f\sec^3(c+dx)}{12ad^2}-\frac{(e+\ldots}{}$$

```
[Out] -3/4*I*(f*x+e)*arctan(exp(I*(d*x+c)))/a/d+3/8*I*f*polylog(2,-I*exp(I*(d*x+c
)))/a/d^2-3/8*I*f*polylog(2,I*exp(I*(d*x+c)))/a/d^2-3/8*f*sec(d*x+c)/a/d^2-
1/12*f*sec(d*x+c)^3/a/d^2-1/4*(f*x+e)*sec(d*x+c)^4/a/d+1/4*f*tan(d*x+c)/a/d
^2+3/8*(f*x+e)*sec(d*x+c)*tan(d*x+c)/a/d+1/4*(f*x+e)*sec(d*x+c)^3*tan(d*x+c
)/a/d+1/12*f*tan(d*x+c)^3/a/d^2
```

**Rubi** [A]
time = 0.14, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4627, 4270, 4266, 2317, 2438, 4494, 3852}

$$\frac{3if\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{8ad^2}-\frac{3if\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{8ad^2}-\frac{3i(e+fx)\text{ArcTan}\left(e^{i(c+dx)}\right)}{4ad}+\frac{f\tan^3(c+dx)}{12ad^2}+\frac{f\tan(c+dx)}{4ad^2}-\frac{f\sec^3(c+dx)}{12ad^2}-\frac{3f\sec(c+dx)}{8ad^2}-\frac{(e+fx)\sec^4(c+dx)}{4ad}+\frac{(e+fx)\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3(e+fx)\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (((-3*I)/4)*(e + f*x)*ArcTan[E^(I*(c + d*x))])/(a*d) + (((3*I)/8)*f*PolyLog
[2, (-I)*E^(I*(c + d*x))])/(a*d^2) - (((3*I)/8)*f*PolyLog[2, I*E^(I*(c + d*
x))])/(a*d^2) - (3*f*Sec[c + d*x])/(8*a*d^2) - (f*Sec[c + d*x]^3)/(12*a*d^2
) - ((e + f*x)*Sec[c + d*x]^4)/(4*a*d) + (f*Tan[c + d*x])/(4*a*d^2) + (3*(e
 + f*x)*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + ((e + f*x)*Sec[c + d*x]^3*Tan[
c + d*x])/(4*a*d) + (f*Tan[c + d*x]^3)/(12*a*d^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4627

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int(e+fx)\sec^5(c+dx)\,dx}{a} - \frac{\int(e+fx)\sec^4(c+dx)\tan(c+dx)\,dx}{a}$$
$$= -\frac{f\sec^3(c+dx)}{12ad^2} - \frac{(e+fx)\sec^4(c+dx)}{4ad} + \frac{(e+fx)\sec^3(c+dx)\tan(c+dx)}{4ad}$$
$$= -\frac{3f\sec(c+dx)}{8ad^2} - \frac{f\sec^3(c+dx)}{12ad^2} - \frac{(e+fx)\sec^4(c+dx)}{4ad} + \frac{3(e+fx)\sec(c}{8}$$
$$= -\frac{3i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} - \frac{3f\sec(c+dx)}{8ad^2} - \frac{f\sec^3(c+dx)}{12ad^2} - \frac{(e+fx)\text{se}}{4a}$$
$$= -\frac{3i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} - \frac{3f\sec(c+dx)}{8ad^2} - \frac{f\sec^3(c+dx)}{12ad^2} - \frac{(e+fx)\text{se}}{4a}$$
$$= -\frac{3i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{3if\text{Li}_2\left(-ie^{i(c+dx)}\right)}{8ad^2} - \frac{3if\text{Li}_2\left(ie^{i(c+dx)}\right)}{8ad^2} - \frac{3f\text{se}}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 853 vs. $2(241) = 482$.
time = 5.45, size = 853, normalized size = 3.54

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/48*(2*(f + 6*d*(e + f*x)) + (6*d*(e + f*x))/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^2 - (4*f*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
- 28*f*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 9*(c + d*x)
*(c*f - d*(2*e + f*x))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 9*d*e*(c +
 d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 - 9*c*f*(c + d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 9*d*e*(c + d*x - 2*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 9
*c*f*(c + d*x - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2 + (9*f*(-2*(-1)^(3/4)*(c + d*x)^2 + Sqrt[2]*((3*I)
*Pi*(c + d*x) + 4*Pi*Log[1 + E^((-I)*(c + d*x))] - 2*(-2*c + Pi - 2*d*x)*Lo
g[1 + I*E^(I*(c + d*x))] - 4*Pi*Log[Cos[(c + d*x)/2]] + 2*Pi*Log[Sin[(2*c -
 Pi + 2*d*x)/4]] - (4*I)*PolyLog[2, (-I)*E^(I*(c + d*x))]))*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2)/(2*Sqrt[2]) + (9*f*(2*(-1)^(1/4)*(c + d*x)^2 + Sq
rt[2]*((-I)*Pi*(c + d*x) - 4*Pi*Log[1 + E^((-I)*(c + d*x))] - 2*(2*c + Pi +
 2*d*x)*Log[1 - I*E^(I*(c + d*x))] + 4*Pi*Log[Cos[(c + d*x)/2]] + 2*Pi*Log[
Sin[(2*c + Pi + 2*d*x)/4]] + (4*I)*PolyLog[2, I*E^(I*(c + d*x))]))*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2)/(2*Sqrt[2]) - (6*d*(e + f*x)*(Cos[(c + d*x
```

```
)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (12*f
*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]))/(a*d^2*(1 + Sin[c + d*x]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(210) = 420.
time = 0.30, size = 483, normalized size = 2.00

| method | result |
|---|---|
| risch | $-\frac{i(if\,e^{i(dx+c)}+9dfx\,e^{5i(dx+c)}-18ide\,e^{2i(dx+c)}+18ide\,e^{4i(dx+c)}+9de\,e^{5i(dx+c)}-18idfx\,e^{2i(dx+c)}+6dfx\,e^{3i(dx+c)}-8if\,e^{3i(dx+c)}-9}{12\left(e^{i(dx+c)}+i\right)^4 d^2 \left(e^{i(dx+}\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I*(I*f*exp(I*(d*x+c))+9*d*f*x*exp(5*I*(d*x+c))-18*I*d*e*exp(2*I*(d*x+
c))+18*I*d*e*exp(4*I*(d*x+c))+9*d*e*exp(5*I*(d*x+c))-18*I*d*f*x*exp(2*I*(d*
x+c))+6*d*f*x*exp(3*I*(d*x+c))-8*I*f*exp(3*I*(d*x+c))-9*I*f*exp(5*I*(d*x+c)
)+6*d*e*exp(3*I*(d*x+c))+18*f*exp(4*I*(d*x+c))+9*d*f*x*exp(I*(d*x+c))+18*I*
d*f*x*exp(4*I*(d*x+c))+9*d*e*exp(I*(d*x+c))+22*f*exp(2*I*(d*x+c))+4*f)/(exp
(I*(d*x+c))+I)^4/d^2/(exp(I*(d*x+c))-I)^2/a-3/8/a/d*e*ln(exp(I*(d*x+c))-I)+
3/8/d/a*ln(exp(I*(d*x+c))+I)*e-3/8/a/d*f*ln(1+I*exp(I*(d*x+c)))*x-3/8/a/d^2
*f*ln(1+I*exp(I*(d*x+c)))*c+3/8*I*f*polylog(2,-I*exp(I*(d*x+c)))/a/d^2+3/8/
d/a*f*ln(1-I*exp(I*(d*x+c)))*x+3/8/d^2/a*f*ln(1-I*exp(I*(d*x+c)))*c-3/8*I*f
*polylog(2,I*exp(I*(d*x+c)))/a/d^2+3/8/a/d^2*f*c*ln(exp(I*(d*x+c))-I)-3/8/d
^2/a*f*c*ln(exp(I*(d*x+c))+I)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 803 vs. 2(210) = 420.
time = 0.42, size = 803, normalized size = 3.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(8*f*cos(d*x + c)^3 - 6*d*f*x + 18*(d*f*x + d*e)*cos(d*x + c)^2 + 14*
f*cos(d*x + c) + 9*(I*f*cos(d*x + c)^2*sin(d*x + c) + I*f*cos(d*x + c)^2)*d
ilog(I*cos(d*x + c) + sin(d*x + c)) + 9*(I*f*cos(d*x + c)^2*sin(d*x + c) +
I*f*cos(d*x + c)^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) + 9*(-I*f*cos(d*x
+ c)^2*sin(d*x + c) - I*f*cos(d*x + c)^2)*dilog(-I*cos(d*x + c) + sin(d*x +
 c)) + 9*(-I*f*cos(d*x + c)^2*sin(d*x + c) - I*f*cos(d*x + c)^2)*dilog(-I*c
os(d*x + c) - sin(d*x + c)) - 6*d*e + 9*((c*f - d*e)*cos(d*x + c)^2*sin(d*x
 + c) + (c*f - d*e)*cos(d*x + c)^2)*log(cos(d*x + c) + I*sin(d*x + c) + I)
- 9*((c*f - d*e)*cos(d*x + c)^2*sin(d*x + c) + (c*f - d*e)*cos(d*x + c)^2)*
log(cos(d*x + c) - I*sin(d*x + c) + I) - 9*((d*f*x + c*f)*cos(d*x + c)^2*si
n(d*x + c) + (d*f*x + c*f)*cos(d*x + c)^2)*log(I*cos(d*x + c) + sin(d*x + c
) + 1) + 9*((d*f*x + c*f)*cos(d*x + c)^2*sin(d*x + c) + (d*f*x + c*f)*cos(d
*x + c)^2)*log(I*cos(d*x + c) - sin(d*x + c) + 1) - 9*((d*f*x + c*f)*cos(d*
x + c)^2*sin(d*x + c) + (d*f*x + c*f)*cos(d*x + c)^2)*log(-I*cos(d*x + c) +
 sin(d*x + c) + 1) + 9*((d*f*x + c*f)*cos(d*x + c)^2*sin(d*x + c) + (d*f*x
+ c*f)*cos(d*x + c)^2)*log(-I*cos(d*x + c) - sin(d*x + c) + 1) + 9*((c*f -
d*e)*cos(d*x + c)^2*sin(d*x + c) + (c*f - d*e)*cos(d*x + c)^2)*log(-cos(d*x
 + c) + I*sin(d*x + c) + I) - 9*((c*f - d*e)*cos(d*x + c)^2*sin(d*x + c) +
(c*f - d*e)*cos(d*x + c)^2)*log(-cos(d*x + c) - I*sin(d*x + c) + I) - 2*(9*
d*f*x - 5*f*cos(d*x + c) + 9*d*e)*sin(d*x + c))/(a*d^2*cos(d*x + c)^2*sin(d
*x + c) + a*d^2*cos(d*x + c)^2)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e\sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f*x*sec(c + d
*x)**3/(sin(c + d*x) + 1), x))/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sec(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

**Mupad** [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.284 $\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=77

$$\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{1}{4d(a+a\sin(c+dx))}$$

```
[Out] 3/8*arctanh(sin(d*x+c))/a/d+1/8/d/(a-a*sin(d*x+c))-1/8*a/d/(a+a*sin(d*x+c))
^2-1/4/d/(a+a*sin(d*x+c))
```

**Rubi** [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$-\frac{a}{8d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{1}{4d(a\sin(c+dx)+a)}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + 1/(8*d*(a - a*Sin[c + d*x])) - a/(8*d*(
a + a*Sin[c + d*x])^2) - 1/(4*d*(a + a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^3\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2} + \frac{1}{4a^2(a+x)^3} + \frac{1}{4a^3(a+x)^2} + \frac{3}{8a^3(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{1}{8d(a-a\sin(c+dx))} - \frac{a}{8d(a+a\sin(c+dx))^2} - \frac{1}{4d(a+a\sin(c+dx))} + \frac{3\text{Subs}}{}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad} + \frac{1}{8d(a-a\sin(c+dx))} - \frac{a}{8d(a+a\sin(c+dx))^2} - \frac{}{4d(a}$$

**Mathematica [A]**
time = 0.07, size = 75, normalized size = 0.97

$$-\frac{\sec^2(c+dx)\left(2-3\sin(c+dx)-3\sin^2(c+dx)+3\tanh^{-1}(\sin(c+dx))(-1+\sin(c+dx))(1+\sin(c+dx))^2\right)}{8ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(Sec[c + d*x]^2*(2 - 3*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 3*ArcTanh[Sin
[c + d*x]]*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^2))/(a*d*(1 + Sin[c + d*x
]))
```

**Maple [A]**
time = 0.15, size = 67, normalized size = 0.87

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{16}-\frac{1}{8(\sin(dx+c)-1)}-\frac{3\ln(\sin(dx+c)-1)}{16}}{da}$ |
| default | $\frac{-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{16}-\frac{1}{8(\sin(dx+c)-1)}-\frac{3\ln(\sin(dx+c)-1)}{16}}{da}$ |
| risch | $-\frac{i\left(6i e^{4i(dx+c)}+3\,e^{5i(dx+c)}-6i e^{2i(dx+c)}+2\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{4\left(e^{i(dx+c)}+i\right)^4\left(e^{i(dx+c)}-i\right)^2 da}+\frac{3\ln\left(e^{i(dx+c)}+i\right)}{8ad}-\frac{3\ln\left(e^{i(dx+c)}-i\right)}{8ad}$ |
| norman | $\frac{\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8ad}+\frac{3\ln\left(\tan\left(\frac{c}{}\right.\right.}{8a}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/8/(1+sin(d*x+c))^2-1/4/(1+sin(d*x+c))+3/16*ln(1+sin(d*x+c))-1/8/(
sin(d*x+c)-1)-3/16*ln(sin(d*x+c)-1))
```

**Maxima [A]**
time = 0.30, size = 91, normalized size = 1.18

$$-\frac{\frac{2\left(3\sin(dx+c)^2+3\sin(dx+c)-2\right)}{a\sin(dx+c)^3+a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{3\log(\sin(dx+c)+1)}{a}+\frac{3\log(\sin(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(
d*x + c)^2 - a*sin(d*x + c) - a) - 3*log(sin(d*x + c) + 1)/a + 3*log(sin(d*
x + c) - 1)/a)/d
```

**Fricas [A]**
time = 0.36, size = 125, normalized size = 1.62

$$-\frac{6\cos(dx+c)^2-3\left(\cos(dx+c)^2\sin(dx+c)+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)+3\left(\cos(dx+c)^2\sin(dx+c)+\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)-6\sin(dx+c)-2}{16\left(ad\cos(dx+c)^2\sin(dx+c)+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*
log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1) - 6*sin(d*x + c) - 2)/(a*d*cos(d*x + c)^2*sin(d*x + c)
 + a*d*cos(d*x + c)^2)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

**Giac [A]**
time = 4.64, size = 96, normalized size = 1.25

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a}-\frac{6\log(|\sin(dx+c)-1|)}{a}+\frac{2(3\sin(dx+c)-5)}{a(\sin(dx+c)-1)}-\frac{9\sin(dx+c)^2+26\sin(dx+c)+21}{a(\sin(dx+c)+1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2*(
3*sin(d*x + c) - 5)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 26*sin(d*x
 + c) + 21)/(a*(sin(d*x + c) + 1)^2))/d
```

**Mupad** [B]
time = 0.10, size = 74, normalized size = 0.96

$$\frac{3\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{8\,a\,d}+\frac{\frac{3\sin(c+d\,x)^2}{8}+\frac{3\,\sin(c+d\,x)}{8}-\frac{1}{4}}{d\,\left(-a\sin\left(c+d\,x\right)^3-a\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atanh(sin(c + d*x)))/(8*a*d) + ((3*sin(c + d*x))/8 + (3*sin(c + d*x)^2)/
8 - 1/4)/(d*(a + a*sin(c + d*x) - a*sin(c + d*x)^2 - a*sin(c + d*x)^3))
```

# 3.285 $\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 23.85, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

**Sympy** [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

[Out] Timed out

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(e + f*x)*(a + a*sin(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^3*(e + f*x)*(a + a*sin(c + d*x))), x)
```

# 3.286 $\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Defer[Int][Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

**Mathematica** [A]
time = 34.79, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
[Out] Integrate[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

**Maple** [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(fx+e)^2\,(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2 + 2*a*f
*x*e + a*e^2)*sin(d*x + c)), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(e + f*x)^2*(a + a*sin(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^3*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.287 $\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)} \, dx$

**Optimal**. Leaf size=449

$$\frac{(e+fx)^{1+m}}{2af(1+m)}+\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{8ad}+\frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1\right.}{8ad}$$

```
[Out] 1/2*(f*x+e)^(1+m)/a/f/(1+m)+1/8*exp(I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-I*d*(
f*x+e)/f)/a/d/((-I*d*(f*x+e)/f)^m)+1/8*(f*x+e)^m*GAMMA(1+m,I*d*(f*x+e)/f)/a
/d/exp(I*(c-d*e/f))/((I*d*(f*x+e)/f)^m)-I*2^(-3-m)*exp(2*I*(c-d*e/f))*(f*x+
e)^m*GAMMA(1+m,-2*I*d*(f*x+e)/f)/a/d/((-I*d*(f*x+e)/f)^m)+I*2^(-3-m)*(f*x+e
)^m*GAMMA(1+m,2*I*d*(f*x+e)/f)/a/d/exp(2*I*(c-d*e/f))/((I*d*(f*x+e)/f)^m)+1
/8*3^(-1-m)*exp(3*I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-3*I*d*(f*x+e)/f)/a/d/((
-I*d*(f*x+e)/f)^m)+1/8*3^(-1-m)*(f*x+e)^m*GAMMA(1+m,3*I*d*(f*x+e)/f)/a/d/ex
p(3*I*(c-d*e/f))/((I*d*(f*x+e)/f)^m)
```

**Rubi** [A]

time = 0.42, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4619, 3393, 3388, 2212, 4491, 3389}

$$\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right)}{8ad}-\frac{i2^{-m-3}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2id(e+fx)}{f}\right)}{ad}+\frac{3^{-m-1}e^{3i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3id(e+fx)}{f}\right)}{8ad}+\frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,\frac{id(e+fx)}{f}\right)}{8ad}+\frac{i2^{-m-3}e^{-2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,\frac{2id(e+fx)}{f}\right)}{ad}+\frac{3^{-m-1}e^{-3i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,\frac{3id(e+fx)}{f}\right)}{8ad}+\frac{(e+fx)^{m+1}}{2af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^m*Cos[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e + f*x)^(1 + m)/(2*a*f*(1 + m)) + (E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[
1 + m, ((-I)*d*(e + f*x))/f])/(8*a*d*(((-I)*d*(e + f*x))/f)^m) + ((e + f*x)
^m*Gamma[1 + m, (I*d*(e + f*x))/f])/(8*a*d*E^(I*(c - (d*e)/f))*((I*d*(e + f
*x))/f)^m) - (I*2^(-3 - m)*E^((2*I)*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m,
 ((-2*I)*d*(e + f*x))/f])/(a*d*(((-I)*d*(e + f*x))/f)^m) + (I*2^(-3 - m)*(e
 + f*x)^m*Gamma[1 + m, ((2*I)*d*(e + f*x))/f])/(a*d*E^((2*I)*(c - (d*e)/f))
*((I*d*(e + f*x))/f)^m) + (3^(-1 - m)*E^((3*I)*(c - (d*e)/f))*(e + f*x)^m*G
amma[1 + m, ((-3*I)*d*(e + f*x))/f])/(8*a*d*(((-I)*d*(e + f*x))/f)^m) + (3^
(-1 - m)*(e + f*x)^m*Gamma[1 + m, ((3*I)*d*(e + f*x))/f])/(8*a*d*E^((3*I)*(
c - (d*e)/f))*((I*d*(e + f*x))/f)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^m \cos^2(c+dx)\,dx}{a} - \frac{\int (e+fx)^m \cos^2(c+dx)\sin(c+dx)\,dx}{a}$$

$$= \frac{\int \left(\frac{1}{2}(e+fx)^m + \frac{1}{2}(e+fx)^m \cos(2c+2dx)\right)\,dx}{a} - \frac{\int \left(\frac{1}{4}(e+fx)^m \sin(c+dx\right.}{}$$

$$= \frac{(e+fx)^{1+m}}{2af(1+m)} - \frac{\int (e+fx)^m \sin(c+dx)\,dx}{4a} - \frac{\int (e+fx)^m \sin(3c+3dx)\,dx}{4a} +$$

$$= \frac{(e+fx)^{1+m}}{2af(1+m)} - \frac{i\int e^{-i(c+dx)}(e+fx)^m\,dx}{8a} + \frac{i\int e^{i(c+dx)}(e+fx)^m\,dx}{8a} - \frac{i\int e^{-}}{}$$

$$= \frac{(e+fx)^{1+m}}{2af(1+m)} + \frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{8ad} + \frac{e^{-}}{}$$

**Mathematica [A]**
time = 2.61, size = 411, normalized size = 0.92

$$\frac{i(e+fx)^m\left(-\frac{12ide}{f+fm}-\frac{12idx}{1+m}-3ie^{i\left(c-\frac{de}{f}\right)}\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)-3ie^{-i\left(c-\frac{de}{f}\right)}\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{id(e+fx)}{f}\right)-3\,2^{-m}e^{2i\left(c-\frac{de}{f}\right)}\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{2id(e+fx)}{f}\right)+3\,2^{-m}e^{-2i\left(c-\frac{de}{f}\right)}\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{2id(e+fx)}{f}\right)-i3^{-m}e^{3i\left(c-\frac{de}{f}\right)}\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{3id(e+fx)}{f}\right)-i3^{-m}e^{-3i\left(c-\frac{de}{f}\right)}\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{3id(e+fx)}{f}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{24ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((I/24)*(e + f*x)^m*(((-12*I)*d*e)/(f + f*m) - ((12*I)*d*x)/(1 + m) - ((3*I
)*E^(I*(c - (d*e)/f))*Gamma[1 + m, ((-I)*d*(e + f*x))/f])/(((-I)*d*(e + f*x
))/f)^m - ((3*I)*Gamma[1 + m, (I*d*(e + f*x))/f])/(E^(I*(c - (d*e)/f))*((I*
d*(e + f*x))/f)^m) - (3*E^((2*I)*(c - (d*e)/f))*Gamma[1 + m, ((-2*I)*d*(e +
 f*x))/f])/(2^m*(((-I)*d*(e + f*x))/f)^m) + (3*Gamma[1 + m, ((2*I)*d*(e + f
*x))/f])/(2^m*E^((2*I)*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m) - (I*E^((3*I)*
(c - (d*e)/f))*Gamma[1 + m, ((-3*I)*d*(e + f*x))/f])/(3^m*(((-I)*d*(e + f*x
))/f)^m) - (I*Gamma[1 + m, ((3*I)*d*(e + f*x))/f])/(3^m*E^((3*I)*(c - (d*e)
/f))*((I*d*(e + f*x))/f)^m))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(a*d*
(1 + Sin[c + d*x]))
```

**Maple [F]**
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m\left(\cos^4(dx+c)\right)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

**Fricas [A]**
time = 0.12, size = 354, normalized size = 0.79

$$\frac{3\,(fm+f)e^{\left(-\frac{fm\log\left(\frac{i\,d}{f}\right)+i\,cf-i\,de}{f}\right)}\Gamma\left(m+1,\frac{i\,dfx+i\,de}{f}\right)-3\,(i\,fm+i\,f)e^{\left(-\frac{fm\log\left(-\frac{2i\,d}{f}\right)-2i\,cf+2i\,de}{f}\right)}\Gamma\left(m+1,-\frac{2\,(i\,dfx+i\,de)}{f}\right)+(fm+f)e^{\left(-\frac{fm\log\left(-\frac{3i\,d}{f}\right)-3i\,cf+3i\,de}{f}\right)}\Gamma\left(m+1,-\frac{3\,(i\,dfx+i\,de)}{f}\right)+3\,(fm+f)e^{\left(-\frac{fm\log\left(-\frac{i\,d}{f}\right)-i\,cf+i\,de}{f}\right)}\Gamma\left(m+1,\frac{-i\,dfx-i\,de}{f}\right)-3\,(-i\,fm-i\,f)e^{\left(-\frac{fm\log\left(\frac{2i\,d}{f}\right)+2i\,cf-2i\,de}{f}\right)}\Gamma\left(m+1,-\frac{2\,(-i\,dfx-i\,de)}{f}\right)+(fm+f)e^{\left(-\frac{fm\log\left(\frac{3i\,d}{f}\right)+3i\,cf-3i\,de}{f}\right)}\Gamma\left(m+1,-\frac{3\,(-i\,dfx-i\,de)}{f}\right)+12\,(dfx+de)(fx+e)^{m}}{24\,(adfm+adf)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(f*m + f)*e^(-(f*m*log(I*d/f) + I*c*f - I*d*e)/f)*gamma(m + 1, (I*d
*f*x + I*d*e)/f) - 3*(I*f*m + I*f)*e^(-(f*m*log(-2*I*d/f) - 2*I*c*f + 2*I*d
*e)/f)*gamma(m + 1, -2*(I*d*f*x + I*d*e)/f) + (f*m + f)*e^(-(f*m*log(-3*I*d
/f) - 3*I*c*f + 3*I*d*e)/f)*gamma(m + 1, -3*(I*d*f*x + I*d*e)/f) + 3*(f*m +
 f)*e^(-(f*m*log(-I*d/f) - I*c*f + I*d*e)/f)*gamma(m + 1, (-I*d*f*x - I*d*e
)/f) - 3*(-I*f*m - I*f)*e^(-(f*m*log(2*I*d/f) + 2*I*c*f - 2*I*d*e)/f)*gamma
(m + 1, -2*(-I*d*f*x - I*d*e)/f) + (f*m + f)*e^(-(f*m*log(3*I*d/f) + 3*I*c*
f - 3*I*d*e)/f)*gamma(m + 1, -3*(-I*d*f*x - I*d*e)/f) + 12*(d*f*x + d*e)*(f
*x + e)^m)/(a*d*f*m + a*d*f)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,{\left(e+f\,x\right)}^m}{a+a\,\sin\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
[Out] int((cos(c + d*x)^4*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

# 3.288 $\int \frac{(e+fx)^m \cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=277

$$-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad}+\frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{id(e}{}\right.}{2ad}$$

```
[Out] -1/2*I*exp(I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-I*d*(f*x+e)/f)/a/d/((-I*d*(f*x
+e)/f)^m)+1/2*I*(f*x+e)^m*GAMMA(1+m,I*d*(f*x+e)/f)/a/d/exp(I*(c-d*e/f))/((I
*d*(f*x+e)/f)^m)+2^(-3-m)*exp(2*I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-2*I*d*(f*
x+e)/f)/a/d/((-I*d*(f*x+e)/f)^m)+2^(-3-m)*(f*x+e)^m*GAMMA(1+m,2*I*d*(f*x+e)
/f)/a/d/exp(2*I*(c-d*e/f))/((I*d*(f*x+e)/f)^m)
```

**Rubi** [A]
time = 0.21, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4619, 3388, 2212, 4491, 12, 3389}

$$-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right)}{2ad}+\frac{2^{-m-3}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2id(e+fx)}{f}\right)}{ad}+\frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,\frac{id(e+fx)}{f}\right)}{2ad}+\frac{2^{-m-3}e^{-2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,\frac{2id(e+fx)}{f}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^m*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-1/2*I)*E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m, ((-I)*d*(e + f*x))/f
])/(a*d*(((-I)*d*(e + f*x))/f)^m) + ((I/2)*(e + f*x)^m*Gamma[1 + m, (I*d*(e
 + f*x))/f])/(a*d*E^(I*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m) + (2^(-3 - m)*
E^((2*I)*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m, ((-2*I)*d*(e + f*x))/f])/(
a*d*(((-I)*d*(e + f*x))/f)^m) + (2^(-3 - m)*(e + f*x)^m*Gamma[1 + m, ((2*I)
*d*(e + f*x))/f])/(a*d*E^((2*I)*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^m\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^m\cos(c+dx)\,dx}{a} - \frac{\int (e+fx)^m\cos(c+dx)\sin(c+dx)\,dx}{a}$$

$$= \frac{\int e^{-i(c+dx)}(e+fx)^m\,dx}{2a} + \frac{\int e^{i(c+dx)}(e+fx)^m\,dx}{2a} - \frac{\int \frac{1}{2}(e+fx)^m\sin(2c+2$$

$$= -\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad} + \frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+$$

$$= -\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad} + \frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+$$

$$= -\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad} + \frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+$$

**Mathematica [A]**

time = 1.21, size = 253, normalized size = 0.91

$$\frac{2^{-3-m}e^{-\frac{2i(de+cf)}{f}}(e+fx)^m\left(\frac{d^2(e+fx)^2}{f^2}\right)^{-m}\left(-i2^{2+m}e^{i\left(3c+\frac{de}{f}\right)}\left(\frac{id(e+fx)}{f}\right)^m\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)+i2^{2+m}e^{i\left(c+\frac{3de}{f}\right)}\left(-\frac{id(e+fx)}{f}\right)^m\Gamma\left(1+m,\frac{id(e+fx)}{f}\right)+e^{4ic}\left(\frac{id(e+fx)}{f}\right)^m\Gamma\left(1+m,-\frac{2id(e+fx)}{f}\right)+e^{\frac{4ide}{f}}\left(-\frac{id(e+fx)}{f}\right)^m\Gamma\left(1+m,\frac{2id(e+fx)}{f}\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2^(-3 - m)*(e + f*x)^m*((-I)*2^(2 + m)*E^(I*(3*c + (d*e)/f))*((I*d*(e + f*
x))/f)^m*Gamma[1 + m, ((-I)*d*(e + f*x))/f] + I*2^(2 + m)*E^(I*(c + (3*d*e)
/f))*(((-I)*d*(e + f*x))/f)^m*Gamma[1 + m, (I*d*(e + f*x))/f] + E^((4*I)*c)
*((I*d*(e + f*x))/f)^m*Gamma[1 + m, ((-2*I)*d*(e + f*x))/f] + E^(((4*I)*d*e
)/f)*(((-I)*d*(e + f*x))/f)^m*Gamma[1 + m, ((2*I)*d*(e + f*x))/f]))/(a*d*E^
(((2*I)*(d*e + c*f))/f)*((d^2*(e + f*x)^2)/f^2)^m)
```

**Maple [F]**
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m\left(\cos^3(dx+c)\right)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

**Fricas [A]**
time = 0.11, size = 197, normalized size = 0.71

$$\frac{4i\,e^{\left(-\frac{fm\log\left(\frac{i\,d}{f}\right)+i\,cf-i\,de}{f}\right)}\Gamma\left(m+1,\frac{i\,dfx+i\,de}{f}\right)+e^{\left(-\frac{fm\log\left(-\frac{2i\,d}{f}\right)-2i\,cf+2i\,de}{f}\right)}\Gamma\left(m+1,-\frac{2\,(i\,dfx+i\,de)}{f}\right)-4i\,e^{\left(-\frac{fm\log\left(-\frac{i\,d}{f}\right)-i\,cf+i\,de}{f}\right)}\Gamma\left(m+1,\frac{-i\,dfx-i\,de}{f}\right)+e^{\left(-\frac{fm\log\left(\frac{2i\,d}{f}\right)+2i\,cf-2i\,de}{f}\right)}\Gamma\left(m+1,-\frac{2\,(-i\,dfx-i\,de)}{f}\right)}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*I*e^(-(f*m*log(I*d/f) + I*c*f - I*d*e)/f)*gamma(m + 1, (I*d*f*x + I*
d*e)/f) + e^(-(f*m*log(-2*I*d/f) - 2*I*c*f + 2*I*d*e)/f)*gamma(m + 1, -2*(I
```

```
*d*f*x + I*d*e)/f) - 4*I*e^(-(f*m*log(-I*d/f) - I*c*f + I*d*e)/f)*gamma(m +
 1, (-I*d*f*x - I*d*e)/f) + e^(-(f*m*log(2*I*d/f) + 2*I*c*f - 2*I*d*e)/f)*g
amma(m + 1, -2*(-I*d*f*x - I*d*e)/f))/(a*d)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*cos(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,{\left(e+f\,x\right)}^m}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
[Out] int((cos(c + d*x)^3*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

# 3.289 $\int \frac{(e+fx)^m \cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=154

$$\frac{(e+fx)^{1+m}}{af(1+m)}+\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad}+\frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\big(}{2ad}$$

```
[Out] (f*x+e)^(1+m)/a/f/(1+m)+1/2*exp(I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-I*d*(f*x+
e)/f)/a/d/((-I*d*(f*x+e)/f)^m)+1/2*(f*x+e)^m*GAMMA(1+m,I*d*(f*x+e)/f)/a/d/e
xp(I*(c-d*e/f))/((I*d*(f*x+e)/f)^m)
```

**Rubi** [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4619, 32, 3389, 2212}

$$\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right)}{2ad}+\frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,\frac{id(e+fx)}{f}\right)}{2ad}+\frac{(e+fx)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^m*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e + f*x)^(1 + m)/(a*f*(1 + m)) + (E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[1
+ m, ((-I)*d*(e + f*x))/f])/(2*a*d*(((-I)*d*(e + f*x))/f)^m) + ((e + f*x)^m
*Gamma[1 + m, (I*d*(e + f*x))/f])/(2*a*d*E^(I*(c - (d*e)/f))*((I*d*(e + f*x
))/f)^m)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4619

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^m \cos^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^m\,dx}{a} - \frac{\int (e+fx)^m \sin(c+dx)\,dx}{a}\\ &= \frac{(e+fx)^{1+m}}{af(1+m)} - \frac{i\int e^{-i(c+dx)}(e+fx)^m\,dx}{2a} + \frac{i\int e^{i(c+dx)}(e+fx)^m\,dx}{2a}\\ &= \frac{(e+fx)^{1+m}}{af(1+m)} + \frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad} + \frac{e^{-}}{}\end{aligned}$$

**Mathematica [A]**
time = 1.90, size = 187, normalized size = 1.21

$$\frac{(e+fx)^m\left(\frac{2d(e+fx)}{f(1+m)}+\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{id(e+fx)}{f}\right)\left(\cos\left(c-\frac{de}{f}\right)-i\sin\left(c-\frac{de}{f}\right)\right)+\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)\left(\cos\left(c-\frac{de}{f}\right)+i\sin\left(c-\frac{de}{f}\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{2ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((e + f*x)^m*((2*d*(e + f*x))/(f*(1 + m)) + (Gamma[1 + m, (I*d*(e + f*x))/f
]*(Cos[c - (d*e)/f] - I*Sin[c - (d*e)/f]))/((I*d*(e + f*x))/f)^m + (Gamma[1
 + m, ((-I)*d*(e + f*x))/f]*(Cos[c - (d*e)/f] + I*Sin[c - (d*e)/f]))/(((-I)
*d*(e + f*x))/f)^m)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(2*a*d*(1 + Si
n[c + d*x]))
```

**Maple [F]**
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m\left(\cos^2(dx+c)\right)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.11, size = 136, normalized size = 0.88

$$\frac{(fm+f)e^{\left(-\frac{fm\log\left(\frac{id}{f}\right)+icf-ide}{f}\right)}\Gamma\left(m+1,\frac{idfx+ide}{f}\right)+(fm+f)e^{\left(-\frac{fm\log\left(-\frac{id}{f}\right)-icf+ide}{f}\right)}\Gamma\left(m+1,\frac{-idfx-ide}{f}\right)+2\,(dfx+de)(fx+e)^m}{2\,(adfm+adf)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((f*m + f)*e^(-(f*m*log(I*d/f) + I*c*f - I*d*e)/f)*gamma(m + 1, (I*d*f*
x + I*d*e)/f) + (f*m + f)*e^(-(f*m*log(-I*d/f) - I*c*f + I*d*e)/f)*gamma(m
+ 1, (-I*d*f*x - I*d*e)/f) + 2*(d*f*x + d*e)*(f*x + e)^m)/(a*d*f*m + a*d*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m\cos^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*cos(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^2 (e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
[Out] int((cos(c + d*x)^2*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

# 3.290 $\int \frac{(e+fx)^m \cos(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=29

$$\mathrm{Int}\left(\frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 5.46, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m\cos(dx+c)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] int((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m \cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*cos(c + d*x)/(sin(c + d*x) + 1), x)/a
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(c+d\,x)\,{(e+f\,x)}^m}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
[Out] int((cos(c + d*x)*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

# 3.291 $\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=23

$$\operatorname{Int}\left(\frac{(e+fx)^m}{a+a\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
[Out] Defer[Int][(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
[Out] Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m/(a+a*sin(d*x+c)),x)
[Out] int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m/(a+a*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m/(sin(c + d*x) + 1), x)/a
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e+f\,x)^m}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(a + a*sin(c + d*x)),x)
[Out] int((e + f*x)^m/(a + a*sin(c + d*x)), x)
```

# 3.292 $\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=29

$$\mathrm{Int}\left(\frac{(e+fx)^m\sec(c+dx)}{a+a\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m\sec(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m\sec(c+dx)}{a+a\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 77.34, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m\sec(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m\sec(dx+c)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m \sec(c+dx)}{\sin(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*sec(c + d*x)/(sin(c + d*x) + 1), x)/a
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(e+f\,x)^m}{\cos\left(c+d\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
[Out] int((e + f*x)^m/(cos(c + d*x)*(a + a*sin(c + d*x))), x)
```

## 3.293 $\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 10.52, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m \left(\sec^2(dx+c)\right)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
[Out] int((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e+fx)^m \sec^2(c+dx)}{\sin(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(e+fx)^m}{\cos(c+dx)^2\,(a+a\sin(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
[Out] int((e + f*x)^m/(cos(c + d*x)^2*(a + a*sin(c + d*x))), x)
```

# 3.294 $\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=432

$$-\frac{i(e+fx)^4}{4bf}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{3if(e+fx)^2\mathrm{Li}_2\left(\frac{\ }{a-}\right.}{bd^2}$$

```
[Out] -1/4*I*(f*x+e)^4/b/f+(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))
/b/d+(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d-3*I*f*(f*x+
e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^2-3*I*f*(f*x+e)^
2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^2+6*f^2*(f*x+e)*pol
ylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^3+6*f^2*(f*x+e)*polylog(
3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^3+6*I*f^3*polylog(4,I*b*exp(I
*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^4+6*I*f^3*polylog(4,I*b*exp(I*(d*x+c))/(
a+(a^2-b^2)^(1/2)))/b/d^4
```

**Rubi** [A]

time = 0.38, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4615, 2221, 2611, 6744, 2320, 6724}

$$\frac{6if^3\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4}+\frac{6if^3\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^4}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^4}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-1/4*I)*(e + f*x)^4)/(b*f) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*d) - ((3*I)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2) - ((3*I)*f*(e + f*x)^2*PolyLog[2,
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2) + (6*f^2*(e + f*x)*P
olyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^3) + (6*f^2*(e
 + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^3) +
((6*I)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^4)
 + ((6*I)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d
^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i(e+fx)^4}{4bf} + \int \frac{e^{i(c+dx)}(e+fx)^3}{a-\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx + \int \frac{e^{i(c+dx)}(e+fx)^3}{a+\sqrt{a^2-b^2}-ibe^{i(c-}}$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ib}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ib}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ib}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ib}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ib}{a+\sqrt{}}\right.}{bd}$$

**Mathematica [A]**
time = 0.14, size = 410, normalized size = 0.95

$$\frac{-\frac{i(e+fx)^4}{f}+\frac{4(e+fx)^3\log\left(1+\frac{ibe^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)}{d}+\frac{4(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}+\frac{12f\left(-id^2(e+fx)^2\mathrm{Li}_2\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)+2f\left(d(e+fx)\mathrm{Li}_3\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)+if\mathrm{Li}_4\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)\right)\right)}{d^4}+\frac{12f\left(-id^2(e+fx)^2\mathrm{Li}_2\left(\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)+2f\left(d(e+fx)\mathrm{Li}_3\left(\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)+if\mathrm{Li}_4\left(\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)\right)\right)}{d^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((-I)*(e + f*x)^4)/f + (4*(e + f*x)^3*Log[1 + (I*b*E^(I*(c + d*x)))/(-a +
Sqrt[a^2 - b^2])])/d + (4*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/d + (12*f*((-I)*d^2*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c +
d*x)))/(a - Sqrt[a^2 - b^2])] + 2*f*(d*(e + f*x)*PolyLog[3, (I*b*E^(I*(c +
d*x)))/(a - Sqrt[a^2 - b^2])] + I*f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])))/d^4 + (12*f*((-I)*d^2*(e + f*x)^2*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])] + 2*f*(d*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])] + I*f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])))/d^4)/(4*b)
```

**Maple [F]**
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3\cos(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1777 vs. 2(377) = 754.
time = 0.56, size = 1777, normalized size = 4.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(-6*I*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*f^3*polylog(4, -(I
*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))/b) + 6*I*f^3*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x
 + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*
I*f^3*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 3*(I*d^2*f^3*x^2 + 2*I*d^2*f^
2*x*e + I*d^2*f*e^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x
+ c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(I*d^2*f^3*
x^2 + 2*I*d^2*f^2*x*e + I*d^2*f*e^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)
 - 3*(-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2)*dilog((-I*a*cos(d*x +
 c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b + 1) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2)*dil
og((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f
*e^2 - d^3*e^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) + 2*I*a) - (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3)*
```

```
log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) - (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3)*log(-2*b*cos(d*x
 + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (c^3*f^3
 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3)*log(-2*b*cos(d*x + c) - 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (d^3*f^3*x^3 + c^3*f^3
+ 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*log(-(I*a*cos(
d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b) + (d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 +
3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (d^3*f^3
*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)
*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x +
 c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*log(-(-I*a*cos(d*x + c) - a*
sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
 b)/b) + 6*(d*f^3*x + d*f^2*e)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x +
c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(d*
f^3*x + d*f^2*e)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(d*f^3*x + d*f^2*
e)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(d*f^3*x + d*f^2*e)*polylog(3,
 -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b))/(b*d^4)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e+f\,x\right)}^3}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
[Out] int((cos(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)), x)
```

# 3.295 $\int \frac{(e+fx)^2 \cos(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=320

$$-\frac{i(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{2if(e+fx)\mathrm{Li}_2\left(\frac{i\cdots}{a-\sqrt{\cdots}}\right.}{bd^2}$$

```
[Out] -1/3*I*(f*x+e)^3/b/f+(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))
/b/d+(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d-2*I*f*(f*x+
e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^2-2*I*f*(f*x+e)*po
lylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^2+2*f^2*polylog(3,I*b*e
xp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^3+2*f^2*polylog(3,I*b*exp(I*(d*x+c))
/(a+(a^2-b^2)^(1/2)))/b/d^3
```

**Rubi** [A]
time = 0.33, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4615, 2221, 2611, 2320, 6724}

$$\frac{2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3}-\frac{2if(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^3}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-1/3*I)*(e + f*x)^3)/(b*f) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*d) - ((2*I)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, (I*
b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2) + (2*f^2*PolyLog[3, (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^3) + (2*f^2*PolyLog[3, (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i(e+fx)^3}{3bf} + \int \frac{e^{i(c+dx)}(e+fx)^2}{a-\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx + \int \frac{e^{i(c+dx)}(e+fx)^2}{a+\sqrt{a^2-b^2}-ibe^{i(c-}}$$

$$= -\frac{i(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ib}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{i(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ib}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{i(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ib}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{i(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ib}{a+\sqrt{}}\right.}{bd}$$

## Mathematica [A]

time = 0.12, size = 302, normalized size = 0.94

$$\frac{-\frac{i(e+fx)^3}{f} + \frac{3(e+fx)^2\log\left(1+\frac{ibe^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)}{d} + \frac{3(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d} + \frac{6f\left(-id(e+fx)\mathrm{Li}_2\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)+f\mathrm{Li}_3\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)\right)}{d^3} + \frac{6f\left(-id(e+fx)\mathrm{Li}_2\left(\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)+f\mathrm{Li}_3\left(\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)\right)}{d^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((-I)*(e + f*x)^3)/f + (3*(e + f*x)^2*Log[1 + (I*b*E^(I*(c + d*x)))/(-a +
Sqrt[a^2 - b^2])])/d + (3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/d + (6*f*((-I)*d*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x))
)/(a - Sqrt[a^2 - b^2])] + f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])]))/d^3 + (6*f*((-I)*d*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])] + f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2])]))/d^3)/(3*b)
```

## Maple [F]

time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2\cos(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1243 vs. 2(279) = 558.
time = 0.53, size = 1243, normalized size = 3.88



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c)
 - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*f^2*polylog(3, -(I*a*co
s(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2))/b) + 2*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) +
 (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*f^2*pol
ylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(I*d*f^2*x + I*d*f*e)*dilog((I*a*cos
(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b + 1) - 2*(I*d*f^2*x + I*d*f*e)*dilog((I*a*cos(d*x + c)
- a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2) - b)/b + 1) - 2*(-I*d*f^2*x - I*d*f*e)*dilog((-I*a*cos(d*x + c) - a*sin(
d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/
b + 1) - 2*(-I*d*f^2*x - I*d*f*e)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c)
 - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
+ 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*log
(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a
) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)
*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
2*I*a) + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*log(-(I*a*cos(d*x
+ c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^
```

```
2)/b^2) - b)/b) + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*log(-(I*a
*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b) + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*
log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x +
c*d*f)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/(b*d^3)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**2*cos(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,(e+f\,x)^2}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
[Out] int((cos(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)), x)
```

# 3.296 $\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=212

$$-\frac{i(e+fx)^2}{2bf}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\mathrm{Li}_2\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-$$

```
[Out] -1/2*I*(f*x+e)^2/b/f+(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b
/d+(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d-I*f*polylog(2,I
*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^2-I*f*polylog(2,I*b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/b/d^2
```

**Rubi** [A]
time = 0.19, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4615, 2221, 2317, 2438}

$$-\frac{if\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^2}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-1/2*I)*(e + f*x)^2)/(b*f) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a +
Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b*d^2) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i(e+fx)^2}{2bf} + \int \frac{e^{i(c+dx)}(e+fx)}{a-\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx + \int \frac{e^{i(c+dx)}(e+fx)}{a+\sqrt{a^2-b^2}-ibe^{i(c+}}$$

$$= -\frac{i(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c-}}{a+\sqrt{a^2}}\right.}{bd}$$

$$= -\frac{i(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c-}}{a+\sqrt{a^2}}\right.}{bd}$$

$$= -\frac{i(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c-}}{a+\sqrt{a^2}}\right.}{bd}$$

**Mathematica [A]**
time = 0.04, size = 197, normalized size = 0.93

$$-\frac{i\left(d(e+fx)\left(de+dfx+2if\log\left(1+\frac{ibe^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)+2if\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)\right)+2f^2\mathrm{Li}_2\left(-\frac{ibe^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)+2f^2\mathrm{Li}_2\left(\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)\right)}{2bd^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-1/2*I)*(d*(e + f*x)*(d*e + d*f*x + (2*I)*f*Log[1 + (I*b*E^(I*(c + d*x)))
/(-a + Sqrt[a^2 - b^2])] + (2*I)*f*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2])]) + 2*f^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 -
b^2])] + 2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]))/(b
*d^2*f)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(187) = 374$.
time = 0.14, size = 1006, normalized size = 4.75

| method | result |
|---|---|
| risch | $-\frac{if\,c^2}{d^2 b}+\frac{iex}{b}+\frac{if\,\mathrm{dilog}\left(\frac{ia+b\,e^{i(dx+c)}-\sqrt{-a^2+b^2}}{ia-\sqrt{-a^2+b^2}}\right)a^2}{d^2 b(-a^2+b^2)}+\frac{if\,\mathrm{dilog}\left(\frac{ia+b\,e^{i(dx+c)}+\sqrt{-a^2+b^2}}{ia+\sqrt{-a^2+b^2}}\right)a^2}{d^2 b(-a^2+b^2)}-\frac{if\,x^2}{2b}-\frac{ibf\,d}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d^2/b*f*c^2-I/d^2*b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^
(1/2))/(I*a+(-a^2+b^2)^(1/2)))+I/b*e*x+I/d^2/b*f/(-a^2+b^2)*dilog((I*a+b*ex
p(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2+I/d^2/b*f/(-a^2+
b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*
a^2-I/d^2*b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a
-(-a^2+b^2)^(1/2)))-1/d/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^
(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2*x-1/d^2/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(
d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2*c-1/2*I/b*f*x^2-2/d/b
*ln(exp(I*(d*x+c)))*e+1/d/b*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*
b)-1/d/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2
+b^2)^(1/2)))*a^2*x-1/d^2/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2
)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*c-2*I/d/b*f*c*x+1/d*b*f/(-a^2+b^2)*ln(
(I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/d^2*b*f
/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2
)))*c+1/d*b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-
a^2+b^2)^(1/2)))*x+1/d^2*b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)
^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-1/d^2/b*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*e
xp(I*(d*x+c))-I*b)+2/d^2/b*f*c*ln(exp(I*(d*x+c)))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(181) = 362$.

time = 0.57, size = 781, normalized size = 3.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*f*dilog((I*a*cos(d*x +
 c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b + 1) + I*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos
(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*f*dilo
g((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (c*f - d*e)*log(2*b*cos(d*x + c) + 2*
I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (c*f - d*e)*log(2*
b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) -
 (c*f - d*e)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
b^2)/b^2) + 2*I*a) - (c*f - d*e)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c)
 + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (d*f*x + c*f)*log(-(I*a*cos(d*x +
c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) - b)/b) + (d*f*x + c*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*
cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (d*f*x +
c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (d*f*x + c*f)*log(-(-I*a*cos(d*x
+ c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2) - b)/b))/(b*d^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)\cos{(c + dx)}}{a + b\sin{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

[Out] integrate((f*x + e)*cos(d*x + c)/(b*sin(d*x + c) + a), x)

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)\,\,(e+f\,x)}{a+b\,\sin (c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)

[Out] int((cos(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)), x)

# 3.297 $\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=18

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

```
[Out] ln(a+b*sin(d*x+c))/b/d
```

**Rubi** [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 31}

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\sin(c+dx))}{bd}\end{aligned}$$

**Mathematica** [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

**Maple [A]**
time = 0.06, size = 19, normalized size = 1.06

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(a+b\sin(dx+c))}{bd}$ | 19 |
| default | $\frac{\ln(a+b\sin(dx+c))}{bd}$ | 19 |
| risch | $-\frac{ix}{b}-\frac{2ic}{bd}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{bd}$ | 54 |
| norman | $\frac{\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{bd}-\frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}$ | 59 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*sin(d*x+c))/b/d
```

**Maxima [A]**
time = 0.29, size = 18, normalized size = 1.00

$$\frac{\log\left(b\sin\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

**Fricas [A]**
time = 0.36, size = 18, normalized size = 1.00

$$\frac{\log\left(b\sin\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

time = 0.33, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x\cos(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{ad} & \text{for } b = 0 \\ \frac{x\cos(c)}{a+b\sin(c)} & \text{for } d = 0 \\ \frac{\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a*d), Eq(b, 0))
, (x*cos(c)/(a + b*sin(c)), Eq(d, 0)), (log(a/b + sin(c + d*x))/(b*d), True
))
```

**Giac** [A]
time = 3.46, size = 19, normalized size = 1.06

$$\frac{\log(|b\sin(dx+c)+a|)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*sin(d*x + c) + a))/(b*d)
```

**Mupad** [B]
time = 0.06, size = 18, normalized size = 1.00

$$\frac{\ln(a+b\sin(c+dx))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x)),x)
```

```
[Out] log(a + b*sin(c + d*x))/(b*d)
```

# 3.298 $\int \frac{(e+fx)^3 \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=618

$$\frac{a(e+fx)^4}{4b^2f}-\frac{6f^2(e+fx)\cos(c+dx)}{bd^3}+\frac{(e+fx)^3\cos(c+dx)}{bd}+\frac{i\sqrt{a^2-b^2}\,(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}$$

```
[Out] 1/4*a*(f*x+e)^4/b^2/f-6*f^2*(f*x+e)*cos(d*x+c)/b/d^3+(f*x+e)^3*cos(d*x+c)/b
/d+6*f^3*sin(d*x+c)/b/d^4-3*f*(f*x+e)^2*sin(d*x+c)/b/d^2+I*(f*x+e)^3*ln(1-I
*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d-I*(f*x+e)^3*ln
(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d+3*f*(f*x+e
)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d
^2-3*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2
)^(1/2)/b^2/d^2+6*I*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(
1/2)))*(a^2-b^2)^(1/2)/b^2/d^3-6*I*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))
/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^3-6*f^3*polylog(4,I*b*exp(I*(d*
x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^4+6*f^3*polylog(4,I*b*exp(
I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^4
```

**Rubi** [A]
time = 0.69, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {4621, 32, 3377, 2717, 3404, 2296, 2221, 2611, 6744, 2320, 6724}

$$-\frac{6f^3\sqrt{a^2-b^2}\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^4}+\frac{6f^3\sqrt{a^2-b^2}\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^4}+\frac{6if^2\sqrt{a^2-b^2}\,(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}-\frac{6if^2\sqrt{a^2-b^2}\,(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^3}+\frac{3f\sqrt{a^2-b^2}\,(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{3f\sqrt{a^2-b^2}\,(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{i\sqrt{a^2-b^2}\,(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{i\sqrt{a^2-b^2}\,(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d}+\frac{a(e+fx)^4}{4b^2f}+\frac{6f^3\sin(c+dx)}{bd^4}-\frac{6f^2(e+fx)\cos(c+dx)}{bd^3}-\frac{3f(e+fx)^2\sin(c+dx)}{bd^2}+\frac{(e+fx)^3\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^4)/(4*b^2*f) - (6*f^2*(e + f*x)*Cos[c + d*x])/(b*d^3) + ((e +
f*x)^3*Cos[c + d*x])/(b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^3*Log[1 - (I*b*E^
(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (I*Sqrt[a^2 - b^2]*(e + f*
x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (3*Sqr
t[a^2 - b^2]*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(b^2*d^2) - (3*Sqrt[a^2 - b^2]*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^2) + ((6*I)*Sqrt[a^2 - b^2]*f^2*
(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^3
) - ((6*I)*Sqrt[a^2 - b^2]*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b^2*d^3) - (6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^4) + (6*Sqrt[a^2 - b^2]*f^3*
PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^4) + (6*f^3
*Sin[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Sin[c + d*x])/(b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int (e+fx)^3\,dx}{b^2} - \frac{\int (e+fx)^3\sin(c+dx)\,dx}{b} - \frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{(2(a^2-b^2))\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,d}{b^2}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{3f(e+fx)^2\sin(c+dx)}{bd^2} + \frac{\left(2i\sqrt{a^2-\right.}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b}}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b}}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b}}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b}}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b}}{}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1588 vs. 2(618) = 1236.
time = 3.38, size = 1588, normalized size = 2.57

---

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + 4*b*d*(e + f*x)*(-6*
f^2 + d^2*(e + f*x)^2)*Cos[c + d*x] - ((4*I)*Sqrt[a^2 - b^2]*((3*I)*Sqrt[a^
2 - b^2]*d^3*e^2*f*x*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*C
os[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*S
in[c]) + (3*I)*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*(Cos[2*c + d*x] + I
*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] -
```

```
a*Sin[c])]*(Cos[c] + I*Sin[c]) + I*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*(
Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c]
 + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^2*f*
(e + f*x)^2*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c
] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[
c]) - 3*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*(Cos[2*c + d*x] + I
*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2]
 + a*Sin[c])]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3
, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)
*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2
- b^2]*d*f^3*x*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Co
s[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*S
in[c]) - 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -((b*(Cos[2*c + d*x] + I*Sin[2*c
+ d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])
)]*(Cos[c] + I*Sin[c]) + 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*(Cos[2*c + d*x
] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c
])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[
1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b
^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 3*Sqrt[a^2
 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)
*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos
[c] + Sin[c]) + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*(Cos[2*c + d*x] + I*
Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2]
+ a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3,
(b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*
(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b
^2]*d*f^3*x*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[
c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + S
in[c]) - (2*I)*d^3*e^3*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqr
t[a^2 - b^2]]*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))/Sqrt[(-a^2 + b^2
)*(Cos[2*c] + I*Sin[2*c])] - 12*b*f*(-2*f^2 + d^2*(e + f*x)^2)*Sin[c + d*x]
)/(4*b^2*d^4)
```

**Maple** [**F**]

time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\cos^2(dx+c))}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2325 vs. 2(552) = 1104.
time = 0.61, size = 2325, normalized size = 3.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^4*f^3*x^4 + 4*a*d^4*f^2*x^3*e + 6*a*d^4*f*x^2*e^2 + 4*a*d^4*x*e^3
+ 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d
*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) -
12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x
 + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12
*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x
+ c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*
I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x +
 c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(I
*b*d^2*f^3*x^2 + 2*I*b*d^2*f^2*x*e + I*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*
dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*b*d^2*f^3*x^2 - 2*I*b*d^2*f^
2*x*e - I*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*s
in(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) - 6*(-I*b*d^2*f^3*x^2 - 2*I*b*d^2*f^2*x*e - I*b*d^2*f*e^2)*sqrt(-
(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(I*b*d^2*f^3*x
^2 + 2*I*b*d^2*f^2*x*e + I*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*
cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*
e^2 - b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*
x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(b*c^3*f^3 - 3*b*c^2*d*f^2
*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x +
c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b*c^3*f^
```

```
3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*l
og(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*
I*a) - 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*sqrt(-
(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^
2 - b^2)/b^2) - 2*I*a) + 2*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*
d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(
-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b) - 2*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x
 + b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^
2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*
d^3*f*x + b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*sqrt(-(a^2 -
b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*(b*d^3*f^3*x^3 + b*c^3*f^3
 + 3*(b*d^3*f*x + b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*sqrt(
-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 12*(b*d*f^3*x + b*d*
f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c
) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(b*
d*f^3*x + b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) +
 a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
))/b) - 12*(b*d*f^3*x + b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a
*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2))/b) + 12*(b*d*f^3*x + b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*po
lylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*(b*d^3*f^3*x^3 + 3*b*d^3*f*x*e^2 -
6*b*d*f^3*x + b*d^3*e^3 + 3*(b*d^3*f^2*x^2 - 2*b*d*f^2)*e)*cos(d*x + c) - 1
2*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*e + b*d^2*f*e^2 - 2*b*f^3)*sin(d*x + c))/(
b^2*d^4)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.299 $\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=460

$$\frac{a(e+fx)^3}{3b^2f}-\frac{2f^2\cos(c+dx)}{bd^3}+\frac{(e+fx)^2\cos(c+dx)}{bd}+\frac{i\sqrt{a^2-b^2}\,(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{i\sqrt{a^2-}}{}$$

```
[Out] 1/3*a*(f*x+e)^3/b^2/f-2*f^2*cos(d*x+c)/b/d^3+(f*x+e)^2*cos(d*x+c)/b/d-2*f*(
f*x+e)*sin(d*x+c)/b/d^2+I*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1
/2)))*(a^2-b^2)^(1/2)/b^2/d-I*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2
)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d+2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a
-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2-2*f*(f*x+e)*polylog(2,I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2+2*I*f^2*polylog(3,I*b
*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^3-2*I*f^2*polylo
g(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^3
```

**Rubi** [A]
time = 0.61, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4621, 32, 3377, 2718, 3404, 2296, 2221, 2611, 2320, 6724}

$$\frac{2if^2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}-\frac{2if^2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^3}+\frac{2f\sqrt{a^2-b^2}\,(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{2f\sqrt{a^2-b^2}\,(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{i\sqrt{a^2-b^2}\,(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{i\sqrt{a^2-b^2}\,(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d}+\frac{a(e+fx)^3}{3b^2f}-\frac{2f^2\cos(c+dx)}{bd^3}-\frac{2f(e+fx)\sin(c+dx)}{bd^2}+\frac{(e+fx)^2\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^3)/(3*b^2*f) - (2*f^2*Cos[c + d*x])/(b*d^3) + ((e + f*x)^2*Cos
[c + d*x])/(b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1
 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (2*Sqrt[a^2 - b^
2]*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^
2*d^2) - (2*Sqrt[a^2 - b^2]*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])])/(b^2*d^2) + ((2*I)*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^3) - ((2*I)*Sqrt[a^2 - b^
2]*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^3) -
 (2*f*(e + f*x)*Sin[c + d*x])/(b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int (e+fx)^2\,dx}{b^2} - \frac{\int (e+fx)^2\sin(c+dx)\,dx}{b} - \frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{(2(a^2-b^2))\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b^2}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{2f(e+fx)\sin(c+dx)}{bd^2} + \frac{\left(2i\sqrt{a^2-b\right.}{}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f^2\cos(c+dx)}{bd^3} + \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}\,(e+fx)}{}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f^2\cos(c+dx)}{bd^3} + \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}\,(e+fx)}{}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f^2\cos(c+dx)}{bd^3} + \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}\,(e+fx)}{}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f^2\cos(c+dx)}{bd^3} + \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}\,(e+fx)}{}$$

**Mathematica** [A]
time = 2.65, size = 782, normalized size = 1.70

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x*(3*e^2 + 3*e*f*x + f^2*x^2) - ((3*I)*Sqrt[a^2 - b^2]*((-I)*(d^2*(Sqrt[
a^2 - b^2]*f*x*(2*e + f*x)*(-Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x])
)/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])] + Log
[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 +
b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])])*(Cos[c] + I*Sin[c]) + 2*e^2*ArcTa
n[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]]*Sqrt[-((a^2 -
b^2)*(Cos[c] + I*Sin[c])^2)]) - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*(Cos[
2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I
*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + 2*Sqrt[a^2 - b^2]*f^2*PolyL
og[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 +
 b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c])) + 2*Sqrt[a^2
 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/
(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c
] + I*Sin[c]) - 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*(Cos[2*c + d*
x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[
c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c])))/(d^3*Sqrt[-((a^2 - b^2)*(Cos[c] +
 I*Sin[c])^2)]) + (3*b*Cos[d*x]*((-2*f^2 + d^2*(e + f*x)^2)*Cos[c] - 2*d*f*
(e + f*x)*Sin[c]))/d^3 - (3*b*(2*d*f*(e + f*x)*Cos[c] + (-2*f^2 + d^2*(e +
f*x)^2)*Sin[c])*Sin[d*x])/d^3)/(3*b^2)
```

**Maple [F]**
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{2} \left(\cos^{2}\left(dx+c\right)\right)}{a+b\sin\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1645 vs. $2(407) = 814$.
time = 0.55, size = 1645, normalized size = 3.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + 6*a*d^3*f*x^2*e + 6*a*d^3*x*e^2 - 6*b*f^2*sqrt(-(a^2
 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*b*f^2*sqrt(-(a^2 - b^
2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*b*f^2*sqrt(-(a^2 - b^2)/b^
2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*b*f^2*sqrt(-(a^2 - b^2)/b^2)*p
olylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(I*b*d*f^2*x + I*b*d*f*e)*sqrt(-(a
^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*b*d*f^2*x - I
*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) -
 (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6
*(-I*b*d*f^2*x - I*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c)
 - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b + 1) - 6*(I*b*d*f^2*x + I*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog(
(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)
*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqr
t(-(a^2 - b^2)/b^2) + 2*I*a) - 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*sqrt
(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a
^2 - b^2)/b^2) - 2*I*a) + 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*sqrt(-(a^
2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
 b^2)/b^2) + 2*I*a) + 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*sqrt(-(a^2 -
b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) - 2*I*a) + 3*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)
*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*(b*d^2*f^2*x^
2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a
*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b) + 3*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*
c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) +
 (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*(b*
d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)
*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 6*(b*d^2*f^2*x^2 + 2*b*d^2*f*x*e + b*
```

```
d^2*e^2 - 2*b*f^2)*cos(d*x + c) - 12*(b*d*f^2*x + b*d*f*e)*sin(d*x + c))/(b
^2*d^3)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
[Out] \text{Hanged}
```

# 3.300 $\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=298

$$\frac{aex}{b^2}+\frac{afx^2}{2b^2}+\frac{(e+fx)\cos(c+dx)}{bd}+\frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1\right.}{b^2d}$$

[Out] a*e*x/b^2+1/2*a*f*x^2/b^2+(f*x+e)*cos(d*x+c)/b/d-f*sin(d*x+c)/b/d^2+I*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d-I*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d+f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2-f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2

**Rubi** [A]

time = 0.34, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4621, 3377, 2717, 3404, 2296, 2221, 2317, 2438}

$$\frac{f\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{f\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d}+\frac{aex}{b^2}+\frac{afx^2}{2b^2}-\frac{f\sin(c+dx)}{bd^2}+\frac{(e+fx)\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*e*x)/b^2 + (a*f*x^2)/(2*b^2) + ((e + f*x)*Cos[c + d*x])/(b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (Sqrt[a^2 - b^2]*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^2) - (Sqrt[a^2 - b^2]*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^2) - (f*Sin[c + d*x])/(b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int (e+fx)\,dx}{b^2} - \frac{\int (e+fx)\sin(c+dx)\,dx}{b} - \frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} - \frac{(2(a^2-b^2))\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b^2}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} - \frac{f\sin(c+dx)}{bd^2} + \frac{\left(2i\sqrt{a^2-b^2}\right)\int \frac{}{2a-2 \cdots}}{b \cdots}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx}}{a-\sqrt{a^2-}}\cdots\right.}{b^2d}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx}}{a-\sqrt{a^2-}}\cdots\right.}{b^2d}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx}}{a-\sqrt{a^2-}}\cdots\right.}{b^2d}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 716 vs. $2(298) = 596$.
time = 5.09, size = 716, normalized size = 2.40

$$\frac{-a(c+dx)(cf-d(2e+fx))+2bd(e+fx)\cos(c+dx)+\frac{2(-a^2+b^2)d(e+fx)\left(\frac{2(de-cf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{-ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}+\frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}+\frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{-b+\sqrt{-a^2+b^2}-a\tan\left(\frac{1}{2}(c+dx)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(i+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}}-\frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b-\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b-\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a+ia\tan\left(\frac{1}{2}(c+dx)\right)}{a+i\left(-b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}\right)}{de-cf+if\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)-if\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}-2bf\sin(c+dx)}{2b^2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x)*(c*f - d*(2*e + f*x))) + 2*b*d*(e + f*x)*Cos[c + d*x] + (2*(
-a^2 + b^2)*d*(e + f*x)*((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b +
 Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] +
PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/S
qrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^
2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 +
 I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] +
(I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-b + Sqrt[-a^2 + b^2] - a*Tan[(c + d
*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]
))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(
c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqr
```

```
t[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-
a^2 + b^2]))]]))/Sqrt[-a^2 + b^2]))/(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)
/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]]) - 2*b*f*Sin[c + d*x])/(2*b^2*d^2)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1122 vs. $2(266) = 532$.
time = 0.40, size = 1123, normalized size = 3.77

| method | result |
|---|---|
| risch | $\frac{af\,x^{2}}{2b^{2}}+\frac{aex}{b^{2}}+\frac{(dxf+de+if)\,\mathrm{e}^{i(dx+c)}}{2b\,d^{2}}+\frac{(dxf+de-if)\,\mathrm{e}^{-i(dx+c)}}{2b\,d^{2}}-\frac{2ia^{2}e\arctan\left(\frac{2ib\,\mathrm{e}^{i(dx+c)}-2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{b^{2}d\sqrt{-a^{2}+b^{2}}}+\frac{ia^{2}f\,\mathrm{dilog}\left(\frac{ia+b\,\mathrm{e}^{i(}}{ia}\right.}{b^{2}d^{2}\sqrt{}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*f*x^2/b^2+a*e*x/b^2+1/2*(d*x*f+I*f+d*e)/b/d^2*exp(I*(d*x+c))+1/2*(d*x
*f-I*f+d*e)/b/d^2*exp(-I*(d*x+c))-2*I/b^2/d*a^2*e/(-a^2+b^2)^(1/2)*arctan(1
/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/d^2*f/(-a^2+b^2)^(1/2)*di
log((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-a^2/b^2
/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2
+b^2)^(1/2)))*x-a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a
^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+a^2/b^2/d*f/(-a^2+b^2)^(1/2)*ln((I
*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+a^2/b^2/d^2
*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b
^2)^(1/2)))*c+1/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1
/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*
x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/d*f/(-a^2+b^2)^(1/2)*ln
((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/d^2*f/
(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)
^(1/2)))*c-I/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^
(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2*I/d^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I
*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+2*I/d*e/(-a^2+b^2)^(1/2)*arctan(1/
2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/b^2/d^2*a^2*f/(-a^2+b^2)^(
1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-
I*a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/
2))/(I*a+(-a^2+b^2)^(1/2)))+2*I/b^2/d^2*a^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2
*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(267) = 534$.
time = 0.56, size = 1040, normalized size = 3.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*f*x^2 + 2*a*d^2*x*e - I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*co
s(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b + 1) + I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x
 + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) - b)/b + 1) + I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c
) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) - I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) -
a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 - b)/b + 1) - 2*b*f*sin(d*x + c) + (b*c*f - b*d*e)*sqrt(-(a^2 - b^2)/b^2)*
log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*
I*a) + (b*c*f - b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*
sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*c*f - b*d*e)*sqrt(-
(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^
2 - b^2)/b^2) + 2*I*a) - (b*c*f - b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*co
s(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b*
d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (
b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x
+ c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) +
 (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d
*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
) - (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*si
n(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b
)/b) + 2*(b*d*f*x + b*d*e)*cos(d*x + c))/(b^2*d^2)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(e+f\,x\right)}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)), x)
```

# 3.301 $\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=70

$$\frac{ax}{b^2}-\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{\cos(c+dx)}{bd}$$

```
[Out] a*x/b^2+cos(d*x+c)/b/d-2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(
a^2-b^2)^(1/2)/b^2/d
```

**Rubi** [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2774, 2814, 2739, 632, 210}

$$-\frac{2\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^2*d) + Cos[c + d*x]/(b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\cos(c+dx)}{bd}+\frac{\int \frac{b+a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{\left(a^2-b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{\left(2\left(a^2-b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}+\frac{\left(4\left(a^2-b^2\right)\right)\operatorname{Subst}\Big(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\big(\frac{1}{2}(c+d}{b^2d}\\
&= \frac{ax}{b^2}-\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{\cos(c+dx)}{bd}
\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 361 vs. 2(70) = 140.
time = 0.91, size = 361, normalized size = 5.16

$$\frac{\cos(c+dx)\left(2(a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{a+b}\left(-2\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\left(2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{a-b}\sqrt{1-\sin(c+dx)}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}\right)\right)\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d\sqrt{1-\sin(c+dx)}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(2*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c + d*x]))
/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*Sqrt[1
- Sin[c + d*x]] + Sqrt[a + b]*(-2*Sqrt[a - b]*ArcTanh[Sqrt[(b*(1 + Sin[c +
d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]]*Sqrt[1 - Sin[c +
```

```
d*x]] + Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*(2*Sqrt[b]*ArcSinh[(Sqrt[
a - b]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])] + Sqrt[a
 - b]*Sqrt[1 - Sin[c + d*x]]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])))/(Sqr
t[a - b]*b*Sqrt[a + b]*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[-((b*(-1 + Sin[c + d*x
]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])
```

**Maple [A]**
time = 0.12, size = 96, normalized size = 1.37

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2\left(-a^2+b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}+\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}}{d}$ |
| default | $\frac{\frac{2\left(-a^2+b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}+\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}}{d}$ |
| risch | $\frac{ax}{b^2}+\frac{e^{i(dx+c)}}{2bd}+\frac{e^{-i(dx+c)}}{2bd}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+c)}-\frac{-ia+\sqrt{-a^2+b^2}}{b}\right)}{d\,b^2}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+\cdots}\right.}{d\,b\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-a^2+b^2)/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*
b)/(a^2-b^2)^(1/2))+2/b^2*(b/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1/2*d*x+
1/2*c))))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.36, size = 214, normalized size = 3.06

$$\left[\frac{2\,adx+2\,b\cos\left(dx+c\right)+\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2+2\,\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2}\right)}{2\,b^2d},\ \frac{adx+b\cos\left(dx+c\right)+\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^2-b^2}\,\cos\left(dx+c\right)}\right)}{b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x + 2*b*cos(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)))/(b^2*d), (a*d*x + b*cos(d*x + c) + sqrt(a^2 - b^2)*arcta
n(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/(b^2*d)]
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1923 vs. $2(58) = 116$.
time = 173.13, size = 1923, normalized size = 27.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*cos(c)**2/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((log(t
an(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(d*tan(c/2 + d*x/2)**2 + d) + log(tan(
c/2 + d*x/2))/(d*tan(c/2 + d*x/2)**2 + d) + 2/(d*tan(c/2 + d*x/2)**2 + d))/
b, Eq(a, 0)), (b*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)**2 + b**2
*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2))
 + b*d*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b*
*2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2*b/(b**2*d*ta
n(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(
b**2)*tan(c/2 + d*x/2)) - d*x*sqrt(b**2)*tan(c/2 + d*x/2)**2/(b**2*d*tan(c/
2 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2
)*tan(c/2 + d*x/2)) - d*x*sqrt(b**2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d -
 b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 2*
sqrt(b**2)*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt
(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)), Eq(a, -sqrt(
b**2))), (b*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d +
b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + b*d
*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*t
an(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2*b/(b**2*d*tan(c/2
 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)
*tan(c/2 + d*x/2)) + d*x*sqrt(b**2)*tan(c/2 + d*x/2)**2/(b**2*d*tan(c/2 + d
*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan
(c/2 + d*x/2)) + d*x*sqrt(b**2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*
sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2*sqrt(
b**2)*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2
)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)), Eq(a, sqrt(b**2))
), ((x*sin(c + d*x)**2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/
(2*d))/a, Eq(b, 0)), (x*cos(c)**2/(a + b*sin(c)), Eq(d, 0)), (-a**2*log(tan
```

```
(c/2 + d*x/2) + b/a - sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqr
t(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) - a**2*log
(tan(c/2 + d*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*
tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + a**2*log(tan(c/2 + d*x/2
) + b/a + sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqrt(-a**2 + b*
*2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + a**2*log(tan(c/2 + d
*x/2) + b/a + sqrt(-a**2 + b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*
x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + a*d*x*sqrt(-a**2 + b**2)*tan(c/2 + d
*x/2)**2/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2
 + b**2)) + a*d*x*sqrt(-a**2 + b**2)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d
*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + b**2*log(tan(c/2 + d*x/2) + b/a - s
qrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2
 + d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + b**2*log(tan(c/2 + d*x/2) + b/a
 - sqrt(-a**2 + b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b
**2*d*sqrt(-a**2 + b**2)) - b**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 +
b**2)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2
 + b**2*d*sqrt(-a**2 + b**2)) - b**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**
2 + b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-
a**2 + b**2)) + 2*b*sqrt(-a**2 + b**2)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 +
 d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)), True))
```

**Giac** [A]
time = 5.33, size = 95, normalized size = 1.36

$$\frac{\frac{(dx+c)a}{b^2} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a/b^2 - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 + 2/((tan(1/
2*d*x + 1/2*c)^2 + 1)*b))/d
```

**Mupad** [B]
time = 3.92, size = 318, normalized size = 4.54

$$\frac{2}{b\,d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2-\frac{64\,a^4}{b^2}}+\frac{64\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4-64\,a^2\,b^2}\right)}{b^2\,d}+\frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{b^2-a^2}}{64\,a^2\,b-\frac{64\,a^4}{b}-128\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{128\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{64\,a^2-\frac{64\,a^4}{b^2}-\frac{128\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{64\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{64\,a^4+128\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b-64\,a^2\,b^2-128\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^3}\right)\,\sqrt{b^2-a^2}}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

```
[Out] 2/(b*d*(tan(c/2 + (d*x)/2)^2 + 1)) + (2*a*atan((64*a^2*tan(c/2 + (d*x)/2))/
(64*a^2 - (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 - 64*a^2*b^2)
```

```
))/(b^2*d) + (2*atanh((64*a^2*(b^2 - a^2)^(1/2))/(64*a^2*b - (64*a^4)/b - 1
28*a^3*tan(c/2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2))/(64*a^2 - (64*a^4)/b^2 - (128*a^3*tan(c/2 + (
d*x)/2))/b + 128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2))/(64*a^4 - 64*a^2*b^2 - 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3
*b*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^2*d)
```

# 3.302 $\int \frac{(e+fx)^3 \cos^3(c+dx)}{a+b\sin(c+dx)} dx$

**Optimal**. Leaf size=737

$$-\frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}+\frac{i(a^2-b^2)(e+fx)^4}{4b^3f}-\frac{6af^3\cos(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\cos(c+dx)}{b^2d^2}-\frac{(a^2-b^2)(e+fx)^3}{}$$

```
[Out] -3/8*f^3*x/b/d^3+1/4*(f*x+e)^3/b/d+3*I*(a^2-b^2)*f*(f*x+e)^2*polylog(2,I*b*
exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^2-6*a*f^3*cos(d*x+c)/b^2/d^4+3*a*
f*(f*x+e)^2*cos(d*x+c)/b^2/d^2-(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/
(a-(a^2-b^2)^(1/2)))/b^3/d-(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(
a^2-b^2)^(1/2)))/b^3/d+1/4*I*(a^2-b^2)*(f*x+e)^4/b^3/f-6*I*(a^2-b^2)*f^3*po
lylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^4-6*(a^2-b^2)*f^2*(f*
x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^3-6*(a^2-b^2)*
f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d^3-6*I*(
a^2-b^2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d^4+3*I*
(a^2-b^2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3
/d^2-6*a*f^2*(f*x+e)*sin(d*x+c)/b^2/d^3+a*(f*x+e)^3*sin(d*x+c)/b^2/d+3/8*f^
3*cos(d*x+c)*sin(d*x+c)/b/d^4-3/4*f*(f*x+e)^2*cos(d*x+c)*sin(d*x+c)/b/d^2+3
/4*f^2*(f*x+e)*sin(d*x+c)^2/b/d^3-1/2*(f*x+e)^3*sin(d*x+c)^2/b/d
```

**Rubi** [A]
time = 0.57, antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4621, 3377, 2718, 4489, 3392, 32, 2715, 8, 4615, 2221, 2611, 6744, 2320, 6724}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) + ((I/4)*(a^2 - b^2)*(e + f*x)^4
)/(b^3*f) - (6*a*f^3*Cos[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cos[c + d
*x])/(b^2*d^2) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b^3*d) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(
c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + ((3*I)*(a^2 - b^2)*f*(e + f*x)
^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^2) + ((3
*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^
2 - b^2])])/(b^3*d^2) - (6*(a^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(
c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^3) - (6*(a^2 - b^2)*f^2*(e + f*x)
*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^3) - ((6*I
)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/
(b^3*d^4) - ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sq
```

```
rt[a^2 - b^2])])/(b^3*d^4) - (6*a*f^2*(e + f*x)*Sin[c + d*x])/(b^2*d^3) + (
a*(e + f*x)^3*Sin[c + d*x])/(b^2*d) + (3*f^3*Cos[c + d*x]*Sin[c + d*x])/(8*
b*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*b*d^2) + (3*f^2*(e
+ f*x)*Sin[c + d*x]^2)/(4*b*d^3) - ((e + f*x)^3*Sin[c + d*x]^2)/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int (e+fx)^3\cos(c+dx)\,dx}{b^2} - \frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)\,dx}{b} - \frac{(a^2}{}$$

$$= \frac{i(a^2-b^2)(e+fx)^4}{4b^3f} + \frac{a(e+fx)^3\sin(c+dx)}{b^2d} - \frac{(e+fx)^3\sin^2(c+dx)}{2bd} - \frac{(}{}$$

$$= \frac{i(a^2-b^2)(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)(e+fx)^3\log\Big(}{b^3d}$$

$$= \frac{(e+fx)^3}{4bd} + \frac{i(a^2-b^2)(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)(e}{}$$

$$= -\frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} + \frac{i(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{6af^3\cos(c+dx)}{b^2d^4} + \frac{3af(e+}{}$$

$$= -\frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} + \frac{i(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{6af^3\cos(c+dx)}{b^2d^4} + \frac{3af(e+}{}$$

$$= -\frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} + \frac{i(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{6af^3\cos(c+dx)}{b^2d^4} + \frac{3af(e+}{}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2452 vs. 2(737) = 1474.
time = 6.58, size = 2452, normalized size = 3.33

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]

[Out] (-32*(a^2 - b^2)*e^3*x*Cot[c] - 48*(a^2 - b^2)*e^2*f*x^2*Cot[c] - 32*(a^2 -
 b^2)*e*f^2*x^3*Cot[c] - 8*(a^2 - b^2)*f^3*x^4*Cot[c] + (16*(a^2 - b^2)*((4
*I)*d^4*e^3*E^((2*I)*c)*x + (6*I)*d^4*e^2*E^((2*I)*c)*f*x^2 + (4*I)*d^4*e*E
^((2*I)*c)*f^2*x^3 + I*d^4*E^((2*I)*c)*f^3*x^4 + (2*I)*d^3*e^3*ArcTan[(2*a*
E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - (2*I)*d^3*e^3*E^((2*I)*c
)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + d^3*e^3*Lo
g[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - d^3*e^3*E
^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2
] + 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)])] - 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x
)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*f^2*x^2*Log[1
 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] -
6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x))
)/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*E^((2*I)*c)*f^3*x
^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*
c)])] + 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a
^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c +
 d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*f^2*x^2*L
og[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
] - 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c)
+ Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d
*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*E^((2*I)*c)*f
^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2
*I)*c)])] + (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I
*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (6*I)*d^2*
(-1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^
(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*e*f^2*PolyLog[3, (I*b*E^(I
*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*e*E^(
(2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)])] + 12*d*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(
I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*E^((2*I)*c)*f^3*x*PolyLog[
3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
+ 12*d*e*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)]))] - 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c +
d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*f^3*x*PolyLo
g[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
)] - 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c)
 + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (12*I)*f^3*PolyLog[4, (I*b*E^(I*(2*c
 + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*E^((2*I)
*c)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)
*E^((2*I)*c)])] + (12*I)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c
) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (12*I)*E^((2*I)*c)*f^3*PolyLog[4, -
```

```
((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/(
d^4*(-1 + E^((2*I)*c))) + (16*a*b*(-6*f^3 - (6*I)*d*f^2*(e + f*x) + 3*d^2*f
*(e + f*x)^2 + I*d^3*(e + f*x)^3)*(Cos[c + d*x] - I*Sin[c + d*x]))/d^4 + (1
6*a*b*(-6*f^3 + (6*I)*d*f^2*(e + f*x) + 3*d^2*f*(e + f*x)^2 - I*d^3*(e + f*
x)^3)*(Cos[c + d*x] + I*Sin[c + d*x]))/d^4 + (b^2*((3*I)*f^3 - 6*d*f^2*(e +
 f*x) - (6*I)*d^2*f*(e + f*x)^2 + 4*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] - I*
Sin[2*(c + d*x)]))/d^4 + (b^2*((-3*I)*f^3 - 6*d*f^2*(e + f*x) + (6*I)*d^2*f
*(e + f*x)^2 + 4*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))/
d^4)/(32*b^3)
```

**Maple** [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\cos^3(dx+c))}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2676 vs. 2(687) = 1374.
time = 0.62, size = 2676, normalized size = 3.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^2*d^3*f^3*x^3 + 6*b^2*d^3*f^2*x^2*e + 6*b^2*d^3*f*x*e^2 - 3*b^2*d
*f^3*x - 24*I*(a^2 - b^2)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c
) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 24*I*(
```

a^2 - b^2)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*I*(a^2 - b^2)*f^3*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*I*(a^2 - b^2)*f^3*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(2*b^2*d^3*f^3*x^3 + 6*b^2*d^3*f*x*e^2 - 3*b^2*d*f^3*x + 2*b^2*d^3*e^3 + 3*(2*b^2*d^3*f^2*x^2 - b^2*d*f^2)*e)*cos(d*x + c)^2 - 24*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*f^2*x*e + a*b*d^2*f*e^2 - 2*a*b*f^3)*cos(d*x + c) + 12*(-I*(a^2 - b^2)*d^2*f^3*x^2 - 2*I*(a^2 - b^2)*d^2*f^2*x*e - I*(a^2 - b^2)*d^2*f*e^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 12*(-I*(a^2 - b^2)*d^2*f^3*x^2 - 2*I*(a^2 - b^2)*d^2*f^2*x*e - I*(a^2 - b^2)*d^2*f*e^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 12*(I*(a^2 - b^2)*d^2*f^3*x^2 + 2*I*(a^2 - b^2)*d^2*f^2*x*e + I*(a^2 - b^2)*d^2*f*e^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 12*(I*(a^2 - b^2)*d^2*f^3*x^2 + 2*I*(a^2 - b^2)*d^2*f^2*x*e + I*(a^2 - b^2)*d^2*f*e^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 4*((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e^2 - (a^2 - b^2)*d^3*e^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 4*((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e^2 - (a^2 - b^2)*d^3*e^3)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 4*((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e^2 - (a^2 - b^2)*d^3*e^3)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 4*((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e^2 - (a^2 - b^2)*d^3*e^3)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 4*((a^2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x + (a^2 - b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*d*f^2)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 4*((a^2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x + (a^2 - b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*d*f^2)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 4*((a^2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x + (a^2 - b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*d*f^2)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 4*((a^2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x + (a^2 - b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*d*f^2)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 24*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*f^2*e)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c

```
))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*f^2
*e)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*((a^2 - b^2)*d*f^3*x + (a^2 -
 b^2)*d*f^2*e)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*((a^2 - b^2)*d*f^
3*x + (a^2 - b^2)*d*f^2*e)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c)
- (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - (8*a*b*d
^3*f^3*x^3 + 24*a*b*d^3*f*x*e^2 - 48*a*b*d*f^3*x + 8*a*b*d^3*e^3 - 3*(2*b^2
*d^2*f^3*x^2 + 4*b^2*d^2*f^2*x*e + 2*b^2*d^2*f*e^2 - b^2*f^3)*cos(d*x + c)
+ 24*(a*b*d^3*f^2*x^2 - 2*a*b*d*f^2)*e)*sin(d*x + c))/(b^3*d^4)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*cos(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
[Out] \text{Hanged}
```

# 3.303 $\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=548

$$\frac{efx}{2bd}+\frac{f^2x^2}{4bd}+\frac{i(a^2-b^2)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cos(c+dx)}{b^2d^2}-\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}-\cdots$$

```
[Out] 1/2*e*f*x/b/d+1/4*f^2*x^2/b/d+1/3*I*(a^2-b^2)*(f*x+e)^3/b^3/f+2*a*f*(f*x+e)
*cos(d*x+c)/b^2/d^2-(a^2-b^2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2
)^(1/2)))/b^3/d-(a^2-b^2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1
/2)))/b^3/d+2*I*(a^2-b^2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/b^3/d^2+2*I*(a^2-b^2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+
(a^2-b^2)^(1/2)))/b^3/d^2-2*(a^2-b^2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(
a^2-b^2)^(1/2)))/b^3/d^3-2*(a^2-b^2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a
^2-b^2)^(1/2)))/b^3/d^3-2*a*f^2*sin(d*x+c)/b^2/d^3+a*(f*x+e)^2*sin(d*x+c)/b
^2/d-1/2*f*(f*x+e)*cos(d*x+c)*sin(d*x+c)/b/d^2+1/4*f^2*sin(d*x+c)^2/b/d^3-1
/2*(f*x+e)^2*sin(d*x+c)^2/b/d
```

**Rubi** [A]
time = 0.47, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4621, 3377, 2717, 4489, 3391, 4615, 2221, 2611, 2320, 6724}

$$-\frac{2f^2(a^2-b^2)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{2f^2(a^2-b^2)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^3}+\frac{2if(a^2-b^2)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{2if(a^2-b^2)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d}+\frac{i(a^2-b^2)(e+fx)^3}{3b^3f}-\frac{2af^2\sin(c+dx)}{b^2d^3}+\frac{2af(e+fx)\cos(c+dx)}{b^2d^2}+\frac{a(e+fx)^2\sin(c+dx)}{b^2d}+\frac{f^2\sin^2(c+dx)}{4bd^3}-\frac{f(e+fx)\sin(c+dx)\cos(c+dx)}{2bd^2}-\frac{(e+fx)^2\sin^2(c+dx)}{2bd}+\frac{efx}{2bd}+\frac{f^2x^2}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) + ((I/3)*(a^2 - b^2)*(e + f*x)^3)/(b^3*
f) + (2*a*f*(e + f*x)*Cos[c + d*x])/(b^2*d^2) - ((a^2 - b^2)*(e + f*x)^2*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d) - ((a^2 - b^2)*
(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d) +
 ((2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b^3*d^2) + ((2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^2) - (2*(a^2 - b^2)*f^2*PolyL
og[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^3) - (2*(a^2 - b
^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^3)
- (2*a*f^2*Sin[c + d*x])/(b^2*d^3) + (a*(e + f*x)^2*Sin[c + d*x])/(b^2*d) -
 (f*(e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*b*d^2) + (f^2*Sin[c + d*x]^2)/(
4*b*d^3) - ((e + f*x)^2*Sin[c + d*x]^2)/(2*b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int (e+fx)^2\cos(c+dx)\,dx}{b^2} - \frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)\,dx}{b} - \frac{(a^2}{}$$

$$= \frac{i(a^2-b^2)(e+fx)^3}{3b^3f} + \frac{a(e+fx)^2\sin(c+dx)}{b^2d} - \frac{(e+fx)^2\sin^2(c+dx)}{2bd} - \frac{(}{}$$

$$= \frac{i(a^2-b^2)(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)(e+fx)^2\log\Big(}{b^3d}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} + \frac{i(a^2-b^2)(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)}{}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} + \frac{i(a^2-b^2)(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)}{}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} + \frac{i(a^2-b^2)(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)}{}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2397 vs. $2(548) = 1096$.
time = 3.14, size = 2397, normalized size = 4.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*I)*a^2*d^3*e^2*E^((2*I)*c)*x - (48*I)*b^2*d^3*e^2*E^((2*I)*c)*x + (48*
I)*a^2*d^3*e*E^((2*I)*c)*f*x^2 - (48*I)*b^2*d^3*e*E^((2*I)*c)*f*x^2 + (16*I
)*a^2*d^3*E^((2*I)*c)*f^2*x^3 - (16*I)*b^2*d^3*E^((2*I)*c)*f^2*x^3 - (48*I)
*a^2*d^2*e^2*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c
+ d*x))))] + (48*I)*b^2*d^2*e^2*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b
*(-1 + E^((2*I)*(c + d*x))))] + (24*I)*a*b*d^2*e^2*E^(I*c)*Cos[d*x] - (24*I
)*a*b*d^2*e^2*E^((3*I)*c)*Cos[d*x] + 48*a*b*d*e*E^(I*c)*f*Cos[d*x] + 48*a*b
*d*e*E^((3*I)*c)*f*Cos[d*x] - (48*I)*a*b*E^(I*c)*f^2*Cos[d*x] + (48*I)*a*b*
E^((3*I)*c)*f^2*Cos[d*x] + (48*I)*a*b*d^2*e*E^(I*c)*f*x*Cos[d*x] - (48*I)*a
*b*d^2*e*E^((3*I)*c)*f*x*Cos[d*x] + 48*a*b*d*E^(I*c)*f^2*x*Cos[d*x] + 48*a*
b*d*E^((3*I)*c)*f^2*x*Cos[d*x] + (24*I)*a*b*d^2*E^(I*c)*f^2*x^2*Cos[d*x] -
(24*I)*a*b*d^2*E^((3*I)*c)*f^2*x^2*Cos[d*x] + 6*b^2*d^2*e^2*Cos[2*d*x] + 6*
```

```
b^2*d^2*e^2*E^((4*I)*c)*Cos[2*d*x] - (6*I)*b^2*d*e*f*Cos[2*d*x] + (6*I)*b^2
*d*e*E^((4*I)*c)*f*Cos[2*d*x] - 3*b^2*f^2*Cos[2*d*x] - 3*b^2*E^((4*I)*c)*f^
2*Cos[2*d*x] + 12*b^2*d^2*e*f*x*Cos[2*d*x] + 12*b^2*d^2*e*E^((4*I)*c)*f*x*C
os[2*d*x] - (6*I)*b^2*d*f^2*x*Cos[2*d*x] + (6*I)*b^2*d*E^((4*I)*c)*f^2*x*Co
s[2*d*x] + 6*b^2*d^2*f^2*x^2*Cos[2*d*x] + 6*b^2*d^2*E^((4*I)*c)*f^2*x^2*Cos
[2*d*x] - 24*a^2*d^2*e^2*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-
1 + E^((2*I)*(c + d*x)))^2] + 24*b^2*d^2*e^2*E^((2*I)*c)*Log[4*a^2*E^((2*I)
*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - 96*a^2*d^2*e*E^((2*I)*c)*
f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)
*c)])] + 96*b^2*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^
(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 48*a^2*d^2*E^((2*I)*c)*f^2*x^2*L
og[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
] + 48*b^2*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*
c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 96*a^2*d^2*e*E^((2*I)*c)*f*x*Log[1
+ (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 9
6*b^2*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])] - 48*a^2*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*
E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 48*b^2
*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[
(-a^2 + b^2)*E^((2*I)*c)])] + (96*I)*(a^2 - b^2)*d*E^((2*I)*c)*f*(e + f*x)*
PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I
)*c)])] + (96*I)*(a^2 - b^2)*d*E^((2*I)*c)*f*(e + f*x)*PolyLog[2, -((b*E^(I
*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 96*a^2*E^
((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2
+ b^2)*E^((2*I)*c)])] + 96*b^2*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c +
d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 96*a^2*E^((2*I)*c)
*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^
((2*I)*c)]))] + 96*b^2*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(
I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 24*a*b*d^2*e^2*E^(I*c)*Si
n[d*x] + 24*a*b*d^2*e^2*E^((3*I)*c)*Sin[d*x] - (48*I)*a*b*d*e*E^(I*c)*f*Sin
[d*x] + (48*I)*a*b*d*e*E^((3*I)*c)*f*Sin[d*x] - 48*a*b*E^(I*c)*f^2*Sin[d*x]
 - 48*a*b*E^((3*I)*c)*f^2*Sin[d*x] + 48*a*b*d^2*e*E^(I*c)*f*x*Sin[d*x] + 48
*a*b*d^2*e*E^((3*I)*c)*f*x*Sin[d*x] - (48*I)*a*b*d*E^(I*c)*f^2*x*Sin[d*x] +
 (48*I)*a*b*d*E^((3*I)*c)*f^2*x*Sin[d*x] + 24*a*b*d^2*E^(I*c)*f^2*x^2*Sin[d
*x] + 24*a*b*d^2*E^((3*I)*c)*f^2*x^2*Sin[d*x] - (6*I)*b^2*d^2*e^2*Sin[2*d*x
] + (6*I)*b^2*d^2*e^2*E^((4*I)*c)*Sin[2*d*x] - 6*b^2*d*e*f*Sin[2*d*x] - 6*b
^2*d*e*E^((4*I)*c)*f*Sin[2*d*x] + (3*I)*b^2*f^2*Sin[2*d*x] - (3*I)*b^2*E^((
4*I)*c)*f^2*Sin[2*d*x] - (12*I)*b^2*d^2*e*f*x*Sin[2*d*x] + (12*I)*b^2*d^2*e
*E^((4*I)*c)*f*x*Sin[2*d*x] - 6*b^2*d*f^2*x*Sin[2*d*x] - 6*b^2*d*E^((4*I)*c
)*f^2*x*Sin[2*d*x] - (6*I)*b^2*d^2*f^2*x^2*Sin[2*d*x] + (6*I)*b^2*d^2*E^((4
*I)*c)*f^2*x^2*Sin[2*d*x])/(48*b^3*d^3*E^((2*I)*c))
```

**Maple** [**F**]

time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 (\cos^3(dx+c))}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1790 vs. 2(508) = 1016.
time = 0.56, size = 1790, normalized size = 3.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*f*x*e + 4*(a^2 - b^2)*f^2*polylog(3, -(I*
a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2))/b) + 4*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x + c) + a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))
/b) + 4*(a^2 - b^2)*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*(a^2 - b^
2)*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - (2*b^2*d^2*f^2*x^2 + 4*b^2*d
^2*f*x*e + 2*b^2*d^2*e^2 - b^2*f^2)*cos(d*x + c)^2 - 8*(a*b*d*f^2*x + a*b*d
*f*e)*cos(d*x + c) + 4*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*d*f*e)*dilog
((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 4*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^
2)*d*f*e)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*
```

```
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 4*(I*(a^2 - b^2)*d*f^2*x
 + I*(a^2 - b^2)*d*f*e)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 4*(I*(a^2
 - b^2)*d*f^2*x + I*(a^2 - b^2)*d*f*e)*dilog((-I*a*cos(d*x + c) - a*sin(d*x
 + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
 1) + 2*((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^2)*d^2*e^2)
*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2
*I*a) + 2*((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^2)*d^2*e^
2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) + 2*((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^2)*d^2*
e^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2
) + 2*I*a) + 2*((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^2)*d
^2*e^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/
b^2) - 2*I*a) + 2*((a^2 - b^2)*d^2*f^2*x^2 - (a^2 - b^2)*c^2*f^2 + 2*((a^2
- b^2)*d^2*f*x + (a^2 - b^2)*c*d*f)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x +
 c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) +
2*((a^2 - b^2)*d^2*f^2*x^2 - (a^2 - b^2)*c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x +
 (a^2 - b^2)*c*d*f)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*((a^2 - b^2)*d
^2*f^2*x^2 - (a^2 - b^2)*c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d
*f)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*((a^2 - b^2)*d^2*f^2*x^2 - (a
^2 - b^2)*c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d*f)*e)*log(-(-I
*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b) - 2*(2*a*b*d^2*f^2*x^2 + 4*a*b*d^2*f*x*e + 2*a*b
*d^2*e^2 - 4*a*b*f^2 - (b^2*d*f^2*x + b^2*d*f*e)*cos(d*x + c))*sin(d*x + c)
)/(b^3*d^3)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cos(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
[Out] \text{Hanged}
```

## 3.304 $\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=351

$$\frac{fx}{4bd}+\frac{i(a^2-b^2)(e+fx)^2}{2b^3f}+\frac{af\cos(c+dx)}{b^2d^2}-\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{(a^2-b^2)(e+fx)\,\mathrm{lo}}{}$$

```
[Out] 1/4*f*x/b/d+1/2*I*(a^2-b^2)*(f*x+e)^2/b^3/f+a*f*cos(d*x+c)/b^2/d^2-(a^2-b^2
)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d-(a^2-b^2)*(f*x
+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d+I*(a^2-b^2)*f*polylo
g(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^2+I*(a^2-b^2)*f*polylog(2
,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d^2+a*(f*x+e)*sin(d*x+c)/b^2/d
-1/4*f*cos(d*x+c)*sin(d*x+c)/b/d^2-1/2*(f*x+e)*sin(d*x+c)^2/b/d
```

**Rubi** [A]
time = 0.26, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4621, 3377, 2718, 4489, 2715, 8, 4615, 2221, 2317, 2438}

$$\frac{if(a^2-b^2)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{if(a^2-b^2)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d}+\frac{i(a^2-b^2)(e+fx)^2}{2b^3f}+\frac{af\cos(c+dx)}{b^2d^2}+\frac{a(e+fx)\sin(c+dx)}{b^2d}-\frac{f\sin(c+dx)\cos(c+dx)}{4bd^2}-\frac{(e+fx)\sin^2(c+dx)}{2bd}+\frac{fx}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (f*x)/(4*b*d) + ((I/2)*(a^2 - b^2)*(e + f*x)^2)/(b^3*f) + (a*f*Cos[c + d*x]
)/(b^2*d^2) - ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(b^3*d) - ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^2) + (I*(a^2 - b^2)*f*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^2) + (a*(e + f*x)*Si
n[c + d*x])/(b^2*d) - (f*Cos[c + d*x]*Sin[c + d*x])/(4*b*d^2) - ((e + f*x)*
Sin[c + d*x]^2)/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int(e+fx)\cos(c+dx)\,dx}{b^2} - \frac{\int(e+fx)\cos(c+dx)\sin(c+dx)\,dx}{b} - \frac{(a^2-b^2}{}$$

$$= \frac{i(a^2-b^2)(e+fx)^2}{2b^3f} + \frac{a(e+fx)\sin(c+dx)}{b^2d} - \frac{(e+fx)\sin^2(c+dx)}{2bd} - \frac{(a^2-}{}$$

$$= \frac{i(a^2-b^2)(e+fx)^2}{2b^3f} + \frac{af\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx}}{a-\sqrt{a^2-}}\right.}{b^3d}$$

$$= \frac{fx}{4bd} + \frac{i(a^2-b^2)(e+fx)^2}{2b^3f} + \frac{af\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)(e+fx)\log\left(1-\frac{}{a-}\right.}{b^3d}$$

$$= \frac{fx}{4bd} + \frac{i(a^2-b^2)(e+fx)^2}{2b^3f} + \frac{af\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)(e+fx)\log\left(1-\frac{}{a-}\right.}{b^3d}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2165 vs. $2(351) = 702$.
time = 13.72, size = 2165, normalized size = 6.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*f*Cos[c + d*x])/(b^2*d^2) + ((d*e - c*f + f*(c + d*x))*Cos[2*(c + d*x)])
/(4*b*d^2) + (a*(d*e - c*f + f*(c + d*x))*Sin[c + d*x])/(b^2*d^2) - (f*Sin[
2*(c + d*x)])/(8*b*d^2) + ((f*(c + d*x)^2 + (2*I)*d*e*Log[Sec[(c + d*x)/2]^
2] - (2*I)*c*f*Log[Sec[(c + d*x)/2]^2] - (2*I)*d*e*Log[Sec[(c + d*x)/2]^2*(
a + b*Sin[c + d*x])] + (2*I)*c*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x]
)] - (4*I)*f*(c + d*x)*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - 2*f*Log[1 + I*
```

```
Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b
- Sqrt[-a^2 + b^2])] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2
 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))] + 2*f*Log[1 -
I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a
 + b + Sqrt[-a^2 + b^2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-
a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + 4*f*PolyLo
g[2, -Cos[c + d*x] + I*Sin[c + d*x]] + 2*f*PolyLog[2, (a*(1 - I*Tan[(c + d*
x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))] - 2*f*PolyLog[2, (a*(1 + I*Tan[(c +
 d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))] + 2*f*PolyLog[2, (a*(I + Tan[(c
+ d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])] - 2*f*PolyLog[2, (a + I*a*Tan[(c
+ d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))])*((e*Cos[c + d*x])/(a + b*Sin[c
 + d*x]) - (a^2*e*Cos[c + d*x])/(b^2*(a + b*Sin[c + d*x])) - (c*f*Cos[c + d
*x])/(d*(a + b*Sin[c + d*x])) + (a^2*c*f*Cos[c + d*x])/(b^2*d*(a + b*Sin[c
+ d*x])) + (f*(c + d*x)*Cos[c + d*x])/(d*(a + b*Sin[c + d*x])) - (a^2*f*(c
+ d*x)*Cos[c + d*x])/(b^2*d*(a + b*Sin[c + d*x]))))/(d*(2*f*(c + d*x) - (4*
I)*f*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - (4*f*Log[1 + Cos[c + d*x] - I*Si
n[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/(-Cos[c + d*x] + I*Sin[c + d*x
]) + (I*f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]
))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[-((b - Sqrt[-a^
2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)
/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c
 + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 - I*Tan
[(c + d*x)/2]) + (I*f*Log[1 - (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt
[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b
- Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]*Sec[
(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] +
 a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 +
 I*Tan[(c + d*x)/2]) + (2*I)*d*e*Tan[(c + d*x)/2] - (2*I)*c*f*Tan[(c + d*x)
/2] + ((2*I)*f*(c + d*x)*Sec[(c + d*x)/2]^2)/(-I + Tan[(c + d*x)/2]) - (f*L
og[1 - (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[(c + d*
x)/2]^2)/(I + Tan[(c + d*x)/2]) + (I*a*f*Log[1 - (a + I*a*Tan[(c + d*x)/2])
/(a + I*(-b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(c + d*x
)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2
 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d
*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[
(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2
]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 +
b^2] + a*Tan[(c + d*x)/2]) - ((2*I)*d*e*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*
Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/
2]))/(a + b*Sin[c + d*x]) + ((2*I)*c*f*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*S
ec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2
]))/(a + b*Sin[c + d*x])))
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1749 vs. $2(320) = 640$.

time = 0.96, size = 1750, normalized size = 4.99

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1750 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^3*a^2*e*ln(exp(I*(d*x+c)))-1/d/b^3*a^2*e*ln(I*b*exp(2*I*(d*x+c))-2*a*
exp(I*(d*x+c))-I*b)+1/16*(2*d*x*f+I*f+2*d*e)/b/d^2*exp(2*I*(d*x+c))+1/16*(2
*d*x*f-I*f+2*d*e)/b/d^2*exp(-2*I*(d*x+c))+I/b*e*x-2/d/b*ln(exp(I*(d*x+c)))*
e+1/d/b*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-1/2*I/b*f*x^2+1/d
/b^3*a^4*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2
+b^2)^(1/2)))*x+1/d^2/b^3*a^4*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b
^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1/d/b^3*a^4*f/(-a^2+b^2)*ln((I*a+b*exp
(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2/b^3*a^4*f/(-a
^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*
c+2*I/d/b^3*a^2*f*c*x-I/d^2/b^3*a^4*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c)
)-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-I/d^2/b^3*a^4*f/(-a^2+b^2)*dilo
g((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+2*I/d^2/b
*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)
^(1/2)))*a^2+2*I/d^2/b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^
(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2+1/2*I*a*(d*x*f-I*f+d*e)/b^2/d^2*exp(-I*(
d*x+c))-I/d^2*b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/
(I*a+(-a^2+b^2)^(1/2)))+1/d*b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b
^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/d^2*b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(
d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1/d*b*f/(-a^2+b^2)*ln((
I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2*b*f/
(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)
))*c-2*I/d/b*f*c*x-I/d^2*b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b
^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-1/d^2/b*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*
exp(I*(d*x+c))-I*b)+2/d^2/b*f*c*ln(exp(I*(d*x+c)))-I/d^2/b*f*c^2-2/d/b*f/(-
a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))
*a^2*x-2/d^2/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a
-(-a^2+b^2)^(1/2)))*a^2*c-2/d/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2
+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*x-2/d^2/b*f/(-a^2+b^2)*ln((I*a+b*e
xp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*c-1/2*I*a*(d*x*
f+I*f+d*e)/b^2/d^2*exp(I*(d*x+c))+I/d^2/b^3*a^2*f*c^2+1/d^2/b^3*a^2*f*c*ln(
I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-2/d^2/b^3*a^2*f*c*ln(exp(I*(d*
x+c)))+1/2*I/b^3*a^2*f*x^2-I/b^3*a^2*e*x
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(320) = 640$.
time = 0.58, size = 1043, normalized size = 2.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d*f*x - 4*a*b*f*cos(d*x + c) - 2*(b^2*d*f*x + b^2*d*e)*cos(d*x +
c)^2 - 2*I*(a^2 - b^2)*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(
d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*I*(a^2
- b^2)*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*I*(a^2 - b^2)*f*dilog((
-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*I*(a^2 - b^2)*f*dilog((-I*a*cos(d*x +
c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) - b)/b + 1) - 2*((a^2 - b^2)*c*f - (a^2 - b^2)*d*e)*log(2*b*cos(d*x +
 c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*((a^2 -
b^2)*c*f - (a^2 - b^2)*d*e)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b
*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*((a^2 - b^2)*c*f - (a^2 - b^2)*d*e)*lo
g(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I
*a) - 2*((a^2 - b^2)*c*f - (a^2 - b^2)*d*e)*log(-2*b*cos(d*x + c) - 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*((a^2 - b^2)*d*f*x +
(a^2 - b^2)*c*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c)
+ I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*((a^2 - b^2)*d*f*x +
 (a^2 - b^2)*c*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c)
 + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*((a^2 - b^2)*d*f*x
+ (a^2 - b^2)*c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*((a^2 - b^2)*d*f*
x + (a^2 - b^2)*c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (4*a*b*d*f*x - b^
2*f*cos(d*x + c) + 4*a*b*d*e)*sin(d*x + c))/(b^3*d^2)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(e + f*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.305 $\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=61

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -(a^2-b^2)*ln(a+b*sin(d*x+c))/b^3/d+a*sin(d*x+c)/b^2/d-1/2*sin(d*x+c)^2/b/d
```

**Rubi** [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^3*d)) + (a*Sin[c + d*x])/(b^2*d)
 - Sin[c + d*x]^2/(2*b*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^3d}\\
&= \frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}\\
&= -\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}
\end{aligned}$$

**Mathematica [A]**
time = 0.07, size = 55, normalized size = 0.90

$$\frac{b^2\cos(2(c+dx))+4(-a^2+b^2)\log(a+b\sin(c+dx))+4ab\sin(c+dx)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^2*Cos[2*(c + d*x)] + 4*(-a^2 + b^2)*Log[a + b*Sin[c + d*x]] + 4*a*b*Sin[
c + d*x])/(4*b^3*d)
```

**Maple [A]**
time = 0.08, size = 54, normalized size = 0.89

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{-\frac{(\sin^2(dx+c))b}{2}+a\sin(dx+c)}{b^2}+\frac{(-a^2+b^2)\ln(a+b\sin(dx+c))}{b^3}}{d}$ |
| default | $\frac{\frac{-\frac{(\sin^2(dx+c))b}{2}+a\sin(dx+c)}{b^2}+\frac{(-a^2+b^2)\ln(a+b\sin(dx+c))}{b^3}}{d}$ |
| risch | $\frac{ixa^2}{b^3}-\frac{ix}{b}+\frac{e^{2i(dx+c)}}{8bd}-\frac{ia\,e^{i(dx+c)}}{2b^2d}+\frac{ia\,e^{-i(dx+c)}}{2b^2d}+\frac{e^{-2i(dx+c)}}{8bd}+\frac{2ia^2c}{b^3d}-\frac{2ic}{bd}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+}}{b}\right.}{b^3d}$ |
| norman | $\frac{-\frac{2(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{bd}-\frac{2(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{bd}+\frac{2a\tan(\frac{dx}{2}+\frac{c}{2})}{b^2d}+\frac{4a(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{b^2d}+\frac{2a(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{b^2d}}{(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))^3}+\frac{(a^2-b^2)\ln\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}{b^3d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*(-1/2*sin(d*x+c)^2*b+a*sin(d*x+c))+(-a^2+b^2)/b^3*ln(a+b*sin(d*x
+c)))
```

**Maxima [A]**
time = 0.29, size = 55, normalized size = 0.90

$$-\frac{\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}+\frac{2\,(a^2-b^2)\log(b\sin(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(b*sin(d
*x + c) + a)/b^3)/d
```

**Fricas** [**A**]
time = 0.38, size = 53, normalized size = 0.87

$$\frac{b^2 \cos (dx + c)^2 + 2\,ab \sin (dx + c) - 2\,(a^2 - b^2) \log (b \sin (dx + c) + a)}{2\,b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) - 2*(a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(b^3*d)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 5.57, size = 56, normalized size = 0.92

$$-\frac{\frac{b \sin(dx+c)^2 - 2\,a \sin(dx+c)}{b^2} + \frac{2\,(a^2-b^2)\log(|b \sin(dx+c)+a|)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(abs(b*s
in(d*x + c) + a))/b^3)/d
```

**Mupad** [**B**]
time = 0.09, size = 55, normalized size = 0.90

$$-\frac{\frac{\sin(c+d\,x)^2}{2\,b} + \frac{\ln(a+b\,\sin(c+d\,x))\,(a^2-b^2)}{b^3} - \frac{a\,\sin(c+d\,x)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^2/(2*b) + (log(a + b*sin(c + d*x))*(a^2 - b^2))/b^3 - (a*sin
(c + d*x))/b^2)/d
```

# 3.306 $\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=937

$$-\frac{2ia(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d}-\frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d}-\frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d}+\frac{b(}{}$$

```
[Out] -6*I*b*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^4+
b*(f*x+e)^3*ln(1+exp(2*I*(d*x+c)))/(a^2-b^2)/d-b*(f*x+e)^3*ln(1-I*b*exp(I*(
d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d-b*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c)
)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d-6*I*b*f^3*polylog(4,I*b*exp(I*(d*x+c))/(
a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^4-3/2*I*b*f*(f*x+e)^2*polylog(2,-exp(2*I*(d
*x+c)))/(a^2-b^2)/d^2+6*I*a*f^3*polylog(4,I*exp(I*(d*x+c)))/(a^2-b^2)/d^4-2
*I*a*(f*x+e)^3*arctan(exp(I*(d*x+c)))/(a^2-b^2)/d+3*I*a*f*(f*x+e)^2*polylog
(2,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^2-6*a*f^2*(f*x+e)*polylog(3,-I*exp(I*(d*x
+c)))/(a^2-b^2)/d^3+6*a*f^2*(f*x+e)*polylog(3,I*exp(I*(d*x+c)))/(a^2-b^2)/d
^3+3/2*b*f^2*(f*x+e)*polylog(3,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^3-6*b*f^2*(f*
x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^3-6*b*f^
2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^3-6
*I*a*f^3*polylog(4,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^4-3*I*a*f*(f*x+e)^2*polyl
og(2,I*exp(I*(d*x+c)))/(a^2-b^2)/d^2+3*I*b*f*(f*x+e)^2*polylog(2,I*b*exp(I*
(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^2+3/4*I*b*f^3*polylog(4,-exp(2*I*
(d*x+c)))/(a^2-b^2)/d^4+3*I*b*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(
a^2-b^2)^(1/2)))/(a^2-b^2)/d^2
```

**Rubi** [A]
time = 1.07, antiderivative size = 937, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4629, 4615, 2221, 2611, 6744, 2320, 6724, 6874, 4266, 3800}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*I)*a*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/((a^2 - b^2)*d) - (b*(e + f*
x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d)
- (b*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^
2 - b^2)*d) + (b*(e + f*x)^3*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d)
+ ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^2
) - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^2)
 + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/((a^2 - b^2)*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c
```

```
+ d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) - (((3*I)/2)*b*f*(e + f*
x)^2*PolyLog[2, -E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) - (6*a*f^2*(e + f*
x)*PolyLog[3, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) + (6*a*f^2*(e + f*x)
*PolyLog[3, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) - (6*b*f^2*(e + f*x)*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^3) - (6
*b*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
(a^2 - b^2)*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, -E^((2*I)*(c + d*x))])/(2*
(a^2 - b^2)*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))])/((a^2 - b
^2)*d^4) + ((6*I)*a*f^3*PolyLog[4, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^4) -
((6*I)*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)*d^4) - ((6*I)*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
- b^2])])/((a^2 - b^2)*d^4) + (((3*I)/4)*b*f^3*PolyLog[4, -E^((2*I)*(c + d*
x))])/((a^2 - b^2)*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4629

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-b^2/(a^2 - b^2), Int[(e + f*
x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3 \sec(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2 \int \frac{(e+fx)^3 \cos(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{ib(e+fx)^4}{4\,(a^2-b^2)\,f} + \frac{\int \left(a(e+fx)^3 \sec(c+dx) - b(e+fx)^3 \tan(c+dx)\right)\,dx}{a^2-b^2} - \frac{b^2 \cdots}{\cdots}$$

$$= \frac{ib(e+fx)^4}{4\,(a^2-b^2)\,f} - \frac{b(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} - \frac{b(e+fx)^3 \log\left(1-\frac{\cdots}{a+\cdots}\right)}{(a^2-b^2)\,d}$$

$$= -\frac{2ia(e+fx)^3 \tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d} - \frac{b(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} - \frac{b(e+\cdots}{\cdots}$$

$$= -\frac{2ia(e+fx)^3 \tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d} - \frac{b(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} - \frac{b(e+\cdots}{\cdots}$$

$$= -\frac{2ia(e+fx)^3 \tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d} - \frac{b(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} - \frac{b(e+\cdots}{\cdots}$$

$$= -\frac{2ia(e+fx)^3 \tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d} - \frac{b(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} - \frac{b(e+\cdots}{\cdots}$$

$$= -\frac{2ia(e+fx)^3 \tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d} - \frac{b(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} - \frac{b(e+\cdots}{\cdots}$$

$$= -\frac{2ia(e+fx)^3 \tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d} - \frac{b(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} - \frac{b(e+\cdots}{\cdots}$$

**<u>Mathematica</u>** **[B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1977 vs. $2(937) = 1874$.
time = 4.64, size = 1977, normalized size = 2.11

---

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*((8*I)*a*d^3*e^3*ArcTan[E^(I*(c + d*x))] + (4*I)*b*d^3*e^3*ArcTan[(2*a
*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - 12*a*d^3*e^2*f*x*Log[1
```

```
- I*E^(I*(c + d*x))] - 12*a*d^3*e*f^2*x^2*Log[1 - I*E^(I*(c + d*x))] - 4*a*
d^3*f^3*x^3*Log[1 - I*E^(I*(c + d*x))] + 12*a*d^3*e^2*f*x*Log[1 + I*E^(I*(c
 + d*x))] + 12*a*d^3*e*f^2*x^2*Log[1 + I*E^(I*(c + d*x))] + 4*a*d^3*f^3*x^3
*Log[1 + I*E^(I*(c + d*x))] - 4*b*d^3*e^3*Log[1 + E^((2*I)*(c + d*x))] - 12
*b*d^3*e^2*f*x*Log[1 + E^((2*I)*(c + d*x))] - 12*b*d^3*e*f^2*x^2*Log[1 + E^
((2*I)*(c + d*x))] - 4*b*d^3*f^3*x^3*Log[1 + E^((2*I)*(c + d*x))] + 2*b*d^3
*e^3*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] + 12
*b*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^
2)*E^((2*I)*c)])] + 12*b*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 4*b*d^3*f^3*x^3*Log[1 + (b*E^(I
*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*b*d^3*e
^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2
*I)*c)])] + 12*b*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) +
 Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 4*b*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c +
d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*a*d^2*f*(e
+ f*x)^2*PolyLog[2, (-I)*E^(I*(c + d*x))] + (12*I)*a*d^2*f*(e + f*x)^2*Poly
Log[2, I*E^(I*(c + d*x))] + (6*I)*b*d^2*e^2*f*PolyLog[2, -E^((2*I)*(c + d*x
))] + (12*I)*b*d^2*e*f^2*x*PolyLog[2, -E^((2*I)*(c + d*x))] + (6*I)*b*d^2*f
^3*x^2*PolyLog[2, -E^((2*I)*(c + d*x))] - (12*I)*b*d^2*e^2*f*PolyLog[2, (I*
b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (24*
I)*b*d^2*e*f^2*x*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a
^2 + b^2)*E^((2*I)*c)])] - (12*I)*b*d^2*f^3*x^2*PolyLog[2, (I*b*E^(I*(2*c +
 d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*b*d^2*e^2*
f*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2
*I)*c)]))] - (24*I)*b*d^2*e*f^2*x*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (12*I)*b*d^2*f^3*x^2*PolyLog[2
, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]
+ 24*a*d*e*f^2*PolyLog[3, (-I)*E^(I*(c + d*x))] + 24*a*d*f^3*x*PolyLog[3, (
-I)*E^(I*(c + d*x))] - 24*a*d*e*f^2*PolyLog[3, I*E^(I*(c + d*x))] - 24*a*d*
f^3*x*PolyLog[3, I*E^(I*(c + d*x))] - 6*b*d*e*f^2*PolyLog[3, -E^((2*I)*(c +
 d*x))] - 6*b*d*f^3*x*PolyLog[3, -E^((2*I)*(c + d*x))] + 24*b*d*e*f^2*PolyL
og[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]
)] + 24*b*d*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-
a^2 + b^2)*E^((2*I)*c)])] + 24*b*d*e*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))
/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 24*b*d*f^3*x*PolyLog[3,
 -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] +
 (24*I)*a*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))] - (24*I)*a*f^3*PolyLog[4, I*
E^(I*(c + d*x))] - (3*I)*b*f^3*PolyLog[4, -E^((2*I)*(c + d*x))] + (24*I)*b*
f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^(
(2*I)*c)])] + (24*I)*b*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c)
+ Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))])/((a - b)*(a + b)*d^4)
```

**Maple** [**F**]

time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \sec(dx+c)}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3065 vs. 2(836) = 1672.
time = 0.69, size = 3065, normalized size = 3.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(-6*I*b*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*b*f^3*polylog(4
, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b) + 6*I*b*f^3*polylog(4, -(-I*a*cos(d*x + c) + a*
sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b) + 6*I*b*f^3*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*(a - b)*f^3*poly
log(4, I*cos(d*x + c) + sin(d*x + c)) - 6*I*(a + b)*f^3*polylog(4, I*cos(d*
x + c) - sin(d*x + c)) + 6*I*(a - b)*f^3*polylog(4, -I*cos(d*x + c) + sin(d
*x + c)) + 6*I*(a + b)*f^3*polylog(4, -I*cos(d*x + c) - sin(d*x + c)) - 3*(
I*b*d^2*f^3*x^2 + 2*I*b*d^2*f^2*x*e + I*b*d^2*f*e^2)*dilog((I*a*cos(d*x + c
) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) - 3*(I*b*d^2*f^3*x^2 + 2*I*b*d^2*f^2*x*e + I*b*d^2*f*e^2)*
```

```
dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(-I*b*d^2*f^3*x^2 - 2*I*b*d^2*f^
2*x*e - I*b*d^2*f*e^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d
*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(-I*b*d^
2*f^3*x^2 - 2*I*b*d^2*f^2*x*e - I*b*d^2*f*e^2)*dilog((-I*a*cos(d*x + c) - a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - 3*(-I*(a - b)*d^2*f^3*x^2 - 2*I*(a - b)*d^2*f^2*x*e - I*(a -
b)*d^2*f*e^2)*dilog(I*cos(d*x + c) + sin(d*x + c)) - 3*(-I*(a + b)*d^2*f^3*
x^2 - 2*I*(a + b)*d^2*f^2*x*e - I*(a + b)*d^2*f*e^2)*dilog(I*cos(d*x + c) -
 sin(d*x + c)) - 3*(I*(a - b)*d^2*f^3*x^2 + 2*I*(a - b)*d^2*f^2*x*e + I*(a
- b)*d^2*f*e^2)*dilog(-I*cos(d*x + c) + sin(d*x + c)) - 3*(I*(a + b)*d^2*f^
3*x^2 + 2*I*(a + b)*d^2*f^2*x*e + I*(a + b)*d^2*f*e^2)*dilog(-I*cos(d*x + c
) - sin(d*x + c)) - (b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*
e^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 + 2*I*a) - (b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*log
(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a
) - (b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*log(-2*b*co
s(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b*
c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*log(-2*b*cos(d*x +
 c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b*d^3*f^3
*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2
*d*f^2)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d^3*f^3*x^3 + b*c^3*f^3 +
 3*(b*d^3*f*x + b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*log(-(I
*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b) + (b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*
c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*log(-(-I*a*cos(d*x + c) -
 a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b) + (b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f)*e^2 + 3*
(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) -
 (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + ((a +
 b)*c^3*f^3 - 3*(a + b)*c^2*d*f^2*e + 3*(a + b)*c*d^2*f*e^2 - (a + b)*d^3*e
^3)*log(cos(d*x + c) + I*sin(d*x + c) + I) - ((a - b)*c^3*f^3 - 3*(a - b)*c
^2*d*f^2*e + 3*(a - b)*c*d^2*f*e^2 - (a - b)*d^3*e^3)*log(cos(d*x + c) - I*
sin(d*x + c) + I) - ((a + b)*d^3*f^3*x^3 + (a + b)*c^3*f^3 + 3*((a + b)*d^3
*f*x + (a + b)*c*d^2*f)*e^2 + 3*((a + b)*d^3*f^2*x^2 - (a + b)*c^2*d*f^2)*e
)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + ((a - b)*d^3*f^3*x^3 + (a - b)*c
^3*f^3 + 3*((a - b)*d^3*f*x + (a - b)*c*d^2*f)*e^2 + 3*((a - b)*d^3*f^2*x^2
 - (a - b)*c^2*d*f^2)*e)*log(I*cos(d*x + c) - sin(d*x + c) + 1) - ((a + b)*
d^3*f^3*x^3 + (a + b)*c^3*f^3 + 3*((a + b)*d^3*f*x + (a + b)*c*d^2*f)*e^2 +
 3*((a + b)*d^3*f^2*x^2 - (a + b)*c^2*d*f^2)*e)*log(-I*cos(d*x + c) + sin(d
*x + c) + 1) + ((a - b)*d^3*f^3*x^3 + (a - b)*c^3*f^3 + 3*((a - b)*d^3*f*x
+ (a - b)*c*d^2*f)*e^2 + 3*((a - b)*d^3*f^2*x^2 - (a - b)*c^2*d*f^2)*e)*log
(-I*cos(d*x + c) - sin(d*x + c) + 1) + ((a + b)*c^3*f^3 - 3*(a + b)*c^2*d*f
^2*e + 3*(a + b)*c*d^2*f*e^2 - (a + b)*d^3*e^3)*log(-cos(d*x + c) + I*sin(d
```

```
*x + c) + I) - ((a - b)*c^3*f^3 - 3*(a - b)*c^2*d*f^2*e + 3*(a - b)*c*d^2*f
*e^2 - (a - b)*d^3*e^3)*log(-cos(d*x + c) - I*sin(d*x + c) + I) + 6*(b*d*f^
3*x + b*d*f^2*e)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(b*d*f^3*x + b*d*
f^2*e)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(b*d*f^3*x + b*d*f^2*e)*pol
ylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(b*d*f^3*x + b*d*f^2*e)*polylog(3, -
(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*...
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**3*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.307 $\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=667

$$-\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d}-\frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d}-\frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d}+\frac{b(}{}$$

```
[Out] -2*I*a*(f*x+e)^2*arctan(exp(I*(d*x+c)))/(a^2-b^2)/d+b*(f*x+e)^2*ln(1+exp(2*
I*(d*x+c)))/(a^2-b^2)/d-b*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1
/2)))/(a^2-b^2)/d-b*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/
(a^2-b^2)/d+2*I*a*f*(f*x+e)*polylog(2,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^2-2*I*
a*f*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/(a^2-b^2)/d^2-I*b*f*(f*x+e)*polylog
(2,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^2+2*I*b*f*(f*x+e)*polylog(2,I*b*exp(I*(d*
x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^2+2*I*b*f*(f*x+e)*polylog(2,I*b*exp(
I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^2-2*a*f^2*polylog(3,-I*exp(I*(d
*x+c)))/(a^2-b^2)/d^3+2*a*f^2*polylog(3,I*exp(I*(d*x+c)))/(a^2-b^2)/d^3+1/2
*b*f^2*polylog(3,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^3-2*b*f^2*polylog(3,I*b*exp
(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^3-2*b*f^2*polylog(3,I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^3
```

**Rubi** [A]
time = 0.77, antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4629, 4615, 2221, 2611, 2320, 6724, 6874, 4266, 3800}

$$-\frac{2af^2\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{d^3(a^2-b^2)}+\frac{2af^2\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{d^3(a^2-b^2)}-\frac{2bf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^3(a^2-b^2)}-\frac{2bf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^3(a^2-b^2)}+\frac{bf^2\text{PolyLog}\left(3,-e^{2i(c+dx)}\right)}{2d^3(a^2-b^2)}+\frac{2iaf(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2(a^2-b^2)}-\frac{2iaf(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2(a^2-b^2)}+\frac{2ibf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2(a^2-b^2)}+\frac{2ibf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2(a^2-b^2)}-\frac{ibf(e+fx)\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{d^2(a^2-b^2)}-\frac{2ia(e+fx)^2\text{ArcTan}\left(e^{i(c+dx)}\right)}{d(a^2-b^2)}-\frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)}-\frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d(a^2-b^2)}+\frac{b(e+fx)^2\log\left(1+e^{2i(c+dx)}\right)}{d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*I)*a*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/((a^2 - b^2)*d) - (b*(e + f*
x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d)
- (b*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^
2 - b^2)*d) + (b*(e + f*x)^2*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d)
+ ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^2)
- ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^2) + (
(2*I)*b*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]
)/((a^2 - b^2)*d^2) + ((2*I)*b*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))
/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) - (I*b*f*(e + f*x)*PolyLog[2, -E
^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) - (2*a*f^2*PolyLog[3, (-I)*E^(I*(c +
 d*x))])/((a^2 - b^2)*d^3) + (2*a*f^2*PolyLog[3, I*E^(I*(c + d*x))])/((a^2
- b^2)*d^3) - (2*b*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2
])])/((a^2 - b^2)*d^3) - (2*b*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqr
```

```
t[a^2 - b^2])])/((a^2 - b^2)*d^3) + (b*f^2*PolyLog[3, -E^((2*I)*(c + d*x))]
)/(2*(a^2 - b^2)*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
```

```
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4629

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-b^2/(a^2 - b^2), Int[(e + f*
x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sec(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{ib(e+fx)^3}{3(a^2-b^2)f} + \frac{\int \left(a(e+fx)^2\sec(c+dx) - b(e+fx)^2\tan(c+dx)\right)\,dx}{a^2-b^2} - \frac{b^2}{}$$

$$= \frac{ib(e+fx)^3}{3(a^2-b^2)f} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+fx)^2\log\left(1-\frac{}{a+}\right.}{(a^2-b^2)d}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+}{}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+}{}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+}{}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+}{}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+}{}$$

**<u>Mathematica</u>** [**A**]
time = 3.42, size = 1085, normalized size = 1.63

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*((2*I)*b*d^2*e^2*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d
*x))))] + (4*I)*a*d^2*e^2*ArcTan[Cos[c + d*x] + I*Sin[c + d*x]] + (8*I)*a*d
^2*e*f*x*ArcTan[Cos[c + d*x] + I*Sin[c + d*x]] + (4*I)*a*d^2*f^2*x^2*ArcTan
[Cos[c + d*x] + I*Sin[c + d*x]] + b*d^2*e^2*Log[4*a^2*E^((2*I)*(c + d*x)) +
 b^2*(-1 + E^((2*I)*(c + d*x)))^2] + 4*b*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d
*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*b*d^2*f^2*x^2*Log
```

```
[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
+ 4*b*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b
^2)*E^((2*I)*c)])] + 2*b*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(
I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*b*d^2*e^2*Log[1 + Cos[2*(c + d*
x)] + I*Sin[2*(c + d*x)]] - 4*b*d^2*e*f*x*Log[1 + Cos[2*(c + d*x)] + I*Sin[
2*(c + d*x)]] - 2*b*d^2*f^2*x^2*Log[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x
)]] - (4*I)*b*d*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) +
 I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (4*I)*b*d*f*(e + f*x)*PolyLog[2, -((b
*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (4*I
)*a*d*e*f*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]] + (4*I)*a*d*f^2*x*PolyL
og[2, I*Cos[c + d*x] - Sin[c + d*x]] - (4*I)*a*d*e*f*PolyLog[2, (-I)*Cos[c
+ d*x] + Sin[c + d*x]] - (4*I)*a*d*f^2*x*PolyLog[2, (-I)*Cos[c + d*x] + Sin
[c + d*x]] + (2*I)*b*d*e*f*PolyLog[2, -Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)
]] + (2*I)*b*d*f^2*x*PolyLog[2, -Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]] + 4
*b*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*
E^((2*I)*c)])] + 4*b*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) +
Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 4*a*f^2*PolyLog[3, I*Cos[c + d*x] - Sin
[c + d*x]] + 4*a*f^2*PolyLog[3, (-I)*Cos[c + d*x] + Sin[c + d*x]] - b*f^2*P
olyLog[3, -Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]])/((a - b)*(a + b)*d^3)
```

**Maple [F]**
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \sec(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2051 vs. 2(597) = 1194.

time = 0.65, size = 2051, normalized size = 3.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*b*f^2*polylog(3, -(I
*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))/b) + 2*b*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x
 + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*
b*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a - b)*f^2*polylog(3, I*co
s(d*x + c) + sin(d*x + c)) - 2*(a + b)*f^2*polylog(3, I*cos(d*x + c) - sin(
d*x + c)) + 2*(a - b)*f^2*polylog(3, -I*cos(d*x + c) + sin(d*x + c)) - 2*(a
 + b)*f^2*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) - 2*(I*b*d*f^2*x + I*b
*d*f*e)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*b*d*f^2*x + I*b*d*f*e
)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*b*d*f^2*x - I*b*d*f*e)*dil
og((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*b*d*f^2*x - I*b*d*f*e)*dilog((
-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*(a - b)*d*f^2*x - I*(a - b)*d*f*e)
*dilog(I*cos(d*x + c) + sin(d*x + c)) - 2*(-I*(a + b)*d*f^2*x - I*(a + b)*d
*f*e)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 2*(I*(a - b)*d*f^2*x + I*(a -
b)*d*f*e)*dilog(-I*cos(d*x + c) + sin(d*x + c)) - 2*(I*(a + b)*d*f^2*x + I*
(a + b)*d*f*e)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + (b*c^2*f^2 - 2*b*c*d
*f*e + b*d^2*e^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^
2 - b^2)/b^2) + 2*I*a) + (b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*log(2*b*cos(
d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b*c^
2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
 + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e
^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 - 2*I*a) + (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*log(-(I
*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b) + (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*
c*d*f)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d^2*f^2*x^2 - b*c^2*f^2 +
2*(b*d^2*f*x + b*c*d*f)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d^2*f^2*
x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*log(-(-I*a*cos(d*x + c) - a*si
n(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b
)/b) - ((a + b)*c^2*f^2 - 2*(a + b)*c*d*f*e + (a + b)*d^2*e^2)*log(cos(d*x
```

```
+ c) + I*sin(d*x + c) + I) + ((a - b)*c^2*f^2 - 2*(a - b)*c*d*f*e + (a - b)
*d^2*e^2)*log(cos(d*x + c) - I*sin(d*x + c) + I) - ((a + b)*d^2*f^2*x^2 - (
a + b)*c^2*f^2 + 2*((a + b)*d^2*f*x + (a + b)*c*d*f)*e)*log(I*cos(d*x + c)
+ sin(d*x + c) + 1) + ((a - b)*d^2*f^2*x^2 - (a - b)*c^2*f^2 + 2*((a - b)*d
^2*f*x + (a - b)*c*d*f)*e)*log(I*cos(d*x + c) - sin(d*x + c) + 1) - ((a + b
)*d^2*f^2*x^2 - (a + b)*c^2*f^2 + 2*((a + b)*d^2*f*x + (a + b)*c*d*f)*e)*lo
g(-I*cos(d*x + c) + sin(d*x + c) + 1) + ((a - b)*d^2*f^2*x^2 - (a - b)*c^2*
f^2 + 2*((a - b)*d^2*f*x + (a - b)*c*d*f)*e)*log(-I*cos(d*x + c) - sin(d*x
+ c) + 1) - ((a + b)*c^2*f^2 - 2*(a + b)*c*d*f*e + (a + b)*d^2*e^2)*log(-co
s(d*x + c) + I*sin(d*x + c) + I) + ((a - b)*c^2*f^2 - 2*(a - b)*c*d*f*e + (
a - b)*d^2*e^2)*log(-cos(d*x + c) - I*sin(d*x + c) + I))/((a^2 - b^2)*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**2*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad** [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.308 $\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=413

$$-\frac{2ia(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d}-\frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d}-\frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d}+\frac{b(e$$

```
[Out] -2*I*a*(f*x+e)*arctan(exp(I*(d*x+c)))/(a^2-b^2)/d+b*(f*x+e)*ln(1+exp(2*I*(d
*x+c)))/(a^2-b^2)/d-b*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/
(a^2-b^2)/d-b*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2
)/d+I*a*f*polylog(2,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^2-I*a*f*polylog(2,I*exp(
I*(d*x+c)))/(a^2-b^2)/d^2-1/2*I*b*f*polylog(2,-exp(2*I*(d*x+c)))/(a^2-b^2)/
d^2+I*b*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^2+I
*b*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^2
```

**Rubi** [A]
time = 0.43, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4629, 4615, 2221, 2317, 2438, 6874, 4266, 3800}

$$\frac{iaf\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}-\frac{iaf\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}+\frac{ibf\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)}+\frac{ibf\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)}-\frac{ibf\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d^2\left(a^2-b^2\right)}-\frac{2ia(e+fx)\text{ArcTan}\left(e^{i(c+dx)}\right)}{d\left(a^2-b^2\right)}-\frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)}-\frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d\left(a^2-b^2\right)}+\frac{b(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*I)*a*(e + f*x)*ArcTan[E^(I*(c + d*x))])/((a^2 - b^2)*d) - (b*(e + f*x)
*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d) - (b
*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^
2)*d) + (b*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d) + (I*a*f
*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^2) - (I*a*f*PolyLog[2, I*
E^(I*(c + d*x))])/((a^2 - b^2)*d^2) + (I*b*f*PolyLog[2, (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) + (I*b*f*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) - ((I/2)*b*f*PolyLog[
2, -E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4629

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-b^2/(a^2 - b^2), Int[(e + f*
x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\sec(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{ib(e+fx)^2}{2(a^2-b^2)f} + \frac{\int (a(e+fx)\sec(c+dx)-b(e+fx)\tan(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int -}{a}$$

$$= \frac{ib(e+fx)^2}{2(a^2-b^2)f} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i}}{a+\sqrt{c}}\right)}{(a^2-b^2)d}$$

$$= -\frac{2ia(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+f}{}$$

$$= -\frac{2ia(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+f}{}$$

$$= -\frac{2ia(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+f}{}$$

$$= -\frac{2ia(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d} - \frac{b(e+f}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2743 vs. $2(413) = 826$.
time = 14.86, size = 2743, normalized size = 6.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((d*e + d*f*x)*(((-I)*b*(d*e + d*f*x)^2)/f + 2*(a - b)*(d*e - c*f)*Log[1 -
Tan[(c + d*x)/2]] - 4*b*(d*e + d*f*x)*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] -
 2*(a + b)*(d*e - c*f)*Log[1 + Tan[(c + d*x)/2]] - (4*I)*b*f*PolyLog[2, -Co
s[c + d*x] + I*Sin[c + d*x]] + (2*I)*(a + b)*f*(Log[1 + I*Tan[(c + d*x)/2]]
*Log[(1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + PolyLog[2, ((1 + I) - (1 - I)*Ta
n[(c + d*x)/2])/2]) - (2*I)*(a + b)*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(1/2
 + I/2)*(1 + Tan[(c + d*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(c + d*x
```

```
)/2])]) + (2*I)*(a - b)*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-1/2 + I/2)*(-1
 + Tan[(c + d*x)/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/2])
 - (2*I)*(a - b)*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[
(c + d*x)/2])] + PolyLog[2, ((1 - I) + (1 + I)*Tan[(c + d*x)/2])/2]))*(a -
b*Sin[c + d*x]))/((a^2 - b^2)*d^2*(-2*a*d*e + 2*a*c*f - (2*I)*a*f*Log[1 - I
*Tan[(c + d*x)/2]] + (2*I)*a*f*Log[1 + I*Tan[(c + d*x)/2]] + 4*b*f*Cos[c +
d*x]*(Log[1 + Cos[c + d*x] - I*Sin[c + d*x]] - Log[(-2*I)/(-I + Tan[(c + d*
x)/2])]) + b*(d*e - c*f + f*(c + d*x))*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2
] + b*d*e*Tan[(c + d*x)/2] - b*c*f*Tan[(c + d*x)/2] - b*f*(c + d*x)*Tan[(c
+ d*x)/2] + (2*I)*b*f*Log[1 - I*Tan[(c + d*x)/2]]*Tan[(c + d*x)/2] - (2*I)*
b*f*Log[1 + I*Tan[(c + d*x)/2]]*Tan[(c + d*x)/2])) + ((f*(c + d*x)^2 + (2*I
)*d*e*Log[Sec[(c + d*x)/2]^2] - (2*I)*c*f*Log[Sec[(c + d*x)/2]^2] - (2*I)*d
*e*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])] + (2*I)*c*f*Log[Sec[(c + d*
x)/2]^2*(a + b*Sin[c + d*x])] - (4*I)*f*(c + d*x)*Log[(-2*I)/(-I + Tan[(c +
 d*x)/2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*
Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + 2*f*Log[1 - I*Tan[(c + d*
x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a
^2 + b^2]))] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] +
a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] - 2*f*Log[1 + I*Tan[(c
 + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt
[-a^2 + b^2])] + 4*f*PolyLog[2, -Cos[c + d*x] + I*Sin[c + d*x]] + 2*f*PolyL
og[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))] - 2*f*Po
lyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))] + 2*f
*PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])] - 2*f*
PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))])*(-(
(b^2*e*Cos[c + d*x])/((a^2 - b^2)*(a + b*Sin[c + d*x]))) + (b^2*c*f*Cos[c +
 d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (b^2*f*(c + d*x)*Cos[c + d*x]
)/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))))/(d*(2*f*(c + d*x) - (4*I)*f*Log[(-
2*I)/(-I + Tan[(c + d*x)/2])] - (4*f*Log[1 + Cos[c + d*x] - I*Sin[c + d*x]]
*(I*Cos[c + d*x] + Sin[c + d*x]))/(-Cos[c + d*x] + I*Sin[c + d*x]) + (I*f*L
og[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c
+ d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[-((b - Sqrt[-a^2 + b^2] +
a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 -
 I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])
/((-I)*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/
2]) + (I*f*Log[1 - (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2
]))]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b - Sqrt[-a^2
 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2
]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c +
 d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c +
 d*x)/2]) + (2*I)*d*e*Tan[(c + d*x)/2] - (2*I)*c*f*Tan[(c + d*x)/2] + ((2*I
)*f*(c + d*x)*Sec[(c + d*x)/2]^2)/(-I + Tan[(c + d*x)/2]) - (f*Log[1 - (a*(
I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(I
 + Tan[(c + d*x)/2]) + (I*a*f*Log[1 - (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b
 + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(c + d*x)/2]) + (a*
```

```
f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a
*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(
b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2
]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*L
og[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Ta
n[(c + d*x)/2]) - ((2*I)*d*e*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(c + d*
x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/(a + b
*Sin[c + d*x]) + ((2*I)*c*f*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(c + d*x
)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/(a + b*
Sin[c + d*x])))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(373) = 746$.
time = 0.21, size = 861, normalized size = 2.08

| method | result |
|---|---|
| risch | $-\frac{eb\ln\left(ib\,\mathrm{e}^{2i(dx+c)}-2a\,\mathrm{e}^{i(dx+c)}-ib\right)}{d(a-b)(a+b)}-\frac{4e\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d(4a+4b)}+\frac{4e\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d(4a-4b)}-\frac{4if\,\mathrm{dilog}\left(-i\mathrm{e}^{i(dx+c)}\right)}{d^2(4a+4b)}-\frac{4if\,\mathrm{dilog}\left(-i(\mathrm{e}^{i(dx}\right.}{d^2(4a-4b)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*e*b/(a-b)/(a+b)*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-4/d*e/
(4*a+4*b)*ln(exp(I*(d*x+c))-I)+4/d*e/(4*a-4*b)*ln(exp(I*(d*x+c))+I)-4*I/d^2
*f/(4*a+4*b)*dilog(-I*exp(I*(d*x+c)))+I/d^2*f*b/(a-b)/(a+b)*dilog((I*b*exp(
I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-4*I/d^2*f/(4*a+4*b)*ln(
-I*(I-exp(I*(d*x+c))))*ln(-I*exp(I*(d*x+c)))-4*I/d^2*f/(4*a-4*b)*dilog(-I*(
exp(I*(d*x+c))+I))-4/d*f/(4*a+4*b)*ln(-I*(I-exp(I*(d*x+c))))*x-4/d^2*f/(4*a
+4*b)*ln(-I*(I-exp(I*(d*x+c))))*c-1/d*f*b/(a-b)/(a+b)*ln(-(I*b*exp(I*(d*x+c
))-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))*x-1/d^2*f*b/(a-b)/(a+b)*ln(-(I*b
*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))*c+4/d*f/(4*a-4*b)*l
n(-I*(exp(I*(d*x+c))+I))*x+4/d^2*f/(4*a-4*b)*ln(-I*(exp(I*(d*x+c))+I))*c-1/
d*f*b/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(
1/2)))*x-1/d^2*f*b/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-
a+(a^2-b^2)^(1/2)))*c+I/d^2*f*b/(a-b)/(a+b)*dilog(-(I*b*exp(I*(d*x+c))-(a^2
-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))+1/d^2*f*c*b/(a-b)/(a+b)*ln(I*b*exp(2*I*
(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+4/d^2*f*c/(4*a+4*b)*ln(exp(I*(d*x+c))-I)-4
/d^2*f*c/(4*a-4*b)*ln(exp(I*(d*x+c))+I)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1193 vs. 2(360) = 720.
time = 0.63, size = 1193, normalized size = 2.89

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(-I*b*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*b*f*dilog((I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) + I*b*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) +
 (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I
*b*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*(a - b)*f*dilog(I*cos(d*x
+ c) + sin(d*x + c)) + I*(a + b)*f*dilog(I*cos(d*x + c) - sin(d*x + c)) - I
*(a - b)*f*dilog(-I*cos(d*x + c) + sin(d*x + c)) - I*(a + b)*f*dilog(-I*cos
(d*x + c) - sin(d*x + c)) - (b*c*f - b*d*e)*log(2*b*cos(d*x + c) + 2*I*b*si
n(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b*c*f - b*d*e)*log(2*b*
cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (
b*c*f - b*d*e)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) + 2*I*a) - (b*c*f - b*d*e)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*
x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b*d*f*x + b*c*f)*log(-(I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f)*log(-(I*a*cos(d*x + c) - a*sin(
d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/
b) + (b*d*f*x + b*c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d*f*x + b*c*
f)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + ((a + b)*c*f - (a + b)*d*e)*log(cos
(d*x + c) + I*sin(d*x + c) + I) - ((a - b)*c*f - (a - b)*d*e)*log(cos(d*x +
 c) - I*sin(d*x + c) + I) - ((a + b)*d*f*x + (a + b)*c*f)*log(I*cos(d*x + c
) + sin(d*x + c) + 1) + ((a - b)*d*f*x + (a - b)*c*f)*log(I*cos(d*x + c) -
sin(d*x + c) + 1) - ((a + b)*d*f*x + (a + b)*c*f)*log(-I*cos(d*x + c) + sin
(d*x + c) + 1) + ((a - b)*d*f*x + (a - b)*c*f)*log(-I*cos(d*x + c) - sin(d*
x + c) + 1) + ((a + b)*c*f - (a + b)*d*e)*log(-cos(d*x + c) + I*sin(d*x + c
) + I) - ((a - b)*c*f - (a - b)*d*e)*log(-cos(d*x + c) - I*sin(d*x + c) + I
))/((a^2 - b^2)*d^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\sec{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.309 $\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=75

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)d}+\frac{\log(1+\sin(c+dx))}{2(a-b)d}-\frac{b\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d+1/2*ln(1+sin(d*x+c))/(a-b)/d-b*ln(a+b*sin(d*x
+c))/(a^2-b^2)/d
```

**Rubi** [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2747, 720, 31, 647}

$$-\frac{b\log(a+b\sin(c+dx))}{d\,(a^2-b^2)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d
) - (b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{b\text{Subst}\left(\int \frac{1}{(a+x)(b^2-x^2)}\,dx,x,b\sin(c+dx)\right)}{d}\\ &= -\frac{b\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sin(c+dx)\right)}{(a^2-b^2)\,d}-\frac{b\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx,x,b\sin(c+dx)\right)}{(a^2-b^2)\,d}\\ &= -\frac{b\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}-\frac{\text{Subst}\left(\int \frac{1}{-b-x}\,dx,x,b\sin(c+dx)\right)}{2(a-b)d}+\frac{\text{Subst}\left(\int \frac{1}{b-x}\,dx\right.}{2(a}\\ &= -\frac{\log(1-\sin(c+dx))}{2(a+b)d}+\frac{\log(1+\sin(c+dx))}{2(a-b)d}-\frac{b\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}\end{aligned}$$

**Mathematica** [A]
time = 0.04, size = 64, normalized size = 0.85

$$\frac{(-a+b)\log(1-\sin(c+dx))+(a+b)\log(1+\sin(c+dx))-2b\log(a+b\sin(c+dx))}{2(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
[Out] ((-a + b)*Log[1 - Sin[c + d*x]] + (a + b)*Log[1 + Sin[c + d*x]] - 2*b*Log[a
 + b*Sin[c + d*x]])/(2*(a - b)*(a + b)*d)
```

**Maple** [A]
time = 0.09, size = 71, normalized size = 0.95

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{b\ln(a+b\sin(dx+c))}{(a-b)(a+b)}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$ |
| default | $\frac{\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{b\ln(a+b\sin(dx+c))}{(a-b)(a+b)}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$ |
| norman | $\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a-b)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d(a+b)}-\frac{b\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{d(a^2-b^2)}$ |
| risch | $\frac{ix}{a+b}+\frac{ic}{d(a+b)}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}+\frac{2ibx}{a^2-b^2}+\frac{2ibc}{d(a^2-b^2)}-\frac{\ln(e^{i(dx+c)}-i)}{d(a+b)}+\frac{\ln(e^{i(dx+c)}+i)}{d(a-b)}-\frac{b\ln\left(e^{2i(dx+c}\right.}{d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(2*a-2*b)*ln(1+sin(d*x+c))-b/(a-b)/(a+b)*ln(a+b*sin(d*x+c))-1/(2*a+2
*b)*ln(sin(d*x+c)-1))
```

**Maxima** [A]
time = 0.30, size = 64, normalized size = 0.85

$$-\frac{\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right)}{a^2-b^2}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a-b}+\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a -
b) + log(sin(d*x + c) - 1)/(a + b))/d
```

**Fricas** [A]
time = 0.36, size = 62, normalized size = 0.83

$$-\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right)-(a+b)\log\left(\sin\left(dx+c\right)+1\right)+(a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^2-b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) + (a - b)
*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 4.14, size = 71, normalized size = 0.95

$$-\frac{\frac{2\,b^2\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^2b-b^3}-\frac{\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a-b}+\frac{\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x +
c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b))/d
```

**Mupad** [B]
time = 0.20, size = 69, normalized size = 0.92

$$\frac{\ln(\sin(c+d\,x)+1)}{2\,d\,(a-b)}-\frac{\ln(\sin(c+d\,x)-1)}{2\,d\,(a+b)}-\frac{b\,\ln(a+b\,\sin(c+d\,x))}{d\,(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(sin(c + d*x) + 1)/(2*d*(a - b)) - log(sin(c + d*x) - 1)/(2*d*(a + b)) -
 (b*log(a + b*sin(c + d*x)))/(d*(a^2 - b^2))
```

# 3.310 $\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=923

$$-\frac{ia(e+fx)^3}{(a^2-b^2)\,d}-\frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d^2}+\frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{ib^2(e+fx)^3\log\left(1-\right.}{(a^2-b^2)^{3/}}$$

```
[Out] -I*a*(f*x+e)^3/(a^2-b^2)/d+6*I*b^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))
/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^3+3*a*f*(f*x+e)^2*ln(1+exp(2*I*(d*x
+c)))/(a^2-b^2)/d^2-6*I*b^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^
2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^3-6*I*b*f*(f*x+e)^2*arctan(exp(I*(d*x+c)))
/(a^2-b^2)/d^2-3*I*a*f^2*(f*x+e)*polylog(2,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^3
-I*b^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/
2)/d+I*b^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)
^(3/2)/d+3*b^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2))
)/(a^2-b^2)^(3/2)/d^2-3*b^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^
2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2-6*b*f^3*polylog(3,-I*exp(I*(d*x+c)))/(a^
2-b^2)/d^4+6*b*f^3*polylog(3,I*exp(I*(d*x+c)))/(a^2-b^2)/d^4+3/2*a*f^3*poly
log(3,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^4-6*I*b*f^2*(f*x+e)*polylog(2,I*exp(I*
(d*x+c)))/(a^2-b^2)/d^3+6*I*b*f^2*(f*x+e)*polylog(2,-I*exp(I*(d*x+c)))/(a^2
-b^2)/d^3-6*b^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-
b^2)^(3/2)/d^4+6*b^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/
(a^2-b^2)^(3/2)/d^4-b*(f*x+e)^3*sec(d*x+c)/(a^2-b^2)/d+a*(f*x+e)^3*tan(d*x+
c)/(a^2-b^2)/d
```

**Rubi** [A]
time = 1.28, antiderivative size = 923, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {4629, 3404, 2296, 2221, 2611, 6744, 2320, 6724, 6874, 4269, 3800, 4494, 4266}

$$-\frac{6b\text{PolyLog}(3,-ie^{i(c+dx)})f^3}{(a^2-b^2)d^4}+\frac{6b\text{PolyLog}(3,ie^{i(c+dx)})f^3}{(a^2-b^2)d^4}+\frac{3a\text{PolyLog}(3,-e^{2i(c+dx)})f^3}{2(a^2-b^2)d^4}-\frac{6b^2\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{(a^2-b^2)^{3/2}d^4}+\frac{6b^2\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f^3}{(a^2-b^2)^{3/2}d^4}+\frac{6ib(e+fx)\text{PolyLog}(2,-ie^{i(c+dx)})f^2}{(a^2-b^2)d^3}-\frac{6ib(e+fx)\text{PolyLog}(2,ie^{i(c+dx)})f^2}{(a^2-b^2)d^3}-\frac{3ia(e+fx)\text{PolyLog}(2,-e^{2i(c+dx)})f^2}{(a^2-b^2)d^3}+\frac{6ib^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^2}{(a^2-b^2)^{3/2}d^3}-\frac{6ib^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f^2}{(a^2-b^2)^{3/2}d^3}-\frac{6ib(e+fx)^2\text{ArcTan}(e^{i(c+dx)})f}{(a^2-b^2)d^2}+\frac{3a(e+fx)^2\log\left(1+e^{2i(c+dx)}\right)f}{(a^2-b^2)d^2}+\frac{3b^2(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f}{(a^2-b^2)^{3/2}d^2}-\frac{3b^2(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f}{(a^2-b^2)^{3/2}d^2}-\frac{ia(e+fx)^3}{(a^2-b^2)d}+\frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d}-\frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d}-\frac{b(e+fx)^3\sec(c+dx)}{(a^2-b^2)d}+\frac{a(e+fx)^3\tan(c+dx)}{(a^2-b^2)d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*a*(e + f*x)^3)/((a^2 - b^2)*d) - ((6*I)*b*f*(e + f*x)^2*ArcTan[E^(I*(
c + d*x))])/((a^2 - b^2)*d^2) + (I*b^2*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (I*b^2*(e + f*x)^3*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) +
 (3*a*f*(e + f*x)^2*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) + ((6*I
)*b*f^2*(e + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) - ((6
*I)*b*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) + (3*b
^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
```

```
(a^2 - b^2)^(3/2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - ((3*I)*a*f^2*(e + f*x
)*PolyLog[2, -E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^3) - (6*b*f^3*PolyLog[3,
 (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^4) + (6*b*f^3*PolyLog[3, I*E^(I*(c +
 d*x))])/((a^2 - b^2)*d^4) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) - ((6*I)*b^2*f^
2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2
- b^2)^(3/2)*d^3) + (3*a*f^3*PolyLog[3, -E^((2*I)*(c + d*x))])/(2*(a^2 - b^
2)*d^4) - (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])
])/((a^2 - b^2)^(3/2)*d^4) + (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^4) - (b*(e + f*x)^3*Sec[c + d*x]
)/((a^2 - b^2)*d) + (a*(e + f*x)^3*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4629

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-b^2/(a^2 - b^2), Int[(e + f*
x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sec^2(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{\int \left(a(e+fx)^3\sec^2(c+dx) - b(e+fx)^3\sec(c+dx)\tan(c+dx)\right)\,dx}{a^2-b^2} - \frac{(2b^2)\cdots}{\cdots}$$

$$= \frac{(2ib^3)\int \frac{e^{i(c+dx)}(e+fx)^3}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{(a^2-b^2)^{3/2}} - \frac{(2ib^3)\int \frac{e^{i(c+dx)}(e+fx)^3}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{(a^2-b^2)^{3/2}} + \cdots$$

$$= \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d} - \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d}\cdots$$

$$= -\frac{ia(e+fx)^3}{(a^2-b^2)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe\cdots}{a-\sqrt{\cdots}}\right)}{(a^2-b^2)^{3/2}d}\cdots$$

$$= -\frac{ia(e+fx)^3}{(a^2-b^2)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe\cdots}{a-\sqrt{\cdots}}\right)}{(a^2-b^2)^{3/2}d}\cdots$$

$$= -\frac{ia(e+fx)^3}{(a^2-b^2)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe\cdots}{a-\sqrt{\cdots}}\right)}{(a^2-b^2)^{3/2}d}\cdots$$

$$= -\frac{ia(e+fx)^3}{(a^2-b^2)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe\cdots}{a-\sqrt{\cdots}}\right)}{(a^2-b^2)^{3/2}d}\cdots$$

$$= -\frac{ia(e+fx)^3}{(a^2-b^2)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe\cdots}{a-\sqrt{\cdots}}\right)}{(a^2-b^2)^{3/2}d}\cdots$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2241 vs. $2(923) = 1846$.
time = 9.37, size = 2241, normalized size = 2.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^3*Sec[c])/((-a^2 + b^2)*d) - (I*b^2*((3*I)*Sqrt[a^2 - b^2]*d^3
*e^2*f*x*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt
```

[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + (3*
I)*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d
*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(
Cos[c] + I*Sin[c]) + I*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*(Cos[2*c + d*
x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])
^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*
PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a
^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 3*Sqrt
[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*(Cos[2*c + d*x] + I*Sin[2*c + d
*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])
]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*(Cos[
2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I
*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*f^3
*x*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[
(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 6*S
qrt[a^2 - b^2]*f^3*PolyLog[4, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*
a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] +
 I*Sin[c]) + 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*(Cos[2*c + d*x] + I*Sin[2*
c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Si
n[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*(Cos[
2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c]
+ I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*
e*f^2*x^2*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] +
Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]
) + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*
x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]
*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*(Cos[2*c
+ d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*
Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*f^3*x*
PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-
a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) - (2*
I)*d^3*e^3*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]
]*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))/((a^2 - b^2)^(3/2)*d^4*Sqrt[
(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]) + (e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[
(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2])/((a + b)*d*(Cos
[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (e^3*Sin[(d*
x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*
x)/2])/((a - b)*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*
x)/2])) + (f*((-6*I)*a*d^3*e^2*x - (6*I)*a*d^3*e*f*x^2 - (2*I)*a*d^3*f^2*x^
3 - (12*I)*b*d^2*e^2*ArcTan[Cos[c + d*x] + I*Sin[c + d*x]] - (24*I)*b*d^2*e
*f*x*ArcTan[Cos[c + d*x] + I*Sin[c + d*x]] - (12*I)*b*d^2*f^2*x^2*ArcTan[Co
s[c + d*x] + I*Sin[c + d*x]] + 6*a*d^2*e^2*Log[1 + Cos[2*(c + d*x)] + I*Sin
[2*(c + d*x)]] + 12*a*d^2*e*f*x*Log[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x
)]] + 6*a*d^2*f^2*x^2*Log[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] - (12*
I)*b*d*f*(e + f*x)*PolyLog[2, I*Cos[c + d*x] - Sin[c + d*x]] + (12*I)*b*d*f

```
*(e + f*x)*PolyLog[2, (-I)*Cos[c + d*x] + Sin[c + d*x]] - (6*I)*a*d*e*f*Pol
yLog[2, -Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]] - (6*I)*a*d*f^2*x*PolyLog[2
, -Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]] + 12*b*f^2*PolyLog[3, I*Cos[c + d
*x] - Sin[c + d*x]] - 12*b*f^2*PolyLog[3, (-I)*Cos[c + d*x] + Sin[c + d*x]]
 + 3*a*f^2*PolyLog[3, -Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]] + 6*a*d^3*e^2
*x*Tan[c] + 6*a*d^3*e*f*x^2*Tan[c] + 2*a*d^3*f^2*x^3*Tan[c]))/(2*(a^2 - b^2
)*d^4)
```

**Maple [F]**
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\sec^2(dx+c))}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4118 vs. 2(824) = 1648.
time = 0.82, size = 4118, normalized size = 4.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2*b - b^3)*d^3*f^3*x^3 + 6*(a^2*b - b^3)*d^3*f^2*x^2*e - 6*I*b^3
*f^3*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(4, -(I*a*cos(d*x + c) + a*
sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b) + 6*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(4, -(I*a*cos(d
*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
```

```
b^2)/b^2))/b) + 6*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(4,
 -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*cos(d*x +
c)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(a^2*b - b^3)*d^3*f*x*e^2 - 6*
(a^3 - a^2*b - a*b^2 + b^3)*f^3*cos(d*x + c)*polylog(3, I*cos(d*x + c) + si
n(d*x + c)) - 6*(a^3 + a^2*b - a*b^2 - b^3)*f^3*cos(d*x + c)*polylog(3, I*c
os(d*x + c) - sin(d*x + c)) - 6*(a^3 - a^2*b - a*b^2 + b^3)*f^3*cos(d*x + c
)*polylog(3, -I*cos(d*x + c) + sin(d*x + c)) - 6*(a^3 + a^2*b - a*b^2 - b^3
)*f^3*cos(d*x + c)*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) + 2*(a^2*b -
b^3)*d^3*e^3 + 3*(I*b^3*d^2*f^3*x^2 + 2*I*b^3*d^2*f^2*x*e + I*b^3*d^2*f*e^2
)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
 c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) + 3*(-I*b^3*d^2*f^3*x^2 - 2*I*b^3*d^2*f^2*x*e - I*b^3*d^2*f*e^2)*sqrt(-(a
^2 - b^2)/b^2)*cos(d*x + c)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(-I*
b^3*d^2*f^3*x^2 - 2*I*b^3*d^2*f^2*x*e - I*b^3*d^2*f*e^2)*sqrt(-(a^2 - b^2)/
b^2)*cos(d*x + c)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(I*b^3*d^2*f^
3*x^2 + 2*I*b^3*d^2*f^2*x*e + I*b^3*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d
*x + c)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (b^3*c^3*f^3 - 3*b^3*c^2*
d*f^2*e + 3*b^3*c*d^2*f*e^2 - b^3*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x +
 c)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
+ 2*I*a) - (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*e + 3*b^3*c*d^2*f*e^2 - b^3*d^3*e
^3)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*
x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b^3*c^3*f^3 - 3*b^3*c^2*d*f
^2*e + 3*b^3*c*d^2*f*e^2 - b^3*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)
*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) +
2*I*a) + (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*e + 3*b^3*c*d^2*f*e^2 - b^3*d^3*e^3
)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b^3*d^3*f^3*x^3 + b^3*c^3*f^
3 + 3*(b^3*d^3*f*x + b^3*c*d^2*f)*e^2 + 3*(b^3*d^3*f^2*x^2 - b^3*c^2*d*f^2)
*e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-(I*a*cos(d*x + c) - a*sin(d*x
+ c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) +
 (b^3*d^3*f^3*x^3 + b^3*c^3*f^3 + 3*(b^3*d^3*f*x + b^3*c*d^2*f)*e^2 + 3*(b^
3*d^3*f^2*x^2 - b^3*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-
(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b) - (b^3*d^3*f^3*x^3 + b^3*c^3*f^3 + 3*(b^3*d^3*
f*x + b^3*c*d^2*f)*e^2 + 3*(b^3*d^3*f^2*x^2 - b^3*c^2*d*f^2)*e)*sqrt(-(a^2
- b^2)/b^2)*cos(d*x + c)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b^3*d^3*f^3*
x^3 + b^3*c^3*f^3 + 3*(b^3*d^3*f*x + b^3*c*d^2*f)*e^2 + 3*(b^3*d^3*f^2*x^2
- b^3*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-(-I*a*cos(d*x
+ c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
```

```
2)/b^2) - b)/b) + 6*(b^3*d*f^3*x + b^3*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*cos(
d*x + c)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(b^3*d*f^3*x + b^3*d*f^2*
e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(3, -(I*a*cos(d*x + c) + a*si
n(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 + 6*(b^3*d*f^3*x + b^3*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylo
g(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(b^3*d*f^3*x + b^3*d*f^2*e)*sqrt(-(a^2
- b^2)/b^2)*cos(d*x + c)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) -
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(-I*(a^3
 - a^2*b - a*b^2 + b^3)*d*f^3*x - I*(a^3 - a^2*b - a*b^2 + b^3)*d*f^2*e)*co
s(d*x + c)*dilog(I*cos(d*x + c) + sin(d*x + c)) + 6*(I*(a^3 + a^2*b - a*b^2
 - b^3)*d*f^3*x + I*(a^3 + a^2*b - a*b^2 - b^3)*d*f^2*e)*cos(d*x + c)*dilog
(I*cos(d*x + c) - sin(d*x + c)) + 6*(I*(a^3 - a^2*b - a*b^2 + b^3)*d*f^3*x
+ I*(a^3 - a^2*b - a*b^2 + b^3)*d*f^2*e)*cos(d*x + c)*dilog(-I*cos(d*x + c)
 + sin(d*x + c)) + 6*(-I*(a^3 + a^2*b - a*b^2 - b^3)*d*f^3*x - I*(a^3 + a^2
*b - a*b^2 - b^3)*d*f^2*e)*cos(d*x + c)*dilog(-...
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**3*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.311 $\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=659

$$-\frac{ia(e+fx)^2}{(a^2-b^2)\,d}-\frac{4ibf(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d^2}+\frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{ib^2(e+fx)^2\log\left(1-\right.}{(a^2-b^2)^3}$$

```
[Out] -I*a*(f*x+e)^2/(a^2-b^2)/d-4*I*b*f*(f*x+e)*arctan(exp(I*(d*x+c)))/(a^2-b^2)
/d^2+2*a*f*(f*x+e)*ln(1+exp(2*I*(d*x+c)))/(a^2-b^2)/d^2+I*b^2*(f*x+e)^2*ln(
1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-I*b^2*(f*x+e)^2
*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+2*I*b*f^2*p
olylog(2,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^3-2*I*b*f^2*polylog(2,I*exp(I*(d*x+
c)))/(a^2-b^2)/d^3-I*a*f^2*polylog(2,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^3+2*b^2
*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2
)/d^2-2*b^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^
2-b^2)^(3/2)/d^2+2*I*b^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2
)))/(a^2-b^2)^(3/2)/d^3-2*I*b^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^
2)^(1/2)))/(a^2-b^2)^(3/2)/d^3-b*(f*x+e)^2*sec(d*x+c)/(a^2-b^2)/d+a*(f*x+e)
^2*tan(d*x+c)/(a^2-b^2)/d
```

**Rubi** [A]

time = 0.97, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4629, 3404, 2296, 2221, 2611, 2320, 6724, 6874, 4269, 3800, 2317, 2438, 4494, 4266}

$$\frac{2ib^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^3(a^2-b^2)^{3/2}}-\frac{2ib^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^3(a^2-b^2)^{3/2}}+\frac{2ibf^2\text{PolyLog}(2,-ie^{i(c+dx)})}{d^3(a^2-b^2)}-\frac{2ibf^2\text{PolyLog}(2,ie^{i(c+dx)})}{d^3(a^2-b^2)}-\frac{iaf^2\text{PolyLog}(2,-e^{2i(c+dx)})}{d^3(a^2-b^2)}+\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2(a^2-b^2)^{3/2}}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2(a^2-b^2)^{3/2}}-\frac{4ibf(e+fx)\text{ArcTan}(e^{i(c+dx)})}{d^2(a^2-b^2)}+\frac{2af(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{d^2(a^2-b^2)}+\frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{3/2}}-\frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d(a^2-b^2)^{3/2}}+\frac{a(e+fx)^2\tan(c+dx)}{d(a^2-b^2)}-\frac{b(e+fx)^2\sec(c+dx)}{d(a^2-b^2)}-\frac{ia(e+fx)^2}{d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*a*(e + f*x)^2)/((a^2 - b^2)*d) - ((4*I)*b*f*(e + f*x)*ArcTan[E^(I*(c
+ d*x))])/((a^2 - b^2)*d^2) + (I*b^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (I*b^2*(e + f*x)^2*Log[
1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) + (
2*a*f*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) + ((2*I)*b*
f^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) - ((2*I)*b*f^2*Poly
Log[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) + (2*b^2*f*(e + f*x)*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (
2*b^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
((a^2 - b^2)^(3/2)*d^2) - (I*a*f^2*PolyLog[2, -E^((2*I)*(c + d*x))])/((a^2
- b^2)*d^3) + ((2*I)*b^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/((a^2 - b^2)^(3/2)*d^3) - ((2*I)*b^2*f^2*PolyLog[3, (I*b*E^(I*(c
```

```
 + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) - (b*(e + f*x)^2*
Sec[c + d*x])/((a^2 - b^2)*d) + (a*(e + f*x)^2*Tan[c + d*x])/((a^2 - b^2)*d
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4629

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-b^2/(a^2 - b^2), Int[(e + f*
x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2 \sec^2(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2 \int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{\int \left(a(e+fx)^2\sec^2(c+dx) - b(e+fx)^2\sec(c+dx)\tan(c+dx)\right)\,dx}{a^2-b^2} - \frac{(2b^2)}{}$$

$$= \frac{(2ib^3)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{(a^2-b^2)^{3/2}} - \frac{(2ib^3)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{(a^2-b^2)^{3/2}} +$$

$$= \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}$$

$$= -\frac{ia(e+fx)^2}{(a^2-b^2)\,d} - \frac{4ibf(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d^2} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(}}{a-\sqrt{a}}\right.}{(a^2-b^2)^{3/2}\,d}$$

$$= -\frac{ia(e+fx)^2}{(a^2-b^2)\,d} - \frac{4ibf(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d^2} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(}}{a-\sqrt{a}}\right.}{(a^2-b^2)^{3/2}\,d}$$

$$= -\frac{ia(e+fx)^2}{(a^2-b^2)\,d} - \frac{4ibf(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d^2} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(}}{a-\sqrt{a}}\right.}{(a^2-b^2)^{3/2}\,d}$$

$$= -\frac{ia(e+fx)^2}{(a^2-b^2)\,d} - \frac{4ibf(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d^2} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(}}{a-\sqrt{a}}\right.}{(a^2-b^2)^{3/2}\,d}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1368 vs. 2(659) = 1318.
time = 7.27, size = 1368, normalized size = 2.08

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^2*Sec[c])/((-a^2 + b^2)*d) + (2*a*e*f*Sec[c]*(Cos[c]*Log[Cos[c
]*Cos[d*x] - Sin[c]*Sin[d*x]] + d*x*Sin[c]))/((a^2 - b^2)*d^2*(Cos[c]^2 + S
in[c]^2)) + ((4*I)*b*e*f*ArcTan[((-I)*Sin[c] - I*Cos[c]*Tan[(d*x)/2])/Sqrt[
Cos[c]^2 + Sin[c]^2]])/((a^2 - b^2)*d^2*Sqrt[Cos[c]^2 + Sin[c]^2]) + (a*f^2
*Csc[c]*((d^2*x^2)/E^(I*ArcTan[Cot[c]]) - (Cot[c]*(I*d*x*(-Pi - 2*ArcTan[Co
t[c]]) - Pi*Log[1 + E^((-2*I)*d*x)] - 2*(d*x - ArcTan[Cot[c]])*Log[1 - E^((
2*I)*(d*x - ArcTan[Cot[c]]))] + Pi*Log[Cos[d*x]] - 2*ArcTan[Cot[c]]*Log[Sin
[d*x - ArcTan[Cot[c]]]] + I*PolyLog[2, E^((2*I)*(d*x - ArcTan[Cot[c]]))]))/
Sqrt[1 + Cot[c]^2])*Sec[c])/((a^2 - b^2)*d^3*Sqrt[Csc[c]^2*(Cos[c]^2 + Sin[
c]^2)]) + (2*b*f^2*(-((Csc[c]*((d*x - ArcTan[Cot[c]])*(Log[1 - E^(I*(d*x -
ArcTan[Cot[c]]))] - Log[1 + E^(I*(d*x - ArcTan[Cot[c]]))]) + I*(PolyLog[2,
-E^(I*(d*x - ArcTan[Cot[c]]))] - PolyLog[2, E^(I*(d*x - ArcTan[Cot[c]]))])))
)/Sqrt[1 + Cot[c]^2]) + (2*ArcTan[Cot[c]]*ArcTanh[(Sin[c] + Cos[c]*Tan[(d*x
)/2])/Sqrt[Cos[c]^2 + Sin[c]^2]])/Sqrt[Cos[c]^2 + Sin[c]^2]))/((a^2 - b^2)*
d^3) - (I*b^2*(2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*(Cos[2*c + d
*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c]
)^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*Po
lyLog[2, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^
2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c]) - I*(-2*Sq
rt[a^2 - b^2]*f^2*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a
*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] +
I*Sin[c]) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c
 + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin
[c])]*(Cos[c] + I*Sin[c]) + d^2*(Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(-Log[1 +
(b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Co
s[c] + I*Sin[c])^2] - a*Sin[c])] + Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c +
 d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c
])])*(Cos[c] + I*Sin[c]) + 2*e^2*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c +
d*x]))/Sqrt[a^2 - b^2]]*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))))/((a^
2 - b^2)^(3/2)*d^3*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]) + (e^2*Sin[(
d*x)/2] + 2*e*f*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2])/((a + b)*d*(Cos[c/2]
 - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (e^2*Sin[(d*x)/2]
 + 2*e*f*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2])/((a - b)*d*(Cos[c/2] + Sin[
c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

**Maple** [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\sec^2(dx+c)\right)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2671 vs. 2(583) = 1166.
time = 0.69, size = 2671, normalized size = 4.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(3, -(I*a*cos(d*
x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))/b) - 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(3, -(
I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2))/b) + 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*pol
ylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*cos(d
*x + c)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2*b - b^3)*d^2*f^2*x^2
 + 4*(a^2*b - b^3)*d^2*f*x*e - 2*I*(a^3 - a^2*b - a*b^2 + b^3)*f^2*cos(d*x
+ c)*dilog(I*cos(d*x + c) + sin(d*x + c)) + 2*I*(a^3 + a^2*b - a*b^2 - b^3)
*f^2*cos(d*x + c)*dilog(I*cos(d*x + c) - sin(d*x + c)) + 2*I*(a^3 - a^2*b -
 a*b^2 + b^3)*f^2*cos(d*x + c)*dilog(-I*cos(d*x + c) + sin(d*x + c)) - 2*I*
(a^3 + a^2*b - a*b^2 - b^3)*f^2*cos(d*x + c)*dilog(-I*cos(d*x + c) - sin(d*
x + c)) + 2*(a^2*b - b^3)*d^2*e^2 + 2*(I*b^3*d*f^2*x + I*b^3*d*f*e)*sqrt(-(
a^2 - b^2)/b^2)*cos(d*x + c)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*
cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(-I
*b^3*d*f^2*x - I*b^3*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog((I*a*
cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + 2*(-I*b^3*d*f^2*x - I*b^3*d*f*e)*sqrt(-(a^2 -
```

```
 b^2)/b^2)*cos(d*x + c)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(I*b^3*
d*f^2*x + I*b^3*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog((-I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b + 1) + (b^3*c^2*f^2 - 2*b^3*c*d*f*e + b^3*d^2*e^2)*sqrt(
-(a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b^3*c^2*f^2 - 2*b^3*c*d*f*e + b^3*d^
2*e^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) - 2*I*b*sin
(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b^3*c^2*f^2 - 2*b^3*c*d*
f*e + b^3*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-2*b*cos(d*x + c
) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b^3*c^2*f^2
 - 2*b^3*c*d*f*e + b^3*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-2*
b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) -
 (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*sqrt(-(a^2
 - b^2)/b^2)*cos(d*x + c)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(
d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b^3*d^2*f^2*
x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*c
os(d*x + c)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b^3*d^2*f^2*x^2 - b^3*c^2*
f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*lo
g(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*
d^2*f*x + b^3*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-(-I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b) + 2*((a^3 + a^2*b - a*b^2 - b^3)*c*f^2 - (a^3 + a^2*b -
 a*b^2 - b^3)*d*f*e)*cos(d*x + c)*log(cos(d*x + c) + I*sin(d*x + c) + I) +
2*((a^3 - a^2*b - a*b^2 + b^3)*c*f^2 - (a^3 - a^2*b - a*b^2 + b^3)*d*f*e)*c
os(d*x + c)*log(cos(d*x + c) - I*sin(d*x + c) + I) - 2*((a^3 + a^2*b - a*b^
2 - b^3)*d*f^2*x + (a^3 + a^2*b - a*b^2 - b^3)*c*f^2)*cos(d*x + c)*log(I*co
s(d*x + c) + sin(d*x + c) + 1) - 2*((a^3 - a^2*b - a*b^2 + b^3)*d*f^2*x + (
a^3 - a^2*b - a*b^2 + b^3)*c*f^2)*cos(d*x + c)*log(I*cos(d*x + c) - sin(d*x
 + c) + 1) - 2*((a^3 + a^2*b - a*b^2 - b^3)*d*f^2*x + (a^3 + a^2*b - a*b^2
 - b^3)*c*f^2)*cos(d*x + c)*log(-I*cos(d*x + c) + sin(d*x + c) + 1) - 2*((a^
3 - a^2*b - a*b^2 + b^3)*d*f^2*x + (a^3 - a^2*b - a*b^2 + b^3)*c*f^2)*cos(d
*x + c)*log(-I*cos(d*x + c) - sin(d*x + c) + 1) + 2*((a^3 + a^2*b - a*b^2 -
 b^3)*c*f^2 - (a^3 + a^2*b - a*b^2 - b^3)*d*f*e)*cos(d*x + c)*log(-cos(d*x
+ c) + I*sin(d*x + c) + I) + 2*((a^3 - a^2*b - a*b^2 + b^3)*c*f^2 - (a^3 -
a^2*b - a*b^2 + b^3)*d*f*e)*cos(d*x + c)*log(-cos(d*x + c) - I*sin(d*x + c)
 + I) - 2*((a^3 - a*b^2)*d^2*f^2*x^2 + 2*(a^3 - a*b^2)*d^2*f*x*e + (a^3 - a
*b^2)*d^2*e^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d^3*cos(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)^2 \sec^2{(c + dx)}}{a + b \sin{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] \text{Hanged}
```

# 3.312 $\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=349

$$\frac{bf\tanh^{-1}(\sin(c+dx))}{(a^2-b^2)\,d^2}+\frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}+\frac{af\log}{(a}$$

```
[Out] b*f*arctanh(sin(d*x+c))/(a^2-b^2)/d^2+a*f*ln(cos(d*x+c))/(a^2-b^2)/d^2+I*b^
2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-I*
b^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+
b^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2
-b^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^
2-b*(f*x+e)*sec(d*x+c)/(a^2-b^2)/d+a*(f*x+e)*tan(d*x+c)/(a^2-b^2)/d
```

**Rubi** [A]
time = 0.56, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {4629, 3404, 2296, 2221, 2317, 2438, 6874, 4269, 3556, 4494, 3855}

$$\frac{b^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2(a^2-b^2)^{3/2}}-\frac{b^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2(a^2-b^2)^{3/2}}+\frac{bf\tanh^{-1}(\sin(c+dx))}{d^2(a^2-b^2)}+\frac{af\log(\cos(c+dx))}{d^2(a^2-b^2)}+\frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{3/2}}-\frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d(a^2-b^2)^{3/2}}+\frac{a(e+fx)\tan(c+dx)}{d(a^2-b^2)}-\frac{b(e+fx)\sec(c+dx)}{d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*f*ArcTanh[Sin[c + d*x]])/((a^2 - b^2)*d^2) + (I*b^2*(e + f*x)*Log[1 - (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (I*b^2*
(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2
)^(3/2)*d) + (a*f*Log[Cos[c + d*x]])/((a^2 - b^2)*d^2) + (b^2*f*PolyLog[2,
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (b^
2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(
3/2)*d^2) - (b*(e + f*x)*Sec[c + d*x])/((a^2 - b^2)*d) + (a*(e + f*x)*Tan[c
 + d*x])/((a^2 - b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4629

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-b^2/(a^2 - b^2), Int[(e + f*
x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{\int \left(a(e+fx)\sec^2(c+dx) - b(e+fx)\sec(c+dx)\tan(c+dx)\right)\,dx}{a^2-b^2} - \frac{(2b^2)\int}{}$$

$$= \frac{(2ib^3)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{(a^2-b^2)^{3/2}} - \frac{(2ib^3)\int \frac{e^{i(c+dx)}(e+fx)}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{(a^2-b^2)^{3/2}} +$$

$$= \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d} - \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d} -$$

$$= \frac{bf\tanh^{-1}(\sin(c+dx))}{(a^2-b^2)d^2} + \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d} - \frac{ib^2(e+fx)}{}$$

$$= \frac{bf\tanh^{-1}(\sin(c+dx))}{(a^2-b^2)d^2} + \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d} - \frac{ib^2(e+fx)}{}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 842 vs. 2(349) = 698.
time = 5.57, size = 842, normalized size = 2.41

$$\frac{\frac{bd(e+fx)}{-a^2+b^2} + \frac{f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b} + \frac{f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a-b} + \frac{b^2d(e+fx)\left(\frac{2(de-cf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{-ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}} + \frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}} + \frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{-b+\sqrt{-a^2+b^2}-a\tan\left(\frac{1}{2}(c+dx)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a\left(i+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}} - \frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b-\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b-\sqrt{-a^2+b^2}}\right)+\mathrm{Li}_2\left(\frac{a+ia\tan\left(\frac{1}{2}(c+dx)\right)}{a+i\left(-b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)\left(de-cf+if\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)-if\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)} + \frac{d(e+fx)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{d(e+fx)\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((b*d*(e + f*x))/(-a^2 + b^2) + (f*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
)/(a + b) + (f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b) + (b^2*d*(
e + f*x)*((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^
2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(
1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]
 + (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c +
 d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x
)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I
*Tan[(c + d*x)/2]]*Log[(-b + Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a -
b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + S
qrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Lo
g[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]
 + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))
/Sqrt[-a^2 + b^2]))/((-a^2 + b^2)*(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/
2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]])) + (d*(e + f*x)*Sin[(c + d*x)/2])/((
a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (d*(e + f*x)*Sin[(c + d*x)/
2])/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/d^2
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1541 vs. $2(319) = 638$.
time = 0.33, size = 1542, normalized size = 4.42

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1542 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(f*x+e)*(-I*a+b*exp(I*(d*x+c)))/d/(-a^2+b^2)/(1+exp(2*I*(d*x+c)))-4/(a^2-
b^2)/d^2*b^2*f/(4*a+4*b)*ln(exp(I*(d*x+c))-I)-4/(a^2-b^2)/d^2*b^2*f/(4*a-4*
b)*ln(exp(I*(d*x+c))+I)-I/(a^2-b^2)^(3/2)/d^2*b^2*f/(a-b)/(a+b)*ln(-(I*b*ex
p(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))*a^2*c-I/(a^2-b^2)^(3/2
)/d^2*b^4*f/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-
b^2)^(1/2)))*c-2/(a^2-b^2)/d^2*a*f*ln(exp(I*(d*x+c)))+1/(a^2-b^2)^(3/2)/d^2
*b^4*f/(a-b)/(a+b)*dilog(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^
2)^(1/2)))-1/(a^2-b^2)^(3/2)/d^2*b^4*f/(a-b)/(a+b)*dilog((I*b*exp(I*(d*x+c)
)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+4/(a^2-b^2)/d^2*a^2*f/(4*a+4*b)*
ln(exp(I*(d*x+c))-I)+4/(a^2-b^2)/d^2*a^2*f/(4*a-4*b)*ln(exp(I*(d*x+c))+I)+2
*I/(a^2-b^2)/d*b^4*e/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(
d*x+c))-2*a)/(-a^2+b^2)^(1/2))-I/(a^2-b^2)^(3/2)/d*b^2*f/(a-b)/(a+b)*ln(-(I
*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))*a^2*x-2*I/(a^2-b^
2)/d*b^2*e/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*
a)/(-a^2+b^2)^(1/2))*a^2-I/(a^2-b^2)^(3/2)/d*b^4*f/(a-b)/(a+b)*ln((I*b*exp(
```

```
I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*x+I/(a^2-b^2)^(3/2)/d*b
^2*f/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1
/2)))*a^2*x+2*I/(a^2-b^2)/d^2*b^2*c*f/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1
/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))*a^2+1/(a^2-b^2)^(3/2)/d^2*b
^2*f/(a-b)/(a+b)*dilog((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)
^(1/2)))*a^2-2*I/(a^2-b^2)/d^2*b^4*c*f/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(
1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/(a^2-b^2)^(3/2)/d^2*b^4*
f/(a-b)/(a+b)*ln(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)
))*c+I/(a^2-b^2)^(3/2)/d^2*b^2*f/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^
2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*c+I/(a^2-b^2)^(3/2)/d*b^4*f/(a-b)/(a+
b)*ln(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))*x-1/(a^2
-b^2)^(3/2)/d^2*b^2*f/(a-b)/(a+b)*dilog(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2
)-a)/(a+(a^2-b^2)^(1/2)))*a^2
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1273 vs. 2(315) = 630.
time = 0.58, size = 1273, normalized size = 3.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog((I*a*cos(d*x + c) -
 a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) - I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog((I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog
((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x
 + c)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(a^2*b - b^3)*d*f*x - (a^
3 + a^2*b - a*b^2 - b^3)*f*cos(d*x + c)*log(sin(d*x + c) + 1) - (a^3 - a^2*
```

```
b - a*b^2 + b^3)*f*cos(d*x + c)*log(-sin(d*x + c) + 1) - (b^3*c*f - b^3*d*e
)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b^3*c*f - b^3*d*e)*sqrt(-(a^2
 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*s
qrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b^3*c*f - b^3*d*e)*sqrt(-(a^2 - b^2)/b^2)
*cos(d*x + c)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
 b^2)/b^2) + 2*I*a) + (b^3*c*f - b^3*d*e)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x +
c)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
- 2*I*a) - (b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-(
I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b) + (b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*
cos(d*x + c)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b^3*d*f*x + b^3*c*f)*sqrt
(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) +
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b^3*d
*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-(-I*a*cos(d*x + c)
 - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b) + 2*(a^2*b - b^3)*d*e - 2*((a^3 - a*b^2)*d*f*x + (a^3 - a*b^2)*
d*e)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d^2*cos(d*x + c))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
[Out] \text{Hanged}
```

# 3.313 $\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=84

$$-\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] -2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-s
ec(d*x+c)*(b-a*sin(d*x+c))/(a^2-b^2)/d
```

**Rubi** [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2775, 12, 2739, 632, 210}

$$-\frac{2b^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{3/2}}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (Sec[c + d*x]*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}+\frac{\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}-\frac{b^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}-\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)\,d}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}+\frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}\right.\right.}{(a^2-b^2)\,d}\\
&= -\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}
\end{aligned}
$$

**Mathematica [A]**
time = 0.19, size = 152, normalized size = 1.81

$$
\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\cos(c+dx)+\sqrt{a^2-b^2}\,(b-b\cos(c+dx)-a\sin(c+dx))}{(-a+b)(a+b)\sqrt{a^2-b^2}\,d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(b - b*Cos[c + d*x] - a*Sin[c + d*x]))/((-a + b)*(a + b)*Sqrt[a
```

```
^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]))
```

**Maple** [A]
time = 0.12, size = 112, normalized size = 1.33

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b^{2}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{(a-b)(a+b)\sqrt{a^{2}-b^{2}}}-\frac{2}{(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$ |
| default | $\frac{-\frac{2}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b^{2}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{(a-b)(a+b)\sqrt{a^{2}-b^{2}}}-\frac{2}{(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$ |
| risch | $\frac{-2ia+2b\,\mathrm{e}^{i(dx+c)}}{d(-a^{2}+b^{2})\left(1+\mathrm{e}^{2i(dx+c)}\right)}-\frac{b^{2}\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)d}+\frac{b^{2}\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia\sqrt{-a^{2}+b^{2}}-a^{2}\cdots}{b\sqrt{-a^{2}+b^{2}}}\right.}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)-2*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/(2*a+2*b)/(tan(1
/2*d*x+1/2*c)-1))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [A]
time = 0.37, size = 305, normalized size = 3.63

$$\left[\frac{\sqrt{-a^{2}+b^{2}}\,b^{2}\cos(dx+c)\log\left(\frac{(2a^{2}-b^{2})\cos(dx+c)^{2}-2ab\sin(dx+c)-a^{2}-b^{2}+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(dx+c)^{2}-2ab\sin(dx+c)-a^{2}-b^{2}}\right)-2a^{2}b+2b^{3}+2(a^{3}-ab^{2})\sin(dx+c)}{2(a^{4}-2a^{2}b^{2}+b^{4})d\cos(dx+c)},\ \frac{\sqrt{a^{2}-b^{2}}\,b^{2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^{2}-b^{2}}\cos(dx+c)}\right)\cos(dx+c)-a^{2}b+b^{3}+(a^{3}-ab^{2})\sin(dx+c)}{(a^{4}-2a^{2}b^{2}+b^{4})d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*b^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac [A]**
time = 5.84, size = 107, normalized size = 1.27

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{(a^2-b^2)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b}{(a^2-b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

**Mupad [B]**
time = 4.23, size = 149, normalized size = 1.77

$$\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\frac{b^2\,\left(2\,a^2\,b-2\,b^3\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}}{2\,b^2}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*b)/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2))/(a^2 - b^2))/(d*(tan(c/2 + (d
*x)/2)^2 - 1)) - (2*b^2*atan(((b^2*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a - b
)^(3/2)) + (2*a*b^2*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)^
(3/2)))/(2*b^2)))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

# 3.314 $\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)} \, dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}, x\right)$$

```
[Out] Unintegrable((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)} \, dx = \int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)} \, dx$$

**Mathematica** [A]
time = 3.42, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m \left(\cos^2(dx+c)\right)}{a+b\sin(dx+c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*cos(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(c+d\,x)^2\,(e+f\,x)^m}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(e + f*x)^m)/(a + b*sin(c + d*x)),x)
[Out] int((cos(c + d*x)^2*(e + f*x)^m)/(a + b*sin(c + d*x)), x)
```

# 3.315 $\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=29

$$\mathrm{Int}\left(\frac{(e+fx)^m\cos(c+dx)}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^m\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 1.88, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m\cos(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*cos(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e+f\,x\right)}^m}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^m)/(a + b*sin(c + d*x)),x)
[Out] int((cos(c + d*x)*(e + f*x)^m)/(a + b*sin(c + d*x)), x)
```

# 3.316 $\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
[Out] Defer[Int][(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
[Out] Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e+f\,x\right)}^m}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(a + b*sin(c + d*x)),x)
[Out] int((e + f*x)^m/(a + b*sin(c + d*x)), x)
```

# 3.317 $\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=29

$$\operatorname{Int}\left(\frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 99.36, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m \sec(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(e+f\,x)^m}{\cos(c+d\,x)\,\left(a+b\,\sin(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
[Out] int((e + f*x)^m/(cos(c + d*x)*(a + b*sin(c + d*x))), x)
```

# 3.318 $\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=31

$$\text{Int}\left(\frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] Defer[Int][((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

**Mathematica** [A]
time = 8.68, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

**Maple** [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^m \left(\sec^2(dx+c)\right)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((f*x + e)^m*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((f*x + e)^m*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)**m*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^m*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(e+f\,x)^m}{\cos(c+d\,x)^2\,\left(a+b\,\sin(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
[Out] int((e + f*x)^m/(cos(c + d*x)^2*(a + b*sin(c + d*x))), x)
```

# 3.319 $\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

**Optimal**. Leaf size=77

$$\frac{2f\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}-\frac{e+fx}{bd(a+b\sin(c+dx))}$$

```
[Out] (-f*x-e)/b/d/(a+b*sin(d*x+c))+2*f*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/b/d^2/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4507, 2739, 632, 210}

$$\frac{2f\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{e+fx}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2]*d
^2) - (e + f*x)/(b*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{e+fx}{bd(a+b\sin(c+dx))} + \frac{f\int \frac{1}{a+b\sin(c+dx)}\,dx}{bd}$$

$$= -\frac{e+fx}{bd(a+b\sin(c+dx))} + \frac{(2f)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd^2}$$

$$= -\frac{e+fx}{bd(a+b\sin(c+dx))} - \frac{(4f)\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+\right.\right.}{bd^2}$$

$$= \frac{2f\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2} - \frac{e+fx}{bd(a+b\sin(c+dx))}$$

**Mathematica [A]**
time = 0.25, size = 73, normalized size = 0.95

$$\frac{\frac{2f\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{d(e+fx)}{a+b\sin(c+dx)}}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (
d*(e + f*x))/(a + b*Sin[c + d*x]))/(b*d^2)
```

**Maple [C]** Result contains complex when optimal does not.
time = 1.38, size = 194, normalized size = 2.52

| method | result |
|---|---|
| risch | $-\frac{2i(fx+e)e^{i(dx+c)}}{bd\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)} - \frac{f\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d^2b} + \frac{f\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d^2b}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(f*x+e)*exp(I*(d*x+c))/b/d/(b*exp(2*I*(d*x+c))-b+2*I*a*exp(I*(d*x+c)))
-1/(-a^2+b^2)^(1/2)*f/d^2/b*ln(exp(I*(d*x+c))+(I*a*(-a^2+b^2)^(1/2)-a^2+b^2
)/b/(-a^2+b^2)^(1/2))+1/(-a^2+b^2)^(1/2)*f/d^2/b*ln(exp(I*(d*x+c))+(I*a*(-a
^2+b^2)^(1/2)+a^2-b^2)/b/(-a^2+b^2)^(1/2))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.37, size = 341, normalized size = 4.43

$$\left[-\frac{2\,(a^2-b^2)dfx+2\,(a^2-b^2)de+(bf\sin\left(dx+c\right)+af)\sqrt{-a^2+b^2}\,\log\left(\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)}{2\,((a^2b^2-b^4)d^2\sin\left(dx+c\right)+(a^3b-ab^3)d^2)},-\frac{(a^2-b^2)dfx+(a^2-b^2)de+(bf\sin\left(dx+c\right)+af)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)}{(a^2b^2-b^4)d^2\sin\left(dx+c\right)+(a^3b-ab^3)d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2 - b^2)*d*f*x + 2*(a^2 - b^2)*d*e + (b*f*sin(d*x + c) + a*f)*s
qrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2)
)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/((a^2*b^2 - b^4)*
d^2*sin(d*x + c) + (a^3*b - a*b^3)*d^2), -((a^2 - b^2)*d*f*x + (a^2 - b^2)*
d*e + (b*f*sin(d*x + c) + a*f)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)
/(sqrt(a^2 - b^2)*cos(d*x + c))))/((a^2*b^2 - b^4)*d^2*sin(d*x + c) + (a^3*
b - a*b^3)*d^2)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)*cos(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x))/(a + b*sin(c + d*x))^2,x)
[Out] \text{Hanged}
```

# 3.320 $\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

**Optimal**. Leaf size=280

$$-\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}+\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}-\frac{2f^2\mathrm{Li}_2\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^3}+\frac{2f^2\mathrm{I}}{}$$

```
[Out] -(f*x+e)^2/b/d/(a+b*sin(d*x+c))-2*I*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a
^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)+2*I*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))
/(a+(a^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)-2*f^2*polylog(2,I*b*exp(I*(d*x+
c))/(a-(a^2-b^2)^(1/2)))/b/d^3/(a^2-b^2)^(1/2)+2*f^2*polylog(2,I*b*exp(I*(d
*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^3/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.34, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$,
Rules used = {4507, 3404, 2296, 2221, 2317, 2438}

$$-\frac{2f^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{2f^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*I)*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
b*Sqrt[a^2 - b^2]*d^2) + ((2*I)*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (2*f^2*PolyLog[2, (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) + (2*f^2*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3
) - (e + f*x)^2/(b*d*(a + b*Sin[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{(e+fx)^2}{bd(a+b\sin(c+dx))} + \frac{(2f)\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sin(c+dx))} + \frac{(4f)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sin(c+dx))} - \frac{(4if)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}\,d} + \frac{(4if)\int \frac{}{2a+2\sqrt{}}}{\sqrt{}}$$

$$= -\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2} + \frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}$$

$$= -\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2} + \frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}$$

$$= -\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2} + \frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}$$

**Mathematica** [A]
time = 3.16, size = 355, normalized size = 1.27

$$\frac{2f\left(\frac{2de\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{e^{ic}f\left(dx\left(\log\left(1+\frac{be^{i(2c+dx)}}{iae^{ic}-\sqrt{(-a^2+b^2)e^{2ic}}}\right)-\log\left(1+\frac{be^{i(2c+dx)}}{iae^{ic}+\sqrt{(-a^2+b^2)e^{2ic}}}\right)\right)-i\mathrm{Li}_2\left(\frac{ibe^{i(2c+dx)}}{ae^{ic}+i\sqrt{(-a^2+b^2)e^{2ic}}}\right)+i\mathrm{Li}_2\left(-\frac{be^{i(2c+dx)}}{iae^{ic}+\sqrt{(-a^2+b^2)e^{2ic}}}\right)\right)}{\sqrt{(-a^2+b^2)e^{2ic}}}\right)}{bd^3} - \frac{(e+fx)^2}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*f*((2*d*e*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (E^(I*c)*f*(d*x*(Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(
-a^2 + b^2)*E^((2*I)*c)])] - Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + S
qrt[(-a^2 + b^2)*E^((2*I)*c)])]) - I*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*
E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + I*PolyLog[2, -((b*E^(I*(2*c
+ d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/Sqrt[(-a^2 + b^
2)*E^((2*I)*c)]))/(b*d^3) - (e + f*x)^2/(b*d*(a + b*Sin[c + d*x]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(250) = 500.
time = 1.19, size = 606, normalized size = 2.16

| method | result |
|---|---|
| risch | $-\frac{2i(x^2f^2+2efx+e^2)\mathrm{e}^{i(dx+c)}}{bd(b\,\mathrm{e}^{2i(dx+c)}-b+2ia\,\mathrm{e}^{i(dx+c)})}+\frac{4ife\arctan\left(\frac{2ib\,\mathrm{e}^{i(dx+c)}-2a}{2\sqrt{-a^2+b^2}}\right)}{d^2b\sqrt{-a^2+b^2}}+\frac{2f^2\ln\left(\frac{ia+b\,\mathrm{e}^{i(dx+c)}-\sqrt{-a^2+b^2}}{ia-\sqrt{-a^2+b^2}}\right)x}{d^2b\sqrt{-a^2+b^2}}+\frac{2f^2\ln\left(\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(f^2*x^2+2*e*f*x+e^2)*exp(I*(d*x+c))/b/d/(b*exp(2*I*(d*x+c))-b+2*I*a*e
xp(I*(d*x+c)))+4*I/d^2/b*f*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+
c))-2*a)/(-a^2+b^2)^(1/2))+2/d^2/b*f^2/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*
x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+2/d^3/b*f^2/(-a^2+b^2)^(1
/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-2/
d^2/b*f^2/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+
(-a^2+b^2)^(1/2)))*x-2/d^3/b*f^2/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+
(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+2*I/d^3/b*f^2/(-a^2+b^2)^(1/2)*
dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/d
^3/b*f^2/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*
a-(-a^2+b^2)^(1/2)))-4*I/d^3/b*f^2*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp
(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1401 vs. 2(245) = 490.
time = 0.58, size = 1401, normalized size = 5.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*f*x*e + (a^2 - b^2)*d^2*e^2 +
 (-I*b^2*f^2*sin(d*x + c) - I*a*b*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*co
```

```
s(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b + 1) + (I*b^2*f^2*sin(d*x + c) + I*a*b*f^2)*sqrt(-(a^2
 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (I*b^2*f^2*sin(d*x +
c) + I*a*b*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x
 + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
 1) + (-I*b^2*f^2*sin(d*x + c) - I*a*b*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-
I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b + 1) + (a*b*c*f^2 - a*b*d*f*e + (b^2*c*f^2 - b^2
*d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (a*b*c*f^2 - a*b*d*f*e
+ (b^2*c*f^2 - b^2*d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(
d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (a*b*
c*f^2 - a*b*d*f*e + (b^2*c*f^2 - b^2*d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) + 2*I*a) - (a*b*c*f^2 - a*b*d*f*e + (b^2*c*f^2 - b^2*d*f*e)*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*s
qrt(-(a^2 - b^2)/b^2) - 2*I*a) + (a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x +
b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*
sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
 b)/b) - (a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x + b^2*c*f^2)*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a*b*d*f^2*x +
a*b*c*f^2 + (b^2*d*f^2*x + b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*
log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*
x + b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c)
 - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b))/((a^2*b^2 - b^4)*d^3*sin(d*x + c) + (a^3*b - a*b^3)*d^3)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cos(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] \text{Hanged}
```

# 3.321 $\int \frac{(e+fx)^3 \cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

**Optimal**. Leaf size=418

$$-\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}+\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}-\frac{6f^2(e+fx)\mathrm{Li}_2\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b}}\right.}{b\sqrt{a^2-b^2}\,d^3}$$

```
[Out] -(f*x+e)^3/b/d/(a+b*sin(d*x+c))-3*I*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-
(a^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)+3*I*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x
+c))/(a+(a^2-b^2)^(1/2)))/b/d^2/(a^2-b^2)^(1/2)-6*f^2*(f*x+e)*polylog(2,I*b
*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^3/(a^2-b^2)^(1/2)+6*f^2*(f*x+e)*po
lylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^3/(a^2-b^2)^(1/2)-6*I*f
^3*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^4/(a^2-b^2)^(1/2)+
6*I*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^4/(a^2-b^2)^(
1/2)
```

**Rubi** [A]

time = 0.57, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4507, 3404, 2296, 2221, 2611, 2320, 6724}

$$-\frac{6if^3\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4\sqrt{a^2-b^2}}+\frac{6if^3\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^4\sqrt{a^2-b^2}}-\frac{6f^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{6f^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-3*I)*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])
/(b*Sqrt[a^2 - b^2]*d^2) + ((3*I)*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (6*f^2*(e + f*x)*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3)
 + (6*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(b*Sqrt[a^2 - b^2]*d^3) - ((6*I)*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) + ((6*I)*f^3*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) - (e + f*x)
^3/(b*d*(a + b*Sin[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= -\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}+\frac{(3f)\int\frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{bd}\\
&= -\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}+\frac{(6f)\int\frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{bd}\\
&= -\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}-\frac{(6if)\int\frac{e^{i(c+dx)}(e+fx)^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}\,d}+\frac{(6if)\int\frac{}{2a+2\sqrt{}}}{\sqrt{}}\\
&= -\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}+\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right.}{b\sqrt{a^2-b^2}\,d^2}\\
&= -\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}+\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right.}{b\sqrt{a^2-b^2}\,d^2}\\
&= -\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}+\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right.}{b\sqrt{a^2-b^2}\,d^2}\\
&= -\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}+\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right.}{b\sqrt{a^2-b^2}\,d^2}
\end{aligned}
$$

**Mathematica** [A]
time = 1.05, size = 703, normalized size = 1.68

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*I)*f*(2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*(Cos[2*c + d*x] +
 I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2]
- a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog
[2, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b
^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c]) - I*(-2*Sqrt[a^
2 - b^2]*f^2*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[
c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin
[c]) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*
x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]
*(Cos[c] + I*Sin[c]) + d^2*(Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(-Log[1 + (b*(C
```

```
os[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c]
+ I*Sin[c])^2] - a*Sin[c])] + Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]
))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])])*
(Cos[c] + I*Sin[c]) + 2*e^2*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x])
)/Sqrt[a^2 - b^2]]*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))))/(b*Sqrt[a
^2 - b^2]*d^4*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]) - (e + f*x)^3/(b*
d*(a + b*Sin[c + d*x]))
```

**Maple** [F]
time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cos(dx+c)}{(a+b\sin(dx+c))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2299 vs. 2(365) = 730.
time = 0.56, size = 2299, normalized size = 5.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 - b^2)*d^3*f^3*x^3 + 6*(a^2 - b^2)*d^3*f^2*x^2*e + 6*(a^2 - b^
2)*d^3*f*x*e^2 + 2*(a^2 - b^2)*d^3*e^3 - 6*(I*a*b*d*f^3*x + I*a*b*d*f^2*e +
 (I*b^2*d*f^3*x + I*b^2*d*f^2*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*dilog
((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a*b*d*f^3*x - I*a*b*d*f^2*e + (-I
```

```
*b^2*d*f^3*x - I*b^2*d*f^2*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*dilog((I
*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a*b*d*f^3*x - I*a*b*d*f^2*e + (-I*b^
2*d*f^3*x - I*b^2*d*f^2*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a
*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1) - 6*(I*a*b*d*f^3*x + I*a*b*d*f^2*e + (I*b^2*d*
f^3*x + I*b^2*d*f^2*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos
(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b + 1) - 3*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*e + a*b*d^2*f*e^2
 + (b^2*c^2*f^3 - 2*b^2*c*d*f^2*e + b^2*d^2*f*e^2)*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b
^2)/b^2) + 2*I*a) - 3*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*e + a*b*d^2*f*e^2 + (b^2
*c^2*f^3 - 2*b^2*c*d*f^2*e + b^2*d^2*f*e^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2
) - 2*I*a) + 3*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*e + a*b*d^2*f*e^2 + (b^2*c^2*f^
3 - 2*b^2*c*d*f^2*e + b^2*d^2*f*e^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*l
og(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*
I*a) + 3*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*e + a*b*d^2*f*e^2 + (b^2*c^2*f^3 - 2*
b^2*c*d*f^2*e + b^2*d^2*f*e^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-2*
b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) +
 3*(a*b*d^2*f^3*x^2 - a*b*c^2*f^3 + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*e + (b^
2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*e)*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos
(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*(a*b*d^2*f
^3*x^2 - a*b*c^2*f^3 + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*e + (b^2*d^2*f^3*x^2
 - b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*e)*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 3*(a*b*d^2*f^3*x^2 - a*b*
c^2*f^3 + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*e + (b^2*d^2*f^3*x^2 - b^2*c^2*f^
3 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*(a*b*d^2*f^3*x^2 - a*b*c^2*f^3 + 2*
(a*b*d^2*f^2*x + a*b*c*d*f^2)*e + (b^2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2*(b^2*d
^2*f^2*x + b^2*c*d*f^2)*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*
cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b) - 6*(b^2*f^3*sin(d*x + c) + a*b*f^3)*sqrt(-(a^2 - b
^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(b^2*f^3*sin(d*x + c) + a
*b*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x +
c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(b^
2*f^3*sin(d*x + c) + a*b*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(
d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2))/b) + 6*(b^2*f^3*sin(d*x + c) + a*b*f^3)*sqrt(-(a^2 - b^2)/b^2)
*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b))/((a^2*b^2 - b^4)*d^4*sin(d*x + c) +
```

```
(a^3*b - a*b^3)*d^4)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cos(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] \text{Hanged}
```

# 3.322 $\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

**Optimal**. Leaf size=116

$$\frac{af\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}+\frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}$$

```
[Out] a*f*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(3/2)/d^2+
1/2*(-f*x-e)/b/d/(a+b*sin(d*x+c))^2+1/2*f*cos(d*x+c)/(a^2-b^2)/d^2/(a+b*sin
(d*x+c))
```

**Rubi** [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4507, 2743, 12, 2739, 632, 210}

$$\frac{af\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}+\frac{f\cos(c+dx)}{2d^2\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)
*d^2) - (e + f*x)/(2*b*d*(a + b*Sin[c + d*x])^2) + (f*Cos[c + d*x])/(2*(a^2
 - b^2)*d^2*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} + \frac{f\int \frac{a}{a+b\sin(c+dx}$$

$$= -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} + \frac{(af)\int \frac{1}{a+b\sin(c}$$

$$= -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} + \frac{(af)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} - \frac{(2af)\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{af\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2} - \frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+d}{2\left(a^2-b^2\right)d^2(a+b\,\mathrm{s}}$$

**Mathematica** [A]

time = 0.74, size = 112, normalized size = 0.97

$$\frac{\frac{2af\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}}+\frac{-\frac{d(e+fx)}{b}+\frac{f\cos(c+dx)(a+b\sin(c+dx))}{(a-b)(a+b)}}{(a+b\sin(c+dx))^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3
/2)) + (-((d*(e + f*x))/b) + (f*Cos[c + d*x]*(a + b*Sin[c + d*x]))/((a - b)
*(a + b)))/(a + b*Sin[c + d*x])^2)/(2*d^2)
```

**Maple** [C] Result contains complex when optimal does not.
time = 2.38, size = 349, normalized size = 3.01

| method | result |
|---|---|
| risch | $\frac{2a^2dfx\,e^{2i(dx+c)}-2b^2dfx\,e^{2i(dx+c)}+2ia^2f\,e^{2i(dx+c)}+ib^2f\,e^{2i(dx+c)}+2a^2de\,e^{2i(dx+c)}+baf\,e^{3i(dx+c)}-2b^2de\,e^{2i(dx+c)}-ib^2f-3abf\,e^{i(}}{\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)^2d^2(a^2-b^2)b}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*a^2*d*f*x*exp(2*I*(d*x+c))-2*b^2*d*f*x*exp(2*I*(d*x+c))+2*I*a^2*f*exp(2*
I*(d*x+c))+I*b^2*f*exp(2*I*(d*x+c))+2*a^2*d*e*exp(2*I*(d*x+c))+b*a*f*exp(3*
I*(d*x+c))-2*b^2*d*e*exp(2*I*(d*x+c))-I*b^2*f-3*a*b*f*exp(I*(d*x+c)))/(b*ex
p(2*I*(d*x+c))-b+2*I*a*exp(I*(d*x+c)))^2/d^2/(a^2-b^2)/b-1/2/(-a^2+b^2)^(1/
2)*f*a/(a+b)/(a-b)/d^2/b*ln(exp(I*(d*x+c))+(I*a*(-a^2+b^2)^(1/2)-a^2+b^2)/b
/(-a^2+b^2)^(1/2))+1/2/(-a^2+b^2)^(1/2)*f*a/(a+b)/(a-b)/d^2/b*ln(exp(I*(d*x
+c))+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/b/(-a^2+b^2)^(1/2))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(108) = 216$.

time = 0.38, size = 627, normalized size = 5.41

$$\left[\frac{2\,(a^4-2\,a^2b^2+b^4)dfx-2\,(a^2b^2-b^4)f\cos(dx+c)\sin(dx+c)-2\,(a^3b-ab^3)f\cos(dx+c)+2\,(a^4-2\,a^2b^2+b^4)de+\left(ab^2f\cos(dx+c)^2-2\,a^2bf\sin(dx+c)-(a^3+ab^2)f\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)}{4\,\left((a^4b^3-2\,a^2b^5+b^7)d^2\cos(dx+c)^2-2\,(a^5b^2-2\,a^3b^4+ab^6)d^2\sin(dx+c)-(a^6b-a^4b^3-a^2b^5+b^7)d^2\right)},\ \frac{(a^4-2\,a^2b^2+b^4)dfx-(a^2b^2-b^4)f\cos(dx+c)\sin(dx+c)-(a^3b-ab^3)f\cos(dx+c)+(a^4-2\,a^2b^2+b^4)de-\left(ab^2f\cos(dx+c)^2-2\,a^2bf\sin(dx+c)-(a^3+ab^2)f\right)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)}{2\,\left((a^4b^3-2\,a^2b^5+b^7)d^2\cos(dx+c)^2-2\,(a^5b^2-2\,a^3b^4+ab^6)d^2\sin(dx+c)-(a^6b-a^4b^3-a^2b^5+b^7)d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^4 - 2*a^2*b^2 + b^4)*d*f*x - 2*(a^2*b^2 - b^4)*f*cos(d*x + c)*si
n(d*x + c) - 2*(a^3*b - a*b^3)*f*cos(d*x + c) + 2*(a^4 - 2*a^2*b^2 + b^4)*d
*e + (a*b^2*f*cos(d*x + c)^2 - 2*a^2*b*f*sin(d*x + c) - (a^3 + a*b^2)*f)*sq
rt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2)
)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/((a^4*b^3 - 2*a^2
*b^5 + b^7)*d^2*cos(d*x + c)^2 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^2*sin(d*
x + c) - (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^2), 1/2*((a^4 - 2*a^2*b^2 + b^
4)*d*f*x - (a^2*b^2 - b^4)*f*cos(d*x + c)*sin(d*x + c) - (a^3*b - a*b^3)*f*
cos(d*x + c) + (a^4 - 2*a^2*b^2 + b^4)*d*e - (a*b^2*f*cos(d*x + c)^2 - 2*a^
2*b*f*sin(d*x + c) - (a^3 + a*b^2)*f)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d^2*c
os(d*x + c)^2 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^2*sin(d*x + c) - (a^6*b -
 a^4*b^3 - a^2*b^5 + b^7)*d^2)]
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

**Mupad [F(-1)]**

time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x))/(a + b*sin(c + d*x))^3,x)
[Out] \text{Hanged}
```

# 3.323 $\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

**Optimal**. Leaf size=357

$$-\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2}-\frac{f^2\log(a+b\sin(c+dx))}{b\left(a^2-b^2\right)d^3}-\frac{a}{}$$

```
[Out] -f^2*ln(a+b*sin(d*x+c))/b/(a^2-b^2)/d^3-I*a*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c
))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2+I*a*f*(f*x+e)*ln(1-I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-a*f^2*polylog(2,I*b*exp
(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3+a*f^2*polylog(2,I*b*
exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-1/2*(f*x+e)^2/b/d
/(a+b*sin(d*x+c))^2+f*(f*x+e)*cos(d*x+c)/(a^2-b^2)/d^2/(a+b*sin(d*x+c))
```

**Rubi** [A]
time = 0.40, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4507, 3405, 3404, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{af^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}+\frac{af^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}-\frac{f^2\log(a+b\sin(c+dx))}{bd^3\left(a^2-b^2\right)}-\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}+\frac{f(e+fx)\cos(c+dx)}{d^2\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-I)*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
b*(a^2 - b^2)^(3/2)*d^2) + (I*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (f^2*Log[a + b*Sin[c + d
*x]])/(b*(a^2 - b^2)*d^3) - (a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (a*f^2*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) - (e + f*x)^2/(
2*b*d*(a + b*Sin[c + d*x])^2) + (f*(e + f*x)*Cos[c + d*x])/((a^2 - b^2)*d^2
*(a + b*Sin[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f\int\frac{e+fx}{(a+b\sin(c+dx))^2}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f(e+fx)\cos(c+dx)}{(a^2-b^2)\,d^2(a+b\sin(c+dx))}+\frac{(af)\int\frac{e+fx}{a+b\sin(c+\ldots}}{b\,(a^2-b^2)\ldots}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f(e+fx)\cos(c+dx)}{(a^2-b^2)\,d^2(a+b\sin(c+dx))}+\frac{(2af)\int\frac{e^{i(\ldots}}{ib+2ae^{i(\ldots}}}{b\,(a^2\ldots}$$

$$= -\frac{f^2\log(a+b\sin(c+dx))}{b\,(a^2-b^2)\,d^3}-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f(e+fx)\cos(c\ldots}{(a^2-b^2)\,d^2(a+b\sin\ldots}$$

$$= -\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d^2}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\ldots\right.}{b\,(a^2-b^2)^{3/2}\,d^2}$$

$$= -\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d^2}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\ldots\right.}{b\,(a^2-b^2)^{3/2}\,d^2}$$

$$= -\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d^2}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\ldots\right.}{b\,(a^2-b^2)^{3/2}\,d^2}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1104 vs. 2(357) = 714.
time = 11.46, size = 1104, normalized size = 3.09

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (f^2*x*Cot[c])/(b*(-a^2 + b^2)*d^2) - ((I/2)*E^(I*c)*f*(4*E^(I*c)*f*x + ((4
*I)*a*e*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]
```

```
*E^(I*c)) - ((4*I)*a*e*E^(I*c)*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 -
b^2]])/Sqrt[a^2 - b^2] + (2*f*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I
)*(c + d*x))))])/(d*E^(I*c)) - (2*E^(I*c)*f*ArcTan[(2*a*E^(I*(c + d*x)))/(b
*(-1 + E^((2*I)*(c + d*x))))])/d - (I*f*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2
*(-1 + E^((2*I)*(c + d*x)))^2])/(d*E^(I*c)) + (I*E^(I*c)*f*Log[4*a^2*E^((2*
I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2])/d + ((2*I)*a*f*x*Log[1 +
 (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqr
t[(-a^2 + b^2)*E^((2*I)*c)] - ((2*I)*a*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c
 + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2
)*E^((2*I)*c)] - ((2*I)*a*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) +
Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + ((2*I)*a
*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 +
b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (2*a*(-1 + E^((2*I)*c
))*f*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^
((2*I)*c)])])/(d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) + (2*a*(-1 + E^((2*I)*c))*
f*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2
*I)*c)]))])/(d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])))/(b*(-a^2 + b^2)*d^2*(-1 +
E^((2*I)*c))) - (f^2*x*Csc[c/2]*Sec[c/2]*(Cos[c/2] - Sin[c/2])*(Cos[c/2] +
Sin[c/2]))/(2*b*(-a + b)*(a + b)*d^2) - (e + f*x)^2/(2*b*d*(a + b*Sin[c + d
*x])^2) + (Csc[c/2]*Sec[c/2]*(-(a*e*f*Cos[c]) - a*f^2*x*Cos[c] - b*e*f*Sin[
d*x] - b*f^2*x*Sin[d*x]))/(2*(a - b)*b*(a + b)*d^2*(a + b*Sin[c + d*x]))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(325) = 650$.
time = 2.31, size = 946, normalized size = 2.65

| method | result |
|---|---|
| risch | $\frac{2a^2 d f^2 x^2 e^{2i(dx+c)} - 2b^2 d f^2 x^2 e^{2i(dx+c)} + 4ia^2 e f e^{2i(dx+c)} + 4ia^2 f^2 x e^{2i(dx+c)} + 4a^2 d e f x e^{2i(dx+c)} + 2ba f^2 x e^{3i(dx+c)} - 4b^2 d e f x e^{2i(dx}}{(b e^{2i(dx+c)} - }$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a^2*d*f^2*x^2*exp(2*I*(d*x+c))-b^2*d*f^2*x^2*exp(2*I*(d*x+c))+2*I*a^2*e*
f*exp(2*I*(d*x+c))+2*I*a^2*f^2*x*exp(2*I*(d*x+c))+2*a^2*d*e*f*x*exp(2*I*(d*
x+c))+b*a*f^2*x*exp(3*I*(d*x+c))-2*b^2*d*e*f*x*exp(2*I*(d*x+c))-I*b^2*f^2*x
+I*b^2*f^2*x*exp(2*I*(d*x+c))+a^2*d*e^2*exp(2*I*(d*x+c))+b*a*e*f*exp(3*I*(d
*x+c))-b^2*d*e^2*exp(2*I*(d*x+c))-I*b^2*e*f-3*a*b*f^2*x*exp(I*(d*x+c))+I*b^
2*e*f*exp(2*I*(d*x+c))-3*a*b*e*f*exp(I*(d*x+c)))/(b*exp(2*I*(d*x+c))-b+2*I*
a*exp(I*(d*x+c)))^2/d^2/(a^2-b^2)/b+1/b/(-a^2+b^2)/d^3*f^2*ln(I*b*exp(2*I*(
d*x+c))-2*a*exp(I*(d*x+c))-I*b)-2/b/(-a^2+b^2)/d^3*f^2*ln(exp(I*(d*x+c)))-I
/b/(-a^2+b^2)^(3/2)/d^3*f^2*a*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))
/(I*a+(-a^2+b^2)^(1/2)))-1/b/(-a^2+b^2)^(3/2)/d^2*f^2*a*ln((I*a+b*exp(I*(d*
x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-1/b/(-a^2+b^2)^(3/2)/d^3*
```

```
f^2*a*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+
I/b/(-a^2+b^2)^(3/2)/d^3*f^2*a*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2)
)/(I*a-(-a^2+b^2)^(1/2)))-2*I/b/(-a^2+b^2)^(3/2)/d^2*f*a*e*arctan(1/2*(2*I*
b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/b/(-a^2+b^2)^(3/2)/d^2*f^2*a*ln((
I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/b/(-a^2+
b^2)^(3/2)/d^3*f^2*a*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+
b^2)^(1/2)))*c+2*I/b/(-a^2+b^2)^(3/2)/d^3*f^2*a*c*arctan(1/2*(2*I*b*exp(I*(
d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2389 vs. 2(321) = 642.
time = 0.66, size = 2389, normalized size = 6.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*((a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d^2*f*
x*e + (a^4 - 2*a^2*b^2 + b^4)*d^2*e^2 - 2*((a^2*b^2 - b^4)*d*f^2*x + (a^2*b
^2 - b^4)*d*f*e)*cos(d*x + c)*sin(d*x + c) - (-I*a*b^3*f^2*cos(d*x + c)^2 +
 2*I*a^2*b^2*f^2*sin(d*x + c) + I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^2 - b^2)/b^
2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (I*a*b^3*f^2*cos(d*x + c)^2 - 2
*I*a^2*b^2*f^2*sin(d*x + c) - I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^2 - b^2)/b^2)
*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (I*a*b^3*f^2*cos(d*x + c)^2 - 2*I
*a^2*b^2*f^2*sin(d*x + c) - I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^2 - b^2)/b^2)*d
ilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (-I*a*b^3*f^2*cos(d*x + c)^2 + 2*I
*a^2*b^2*f^2*sin(d*x + c) + I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^2 - b^2)/b^2)*d
ilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + ((a^3*b + a*b^3)*d*f^2*x + (a^3*b
```

```
+ a*b^3)*c*f^2 - (a*b^3*d*f^2*x + a*b^3*c*f^2)*cos(d*x + c)^2 + 2*(a^2*b^2*
d*f^2*x + a^2*b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos
(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b) - ((a^3*b + a*b^3)*d*f^2*x + (a^3*b + a*b^3)*c*f^2 - (
a*b^3*d*f^2*x + a*b^3*c*f^2)*cos(d*x + c)^2 + 2*(a^2*b^2*d*f^2*x + a^2*b^2*
c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(
d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/
b) + ((a^3*b + a*b^3)*d*f^2*x + (a^3*b + a*b^3)*c*f^2 - (a*b^3*d*f^2*x + a*
b^3*c*f^2)*cos(d*x + c)^2 + 2*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos
(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - ((a^3*b + a*
b^3)*d*f^2*x + (a^3*b + a*b^3)*c*f^2 - (a*b^3*d*f^2*x + a*b^3*c*f^2)*cos(d*
x + c)^2 + 2*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*((a^3*b - a*b^3)*d*f^2*x +
(a^3*b - a*b^3)*d*f*e)*cos(d*x + c) - ((a^2*b^2 - b^4)*f^2*cos(d*x + c)^2 -
 2*(a^3*b - a*b^3)*f^2*sin(d*x + c) - (a^4 - b^4)*f^2 - ((a^3*b + a*b^3)*c*
f^2 - (a^3*b + a*b^3)*d*f*e - (a*b^3*c*f^2 - a*b^3*d*f*e)*cos(d*x + c)^2 +
2*(a^2*b^2*c*f^2 - a^2*b^2*d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log
(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a
) - ((a^2*b^2 - b^4)*f^2*cos(d*x + c)^2 - 2*(a^3*b - a*b^3)*f^2*sin(d*x + c
) - (a^4 - b^4)*f^2 - ((a^3*b + a*b^3)*c*f^2 - (a^3*b + a*b^3)*d*f*e - (a*b
^3*c*f^2 - a*b^3*d*f*e)*cos(d*x + c)^2 + 2*(a^2*b^2*c*f^2 - a^2*b^2*d*f*e)*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - ((a^2*b^2 - b^4)*f^2*cos(d*x +
 c)^2 - 2*(a^3*b - a*b^3)*f^2*sin(d*x + c) - (a^4 - b^4)*f^2 + ((a^3*b + a*
b^3)*c*f^2 - (a^3*b + a*b^3)*d*f*e - (a*b^3*c*f^2 - a*b^3*d*f*e)*cos(d*x +
c)^2 + 2*(a^2*b^2*c*f^2 - a^2*b^2*d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 + 2*I*a) - ((a^2*b^2 - b^4)*f^2*cos(d*x + c)^2 - 2*(a^3*b - a*b^3)*f^2*sin
(d*x + c) - (a^4 - b^4)*f^2 + ((a^3*b + a*b^3)*c*f^2 - (a^3*b + a*b^3)*d*f*
e - (a*b^3*c*f^2 - a*b^3*d*f*e)*cos(d*x + c)^2 + 2*(a^2*b^2*c*f^2 - a^2*b^2
*d*f*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b
*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b^3 - 2*a^2*b^5
+ b^7)*d^3*cos(d*x + c)^2 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^3*sin(d*x + c
) - (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^3)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)^2*cos(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x))^3,x)
[Out] \text{Hanged}
```

# 3.324 $\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

**Optimal**. Leaf size=753

$$\frac{3if(e+fx)^2}{2b(a^2-b^2)d^2}-\frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^3}-\frac{3iaf(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{3/2}d^2}-\frac{3f^2(e+fx)\ldots}{b\ldots}$$

```
[Out] 3/2*I*f*(f*x+e)^2/b/(a^2-b^2)/d^2-3*f^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-
(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^3-3/2*I*a*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c
))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-3*f^2*(f*x+e)*ln(1-I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^3+3/2*I*a*f*(f*x+e)^2*ln(1-I*b*
exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2+3*I*f^3*polylog(2
,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^4-3*a*f^2*(f*x+e)*po
lylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3+3*I*f
^3*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^4+3*a*f^
2*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/
2)/d^3-3*I*a*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b
^2)^(3/2)/d^4+3*I*a*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b
/(a^2-b^2)^(3/2)/d^4-1/2*(f*x+e)^3/b/d/(a+b*sin(d*x+c))^2+3/2*f*(f*x+e)^2*c
os(d*x+c)/(a^2-b^2)/d^2/(a+b*sin(d*x+c))
```

**Rubi** [A]
time = 0.84, antiderivative size = 753, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {4507, 3405, 3404, 2296, 2221, 2611, 2320, 6724, 4615, 2317, 2438}

$$\frac{3if^3\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4(a^2-b^2)}+\frac{3if^3\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^4(a^2-b^2)}-\frac{3iaf^3\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4(a^2-b^2)^{3/2}}+\frac{3iaf^3\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^4(a^2-b^2)^{3/2}}-\frac{3af^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3(a^2-b^2)^{3/2}}+\frac{3af^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3(a^2-b^2)^{3/2}}-\frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3(a^2-b^2)}-\frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3(a^2-b^2)}-\frac{3iaf(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd^2(a^2-b^2)^{3/2}}+\frac{3iaf(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd^2(a^2-b^2)^{3/2}}+\frac{3f(e+fx)^2\cos(c+dx)}{2d^2(a^2-b^2)(a+b\sin(c+dx))}+\frac{3if(e+fx)^2}{2bd^2(a^2-b^2)}-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((3*I)/2)*f*(e + f*x)^2)/(b*(a^2 - b^2)*d^2) - (3*f^2*(e + f*x)*Log[1 - (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (((3*I)/2
)*a*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
(a^2 - b^2)^(3/2)*d^2) - (3*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (((3*I)/2)*a*f*(e + f*x)^2*Log[1
 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2)
+ ((3*I)*f^3*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a
^2 - b^2)*d^4) - (3*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + ((3*I)*f^3*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + (3*a*f^2*(e +
f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2
)^(3/2)*d^3) - ((3*I)*a*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
```

```
- b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) + ((3*I)*a*f^3*PolyLog[3, (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) - (e + f*x)^3/(2
*b*d*(a + b*Sin[c + d*x])^2) + (3*f*(e + f*x)^2*Cos[c + d*x])/(2*(a^2 - b^2
)*d^2*(a + b*Sin[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2} + \frac{(3f)\int \frac{(e+fx)^2}{(a+b\sin(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2} + \frac{3f(e+fx)^2\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} + \frac{(3af)\int \frac{(e+}{a+b\sin}}{2b\left(a^2-\right.}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2} + \frac{3f(e+fx)^2\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3f^2(e+fx)\log\left(1\right.}{b\left(a^2-\right.}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3iaf(e+fx)^2\log\left(\right.}{2b\left(a^2-\right.}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3iaf(e+fx)^2\log\left(\right.}{2b\left(a^2-\right.}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3iaf(e+fx)^2\log\left(\right.}{2b\left(a^2-\right.}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3iaf(e+fx)^2\log\left(\right.}{2b\left(a^2-\right.}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2311 vs. 2(753) = 1506.
time = 15.78, size = 2311, normalized size = 3.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-3*I)*E^(I*c)*f*(2*e*E^(I*c)*f*x + E^(I*c)*f^2*x^2 + (I*a*e^2*ArcTan[(I*a
 + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*E^(I*c)) - (I*a*e^
2*E^(I*c)*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2
```

```
] + (2*a*e*f*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqrt[a^2 -
 b^2]*d*E^(I*c)) + (e*f*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c +
 d*x))))])/(d*E^(I*c)) - (e*E^(I*c)*f*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 +
 E^((2*I)*(c + d*x))))])/d + ((2*I)*a*e*f*ArcTanh[(-a + I*b*E^(I*(c + d*x))
)/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*d*E^(I*c)) - ((I/2)*e*f*Log[4*a^2*E^((
2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2])/(d*E^(I*c)) + ((I/2)*e
*E^(I*c)*f*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2
])/d + (I*a*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (I*a*e*E^((2*I)*c)*f
*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*
c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (I*f^2*x*Log[1 + (b*E^(I*(2*c + d*x
)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/(d*E^(I*c)) + (I*E^(I*
c)*f^2*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^(
(2*I)*c)])])/d + ((I/2)*a*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c
) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - ((I/
2)*a*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[
(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (I*a*e*f*x*Lo
g[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
)/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + (I*a*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(
2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 +
b^2)*E^((2*I)*c)] - (I*f^2*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + S
qrt[(-a^2 + b^2)*E^((2*I)*c)])])/(d*E^(I*c)) + (I*E^(I*c)*f^2*x*Log[1 + (b*
E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/d - ((I
/2)*a*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2
)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + ((I/2)*a*E^((2*I)*c)*f^2
*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I
)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - ((-1 + E^((2*I)*c))*f*(-(Sqrt[(-a
^2 + b^2)*E^((2*I)*c)]*f) + a*d*E^(I*c)*(e + f*x))*PolyLog[2, (I*b*E^(I*(2*
c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/(d^2*E^(I*c)*Sq
rt[(-a^2 + b^2)*E^((2*I)*c)]) + ((-1 + E^((2*I)*c))*f*(Sqrt[(-a^2 + b^2)*E^
((2*I)*c)]*f + a*d*E^(I*c)*(e + f*x))*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I
*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))])/(d^2*E^(I*c)*Sqrt[(-a^2 + b
^2)*E^((2*I)*c)]) + (I*a*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c)
+ I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
- (I*a*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) - (I*a
*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^
((2*I)*c)]))])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) + (I*a*E^((2*I)*c)*f^2*
PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I
)*c)]))])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])))/(b*(-a^2 + b^2)*d^2*(-1 +
E^((2*I)*c))) - (e + f*x)^3/(2*b*d*(a + b*Sin[c + d*x])^2) - (3*Csc[c/2]*Se
c[c/2]*(a*e^2*f*Cos[c] + 2*a*e*f^2*x*Cos[c] + a*f^3*x^2*Cos[c] + b*e^2*f*Si
n[d*x] + 2*b*e*f^2*x*Sin[d*x] + b*f^3*x^2*Sin[d*x]))/(4*(a - b)*b*(a + b)*d
^2*(a + b*Sin[c + d*x]))
```

**Maple** [F]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cos(dx+c)}{(a+b\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4952 vs. 2(662) = 1324.
time = 0.74, size = 4952, normalized size = 6.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^4 - 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 6*(a^4 - 2*a^2*b^2 + b^4)*d^3*
f^2*x^2*e + 6*(a^4 - 2*a^2*b^2 + b^4)*d^3*f*x*e^2 + 2*(a^4 - 2*a^2*b^2 + b^
4)*d^3*e^3 - 6*((a^2*b^2 - b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 - b^4)*d^2*f^2*x*e
 + (a^2*b^2 - b^4)*d^2*f*e^2)*cos(d*x + c)*sin(d*x + c) + 6*(a*b^3*f^3*cos(
d*x + c)^2 - 2*a^2*b^2*f^3*sin(d*x + c) - (a^3*b + a*b^3)*f^3)*sqrt(-(a^2 -
 b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c)
 - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(a*b^3*f^3*cos(d*x + c)
^2 - 2*a^2*b^2*f^3*sin(d*x + c) - (a^3*b + a*b^3)*f^3)*sqrt(-(a^2 - b^2)/b^
2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(a*b^3*f^3*cos(d*x + c)^2 - 2*a
^2*b^2*f^3*sin(d*x + c) - (a^3*b + a*b^3)*f^3)*sqrt(-(a^2 - b^2)/b^2)*polyl
og(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x
```

```
+ c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(a*b^3*f^3*cos(d*x + c)^2 - 2*a^2*b^2*
f^3*sin(d*x + c) - (a^3*b + a*b^3)*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -
(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2))/b) - 6*((a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(a^3*b - a*b
^3)*d^2*f^2*x*e + (a^3*b - a*b^3)*d^2*f*e^2)*cos(d*x + c) + 6*(I*(a^2*b^2 -
 b^4)*f^3*cos(d*x + c)^2 - 2*I*(a^3*b - a*b^3)*f^3*sin(d*x + c) - I*(a^4 -
b^4)*f^3 + (-I*(a^3*b + a*b^3)*d*f^3*x - I*(a^3*b + a*b^3)*d*f^2*e + (I*a*b
^3*d*f^3*x + I*a*b^3*d*f^2*e)*cos(d*x + c)^2 + 2*(-I*a^2*b^2*d*f^3*x - I*a^
2*b^2*d*f^2*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a*cos(d*x + c
) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) + 6*(I*(a^2*b^2 - b^4)*f^3*cos(d*x + c)^2 - 2*I*(a^3*b - a
*b^3)*f^3*sin(d*x + c) - I*(a^4 - b^4)*f^3 + (I*(a^3*b + a*b^3)*d*f^3*x + I
*(a^3*b + a*b^3)*d*f^2*e + (-I*a*b^3*d*f^3*x - I*a*b^3*d*f^2*e)*cos(d*x + c
)^2 + 2*(I*a^2*b^2*d*f^3*x + I*a^2*b^2*d*f^2*e)*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(-I*(a^2*b^2 - b^4)*f
^3*cos(d*x + c)^2 + 2*I*(a^3*b - a*b^3)*f^3*sin(d*x + c) + I*(a^4 - b^4)*f^
3 + (I*(a^3*b + a*b^3)*d*f^3*x + I*(a^3*b + a*b^3)*d*f^2*e + (-I*a*b^3*d*f^
3*x - I*a*b^3*d*f^2*e)*cos(d*x + c)^2 + 2*(I*a^2*b^2*d*f^3*x + I*a^2*b^2*d*
f^2*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x + c) - a*s
in(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) + 6*(-I*(a^2*b^2 - b^4)*f^3*cos(d*x + c)^2 + 2*I*(a^3*b - a*b^3)*
f^3*sin(d*x + c) + I*(a^4 - b^4)*f^3 + (-I*(a^3*b + a*b^3)*d*f^3*x - I*(a^3
*b + a*b^3)*d*f^2*e + (I*a*b^3*d*f^3*x + I*a*b^3*d*f^2*e)*cos(d*x + c)^2 +
2*(-I*a^2*b^2*d*f^3*x - I*a^2*b^2*d*f^2*e)*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(2*(a^4 - b^4)*c*f^3 - 2*
(a^4 - b^4)*d*f^2*e - 2*((a^2*b^2 - b^4)*c*f^3 - (a^2*b^2 - b^4)*d*f^2*e)*c
os(d*x + c)^2 + 4*((a^3*b - a*b^3)*c*f^3 - (a^3*b - a*b^3)*d*f^2*e)*sin(d*x
 + c) + ((a^3*b + a*b^3)*c^2*f^3 - 2*(a^3*b + a*b^3)*c*d*f^2*e + (a^3*b + a
*b^3)*d^2*f*e^2 - (a*b^3*c^2*f^3 - 2*a*b^3*c*d*f^2*e + a*b^3*d^2*f*e^2)*cos
(d*x + c)^2 + 2*(a^2*b^2*c^2*f^3 - 2*a^2*b^2*c*d*f^2*e + a^2*b^2*d^2*f*e^2)
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 3*(2*(a^4 - b^4)*c*f^3 - 2*(a
^4 - b^4)*d*f^2*e - 2*((a^2*b^2 - b^4)*c*f^3 - (a^2*b^2 - b^4)*d*f^2*e)*cos
(d*x + c)^2 + 4*((a^3*b - a*b^3)*c*f^3 - (a^3*b - a*b^3)*d*f^2*e)*sin(d*x +
 c) + ((a^3*b + a*b^3)*c^2*f^3 - 2*(a^3*b + a*b^3)*c*d*f^2*e + (a^3*b + a*b
^3)*d^2*f*e^2 - (a*b^3*c^2*f^3 - 2*a*b^3*c*d*f^2*e + a*b^3*d^2*f*e^2)*cos(d
*x + c)^2 + 2*(a^2*b^2*c^2*f^3 - 2*a^2*b^2*c*d*f^2*e + a^2*b^2*d^2*f*e^2)*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 3*(2*(a^4 - b^4)*c*f^3 - 2*(a^4
 - b^4)*d*f^2*e - 2*((a^2*b^2 - b^4)*c*f^3 - (a^2*b^2 - b^4)*d*f^2*e)*cos(d
*x + c)^2 + 4*((a^3*b - a*b^3)*c*f^3 - (a^3*b - a*b^3)*d*f^2*e)*sin(d*x + c
) - ((a^3*b + a*b^3)*c^2*f^3 - 2*(a^3*b + a*b^3)*c*d*f^2*e + (a^3*b + a*b^3
)*d^2*f*e^2 - (a*b^3*c^2*f^3 - 2*a*b^3*c*d*f^2*e + a*b^3*d^2*f*e^2)*cos(d*x
```

```
 + c)^2 + 2*(a^2*b^2*c^2*f^3 - 2*a^2*b^2*c*d*f^2*e + a^2*b^2*d^2*f*e^2)*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x +
c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 3*(2*(a^4 - b^4)*c*f^3 - 2*(a^4
- b^4)*d*f^2*e - 2*((a^2*b^2 - b^4)*c*f^3 - (a^2*b^2 - b^4)*d*f^2*e)*cos(d*
x + c)^2 + 4*((a^3*b - a*b^3)*c*f^3 - (a^3*b - a*b^3)*d*f^2*e)*sin(d*x + c)
 - ((a^3*b + a*b^3)*c^2*f^3 - 2*(a^3*b + a*b^3)*c*d*f^2*e + (a^3*b + a*b^3)
*d^2*f*e^2 - (a*b^3*c^2*f^3 - 2*a*b^3*c*d*f^2*e + a*b^3*d^2*f*e^2)*cos(d*x
+ c)^2 + 2*(a^2*b^2*c^2*f^3 - 2*a^2*b^2*c*d*f^2*e + a^2*b^2*d^2*f*e^2)*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c
) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 3*(2*...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)^3*cos(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x))^3,x)
[Out] \text{Hanged}
```

# 3.325 $\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=765

$$-\frac{(e+fx)^4}{4bf}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{i\sqrt{a^2-b^2}\,(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd}+\frac{i\sqrt{a^2-b^2}\,(e+f}{\phantom{x}}$$

```
[Out] -1/4*(f*x+e)^4/b/f-2*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a/d-6*I*f^3*polylog(
4,-exp(I*(d*x+c)))/a/d^4+6*I*f^3*polylog(4,exp(I*(d*x+c)))/a/d^4-6*f^2*(f*x
+e)*polylog(3,-exp(I*(d*x+c)))/a/d^3+6*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c))
)/a/d^3-6*I*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(
a^2-b^2)^(1/2)/a/b/d^3+3*I*f*(f*x+e)^2*polylog(2,-exp(I*(d*x+c)))/a/d^2+I*(
f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d
-3*I*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a/d^2-3*f*(f*x+e)^2*polylog(2,I*
b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d^2+3*f*(f*x+e)^2
*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d^2+
6*I*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)
^(1/2)/a/b/d^3-I*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^
2-b^2)^(1/2)/a/b/d+6*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*
(a^2-b^2)^(1/2)/a/b/d^4-6*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/
2)))*(a^2-b^2)^(1/2)/a/b/d^4
```

**Rubi** [A]
time = 0.92, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4639, 4493, 3377, 2717, 4268, 2611, 6744, 2320, 6724, 4621, 32, 3404, 2296, 2221}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(e + f*x)^4/(b*f) - (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) - (
I*Sqrt[a^2 - b^2]*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(a*b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*
x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(
I*(c + d*x))])/(a*d^2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/
(a*d^2) - (3*Sqrt[a^2 - b^2]*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/(a*b*d^2) + (3*Sqrt[a^2 - b^2]*f*(e + f*x)^2*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^2) - (6*f^2*(e +
 f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, E
^(I*(c + d*x))])/(a*d^3) - ((6*I)*Sqrt[a^2 - b^2]*f^2*(e + f*x)*PolyLog[3,
```

```
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b*d^3) + ((6*I)*Sqrt[a^2 -
 b^2]*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(a*b*d^3) - ((6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3
*PolyLog[4, E^(I*(c + d*x))])/(a*d^4) + (6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b*d^4) - (6*Sqrt[a^2 - b^2]
*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
```

```
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^3\csc(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{(e}{a+b\mathrm{s}}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\int \frac{}{ib}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{3if(e+fx)^2\mathrm{Li}_2(-e}{ad^2}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)^3}{}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)^3}{}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)^3}{}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)^3}{}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)^3}{}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1897 vs. $2(765) = 1530$.
time = 1.54, size = 1897, normalized size = 2.48

---

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/b + (-2*d^3*e^3*ArcTan
h[E^(I*(c + d*x))] + 3*d^3*e^2*f*x*Log[1 - E^(I*(c + d*x))] + 3*d^3*e*f^2*x
^2*Log[1 - E^(I*(c + d*x))] + d^3*f^3*x^3*Log[1 - E^(I*(c + d*x))] - 3*d^3*
e^2*f*x*Log[1 + E^(I*(c + d*x))] - 3*d^3*e*f^2*x^2*Log[1 + E^(I*(c + d*x))]
```

```
 - d^3*f^3*x^3*Log[1 + E^(I*(c + d*x))] + (3*I)*d^2*f*(e + f*x)^2*PolyLog[2
, -E^(I*(c + d*x))] - (3*I)*d^2*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))] -
 6*d*e*f^2*PolyLog[3, -E^(I*(c + d*x))] - 6*d*f^3*x*PolyLog[3, -E^(I*(c + d
*x))] + 6*d*e*f^2*PolyLog[3, E^(I*(c + d*x))] + 6*d*f^3*x*PolyLog[3, E^(I*(
c + d*x))] - (6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))] + (6*I)*f^3*PolyLog[4,
E^(I*(c + d*x))])/(a*d^4) + (I*Sqrt[a^2 - b^2]*((3*I)*Sqrt[a^2 - b^2]*d^3*e
^2*f*x*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(
-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + (3*I)
*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x
]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Co
s[c] + I*Sin[c]) + I*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*(Cos[2*c + d*x]
 + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2
] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*Po
lyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2
 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 3*Sqrt[a
^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x
]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*
(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*(Cos[2*
c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*S
in[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*f^3*x
*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-
a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 6*Sqr
t[a^2 - b^2]*f^3*PolyLog[4, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*
Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I
*Sin[c]) + 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*(Cos[2*c + d*x] + I*Sin[2*c
+ d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[
c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*(Cos[2*
c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] +
I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e*
f^2*x^2*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sq
rt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c])
+ Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]
))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(
(-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*(Cos[2*c +
d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Si
n[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*f^3*x*Po
lyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^
2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) - (2*I)
*d^3*e^3*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]]*
Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))/(a*b*d^4*Sqrt[(-a^2 + b^2)*(Co
s[2*c] + I*Sin[2*c])])
```

**Maple** [**F**]

time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cos (dx+c) \cot (dx+c)}{a+b\sin (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3082 vs. 2(677) = 1354.
time = 0.68, size = 3082, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/4*(a*d^4*f^3*x^4 + 4*a*d^4*f^2*x^3*e + 6*a*d^4*f*x^2*e^2 + 4*a*d^4*x*e^3
 + 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(
d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) -
 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*
x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 1
2*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x
 + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12
*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x
+ c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*
I*b*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c)) + 12*I*b*f^3*polylog(4, c
os(d*x + c) - I*sin(d*x + c)) - 12*I*b*f^3*polylog(4, -cos(d*x + c) + I*sin
```

```
(d*x + c)) + 12*I*b*f^3*polylog(4, -cos(d*x + c) - I*sin(d*x + c)) + 6*(-I*
b*d^2*f^3*x^2 - 2*I*b*d^2*f^2*x*e - I*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*d
ilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(I*b*d^2*f^3*x^2 + 2*I*b*d^2*f^2*
x*e + I*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin
(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)
/b + 1) + 6*(I*b*d^2*f^3*x^2 + 2*I*b*d^2*f^2*x*e + I*b*d^2*f*e^2)*sqrt(-(a^
2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(-I*b*d^2*f^3*x^2
 - 2*I*b*d^2*f^2*x*e - I*b*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*co
s(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b + 1) + 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^
2 - b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*e
 + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c)
 - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b*c^3*f^3
- 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log
(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*
a) - 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*sqrt(-(a
^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) - 2*I*a) + 2*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^
2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(
I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b) - 2*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x +
 b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)
*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^
3*f*x + b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*sqrt(-(a^2 - b^
2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*(b*d^3*f^3*x^3 + b*c^3*f^3 +
 3*(b*d^3*f*x + b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*sqrt(-(
a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c)
- I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 12*(b*d*f^3*x + b*d*f^
2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c)
+ (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(b*d*
f^3*x + b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))
/b) - 12*(b*d*f^3*x + b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*c
os(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))/b) + 12*(b*d*f^3*x + b*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*poly
log(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(I*b*d^2*f^3*x^2 + 2*I*b*d^2*f^2*x*e
+ I*b*d^2*f*e^2)*dilog(cos(d*x + c) + I*sin(d*x + c)) + 6*(-I*b*d^2*f^3*x^2
 - 2*I*b*d^2*f^2*x*e - I*b*d^2*f*e^2)*dilog(cos(d*x + c) - I*sin(d*x + c))
+ 6*(I*b*d^2*f^3*x^2 + 2*I*b*d^2*f^2*x*e + I*b*d^2*f*e^2)*dilog(-cos(d*x +
```

```
c) + I*sin(d*x + c)) + 6*(-I*b*d^2*f^3*x^2 - 2*I*b*d^2*f^2*x*e - I*b*d^2*f*
e^2)*dilog(-cos(d*x + c) - I*sin(d*x + c)) + 2*(b*d^3*f^3*x^3 + 3*b*d^3*f^2
*x^2*e + 3*b*d^3*f*x*e^2 + b*d^3*e^3)*log(cos(d*x + c) + I*sin(d*x + c) + 1
) + 2*(b*d^3*f^3*x^3 + 3*b*d^3*f^2*x^2*e + 3*b*d^3*f*x*e^2 + b*d^3*e^3)*log
(cos(d*x + c) - I*sin(d*x + c) + 1) + 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*
c*d^2*f*e^2 - b*d^3*e^3)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)
+ 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*log(-1/2*co
s(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - 2*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(
b*d^3*f*x + b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*e)*log(-cos(d*
x + c) + I*sin(d*x + c) + 1) - 2*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x
+ b*c*d^2*f)*e^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f...
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \cos(c+dx) \cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.326 $\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=557

$$-\frac{(e+fx)^3}{3bf}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{i\sqrt{a^2-b^2}\,(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd}+\frac{i\sqrt{a^2-b^2}\,(e+\ldots}{}$$

```
[Out] -1/3*(f*x+e)^3/b/f-2*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d+2*I*f*(f*x+e)*po
lylog(2,-exp(I*(d*x+c)))/a/d^2-2*I*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^
2-2*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3+2*f^2*polylog(3,exp(I*(d*x+c)))/a/
d^3-I*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2
)/a/b/d+I*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^
(1/2)/a/b/d-2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(
a^2-b^2)^(1/2)/a/b/d^2+2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2
)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d^2-2*I*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(
a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d^3+2*I*f^2*polylog(3,I*b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d^3
```

**Rubi** [A]

time = 0.76, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {4639, 4493, 3377, 2718, 4268, 2611, 2320, 6724, 4621, 32, 3404, 2296, 2221}

$$-\frac{2if^2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^3}+\frac{2if^2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd^3}-\frac{2f\sqrt{a^2-b^2}\,(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^2}+\frac{2f\sqrt{a^2-b^2}\,(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd^2}-\frac{2f^2\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{2f^2\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}+\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{i\sqrt{a^2-b^2}\,(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd}+\frac{i\sqrt{a^2-b^2}\,(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+fx)^3}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(e + f*x)^3/(b*f) - (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) - (
I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(a*b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*
x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*
(c + d*x))])/(a*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d
^2) - (2*Sqrt[a^2 - b^2]*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a -
Sqrt[a^2 - b^2])])/(a*b*d^2) + (2*Sqrt[a^2 - b^2]*f*(e + f*x)*PolyLog[2, (I
*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^2) - (2*f^2*PolyLog[3, -
E^(I*(c + d*x))])/(a*d^3) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) - (
(2*I)*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(a*b*d^3) + ((2*I)*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (I*b*E^(I*(c + d*
x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
```

```
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^2\csc(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{(e}{a+b\,\mathrm{s}}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\int \frac{}{ib}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{2if(e+fx)\mathrm{Li}_2\left(-e^{i}\right.}{ad^2}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)^2}{}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)^2}{}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)^2}{}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)^2}{}$$

**Mathematica** [A]
time = 1.09, size = 917, normalized size = 1.65

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(x*(3*e^2 + 3*e*f*x + f^2*x^2))/b + (-2*d^2*e^2*ArcTanh[E^(I*(c + d*x)
)] + 2*d^2*e*f*x*Log[1 - E^(I*(c + d*x))] + d^2*f^2*x^2*Log[1 - E^(I*(c + d
*x))] - 2*d^2*e*f*x*Log[1 + E^(I*(c + d*x))] - d^2*f^2*x^2*Log[1 + E^(I*(c
+ d*x))] + (2*I)*d*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))] - (2*I)*d*f*(e
+ f*x)*PolyLog[2, E^(I*(c + d*x))] - 2*f^2*PolyLog[3, -E^(I*(c + d*x))] + 2
*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) + (I*Sqrt[a^2 - b^2]*(2*Sqrt[a^2
- b^2]*d*f*(e + f*x)*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(
I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c]
```

```
 + I*Sin[c]) - 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*(Cos[2*c + d*x
] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c
])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c]) - I*(-2*Sqrt[a^2 - b^2]*f^2*PolyLog[
3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2
)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + 2*Sqrt[a^2 - b
^2]*f^2*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] +
 Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c])
+ d^2*(Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(-Log[1 + (b*(Cos[2*c + d*x] + I*Sin
[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Si
n[c])] + Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + S
qrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])])*(Cos[c] + I*Sin[c]) +
 2*e^2*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^2]]*Sq
rt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))))/(a*b*d^3*Sqrt[(-a^2 + b^2)*(Co
s[2*c] + I*Sin[2*c])])
```

**Maple [F]**
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \cos(dx+c)\cot(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2123 vs. 2(489) = 978.
time = 0.63, size = 2123, normalized size = 3.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/6*(2*a*d^3*f^2*x^3 + 6*a*d^3*f*x^2*e + 6*a*d^3*x*e^2 - 6*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*b*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c)) - 6*b*f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c)) + 6*b*f^2*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) + 6*b*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) + 6*(-I*b*d*f^2*x - I*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(I*b*d*f^2*x + I*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(I*b*d*f^2*x + I*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(-I*b*d*f^2*x - I*b*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 3*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 3*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 6*(I*b*d*f^2*x + I*b*d*f*e)*dilog(cos(d*x + c) + I*sin(d*x + c)) + 6*(-I*b*d*f^2*x - I*b*d*f*e)*dilog(cos(d*x + c) - I*sin(d*x + c)) + 6*(I*b*d*f^2*x + I*b*d*f*e)*dilog(-cos(d*x + c) + I*sin(d*x + c)) + 6*(-I*b*d*f^2*x - I*b*d*f*e)*dilog(-cos(d*x + c) - I*sin(d*x + c)) + 3*(b

```
*d^2*f^2*x^2 + 2*b*d^2*f*x*e + b*d^2*e^2)*log(cos(d*x + c) + I*sin(d*x + c)
 + 1) + 3*(b*d^2*f^2*x^2 + 2*b*d^2*f*x*e + b*d^2*e^2)*log(cos(d*x + c) - I*
sin(d*x + c) + 1) - 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*log(-1/2*cos(d*
x + c) + 1/2*I*sin(d*x + c) + 1/2) - 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2
)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - 3*(b*d^2*f^2*x^2 - b*
c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*log(-cos(d*x + c) + I*sin(d*x + c) + 1
) - 3*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*e)*log(-cos(d*x
+ c) - I*sin(d*x + c) + 1))/(a*b*d^3)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.327 $\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=351

$$-\frac{ex}{b}-\frac{fx^2}{2b}-\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd}+\frac{i\sqrt{a^2-b^2}\,(e+fx)\,\mathrm{l}}{}$$

```
[Out] -e*x/b-1/2*f*x^2/b-2*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d+I*f*polylog(2,-exp
(I*(d*x+c)))/a/d^2-I*f*polylog(2,exp(I*(d*x+c)))/a/d^2-I*(f*x+e)*ln(1-I*b*e
xp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d+I*(f*x+e)*ln(1-I*b
*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d-f*polylog(2,I*b*
exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d^2+f*polylog(2,I*b
*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/a/b/d^2
```

**Rubi** [A]
time = 0.44, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {4639, 4493, 3377, 2717, 4268, 2317, 2438, 4621, 3404, 2296, 2221}

$$-\frac{f\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^2}+\frac{f\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd^2}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd}+\frac{i\sqrt{a^2-b^2}\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd}-\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{ex}{b}-\frac{fx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((e*x)/b) - (f*x^2)/(2*b) - (2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) -
 (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(a*b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a*b*d) + (I*f*PolyLog[2, -E^(I*(c + d*x))])/(a*
d^2) - (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (Sqrt[a^2 - b^2]*f*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b*d^2) + (Sqrt[a^2 -
 b^2]*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
```

```
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)\csc(c+dx)\,dx}{a} - \frac{\int (e+fx)\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{e+}{a+b\sin(}$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\int \frac{}{ib+}$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{\left(2i\sqrt{a^2-b^2}\right)\int \frac{}{2a-2}}{}$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)\,\mathrm{lo}}{ab}$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)\,\mathrm{lo}}{ab}$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}\,(e+fx)\,\mathrm{lo}}{ab}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 812 vs. $2(351) = 702$.

time = 4.03, size = 812, normalized size = 2.31

$$\frac{\frac{(c+dx)(cf-d(2e+fx))}{b}+\frac{2de\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a}-\frac{2cf\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a}+\frac{2f\left((c+dx)\left(\log\left(1-e^{i(c+dx)}\right)-\log\left(1+e^{i(c+dx)}\right)\right)+i\left(\text{Li}_2\left(-e^{i(c+dx)}\right)-\text{Li}_2\left(e^{i(c+dx)}\right)\right)\right)}{a}+\frac{2(a^2-b^2)d(e+fx)\left(\frac{2(de-cf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{-ia+b+\sqrt{-a^2+b^2}}\right)+\text{Li}_2\left(\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}+\frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b+\sqrt{-a^2+b^2}}\right)+\text{Li}_2\left(\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}+\frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{-b+\sqrt{-a^2+b^2}-a\tan\left(\frac{1}{2}(c+dx)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)+\text{Li}_2\left(\frac{a\left(i+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ia-b+\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}}-\frac{if\left(\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b-\sqrt{-a^2+b^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{ia+b-\sqrt{-a^2+b^2}}\right)+\text{Li}_2\left(\frac{a+ia\tan\left(\frac{1}{2}(c+dx)\right)}{a+i\left(-b+\sqrt{-a^2+b^2}\right)}\right)\right)}{\sqrt{-a^2+b^2}}\right)}{ab\left(de-cf+if\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)-if\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((c + d*x)*(c*f - d*(2*e + f*x)))/b + (2*d*e*Log[Tan[(c + d*x)/2]])/a - (2
*c*f*Log[Tan[(c + d*x)/2]])/a + (2*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] -
 Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E
^(I*(c + d*x))])))/a + (2*(a^2 - b^2)*d*(e + f*x)*((2*(d*e - c*f)*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*
Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a +
 b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b
 + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c + d*x)/2
]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b
^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2
]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-b + Sqrt[
-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] + PolyLog[2
, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^
2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c
 + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*
x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/(a*b*(d*e - c*
f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]])))/(2
*d^2)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1206 vs. 2(311) = 622.
time = 0.25, size = 1207, normalized size = 3.44

| method | result |
|---|---|
| risch | $-\frac{f x^2}{2b}-\frac{ex}{b}+\frac{e\ln\left(e^{i(dx+c)}-1\right)}{da}-\frac{e\ln\left(e^{i(dx+c)}+1\right)}{da}+\frac{iaf\,\text{dilog}\left(\frac{ia+b\,e^{i(dx+c)}+\sqrt{-a^2+b^2}}{ia+\sqrt{-a^2+b^2}}\right)}{b\,d^2\sqrt{-a^2+b^2}}+\frac{ifb\,\text{dilog}\left(\frac{ia+b\,e^{i(dx+}}{ia-\sqrt{}}\right.}{d^2a\sqrt{-}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f*x^2/b-e*x/b+1/d/a*e*ln(exp(I*(d*x+c))-1)-1/d/a*e*ln(exp(I*(d*x+c))+1
)+2*I/b/d*a*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+
b^2)^(1/2))+I/d^2*f/a*dilog(exp(I*(d*x+c))+1)+I/d^2*f*dilog(exp(I*(d*x+c)))
/a-1/d^2/a*f*c*ln(exp(I*(d*x+c))-1)-I/d^2*f*b/a/(-a^2+b^2)^(1/2)*dilog((I*a
+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+2*I/d^2*f*c/a*b
/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-1
```

```
/d/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(
-a^2+b^2)^(1/2)))*x-1/d^2/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(
-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2*I/b/d^2*a*f*c/(-a^2+b^2)^(1/2)
*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/d/b*a*f/(-a^2+b^
2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))
*x+1/d^2/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/
(I*a-(-a^2+b^2)^(1/2)))*c-2*I/d*e/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*ex
p(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/b/d^2*a*f/(-a^2+b^2)^(1/2)*dilog((I*a
+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-1/d/a*ln(exp(I*
(d*x+c))+1)*f*x-1/d*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b
^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-1/d^2*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b
*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-I/d^2/b*a*f/(-a
^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)
^(1/2)))+1/d*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/
2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*
(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/d^2*f*b/a/(-a^2+b^2)
^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2))
)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(308) = 616$.
time = 0.65, size = 1280, normalized size = 3.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*(a*d^2*f*x^2 + 2*a*d^2*x*e - I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*c
os(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) - b)/b + 1) + I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*
```

```
x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) - b)/b + 1) + I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x +
c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) - b)/b + 1) - I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) -
 a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) + I*b*f*dilog(cos(d*x + c) + I*sin(d*x + c)) - I*b*f*dilog(co
s(d*x + c) - I*sin(d*x + c)) + I*b*f*dilog(-cos(d*x + c) + I*sin(d*x + c))
- I*b*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) + (b*c*f - b*d*e)*sqrt(-(a^2
- b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) + 2*I*a) + (b*c*f - b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x +
 c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*c*f - b
*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2
*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b*c*f - b*d*e)*sqrt(-(a^2 - b^2)/b^2)
*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
2*I*a) + (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) -
a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 - b)/b) - (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c)
- a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2) - b)/b) + (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x +
c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) - b)/b) - (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x
 + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) - b)/b) + (b*d*f*x + b*d*e)*log(cos(d*x + c) + I*sin(d*x + c) + 1)
 + (b*d*f*x + b*d*e)*log(cos(d*x + c) - I*sin(d*x + c) + 1) + (b*c*f - b*d*
e)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + (b*c*f - b*d*e)*log(
-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - (b*d*f*x + b*c*f)*log(-cos(
d*x + c) + I*sin(d*x + c) + 1) - (b*d*f*x + b*c*f)*log(-cos(d*x + c) - I*si
n(d*x + c) + 1))/(a*b*d^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)\cos(c + dx)\cot(c + dx)}{a + b\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] Timed out
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.328 $\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=75

$$-\frac{x}{b}+\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -x/b-arctanh(cos(d*x+c))/a/d+2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))*(a^2-b^2)^(1/2)/a/b/d
```

**Rubi** [A]
time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2968, 3137, 2739, 632, 210, 3855}

$$\frac{2\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x/b) + (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(a*b*d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[C*(x
/(b*d)), x] + (Dist[(A*b^2 + a^2*C)/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f
*x]), x], x] - Dist[(c^2*C + A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f
*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx\\ &= -\frac{x}{b}+\frac{\int \csc(c+dx)\,dx}{a}-\left(-\frac{a}{b}+\frac{b}{a}\right)\int \frac{1}{a+b\sin(c+dx)}\,dx\\ &= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c\right.\right.}{d}\\ &= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\left(4\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2\right.}{d}\\ &= -\frac{x}{b}+\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}\end{aligned}$$

**Mathematica [A]**
time = 0.09, size = 90, normalized size = 1.20

$$-\frac{ac+adx-2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*c + a*d*x - 2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]] + b*Log[Cos[(c + d*x)/2]] - b*Log[Sin[(c + d*x)/2]])/(a*b*d))
```

**Maple [A]**
time = 0.16, size = 94, normalized size = 1.25

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\left(2a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ab\sqrt{a^2-b^2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$ |
| default | $\frac{\frac{\left(2a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ab\sqrt{a^2-b^2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$ |
| risch | $-\frac{x}{b}+\frac{i\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}+\frac{i\left(a+\sqrt{a^2-b^2}\right)}{b}\right)}{dba}-\frac{i\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}-\frac{i\left(-a+\sqrt{a^2-b^2}\right)}{b}\right)}{dba}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((2*a^2-2*b^2)/a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2))+1/a*ln(tan(1/2*d*x+1/2*c))-2/b*arctan(tan(1/2*d*x+1/2*
c)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.40, size = 262, normalized size = 3.49

$$\left[-\frac{2\,adx+b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2-2\,\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2}\right)}{2\,abd},\ -\frac{2\,adx+b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2\,\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^2-b^2}\,\cos\left(dx+c\right)}\right)}{2\,abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) - sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b*d),
 -1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(
d*x + c))))/(a*b*d)]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(c + dx \right)} \cot{\left(c + dx \right)}}{a + b \sin{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 5.24, size = 94, normalized size = 1.25

$$-\frac{\frac{dx+c}{b} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/b - log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*
sqrt(a^2 - b^2)/(a*b))/d
```

**Mupad** [B]
time = 5.39, size = 896, normalized size = 11.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + (2*atan((64*a^3)/(64*a^2*b - 64*b^3 + 64*a^
3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2)) - (64*a*b^2)/(64*a^2*b
- 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2)) + (64*b
^3*tan(c/2 + (d*x)/2))/(64*a^2*b - 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*
```

a*b^2*tan(c/2 + (d*x)/2)) - (64*a^2*b*tan(c/2 + (d*x)/2))/(64*a^2*b - 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2))))/(b*d) - (2*atanh((64*a^2*(b^2 - a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a^3*tan(c/2 + (d*x)/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 + (d*x)/2))/a + (1024*b^6*tan(c/2 + (d*x)/2))/a^3) - (512*b^2*(b^2 - a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a^3*tan(c/2 + (d*x)/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 + (d*x)/2))/a + (1024*b^6*tan(c/2 + (d*x)/2))/a^3) + (512*b^4*(b^2 - a^2)^(1/2))/(256*a^4*b + 512*b^5 - 768*a^2*b^3 - 64*a^5*tan(c/2 + (d*x)/2) - 1792*a*b^4*tan(c/2 + (d*x)/2) + 832*a^3*b^2*tan(c/2 + (d*x)/2) + (1024*b^6*tan(c/2 + (d*x)/2))/a) - (1280*b^3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(256*a^3*b - 768*a*b^3 + (512*b^5)/a - 64*a^4*tan(c/2 + (d*x)/2) - 1792*b^4*tan(c/2 + (d*x)/2) + 832*a^2*b^2*tan(c/2 + (d*x)/2) + (1024*b^6*tan(c/2 + (d*x)/2))/a^2) + (1024*b^5*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(512*a*b^5 + 256*a^5*b - 768*a^3*b^3 - 64*a^6*tan(c/2 + (d*x)/2) + 1024*b^6*tan(c/2 + (d*x)/2) - 1792*a^2*b^4*tan(c/2 + (d*x)/2) + 832*a^4*b^2*tan(c/2 + (d*x)/2)) + (320*a*b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a^3*tan(c/2 + (d*x)/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 + (d*x)/2))/a + (1024*b^6*tan(c/2 + (d*x)/2))/a^3))*(b^2 - a^2)^(1/2))/(a*b*d)

# 3.329 $\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=763

$$-\frac{i(e+fx)^4}{4af}-\frac{i(a^2-b^2)(e+fx)^4}{4ab^2f}+\frac{6f^3\cos(c+dx)}{bd^4}-\frac{3f(e+fx)^2\cos(c+dx)}{bd^2}+\frac{(a^2-b^2)(e+fx)^3\log\Big(1-}{ab^2d}$$

```
[Out] -3*I*(a^2-b^2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2))
)/a/b^2/d^2-3/2*I*f*(f*x+e)^2*polylog(2,exp(2*I*(d*x+c)))/a/d^2+6*f^3*cos(d
*x+c)/b/d^4-3*f*(f*x+e)^2*cos(d*x+c)/b/d^2+(f*x+e)^3*ln(1-exp(2*I*(d*x+c)))
/a/d+(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2
/d+(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d
+6*I*(a^2-b^2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/
d^4+3/4*I*f^3*polylog(4,exp(2*I*(d*x+c)))/a/d^4+6*I*(a^2-b^2)*f^3*polylog(4
,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d^4+3/2*f^2*(f*x+e)*polylog(
3,exp(2*I*(d*x+c)))/a/d^3+6*(a^2-b^2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+
c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d^3+6*(a^2-b^2)*f^2*(f*x+e)*polylog(3,I*b*ex
p(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d^3-1/4*I*(f*x+e)^4/a/f-3*I*(a^2-b^
2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d^2-
1/4*I*(a^2-b^2)*(f*x+e)^4/a/b^2/f+6*f^2*(f*x+e)*sin(d*x+c)/b/d^3-(f*x+e)^3*
sin(d*x+c)/b/d
```

**Rubi** [A]
time = 0.91, antiderivative size = 763, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4639, 4493, 4489, 3392, 32, 2715, 8, 3798, 2221, 2611, 6744, 2320, 6724, 4621, 3377, 2718, 4615}

$$\frac{6if^3(a^2-b^2)\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^4}+\frac{6if^3(a^2-b^2)\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^4}+\frac{6f^2(a^2-b^2)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^3}+\frac{6f^2(a^2-b^2)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^3}-\frac{3if(a^2-b^2)(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2}-\frac{3if(a^2-b^2)(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^2}+\frac{3if^3\text{PolyLog}(4,e^{2i(c+dx)})}{4ad^4}+\frac{3f^2(e+fx)\text{PolyLog}(3,e^{2i(c+dx)})}{2ad^3}-\frac{3if(e+fx)^2\text{PolyLog}(2,e^{2i(c+dx)})}{2ad^2}+\frac{(a^2-b^2)(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d}+\frac{(a^2-b^2)(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d}-\frac{i(a^2-b^2)(e+fx)^4}{4ab^2f}+\frac{(e+fx)^3\log(1-e^{2i(c+dx)})}{ad}-\frac{i(e+fx)^4}{4af}+\frac{6f^3\cos(c+dx)}{bd^4}+\frac{6f^2(e+fx)\sin(c+dx)}{bd^3}-\frac{3f(e+fx)^2\cos(c+dx)}{bd^2}-\frac{(e+fx)^3\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-1/4*I)*(e + f*x)^4)/(a*f) - ((I/4)*(a^2 - b^2)*(e + f*x)^4)/(a*b^2*f) +
(6*f^3*Cos[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x])/(b*d^2) + ((a
^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])
/(a*b^2*d) + ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/(a*b^2*d) + ((e + f*x)^3*Log[1 - E^((2*I)*(c + d*x))])/(a*
d) - ((3*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a -
 Sqrt[a^2 - b^2])])/(a*b^2*d^2) - ((3*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[
2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^2) - (((3*I)/2)*f
*(e + f*x)^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^2) + (6*(a^2 - b^2)*f^2*
(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d
```

```
^3) + (6*(a^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/(a*b^2*d^3) + (3*f^2*(e + f*x)*PolyLog[3, E^((2*I)*(c + d*
x))])/(2*a*d^3) + ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])/(a*b^2*d^4) + ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^4) + (((3*I)/4)*f^3*Poly
Log[4, E^((2*I)*(c + d*x))])/(a*d^4) + (6*f^2*(e + f*x)*Sin[c + d*x])/(b*d^
3) - ((e + f*x)^3*Sin[c + d*x])/(b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
```

```
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos^2(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^3\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\cos(c+dx)\,dx}{b} + \left(\frac{a}{b} -\right.$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i(a^2-b^2)(e+fx)^4}{4ab^2f} - \frac{(e+fx)^3\sin(c+dx)}{bd} - \frac{(2}{}$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i(a^2-b^2)(e+fx)^4}{4ab^2f} - \frac{3f(e+fx)^2\cos(c+dx)}{bd^2} +$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i(a^2-b^2)(e+fx)^4}{4ab^2f} - \frac{3f(e+fx)^2\cos(c+dx)}{bd^2} +$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i(a^2-b^2)(e+fx)^4}{4ab^2f} + \frac{6f^3\cos(c+dx)}{bd^4} - \frac{3f(e+}{}$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i(a^2-b^2)(e+fx)^4}{4ab^2f} + \frac{6f^3\cos(c+dx)}{bd^4} - \frac{3f(e+}{}$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i(a^2-b^2)(e+fx)^4}{4ab^2f} + \frac{6f^3\cos(c+dx)}{bd^4} - \frac{3f(e+}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3808 vs. $2(763) = 1526$.
time = 8.56, size = 3808, normalized size = 4.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(e*f^2*Csc[c]*(2*d^2*x^2*(2*d*E^((2*I)*c)*x + (3*I)*(-1 + E^((2*I)*c))
*Log[1 - E^((2*I)*(c + d*x))]) + 6*d*(-1 + E^((2*I)*c))*x*PolyLog[2, E^((2*
I)*(c + d*x))] + (3*I)*(-1 + E^((2*I)*c))*PolyLog[3, E^((2*I)*(c + d*x))]))
/(a*d^3*E^(I*c)) - (E^(I*c)*f^3*Csc[c]*(x^4 + (-1 + E^((-2*I)*c))*x^4 + ((-
1 + E^((2*I)*c))*(2*d^4*x^4 + (4*I)*d^3*x^3*Log[1 - E^((2*I)*(c + d*x))] +
6*d^2*x^2*PolyLog[2, E^((2*I)*(c + d*x))] + (6*I)*d*x*PolyLog[3, E^((2*I)*(
```

```
c + d*x))] - 3*PolyLog[4, E^((2*I)*(c + d*x))]))/(2*d^4*E^((2*I)*c))))/(4*a
) + ((a^2 - b^2)*((-4*I)*d^4*e^3*E^((2*I)*c)*x - (6*I)*d^4*e^2*E^((2*I)*c)*
f*x^2 - (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 - I*d^4*E^((2*I)*c)*f^3*x^4 - (2*I)
*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + (2*
I)*d^3*e^3*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c +
d*x))))] - d^3*e^3*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c +
d*x)))^2] + d^3*e^3*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E
^((2*I)*(c + d*x)))^2] - 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1
 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] -
6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b
^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x
)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*f^3*x^3*Log[1 +
 (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*
d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(
-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I
*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e^2*E^((2*I)*c)*f*x*L
og[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
] - 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2
 + b^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c +
 d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*f^3*x^3*Log
[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
+ 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])] - (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)^2
*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*
I)*c)])] - (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(I*
(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*e*f^2
*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*
I)*c)])] + 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(
I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*f^3*x*PolyLog[3, (I*b*E^(I
*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*E^((2
*I)*c)*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)])] - 12*d*e*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*e*E^((2*I)*c)*f^2*PolyLog
[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))
] - 12*d*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2
 + b^2)*E^((2*I)*c)]))] + 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, -((b*E^(I*(2*c
+ d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (12*I)*f^3*Poly
Log[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)
])] + (12*I)*E^((2*I)*c)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c)
+ I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*f^3*PolyLog[4, -((b*E^(I*(2*c
 + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (12*I)*E^((2*I
)*c)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2
)*E^((2*I)*c)]))]))/(2*a*b^2*d^4*(-1 + E^((2*I)*c))) + (e^3*Csc[c]*(-(d*x*C
os[c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*d*(Cos[c]^2 + S
```

```
in[c]^2)) + Csc[c]*(Cos[c + d*x]/(8*b^2*d^4) - ((I/8)*Sin[c + d*x])/(b^2*d^
4))*(4*a*d^4*e^3*x*Cos[d*x] + 6*a*d^4*e^2*f*x^2*Cos[d*x] + 4*a*d^4*e*f^2*x^
3*Cos[d*x] + a*d^4*f^3*x^4*Cos[d*x] + 4*a*d^4*e^3*x*Cos[2*c + d*x] + 6*a*d^
4*e^2*f*x^2*Cos[2*c + d*x] + 4*a*d^4*e*f^2*x^3*Cos[2*c + d*x] + a*d^4*f^3*x
^4*Cos[2*c + d*x] - 2*b*d^3*e^3*Cos[c + 2*d*x] - (6*I)*b*d^2*e^2*f*Cos[c +
2*d*x] + 12*b*d*e*f^2*Cos[c + 2*d*x] + (12*I)*b*f^3*Cos[c + 2*d*x] - 6*b*d^
3*e^2*f*x*Cos[c + 2*d*x] - (12*I)*b*d^2*e*f^2*x*Cos[c + 2*d*x] + 12*b*d*f^3
*x*Cos[c + 2*d*x] - 6*b*d^3*e*f^2*x^2*Cos[c + 2*d*x] - (6*I)*b*d^2*f^3*x^2*
Cos[c + 2*d*x] - 2*b*d^3*f^3*x^3*Cos[c + 2*d*x] + 2*b*d^3*e^3*Cos[3*c + 2*d
*x] + (6*I)*b*d^2*e^2*f*Cos[3*c + 2*d*x] - 12*b*d*e*f^2*Cos[3*c + 2*d*x] -
(12*I)*b*f^3*Cos[3*c + 2*d*x] + 6*b*d^3*e^2*f*x*Cos[3*c + 2*d*x] + (12*I)*b
*d^2*e*f^2*x*Cos[3*c + 2*d*x] - 12*b*d*f^3*x*Cos[3*c + 2*d*x] + 6*b*d^3*e*f
^2*x^2*Cos[3*c + 2*d*x] + (6*I)*b*d^2*f^3*x^2*Cos[3*c + 2*d*x] + 2*b*d^3*f^
3*x^3*Cos[3*c + 2*d*x] - (4*I)*b*d^3*e^3*Sin[c] - 12*b*d^2*e^2*f*Sin[c] + (
24*I)*b*d*e*f^2*Sin[c] + 24*b*f^3*Sin[c] - (12*I)*b*d^3*e^2*f*x*Sin[c] - 24
*b*d^2*e*f^2*x*Sin[c] + (24*I)*b*d*f^3*x*Sin[c]...
```

**Maple** [**F**]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\cos^2(dx+c)) \cot(dx+c)}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3448 vs. 2(702) = 1404.
time = 0.76, size = 3448, normalized size = 4.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*(6*I*b^2*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c)) - 6*I*b^2*f^3*po
lylog(4, cos(d*x + c) - I*sin(d*x + c)) - 6*I*b^2*f^3*polylog(4, -cos(d*x +
 c) + I*sin(d*x + c)) + 6*I*b^2*f^3*polylog(4, -cos(d*x + c) - I*sin(d*x +
c)) - 6*I*(a^2 - b^2)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) +
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*(a^2 -
 b^2)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c)
- I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*I*(a^2 - b^2)*f^3*polylo
g(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*I*(a^2 - b^2)*f^3*polylog(4, -(-I*a*cos
(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2))/b) - 6*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*f^2*x*e + a*b*d^2*f*e^2 -
 2*a*b*f^3)*cos(d*x + c) - 3*(I*(a^2 - b^2)*d^2*f^3*x^2 + 2*I*(a^2 - b^2)*d
^2*f^2*x*e + I*(a^2 - b^2)*d^2*f*e^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
 c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) - 3*(I*(a^2 - b^2)*d^2*f^3*x^2 + 2*I*(a^2 - b^2)*d^2*f^2*x*e + I*(a^2 - b
^2)*d^2*f*e^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(-I*(a^2 - b^2)*d
^2*f^3*x^2 - 2*I*(a^2 - b^2)*d^2*f^2*x*e - I*(a^2 - b^2)*d^2*f*e^2)*dilog((
-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(-I*(a^2 - b^2)*d^2*f^3*x^2 - 2*I*(a^2
 - b^2)*d^2*f^2*x*e - I*(a^2 - b^2)*d^2*f*e^2)*dilog((-I*a*cos(d*x + c) - a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - 3*(I*b^2*d^2*f^3*x^2 + 2*I*b^2*d^2*f^2*x*e + I*b^2*d^2*f*e^2)
*dilog(cos(d*x + c) + I*sin(d*x + c)) - 3*(-I*b^2*d^2*f^3*x^2 - 2*I*b^2*d^2
*f^2*x*e - I*b^2*d^2*f*e^2)*dilog(cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*b^
2*d^2*f^3*x^2 - 2*I*b^2*d^2*f^2*x*e - I*b^2*d^2*f*e^2)*dilog(-cos(d*x + c)
+ I*sin(d*x + c)) - 3*(I*b^2*d^2*f^3*x^2 + 2*I*b^2*d^2*f^2*x*e + I*b^2*d^2*
f*e^2)*dilog(-cos(d*x + c) - I*sin(d*x + c)) - ((a^2 - b^2)*c^3*f^3 - 3*(a^
2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e^2 - (a^2 - b^2)*d^3*e^3)*log
(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a
) - ((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*
f*e^2 - (a^2 - b^2)*d^3*e^3)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - ((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^
2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e^2 - (a^2 - b^2)*d^3*e^3)*log(-2*b*cos(d
*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - ((a^2
- b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e^2 - (a
^2 - b^2)*d^3*e^3)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(
a^2 - b^2)/b^2) - 2*I*a) + ((a^2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*c^3*f^3 +
 3*((a^2 - b^2)*d^3*f*x + (a^2 - b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2)*d^3*f^2
*x^2 - (a^2 - b^2)*c^2*d*f^2)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) +
```

```
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + ((a^2
- b^2)*d^3*f^3*x^3 + (a^2 - b^2)*c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x + (a^2 -
b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*d*f^2)*e)*
log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + ((a^2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*
c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x + (a^2 - b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2
)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*d*f^2)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d
*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
) + ((a^2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x
 + (a^2 - b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*
d*f^2)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3
*f^2*x^2*e + 3*b^2*d^3*f*x*e^2 + b^2*d^3*e^3)*log(cos(d*x + c) + I*sin(d*x
+ c) + 1) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*f^2*x^2*e + 3*b^2*d^3*f*x*e^2 + b^
2*d^3*e^3)*log(cos(d*x + c) - I*sin(d*x + c) + 1) - (b^2*c^3*f^3 - 3*b^2*c^
2*d*f^2*e + 3*b^2*c*d^2*f*e^2 - b^2*d^3*e^3)*log(-1/2*cos(d*x + c) + 1/2*I*
sin(d*x + c) + 1/2) - (b^2*c^3*f^3 - 3*b^2*c^2*d*f^2*e + 3*b^2*c*d^2*f*e^2
- b^2*d^3*e^3)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + (b^2*d^3
*f^3*x^3 + b^2*c^3*f^3 + 3*(b^2*d^3*f*x + b^2*c*d^2*f)*e^2 + 3*(b^2*d^3*f^2
*x^2 - b^2*c^2*d*f^2)*e)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + (b^2*d^3
*f^3*x^3 + b^2*c^3*f^3 + 3*(b^2*d^3*f*x + b^2*c*d^2*f)*e^2 + 3*(b^2*d^3*f^2
*x^2 - b^2*c^2*d*f^2)*e)*log(-cos(d*x + c) - I*sin(d*x + c) + 1) + 6*((a^2
- b^2)*d*f^3*x + (a^2 - b^2)*d*f^2*e)*polylog(3, -(I*a*cos(d*x + c) + a*sin
(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
+ 6*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*f^2*e)*polylog(3, -(I*a*cos(d*x +
c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b) + 6*((a^2 - b^2)*d*f^3*x + (a^2 - b^2...
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

## 3.330 $\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=566

$$-\frac{i(e+fx)^3}{3af}-\frac{i(a^2-b^2)(e+fx)^3}{3ab^2f}-\frac{2f(e+fx)\cos(c+dx)}{bd^2}+\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d}+$$

```
[Out] -1/3*I*(f*x+e)^3/a/f-1/3*I*(a^2-b^2)*(f*x+e)^3/a/b^2/f-2*f*(f*x+e)*cos(d*x+
c)/b/d^2+(f*x+e)^2*ln(1-exp(2*I*(d*x+c)))/a/d+(a^2-b^2)*(f*x+e)^2*ln(1-I*b*
exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d+(a^2-b^2)*(f*x+e)^2*ln(1-I*b*ex
p(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d-I*f*(f*x+e)*polylog(2,exp(2*I*(d*
x+c)))/a/d^2-2*I*(a^2-b^2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b
^2)^(1/2)))/a/b^2/d^2-2*I*(a^2-b^2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/a/b^2/d^2+1/2*f^2*polylog(3,exp(2*I*(d*x+c)))/a/d^3+2*
(a^2-b^2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d^3+2
*(a^2-b^2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d^3+
2*f^2*sin(d*x+c)/b/d^3-(f*x+e)^2*sin(d*x+c)/b/d
```

**Rubi** [A]
time = 0.72, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {4639, 4493, 4489, 3391, 3798, 2221, 2611, 2320, 6724, 4621, 3377, 2717, 4615}

$$\frac{2f^2(a^2-b^2)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^3}+\frac{2f^2(a^2-b^2)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^3}-\frac{2if(a^2-b^2)(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2}-\frac{2if(a^2-b^2)(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^2}+\frac{f^2\operatorname{PolyLog}(3,e^{2i(c+dx)})}{2ad^3}-\frac{if(e+fx)\operatorname{PolyLog}(2,e^{2i(c+dx)})}{ad^2}+\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d}+\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d}-\frac{i(a^2-b^2)(e+fx)^3}{3ab^2f}+\frac{(e+fx)^2\log\left(1-e^{2i(c+dx)}\right)}{ad}-\frac{i(e+fx)^3}{3af}+\frac{2f^2\sin(c+dx)}{bd^3}-\frac{2f(e+fx)\cos(c+dx)}{bd^2}-\frac{(e+fx)^2\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-1/3*I)*(e + f*x)^3)/(a*f) - ((I/3)*(a^2 - b^2)*(e + f*x)^3)/(a*b^2*f) -
(2*f*(e + f*x)*Cos[c + d*x])/(b*d^2) + ((a^2 - b^2)*(e + f*x)^2*Log[1 - (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d) + ((a^2 - b^2)*(e + f*
x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d) + ((e
+ f*x)^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d) - ((2*I)*(a^2 - b^2)*f*(e + f*
x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d^2) - (
(2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^
2 - b^2])])/(a*b^2*d^2) - (I*f*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x))])/(
a*d^2) + (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(a*b^2*d^3) + (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))
/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^3) + (f^2*PolyLog[3, E^((2*I)*(c + d*x))]
)/(2*a*d^3) + (2*f^2*Sin[c + d*x])/(b*d^3) - ((e + f*x)^2*Sin[c + d*x])/(b*
d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos^2(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^2\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\cos(c+dx)\,dx}{b} + \left(\frac{a}{b}\right.$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i(a^2-b^2)(e+fx)^3}{3ab^2f} - \frac{(e+fx)^2\sin(c+dx)}{bd} -$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i(a^2-b^2)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cos(c+dx)}{bd^2} -$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i(a^2-b^2)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cos(c+dx)}{bd^2} -$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i(a^2-b^2)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cos(c+dx)}{bd^2} -$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i(a^2-b^2)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cos(c+dx)}{bd^2} -$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1740 vs. 2(566) = 1132.
time = 7.99, size = 1740, normalized size = 3.07

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/12*(f^2*Csc[c]*(2*d^2*x^2*(2*d*E^((2*I)*c)*x + (3*I)*(-1 + E^((2*I)*c))*
Log[1 - E^((2*I)*(c + d*x))]) + 6*d*(-1 + E^((2*I)*c))*x*PolyLog[2, E^((2*I
)*(c + d*x))] + (3*I)*(-1 + E^((2*I)*c))*PolyLog[3, E^((2*I)*(c + d*x))]))/
(a*d^3*E^(I*c)) + ((a^2 - b^2)*((-12*I)*d^3*e^2*E^((2*I)*c)*x - (12*I)*d^3*
e*E^((2*I)*c)*f*x^2 - (4*I)*d^3*E^((2*I)*c)*f^2*x^3 - (6*I)*d^2*e^2*ArcTan[
(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + (6*I)*d^2*e^2*E^((2
```

```
*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - 3*d^2
*e^2*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] + 3*
d^2*e^2*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c +
 d*x)))^2] - 12*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt
[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*
c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^2*f^2*x^2*
Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]
)] + 6*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) -
 Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x
)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d^2*e*E^((2*I)*c)*
f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)
*c)])] - 6*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-
a^2 + b^2)*E^((2*I)*c)])] + 6*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c
+ d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*d*(-1 + E
^((2*I)*c))*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*S
qrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*d*(-1 + E^((2*I)*c))*f*(e + f*x)*P
olyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)
*c)]))] - 12*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a
^2 + b^2)*E^((2*I)*c)])] + 12*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d
*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*f^2*PolyLog[3, -
((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 1
2*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-
a^2 + b^2)*E^((2*I)*c)]))]))/(6*a*b^2*d^3*(-1 + E^((2*I)*c))) + (a*x*(3*e^2
 + 3*e*f*x + f^2*x^2)*Cos[c]*Csc[c/2]*Sec[c/2])/(6*b^2) - (Cos[d*x]*(2*d*e*
f*Cos[c] + 2*d*f^2*x*Cos[c] + d^2*e^2*Sin[c] - 2*f^2*Sin[c] + 2*d^2*e*f*x*S
in[c] + d^2*f^2*x^2*Sin[c]))/(b*d^3) + (e^2*Csc[c]*(-(d*x*Cos[c]) + Log[Cos
[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*d*(Cos[c]^2 + Sin[c]^2)) - ((d^
2*e^2*Cos[c] - 2*f^2*Cos[c] + 2*d^2*e*f*x*Cos[c] + d^2*f^2*x^2*Cos[c] - 2*d
*e*f*Sin[c] - 2*d*f^2*x*Sin[c])*Sin[d*x])/(b*d^3) - (e*f*Csc[c]*Sec[c]*(d^2
*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi + 2*ArcTan[Tan[c]]) - Pi*Log[1 + E
^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*Log[1 - E^((2*I)*(d*x + ArcTan[Ta
n[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan[c]]*Log[Sin[d*x + ArcTan[Tan[c]]
]] + I*PolyLog[2, E^((2*I)*(d*x + ArcTan[Tan[c]]))]))*Tan[c])/Sqrt[1 + Tan[c
]^2]))/(a*d^2*Sqrt[Sec[c]^2*(Cos[c]^2 + Sin[c]^2)])
```

**Maple [F]**

time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 (\cos^2(dx+c)) \cot(dx+c)}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2275 vs. 2(521) = 1042.
time = 0.68, size = 2275, normalized size = 4.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*(2*b^2*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 2*b^2*f^2*polylo
g(3, cos(d*x + c) - I*sin(d*x + c)) + 2*b^2*f^2*polylog(3, -cos(d*x + c) +
I*sin(d*x + c)) + 2*b^2*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) + 2*
(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2 - b^2)*f^2*po
lylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2 - b^2)*f^2*polylog(3, -(-I*a*co
s(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2))/b) + 2*(a^2 - b^2)*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*si
n(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 - 4*(a*b*d*f^2*x + a*b*d*f*e)*cos(d*x + c) - 2*(I*(a^2 - b^2)*d*f^2*x + I*
(a^2 - b^2)*d*f*e)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x +
c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*(a^2 - b^2
)*d*f^2*x + I*(a^2 - b^2)*d*f*e)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) -
 (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2
*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*d*f*e)*dilog((-I*a*cos(d*x + c) -
a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 - b)/b + 1) - 2*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*d*f*e)*dilog((-I*a
*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*b^2*d*f^2*x + I*b^2*d*f*e)*dilog(cos(d*
x + c) + I*sin(d*x + c)) - 2*(-I*b^2*d*f^2*x - I*b^2*d*f*e)*dilog(cos(d*x +
```

```
 c) - I*sin(d*x + c)) - 2*(-I*b^2*d*f^2*x - I*b^2*d*f*e)*dilog(-cos(d*x + c
) + I*sin(d*x + c)) - 2*(I*b^2*d*f^2*x + I*b^2*d*f*e)*dilog(-cos(d*x + c) -
 I*sin(d*x + c)) + ((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^
2)*d^2*e^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) + 2*I*a) + ((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^
2)*d^2*e^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) - 2*I*a) + ((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^
2)*d^2*e^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b
^2)/b^2) + 2*I*a) + ((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b
^2)*d^2*e^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
b^2)/b^2) - 2*I*a) + ((a^2 - b^2)*d^2*f^2*x^2 - (a^2 - b^2)*c^2*f^2 + 2*((a
^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d*f)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*
x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)
 + ((a^2 - b^2)*d^2*f^2*x^2 - (a^2 - b^2)*c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x
+ (a^2 - b^2)*c*d*f)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + ((a^2 - b^2)*d^
2*f^2*x^2 - (a^2 - b^2)*c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d*
f)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + ((a^2 - b^2)*d^2*f^2*x^2 - (a^2
- b^2)*c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d*f)*e)*log(-(-I*a*
cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*f*x*e + b^2*d^2*e^2)
*log(cos(d*x + c) + I*sin(d*x + c) + 1) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*f*x*
e + b^2*d^2*e^2)*log(cos(d*x + c) - I*sin(d*x + c) + 1) + (b^2*c^2*f^2 - 2*
b^2*c*d*f*e + b^2*d^2*e^2)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2
) + (b^2*c^2*f^2 - 2*b^2*c*d*f*e + b^2*d^2*e^2)*log(-1/2*cos(d*x + c) - 1/2
*I*sin(d*x + c) + 1/2) + (b^2*d^2*f^2*x^2 - b^2*c^2*f^2 + 2*(b^2*d^2*f*x +
b^2*c*d*f)*e)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + (b^2*d^2*f^2*x^2 -
b^2*c^2*f^2 + 2*(b^2*d^2*f*x + b^2*c*d*f)*e)*log(-cos(d*x + c) - I*sin(d*x
+ c) + 1) - 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*f*x*e + a*b*d^2*e^2 - 2*a*b*f^2)
*sin(d*x + c))/(a*b^2*d^3)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.331 $\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=379

$$-\frac{i(e+fx)^2}{2af}-\frac{i(a^2-b^2)\,(e+fx)^2}{2ab^2f}-\frac{f\cos(c+dx)}{bd^2}+\frac{(a^2-b^2)\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d}+\frac{(a^2-b^2)\,(e$$

```
[Out] -1/2*I*(f*x+e)^2/a/f-1/2*I*(a^2-b^2)*(f*x+e)^2/a/b^2/f-f*cos(d*x+c)/b/d^2+(
f*x+e)*ln(1-exp(2*I*(d*x+c)))/a/d+(a^2-b^2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))
/(a-(a^2-b^2)^(1/2)))/a/b^2/d+(a^2-b^2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+
(a^2-b^2)^(1/2)))/a/b^2/d-1/2*I*f*polylog(2,exp(2*I*(d*x+c)))/a/d^2-I*(a^2-
b^2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d^2-I*(a^2-b
^2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d^2-(f*x+e)*s
in(d*x+c)/b/d
```

**Rubi** [A]
time = 0.41, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {4639, 4493, 4489, 2715, 8, 3798, 2221, 2317, 2438, 4621, 3377, 2718, 4615}

$$-\frac{if(a^2-b^2)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2}-\frac{if(a^2-b^2)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^2}-\frac{if\text{PolyLog}(2,e^{2i(c+dx)})}{2ad^2}+\frac{(a^2-b^2)\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d}+\frac{(a^2-b^2)\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d}-\frac{i(a^2-b^2)\,(e+fx)^2}{2ab^2f}+\frac{(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{ad}-\frac{i(e+fx)^2}{2af}-\frac{f\cos(c+dx)}{bd^2}-\frac{(e+fx)\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-1/2*I)*(e + f*x)^2)/(a*f) - ((I/2)*(a^2 - b^2)*(e + f*x)^2)/(a*b^2*f) -
(f*Cos[c + d*x])/(b*d^2) + ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d) + ((a^2 - b^2)*(e + f*x)*Log[1 - (I*
b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d) + ((e + f*x)*Log[1 - E
^((2*I)*(c + d*x))])/(a*d) - (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d^2) - (I*(a^2 - b^2)*f*PolyLog[2, (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^2) - ((I/2)*f*PolyLog[2,
E^((2*I)*(c + d*x))])/(a*d^2) - ((e + f*x)*Sin[c + d*x])/(b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos^2(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)\cos(c+dx)\,dx}{b} + \left(\frac{a}{b} - \frac{b}{a}\right.$$

$$= -\frac{i(e+fx)^2}{2af} - \frac{i(a^2-b^2)(e+fx)^2}{2ab^2f} - \frac{(e+fx)\sin(c+dx)}{bd} - \frac{(2}{}$$

$$= -\frac{i(e+fx)^2}{2af} - \frac{i(a^2-b^2)(e+fx)^2}{2ab^2f} - \frac{f\cos(c+dx)}{bd^2} + \frac{(a^2-b^2)}{}$$

$$= -\frac{i(e+fx)^2}{2af} - \frac{i(a^2-b^2)(e+fx)^2}{2ab^2f} - \frac{f\cos(c+dx)}{bd^2} + \frac{(a^2-b^2)}{}$$

$$= -\frac{i(e+fx)^2}{2af} - \frac{i(a^2-b^2)(e+fx)^2}{2ab^2f} - \frac{f\cos(c+dx)}{bd^2} + \frac{(a^2-b^2)}{}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2209 vs. 2(379) = 758.
time = 13.60, size = 2209, normalized size = 5.83

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((f*Cos[c + d*x])/(b*d^2)) + (e*Log[Sin[c + d*x]])/(a*d) - (c*f*Log[Sin[c
+ d*x]])/(a*d^2) + (f*((c + d*x)*Log[1 - E^((2*I)*(c + d*x))] - (I/2)*((c +
 d*x)^2 + PolyLog[2, E^((2*I)*(c + d*x))])))/(a*d^2) - ((d*e - c*f + f*(c +
 d*x))*Sin[c + d*x])/(b*d^2) + ((f*(c + d*x)^2 + (2*I)*d*e*Log[Sec[(c + d*x
)/2]^2] - (2*I)*c*f*Log[Sec[(c + d*x)/2]^2] - (2*I)*d*e*Log[Sec[(c + d*x)/2
]^2*(a + b*Sin[c + d*x])] + (2*I)*c*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c +
 d*x])] - (4*I)*f*(c + d*x)*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - 2*f*Log[1
 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a
 + b - Sqrt[-a^2 + b^2])] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt
[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))] + 2*f*Log
[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((
-I)*a + b + Sqrt[-a^2 + b^2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + S
qrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + 4*f*P
```

```
olyLog[2, -Cos[c + d*x] + I*Sin[c + d*x]] + 2*f*PolyLog[2, (a*(1 - I*Tan[(c
 + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))] - 2*f*PolyLog[2, (a*(1 + I*Tan
[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))] + 2*f*PolyLog[2, (a*(I + Ta
n[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])] - 2*f*PolyLog[2, (a + I*a*Ta
n[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]])*((a*e*Cos[c + d*x])/(b*(a
 + b*Sin[c + d*x])) - (b*e*Cos[c + d*x])/(a*(a + b*Sin[c + d*x])) - (a*c*f*
Cos[c + d*x])/(b*d*(a + b*Sin[c + d*x])) + (b*c*f*Cos[c + d*x])/(a*d*(a + b
*Sin[c + d*x])) + (a*f*(c + d*x)*Cos[c + d*x])/(b*d*(a + b*Sin[c + d*x])) -
 (b*f*(c + d*x)*Cos[c + d*x])/(a*d*(a + b*Sin[c + d*x]))))/(d*(2*f*(c + d*x
) - (4*I)*f*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - (4*f*Log[1 + Cos[c + d*x]
 - I*Sin[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/(-Cos[c + d*x] + I*Sin[
c + d*x]) + (I*f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2
 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[-((b - S
qrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c
 + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a
*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1
- I*Tan[(c + d*x)/2]) + (I*f*Log[1 - (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b
 + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*
Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2]
)]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 +
 b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^
2)/(1 + I*Tan[(c + d*x)/2]) + (2*I)*d*e*Tan[(c + d*x)/2] - (2*I)*c*f*Tan[(c
 + d*x)/2] + ((2*I)*f*(c + d*x)*Sec[(c + d*x)/2]^2)/(-I + Tan[(c + d*x)/2])
 - (f*Log[1 - (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[
(c + d*x)/2]^2)/(I + Tan[(c + d*x)/2]) + (I*a*f*Log[1 - (a + I*a*Tan[(c + d
*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(
c + d*x)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sq
rt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec
[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) + (a*f*Log[1 -
 I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c +
 d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[
-a^2 + b^2] + a*Tan[(c + d*x)/2]) - ((2*I)*d*e*Cos[(c + d*x)/2]^2*(b*Cos[c
+ d*x]*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c
+ d*x)/2]))/(a + b*Sin[c + d*x]) + ((2*I)*c*f*Cos[(c + d*x)/2]^2*(b*Cos[c +
 d*x]*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c +
 d*x)/2]))/(a + b*Sin[c + d*x])))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1720 vs. $2(344) = 688$.
time = 1.68, size = 1721, normalized size = 4.54

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1721 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] I/d^2*f*dilog(exp(I*(d*x+c)))/a-1/d^2/a*f*c*ln(exp(I*(d*x+c))-1)+1/d/a*ln(e
xp(I*(d*x+c))+1)*f*x+1/d/a*e*ln(exp(I*(d*x+c))-1)+1/d/a*e*ln(exp(I*(d*x+c))
+1)+2/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^
2+b^2)^(1/2)))*a*c+2/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/
2))/(I*a+(-a^2+b^2)^(1/2)))*a*x+2/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))
+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a*c-1/d^2/b^2*a*f*c*ln(I*b*exp(2
*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+2/d^2/b^2*a*f*c*ln(exp(I*(d*x+c)))-I/d^
2/b^2*a*f*c^2-2*I/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(
1/2))/(I*a-(-a^2+b^2)^(1/2)))*a-2*I/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*
x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a+1/2*I*(d*x*f+I*f+d*e)/b/d
^2*exp(I*(d*x+c))+2/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2
))/(I*a-(-a^2+b^2)^(1/2)))*a*x+I*a/b^2*e*x+1/d/b^2*a*e*ln(I*b*exp(2*I*(d*x+
c))-2*a*exp(I*(d*x+c))-I*b)-2/d/b^2*a*e*ln(exp(I*(d*x+c)))-I/d^2*f/a*dilog(
exp(I*(d*x+c))+1)-1/2*I*a/b^2*f*x^2-1/2*I*(d*x*f-I*f+d*e)/b/d^2*exp(-I*(d*x
+c))-1/d*e/a*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-1/d^2*b^2*f/a/
(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)
))*c-1/d/b^2*a^3*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I
*a-(-a^2+b^2)^(1/2)))*x-1/d^2/b^2*a^3*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))
-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/d/b^2*a^3*f/(-a^2+b^2)*ln((I
*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/d^2/b^2*a
^3*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^
(1/2)))*c-1/d*b^2*f/a/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))
/(I*a+(-a^2+b^2)^(1/2)))*x-1/d^2*b^2*f/a/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c)
)+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-1/d*b^2*f/a/(-a^2+b^2)*ln((I*
a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+I/d^2/b^2*a^
3*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2
)^(1/2)))+I/d^2/b^2*a^3*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)
^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+I/d^2*b^2*f/a/(-a^2+b^2)*dilog((I*a+b*exp(I
*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+I/d^2*b^2*f/a/(-a^2+b^2
)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2*I
/d/b^2*a*f*c*x+1/d^2*f*c/a*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1296 vs. 2(344) = 688.
time = 0.63, size = 1296, normalized size = 3.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*a*b*f*cos(d*x + c) + I*b^2*f*dilog(cos(d*x + c) + I*sin(d*x + c)) -
 I*b^2*f*dilog(cos(d*x + c) - I*sin(d*x + c)) - I*b^2*f*dilog(-cos(d*x + c)
 + I*sin(d*x + c)) + I*b^2*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) + I*(a^2
 - b^2)*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*(a^2 - b^2)*f*dilog((I
*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1) - I*(a^2 - b^2)*f*dilog((-I*a*cos(d*x + c) -
 a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) - I*(a^2 - b^2)*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) -
 (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (
(a^2 - b^2)*c*f - (a^2 - b^2)*d*e)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c
) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + ((a^2 - b^2)*c*f - (a^2 - b^2)*d*
e)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) + ((a^2 - b^2)*c*f - (a^2 - b^2)*d*e)*log(-2*b*cos(d*x + c) + 2*I*b
*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + ((a^2 - b^2)*c*f - (a
^2 - b^2)*d*e)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) - 2*I*a) - ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-(I*a*cos(
d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b) - ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-(I*a*cos(d
*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b) - ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-(-I*a*cos(d
*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b) - ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-(-I*a*cos(d
*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b) - (b^2*d*f*x + b^2*d*e)*log(cos(d*x + c) + I*sin(d*x + c
) + 1) - (b^2*d*f*x + b^2*d*e)*log(cos(d*x + c) - I*sin(d*x + c) + 1) + (b^
2*c*f - b^2*d*e)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + (b^2*c
*f - b^2*d*e)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - (b^2*d*f*
x + b^2*c*f)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) - (b^2*d*f*x + b^2*c*f
)*log(-cos(d*x + c) - I*sin(d*x + c) + 1) + 2*(a*b*d*f*x + a*b*d*e)*sin(d*x
 + c))/(a*b^2*d^2)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)^2*cot(d*x + c)/(b*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.332 $\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=59

$$\frac{\log(\sin(c+dx))}{ad}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}-\frac{\sin(c+dx)}{bd}$$

```
[Out] ln(sin(d*x+c))/a/d+(a^2-b^2)*ln(a+b*sin(d*x+c))/a/b^2/d-sin(d*x+c)/b/d
```

**Rubi** [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a*b^2*d) -
 Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b(b^2-x^2)}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\log(\sin(c+dx))}{ad} + \frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d} - \frac{\sin(c+dx)}{bd}$$

**Mathematica** [A]
time = 0.05, size = 53, normalized size = 0.90

$$\frac{b^2\log(\sin(c+dx)) + (a^2-b^2)\log(a+b\sin(c+dx)) - ab\sin(c+dx)}{ab^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] (b^2*Log[Sin[c + d*x]] + (a^2 - b^2)*Log[a + b*Sin[c + d*x]] - a*b*Sin[c + d*x])/(a*b^2*d)
```

**Maple** [A]
time = 0.18, size = 55, normalized size = 0.93

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{\sin(dx+c)}{b}+\frac{\ln(\sin(dx+c))}{a}+\frac{(a^2-b^2)\ln(a+b\sin(dx+c))}{b^2a}}{d}$ |
| default | $\frac{-\frac{\sin(dx+c)}{b}+\frac{\ln(\sin(dx+c))}{a}+\frac{(a^2-b^2)\ln(a+b\sin(dx+c))}{b^2a}}{d}$ |
| risch | $-\frac{iax}{b^2}+\frac{i e^{i(dx+c)}}{2bd}-\frac{i e^{-i(dx+c)}}{2bd}-\frac{2iac}{b^2d}+\frac{a\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{b^2d}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{ad}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/b*sin(d*x+c)+1/a*ln(sin(d*x+c))+1/b^2*(a^2-b^2)/a*ln(a+b*sin(d*x+c)
))
```

**Maxima** [A]
time = 0.27, size = 54, normalized size = 0.92

$$\frac{\frac{\log(\sin(dx+c))}{a} - \frac{\sin(dx+c)}{b} + \frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{ab^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c))/a - sin(d*x + c)/b + (a^2 - b^2)*log(b*sin(d*x + c) + a)
/(a*b^2))/d
```

**Fricas** [A]
time = 0.38, size = 55, normalized size = 0.93

$$\frac{b^2 \log \left(-\frac{1}{2} \sin \left(dx+c\right)\right) - ab \sin \left(dx+c\right) + \left(a^2-b^2\right) \log \left(b \sin \left(dx+c\right)+a\right)}{ab^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b^2*log(-1/2*sin(d*x + c)) - a*b*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(a*b^2*d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2 \left(c+dx\right) \cot \left(c+dx\right)}{a+b \sin \left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 7.95, size = 56, normalized size = 0.95

$$\frac{\frac{\log(|\sin(dx+c)|)}{a} - \frac{\sin(dx+c)}{b} + \frac{(a^2-b^2)\log(|b\sin(dx+c)+a|)}{ab^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(sin(d*x + c)))/a - sin(d*x + c)/b + (a^2 - b^2)*log(abs(b*sin(d*x
+ c) + a))/(a*b^2))/d
```

**Mupad** [B]
time = 4.69, size = 98, normalized size = 1.66

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{\sin\left(c+d\,x\right)}{b\,d} + \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(\frac{a}{b^2}-\frac{1}{a}\right)}{d} - \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) - sin(c + d*x)/(b*d) + (log(a + 2*b*tan(c/2 +
 (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a/b^2 - 1/a))/d - (a*log(tan(c/2 + (d*
x)/2)^2 + 1))/(b^2*d)
```

# 3.333 $\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=1138

$$\frac{3ef^2x}{4bd^2}+\frac{3f^3x^2}{8bd^2}-\frac{(e+fx)^4}{8bf}+\frac{(a^2-b^2)\,(e+fx)^4}{4b^3f}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}-6($$

```
[Out] -6*(a^2-b^2)^(3/2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/
b^3/d^4+6*(a^2-b^2)^(3/2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/
2)))/a/b^3/d^4+1/4*(a^2-b^2)*(f*x+e)^4/b^3/f-1/8*(f*x+e)^4/b/f+3/4*e*f^2*x/
b/d^2-1/2*(f*x+e)^3*cos(d*x+c)*sin(d*x+c)/b/d-6*f^2*(f*x+e)*polylog(3,-exp(
I*(d*x+c)))/a/d^3+6*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a/d^3-6*I*f^3*pol
ylog(4,-exp(I*(d*x+c)))/a/d^4-6*f^2*(f*x+e)*cos(d*x+c)/a/d^3-3*f*(f*x+e)^2*
sin(d*x+c)/a/d^2+3/8*f^3*cos(d*x+c)^2/b/d^4+(a^2-b^2)*(f*x+e)^3*cos(d*x+c)/
a/b^2/d+6*(a^2-b^2)*f^3*sin(d*x+c)/a/b^2/d^4-3/4*f*(f*x+e)^2*cos(d*x+c)^2/b
/d^2+I*(a^2-b^2)^(3/2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)
))/a/b^3/d+3*(a^2-b^2)^(3/2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a
^2-b^2)^(1/2)))/a/b^3/d^2-3*(a^2-b^2)^(3/2)*f*(f*x+e)^2*polylog(2,I*b*exp(I
*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d^2-I*(a^2-b^2)^(3/2)*(f*x+e)^3*ln(1-I
*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d-6*(a^2-b^2)*f^2*(f*x+e)*cos(
d*x+c)/a/b^2/d^3+3/8*f^3*x^2/b/d^2+3/4*f^2*(f*x+e)*cos(d*x+c)*sin(d*x+c)/b/
d^3+(f*x+e)^3*cos(d*x+c)/a/d+6*f^3*sin(d*x+c)/a/d^4-2*(f*x+e)^3*arctanh(exp
(I*(d*x+c)))/a/d-6*I*(a^2-b^2)^(3/2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/a/b^3/d^3+6*I*(a^2-b^2)^(3/2)*f^2*(f*x+e)*polylog(3
,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d^3+6*I*f^3*polylog(4,exp(I*
(d*x+c)))/a/d^4-3*I*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a/d^2-3*(a^2-b^2)
*f*(f*x+e)^2*sin(d*x+c)/a/b^2/d^2+3*I*f*(f*x+e)^2*polylog(2,-exp(I*(d*x+c))
)/a/d^2
```

**Rubi** [A]
time = 1.34, antiderivative size = 1138, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 18, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {4639, 4493, 4490, 3392, 3377, 2717, 2713, 4268, 2611, 6744, 2320, 6724, 4621, 32, 3391, 3404, 2296, 2221}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*e*f^2*x)/(4*b*d^2) + (3*f^3*x^2)/(8*b*d^2) - (e + f*x)^4/(8*b*f) + ((a^2
 - b^2)*(e + f*x)^4)/(4*b^3*f) - (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(
a*d) - (6*f^2*(e + f*x)*Cos[c + d*x])/(a*d^3) - (6*(a^2 - b^2)*f^2*(e + f*x
```

```
)*Cos[c + d*x])/(a*b^2*d^3) + ((e + f*x)^3*Cos[c + d*x])/(a*d) + ((a^2 - b^
2)*(e + f*x)^3*Cos[c + d*x])/(a*b^2*d) + (3*f^3*Cos[c + d*x]^2)/(8*b*d^4) -
 (3*f*(e + f*x)^2*Cos[c + d*x]^2)/(4*b*d^2) + (I*(a^2 - b^2)^(3/2)*(e + f*x
)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d) - (I*(a
^2 - b^2)^(3/2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(a*b^3*d) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^
2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (3*(a^2 -
b^2)^(3/2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*b^3*d^2) - (3*(a^2 - b^2)^(3/2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d^2) - (6*f^2*(e + f*x)*PolyLo
g[3, -E^(I*(c + d*x))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x
))])/(a*d^3) + ((6*I)*(a^2 - b^2)^(3/2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^3) - ((6*I)*(a^2 - b^2)^(3/2)*
f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b
^3*d^3) - ((6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*Pol
yLog[4, E^(I*(c + d*x))])/(a*d^4) - (6*(a^2 - b^2)^(3/2)*f^3*PolyLog[4, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^4) + (6*(a^2 - b^2)^(3/
2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d^4)
 + (6*f^3*Sin[c + d*x])/(a*d^4) + (6*(a^2 - b^2)*f^3*Sin[c + d*x])/(a*b^2*d
^4) - (3*f*(e + f*x)^2*Sin[c + d*x])/(a*d^2) - (3*(a^2 - b^2)*f*(e + f*x)^2
*Sin[c + d*x])/(a*b^2*d^2) + (3*f^2*(e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(4
*b*d^3) - ((e + f*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
```

```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos^3(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^3\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\cos^2(c+d}{b}$$

$$= -\frac{3f(e+fx)^2\cos^2(c+dx)}{4bd^2} - \frac{(e+fx)^3\cos(c+dx)\sin(c+dx)}{2bd}$$

$$= -\frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right.}{ad}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)}{}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)}{}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)}{}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)}{}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)}{}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2489 vs. 2(1138) = 2276.
time = 5.89, size = 2489, normalized size = 2.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 - 3*b^2)*e^3*x)/(2*b^3) + (3*(2*a^2 - 3*b^2)*e^2*f*x^2)/(4*b^3) + (
(2*a^2 - 3*b^2)*e*f^2*x^3)/(2*b^3) + ((2*a^2 - 3*b^2)*f^3*x^4)/(8*b^3) + (-
2*d^3*e^3*ArcTanh[E^(I*(c + d*x))] + 3*d^3*e^2*f*x*Log[1 - E^(I*(c + d*x))]
 + 3*d^3*e*f^2*x^2*Log[1 - E^(I*(c + d*x))] + d^3*f^3*x^3*Log[1 - E^(I*(c +
 d*x))] - 3*d^3*e^2*f*x*Log[1 + E^(I*(c + d*x))] - 3*d^3*e*f^2*x^2*Log[1 +
E^(I*(c + d*x))] - d^3*f^3*x^3*Log[1 + E^(I*(c + d*x))] + (3*I)*d^2*f*(e +
f*x)^2*PolyLog[2, -E^(I*(c + d*x))] - (3*I)*d^2*f*(e + f*x)^2*PolyLog[2, E^
(I*(c + d*x))] - 6*d*e*f^2*PolyLog[3, -E^(I*(c + d*x))] - 6*d*f^3*x*PolyLog
[3, -E^(I*(c + d*x))] + 6*d*e*f^2*PolyLog[3, E^(I*(c + d*x))] + 6*d*f^3*x*P
olyLog[3, E^(I*(c + d*x))] - (6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))] + (6*I)
*f^3*PolyLog[4, E^(I*(c + d*x))])/(a*d^4) + ((2*I)*(a^2 - b^2)^(3/2)*d^3*e^
3*Sqrt[-((a^2 - b^2)^2*E^((4*I)*c))]*ArcTanh[(-a + I*b*E^(I*(c + d*x)))/Sqr
t[a^2 - b^2]] + (3*I)*(a^2 - b^2)^2*d^3*e^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2
*I)*c)]*f*x*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E
^((2*I)*c)])] + I*(a^2 - b^2)^2*d^3*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*
f^3*x^3*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2
*I)*c)])] - (3*I)*(a^2 - b^2)^2*d^3*e^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*
c)]*f*x*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2
*I)*c)])] - I*(a^2 - b^2)^2*d^3*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f^3*
x^3*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*
c)])] - (3*d^3*e*((a^2 - b^2)*E^((2*I)*c))^(5/2)*f^2*x^2*Log[1 + (b*E^(I*(2
*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/E^((3*I)*c) +
(3*d^3*e*((a^2 - b^2)*E^((2*I)*c))^(5/2)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x
)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/E^((3*I)*c) + 3*(a^2 -
 b^2)^2*d^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f*(e^2 + f^2*x^2)*PolyLo
g[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] -
 3*(a^2 - b^2)^2*d^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f*(e^2 + f^2*x^
2)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I
)*c)])] + ((6*I)*d^2*e*((a^2 - b^2)*E^((2*I)*c))^(5/2)*f^2*x*PolyLog[2, (I*
b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/E^((3
*I)*c) - ((6*I)*d^2*e*((a^2 - b^2)*E^((2*I)*c))^(5/2)*f^2*x*PolyLog[2, -((b
*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))])/E^((3
*I)*c) + (6*I)*(a^2 - b^2)^2*d*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f^3*x
*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*
c)])] - (6*I)*(a^2 - b^2)^2*d*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f^3*x*
PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c
)])] - (6*d*e*((a^2 - b^2)*E^((2*I)*c))^(5/2)*f^2*PolyLog[3, (I*b*E^(I*(2*c
 + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/E^((3*I)*c) + (6
*d*e*((a^2 - b^2)*E^((2*I)*c))^(5/2)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))
/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))])/E^((3*I)*c) - 6*(a^2 - b
^2)^2*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f^3*PolyLog[4, (I*b*E^(I*(2*c
+ d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 6*(a^2 - b^2)^2*E^(
I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/
(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])])/(a*b^3*d^4*Sqrt[-((a^2 - b^2)
^2*E^((4*I)*c))]) + (a*((6*I)*f^3 - 6*d*f^2*(e + f*x) - (3*I)*d^2*f*(e + f*
```

```
x)^2 + d^3*(e + f*x)^3)*(Cos[c + d*x] - I*Sin[c + d*x]))/(2*b^2*d^4) + (a*(
(-6*I)*f^3 - 6*d*f^2*(e + f*x) + (3*I)*d^2*f*(e + f*x)^2 + d^3*(e + f*x)^3)
*(Cos[c + d*x] + I*Sin[c + d*x]))/(2*b^2*d^4) + ((3*f^3 + (6*I)*d*f^2*(e +
f*x) - 6*d^2*f*(e + f*x)^2 - (4*I)*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] - I*S
in[2*(c + d*x)]))/(32*b*d^4) + ((3*f^3 - (6*I)*d*f^2*(e + f*x) - 6*d^2*f*(e
 + f*x)^2 + (4*I)*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))
/(32*b*d^4)
```

**Maple** [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\cos^3(dx+c)) \cot(dx+c)}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4197 vs. 2(1047) = 2094.
time = 0.91, size = 4197, normalized size = 3.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/8*((2*a^3 - 3*a*b^2)*d^4*f^3*x^4 + 3*a*b^2*d^2*f^3*x^2 + 6*(2*a^3 - 3*a*b
^2)*d^4*f*x^2*e^2 + 4*(2*a^3 - 3*a*b^2)*d^4*x*e^3 + 24*I*b^3*f^3*polylog(4,
 cos(d*x + c) + I*sin(d*x + c)) - 24*I*b^3*f^3*polylog(4, cos(d*x + c) - I*
```

```
sin(d*x + c)) + 24*I*b^3*f^3*polylog(4, -cos(d*x + c) + I*sin(d*x + c)) - 2
4*I*b^3*f^3*polylog(4, -cos(d*x + c) - I*sin(d*x + c)) + 24*I*(a^2*b - b^3)
*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c)
+ (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 24*I*(a^
2*b - b^3)*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin
(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
- 24*I*(a^2*b - b^3)*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x +
 c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))/b) + 24*I*(a^2*b - b^3)*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*
a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2))/b) - 3*(2*a*b^2*d^2*f^3*x^2 + 4*a*b^2*d^2*f^2*x*e + 2*a*
b^2*d^2*f*e^2 - a*b^2*f^3)*cos(d*x + c)^2 - 12*(I*(a^2*b - b^3)*d^2*f^3*x^2
 + 2*I*(a^2*b - b^3)*d^2*f^2*x*e + I*(a^2*b - b^3)*d^2*f*e^2)*sqrt(-(a^2 -
b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 12*(-I*(a^2*b - b^3)*d^2
*f^3*x^2 - 2*I*(a^2*b - b^3)*d^2*f^2*x*e - I*(a^2*b - b^3)*d^2*f*e^2)*sqrt(
-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 12*(-I*(a^2*b -
b^3)*d^2*f^3*x^2 - 2*I*(a^2*b - b^3)*d^2*f^2*x*e - I*(a^2*b - b^3)*d^2*f*e^
2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 12*(I*(
a^2*b - b^3)*d^2*f^3*x^2 + 2*I*(a^2*b - b^3)*d^2*f^2*x*e + I*(a^2*b - b^3)*
d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c)
 - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 4*((a^2*b - b^3)*c^3*f^3 - 3*(a^2*b - b^3)*c^2*d*f^2*e + 3*(a^2*b - b^3)*c
*d^2*f*e^2 - (a^2*b - b^3)*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x
+ c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 4*((a^2*b
 - b^3)*c^3*f^3 - 3*(a^2*b - b^3)*c^2*d*f^2*e + 3*(a^2*b - b^3)*c*d^2*f*e^2
 - (a^2*b - b^3)*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I
*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 4*((a^2*b - b^3)*c^
3*f^3 - 3*(a^2*b - b^3)*c^2*d*f^2*e + 3*(a^2*b - b^3)*c*d^2*f*e^2 - (a^2*b
- b^3)*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*
x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 4*((a^2*b - b^3)*c^3*f^3 - 3
*(a^2*b - b^3)*c^2*d*f^2*e + 3*(a^2*b - b^3)*c*d^2*f*e^2 - (a^2*b - b^3)*d^
3*e^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 4*((a^2*b - b^3)*d^3*f^3*x^3 + (a^2*b
 - b^3)*c^3*f^3 + 3*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*e^2 + 3
*((a^2*b - b^3)*d^3*f^2*x^2 - (a^2*b - b^3)*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)
/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 4*((a^2*b - b^3)*d^3*f^3*x^3 + (a
^2*b - b^3)*c^3*f^3 + 3*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*e^2
 + 3*((a^2*b - b^3)*d^3*f^2*x^2 - (a^2*b - b^3)*c^2*d*f^2)*e)*sqrt(-(a^2 -
b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 4*((a^2*b - b^3)*d^3*f^3*x^3
+ (a^2*b - b^3)*c^3*f^3 + 3*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)
```

```
*e^2 + 3*((a^2*b - b^3)*d^3*f^2*x^2 - (a^2*b - b^3)*c^2*d*f^2)*e)*sqrt(-(a^
2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 4*((a^2*b - b^3)*d^3*f^3
*x^3 + (a^2*b - b^3)*c^3*f^3 + 3*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d
^2*f)*e^2 + 3*((a^2*b - b^3)*d^3*f^2*x^2 - (a^2*b - b^3)*c^2*d*f^2)*e)*sqrt
(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 24*((a^2*b - b^3)*d
*f^3*x + (a^2*b - b^3)*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos
(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2))/b) + 24*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*f^2*e)*sqrt(
-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d
*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 24*((a^2*b - b^3)*
d*f^3*x + (a^2*b - b^3)*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*c
os(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))/b) + 24*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*f^2*e)*sqr
t(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 8*(a^2*b*d^3*f^
3*x^3 + 3*a^2*b*d^3*f*x*e^2 - 6*a^2*b*d*f^3*x + a^2*b*d^3*e^3 + 3*(a^2*b*d^
3*f^2*x^2 - 2*a^2*b*d*f^2)*e)*cos(d*x + c) - 12...
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cos(c + d*x)**3*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
[Out] \text{Hanged}
```

# 3.334 $\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=825

$$\frac{f^2x}{4bd^2}-\frac{(e+fx)^3}{6bf}+\frac{(a^2-b^2)(e+fx)^3}{3b^3f}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{2f^2\cos(c+dx)}{ad^3}-\frac{2(a^2-b^2)f^2\cos(c}{ab^2d^3}$$

```
[Out] 1/4*f^2*x/b/d^2-1/6*(f*x+e)^3/b/f+1/3*(a^2-b^2)*(f*x+e)^3/b^3/f-2*(f*x+e)^2
*arctanh(exp(I*(d*x+c)))/a/d-2*f^2*cos(d*x+c)/a/d^3-2*(a^2-b^2)*f^2*cos(d*x
+c)/a/b^2/d^3+(f*x+e)^2*cos(d*x+c)/a/d+(a^2-b^2)*(f*x+e)^2*cos(d*x+c)/a/b^2
/d-1/2*f*(f*x+e)*cos(d*x+c)^2/b/d^2-2*I*(a^2-b^2)^(3/2)*f^2*polylog(3,I*b*e
xp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d^3+2*I*(a^2-b^2)^(3/2)*f^2*polylo
g(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d^3+I*(a^2-b^2)^(3/2)*(f*
x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d+2*I*f*(f*x+e)*p
olylog(2,-exp(I*(d*x+c)))/a/d^2+2*(a^2-b^2)^(3/2)*f*(f*x+e)*polylog(2,I*b*e
xp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d^2-2*(a^2-b^2)^(3/2)*f*(f*x+e)*po
lylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d^2-2*f^2*polylog(3,-
exp(I*(d*x+c)))/a/d^3+2*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3-I*(a^2-b^2)^(3/
2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d-2*I*f*(f*
x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2-2*f*(f*x+e)*sin(d*x+c)/a/d^2-2*(a^2-b^
2)*f*(f*x+e)*sin(d*x+c)/a/b^2/d^2+1/4*f^2*cos(d*x+c)*sin(d*x+c)/b/d^3-1/2*(
f*x+e)^2*cos(d*x+c)*sin(d*x+c)/b/d
```

**Rubi** [A]
time = 1.05, antiderivative size = 825, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 18, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {4639, 4493, 4490, 3391, 3377, 2718, 4268, 2611, 2320, 6724, 4621, 3392, 32, 2715, 8, 3404, 2296, 2221}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (f^2*x)/(4*b*d^2) - (e + f*x)^3/(6*b*f) + ((a^2 - b^2)*(e + f*x)^3)/(3*b^3*
f) - (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) - (2*f^2*Cos[c + d*x])/
(a*d^3) - (2*(a^2 - b^2)*f^2*Cos[c + d*x])/(a*b^2*d^3) + ((e + f*x)^2*Cos[c
 + d*x])/(a*d) + ((a^2 - b^2)*(e + f*x)^2*Cos[c + d*x])/(a*b^2*d) - (f*(e +
 f*x)*Cos[c + d*x]^2)/(2*b*d^2) + (I*(a^2 - b^2)^(3/2)*(e + f*x)^2*Log[1 -
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d) - (I*(a^2 - b^2)^(3
/2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^
3*d) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - ((2*I)*f*
(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (2*(a^2 - b^2)^(3/2)*f*(e
```

```
+ f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^2)
 - (2*(a^2 - b^2)^(3/2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(a*b^3*d^2) - (2*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3
) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) + ((2*I)*(a^2 - b^2)^(3/2)*
f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^3) -
((2*I)*(a^2 - b^2)^(3/2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(a*b^3*d^3) - (2*f*(e + f*x)*Sin[c + d*x])/(a*d^2) - (2*(a^2 - b
^2)*f*(e + f*x)*Sin[c + d*x])/(a*b^2*d^2) + (f^2*Cos[c + d*x]*Sin[c + d*x])
/(4*b*d^3) - ((e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos^3(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^2\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\cos^2(c+d}{b}$$

$$= -\frac{f(e+fx)\cos^2(c+dx)}{2bd^2} - \frac{(e+fx)^2\cos(c+dx)\sin(c+dx)}{2bd} +$$

$$= -\frac{(e+fx)^3}{6bf} + \frac{(a^2-b^2)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right.}{ad}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{(a^2-b^2)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e\right.}{ad}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{(a^2-b^2)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e\right.}{ad}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{(a^2-b^2)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e\right.}{ad}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{(a^2-b^2)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e\right.}{ad}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{(a^2-b^2)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e\right.}{ad}$$

**Mathematica [A]**
time = 3.11, size = 1221, normalized size = 1.48

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((24*(-2*d^2*e^2*ArcTanh[E^(I*(c + d*x))] + 2*d^2*e*f*x*Log[1 - E^(I*(c + d
*x))] + d^2*f^2*x^2*Log[1 - E^(I*(c + d*x))] - 2*d^2*e*f*x*Log[1 + E^(I*(c
+ d*x))] - d^2*f^2*x^2*Log[1 + E^(I*(c + d*x))] + (2*I)*d*f*(e + f*x)*PolyL
```

```
og[2, -E^(I*(c + d*x))] - (2*I)*d*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))] -
 2*f^2*PolyLog[3, -E^(I*(c + d*x))] + 2*f^2*PolyLog[3, E^(I*(c + d*x))]))/a
 + (24*((2*I)*(a^2 - b^2)^(3/2)*d^2*e^2*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*ArcTa
nh[(-a + I*b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + (2*I)*(a^2 - b^2)^2*d^2*e*
E^(I*c)*f*x*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E
^((2*I)*c)])] + I*(a^2 - b^2)^2*d^2*E^(I*c)*f^2*x^2*Log[1 - (I*b*E^(I*(2*c
+ d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (2*I)*(a^2 - b^2)^2
*d^2*e*E^(I*c)*f*x*Log[1 - (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 -
 b^2)*E^((2*I)*c)])] - I*(a^2 - b^2)^2*d^2*E^(I*c)*f^2*x^2*Log[1 - (I*b*E^(
I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 2*(a^2 - b^2
)^2*d*E^(I*c)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) - S
qrt[(a^2 - b^2)*E^((2*I)*c)])] - 2*(a^2 - b^2)^2*d*E^(I*c)*f*(e + f*x)*Poly
Log[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
 + (2*I)*(a^2 - b^2)^2*E^(I*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^
(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (2*I)*(a^2 - b^2)^2*E^(I*c)*f^2*P
olyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)
])]))/(a*b^3*Sqrt[(a^2 - b^2)*E^((2*I)*c)]) - (-24*a^2*d^3*e^2*x + 36*b^2*d
^3*e^2*x - 24*a^2*d^3*e*f*x^2 + 36*b^2*d^3*e*f*x^2 - 8*a^2*d^3*f^2*x^3 + 12
*b^2*d^3*f^2*x^3 - 24*a*b*(-2*f^2 + d^2*(e + f*x)^2)*Cos[c + d*x] + 6*b^2*d
*f*(e + f*x)*Cos[2*(c + d*x)] + 48*a*b*d*e*f*Sin[c + d*x] + 48*a*b*d*f^2*x*
Sin[c + d*x] + 6*b^2*d^2*e^2*Sin[2*(c + d*x)] - 3*b^2*f^2*Sin[2*(c + d*x)]
+ 12*b^2*d^2*e*f*x*Sin[2*(c + d*x)] + 6*b^2*d^2*f^2*x^2*Sin[2*(c + d*x)])/b
^3)/(24*d^3)
```

**Maple** [**F**]
time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 (\cos^3(dx+c)) \cot(dx+c)}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2802 vs. 2(754) = 1508.
time = 0.73, size = 2802, normalized size = 3.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/12*(2*(2*a^3 - 3*a*b^2)*d^3*f^2*x^3 + 6*(2*a^3 - 3*a*b^2)*d^3*f*x^2*e + 3
*a*b^2*d*f^2*x + 6*(2*a^3 - 3*a*b^2)*d^3*x*e^2 + 12*b^3*f^2*polylog(3, cos(
d*x + c) + I*sin(d*x + c)) + 12*b^3*f^2*polylog(3, cos(d*x + c) - I*sin(d*x
 + c)) - 12*b^3*f^2*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) - 12*b^3*f^2
*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) - 12*(a^2*b - b^3)*f^2*sqrt(-(a
^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a^2*b - b^3)*f^2*
sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*
cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*(a^2*b - b
^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x +
 c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(
a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*
sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b) - 6*(a*b^2*d*f^2*x + a*b^2*d*f*e)*cos(d*x + c)^2 - 12*(I*(a^2*b - b^3)*d
*f^2*x + I*(a^2*b - b^3)*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x +
 c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b + 1) - 12*(-I*(a^2*b - b^3)*d*f^2*x - I*(a^2*b - b^3)*d*f*e)*
sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 12*(-I*(a^2
*b - b^3)*d*f^2*x - I*(a^2*b - b^3)*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I
*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1) - 12*(I*(a^2*b - b^3)*d*f^2*x + I*(a^2*b - b
^3)*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c)
 - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) -
 6*((a^2*b - b^3)*c^2*f^2 - 2*(a^2*b - b^3)*c*d*f*e + (a^2*b - b^3)*d^2*e^2
)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) + 2*I*a) - 6*((a^2*b - b^3)*c^2*f^2 - 2*(a^2*b - b^3)*
c*d*f*e + (a^2*b - b^3)*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c
) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 6*((a^2*b -
b^3)*c^2*f^2 - 2*(a^2*b - b^3)*c*d*f*e + (a^2*b - b^3)*d^2*e^2)*sqrt(-(a^2
- b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b
^2)/b^2) + 2*I*a) + 6*((a^2*b - b^3)*c^2*f^2 - 2*(a^2*b - b^3)*c*d*f*e + (a
```

```
^2*b - b^3)*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 6*((a^2*b - b^3)*d^2*f^
2*x^2 - (a^2*b - b^3)*c^2*f^2 + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*c*
d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 6*((a^2*
b - b^3)*d^2*f^2*x^2 - (a^2*b - b^3)*c^2*f^2 + 2*((a^2*b - b^3)*d^2*f*x + (
a^2*b - b^3)*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*si
n(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b
)/b) + 6*((a^2*b - b^3)*d^2*f^2*x^2 - (a^2*b - b^3)*c^2*f^2 + 2*((a^2*b - b
^3)*d^2*f*x + (a^2*b - b^3)*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos
(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b) - 6*((a^2*b - b^3)*d^2*f^2*x^2 - (a^2*b - b^3)*c^2*f^2
 + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2
)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 12*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*
f*x*e + a^2*b*d^2*e^2 - 2*a^2*b*f^2)*cos(d*x + c) - 12*(I*b^3*d*f^2*x + I*b
^3*d*f*e)*dilog(cos(d*x + c) + I*sin(d*x + c)) - 12*(-I*b^3*d*f^2*x - I*b^3
*d*f*e)*dilog(cos(d*x + c) - I*sin(d*x + c)) - 12*(I*b^3*d*f^2*x + I*b^3*d*
f*e)*dilog(-cos(d*x + c) + I*sin(d*x + c)) - 12*(-I*b^3*d*f^2*x - I*b^3*d*f
*e)*dilog(-cos(d*x + c) - I*sin(d*x + c)) - 6*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*
f*x*e + b^3*d^2*e^2)*log(cos(d*x + c) + I*sin(d*x + c) + 1) - 6*(b^3*d^2*f^
2*x^2 + 2*b^3*d^2*f*x*e + b^3*d^2*e^2)*log(cos(d*x + c) - I*sin(d*x + c) +
1) + 6*(b^3*c^2*f^2 - 2*b^3*c*d*f*e + b^3*d^2*e^2)*log(-1/2*cos(d*x + c) +
1/2*I*sin(d*x + c) + 1/2) + 6*(b^3*c^2*f^2 - 2*b^3*c*d*f*e + b^3*d^2*e^2)*l
og(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + 6*(b^3*d^2*f^2*x^2 - b^3
*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*log(-cos(d*x + c) + I*sin(d*x + c
) + 1) + 6*(b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*
log(-cos(d*x + c) - I*sin(d*x + c) + 1) - 3*(8*a^2*b*d*f^2*x + 8*a^2*b*d*f*
e + (2*a*b^2*d^2*f^2*x^2 + 4*a*b^2*d^2*f*x*e + 2*a*b^2*d^2*e^2 - a*b^2*f^2)
*cos(d*x + c))*sin(d*x + c))/(a*b^3*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cos(c + d*x)**3*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.335 $\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=524

$$-\frac{ex}{2b}+\frac{(a^2-b^2)\,ex}{b^3}-\frac{fx^2}{4b}+\frac{(a^2-b^2)\,fx^2}{2b^3}-\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{(a^2-b^2)\,(e+}{ab}$$

[Out] -1/2*e*x/b+(a^2-b^2)*e*x/b^3-1/4*f*x^2/b+1/2*(a^2-b^2)*f*x^2/b^3-2*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)*cos(d*x+c)/a/d+(a^2-b^2)*(f*x+e)*cos(d*x+c)/a/b^2/d-1/4*f*cos(d*x+c)^2/b/d^2-I*f*polylog(2,exp(I*(d*x+c)))/a/d^2-I*(a^2-b^2)^(3/2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d+I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2+I*(a^2-b^2)^(3/2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d+(a^2-b^2)^(3/2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d^2-(a^2-b^2)^(3/2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d^2-f*sin(d*x+c)/a/d^2-(a^2-b^2)*f*sin(d*x+c)/a/b^2/d^2-1/2*(f*x+e)*cos(d*x+c)*sin(d*x+c)/b/d

**Rubi** [A]
time = 0.59, antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4639, 4493, 4490, 2713, 3377, 2717, 4268, 2317, 2438, 4621, 3391, 3404, 2296, 2221}

$$\frac{f(a^2-b^2)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^3d^2}-\frac{f(a^2-b^2)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^3d^2}+\frac{if\text{PolyLog}(2,-e^{i(c+dx)})}{ad^2}-\frac{if\text{PolyLog}(2,e^{i(c+dx)})}{ad^2}-\frac{f(a^2-b^2)\sin(c+dx)}{ab^2d^2}+\frac{(a^2-b^2)(e+fx)\cos(c+dx)}{ab^2d}+\frac{i(a^2-b^2)^{3/2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^3d}-\frac{i(a^2-b^2)^{3/2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^3d}+\frac{ex(a^2-b^2)}{b^3}+\frac{fx^2(a^2-b^2)}{2b^3}-\frac{f\sin(c+dx)}{ad^2}+\frac{(e+fx)\cos(c+dx)}{ad}-\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{f\cos^2(c+dx)}{4bd^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2bd}-\frac{ex}{2b}-\frac{fx^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(e*x)/b + ((a^2 - b^2)*e*x)/b^3 - (f*x^2)/(4*b) + ((a^2 - b^2)*f*x^2)/(2*b^3) - (2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)*Cos[c + d*x])/(a*d) + ((a^2 - b^2)*(e + f*x)*Cos[c + d*x])/(a*b^2*d) - (f*Cos[c + d*x]^2)/(4*b*d^2) + (I*(a^2 - b^2)^(3/2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d) - (I*(a^2 - b^2)^(3/2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d) + (I*f*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + ((a^2 - b^2)^(3/2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^2) - ((a^2 - b^2)^(3/2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d^2) - (f*Sin[c + d*x])/(a*d^2) - ((a^2 - b^2)*f*Sin[c + d*x])/(a*b^2*d^2) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
```

```
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos^3(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)\cos^2(c+dx)\,c}{b}$$

$$= -\frac{f\cos^2(c+dx)}{4bd^2} - \frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int (e+fx}{}$$

$$= -\frac{ex}{2b} + \frac{(a^2-b^2)\,ex}{b^3} - \frac{fx^2}{4b} + \frac{(a^2-b^2)\,fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}}{ad}$$

$$= -\frac{ex}{2b} + \frac{(a^2-b^2)\,ex}{b^3} - \frac{fx^2}{4b} + \frac{(a^2-b^2)\,fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}}{ad}$$

$$= -\frac{ex}{2b} + \frac{(a^2-b^2)\,ex}{b^3} - \frac{fx^2}{4b} + \frac{(a^2-b^2)\,fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}}{ad}$$

$$= -\frac{ex}{2b} + \frac{(a^2-b^2)\,ex}{b^3} - \frac{fx^2}{4b} + \frac{(a^2-b^2)\,fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}}{ad}$$

$$= -\frac{ex}{2b} + \frac{(a^2-b^2)\,ex}{b^3} - \frac{fx^2}{4b} + \frac{(a^2-b^2)\,fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}}{ad}$$

**Mathematica [A]**
time = 9.54, size = 934, normalized size = 1.78

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*((-2*a^2 + 3*b^2)*(c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(b^3*d^2) +
 (a*(d*e - c*f + f*(c + d*x))*Cos[c + d*x])/(b^2*d^2) - (f*Cos[2*(c + d*x)]
)/(8*b*d^2) + (e*Log[Tan[(c + d*x)/2]])/(a*d) - (c*f*Log[Tan[(c + d*x)/2]])
/(a*d^2) + (f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x)
```

```
)]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d
^2) - ((a^2 - b^2)^2*(d*e + d*f*x)*((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*Tan[(c + d*x)/2
]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2
+ b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 +
b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqr
t[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog
[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^
2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] +
a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))] + PolyLog[2, (a*(I + Tan
[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*f*(Lo
g[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(
I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I
*(-b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/(a*b^3*d^2*(d*e - c*f + I*f
*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]])) - (a*f*Sin
[c + d*x])/(b^2*d^2) - ((d*e - c*f + f*(c + d*x))*Sin[2*(c + d*x)])/(4*b*d^
2)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1876 vs. 2(476) = 952.
time = 1.72, size = 1877, normalized size = 3.58

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1877 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] -2*I/d^2/b*a*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2
))/(I*a-(-a^2+b^2)^(1/2)))+2*I/b/d^2*a*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(
I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+4*I/b/d*a*e/(-a^2+b^2)
^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/d^2*f*dilo
g(exp(I*(d*x+c)))/a+I/d^2*f/a*dilog(exp(I*(d*x+c))+1)-3/2*e*x/b+2*I/d^2*f*c
/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2
))+a^2*e*x/b^3-1/8*f/b/d^2*cos(2*d*x+2*c)+2*I*a^3/b^3/d^2*f*c/(-a^2+b^2)^(1
/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-1/d^2/a*f*c*ln(
exp(I*(d*x+c))-1)-1/d/a*ln(exp(I*(d*x+c))+1)*f*x-4*I/b/d^2*a*f*c/(-a^2+b^2)
^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/d/a*e*ln(e
xp(I*(d*x+c))-1)-1/d/a*e*ln(exp(I*(d*x+c))+1)-3/4*f*x^2/b+2/d/b*a*f/(-a^2+b
^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2))
)*x+2/d^2/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))
/(I*a-(-a^2+b^2)^(1/2)))*c-2/d/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+
c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-2/d^2/b*a*f/(-a^2+b^2)^(1/2
)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-1/4*
```

```
(f*x+e)/d/b*sin(2*d*x+2*c)+1/2*a*(d*x*f+I*f+d*e)/b^2/d^2*exp(I*(d*x+c))+1/2
*a*(d*x*f-I*f+d*e)/b^2/d^2*exp(-I*(d*x+c))+I*a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)
*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2*I*
a^3/b^3/d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^
2)^(1/2))-I*a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^
2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-a^3/b^3/d*f/(-a^2+b^2)^(1/2)*ln((I*a+
b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-a^3/b^3/d^2*f/
(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)
^(1/2)))*c+a^3/b^3/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)
^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*
exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-1/d*f*b/a/(-a^2+
b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)
))*x-1/d^2*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2)
)/(I*a-(-a^2+b^2)^(1/2)))*c+1/d*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x
+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2*f*b/a/(-a^2+b^2)^(1/
2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/d
^2*f*b/a/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*
a-(-a^2+b^2)^(1/2)))-2*I/d*e/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(
d*x+c))-2*a)/(-a^2+b^2)^(1/2))-I/d^2*f*b/a/(-a^2+b^2)^(1/2)*dilog((I*a+b*ex
p(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+1/2*a^2*f*x^2/b^3
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1622 vs. 2(472) = 944.
time = 0.68, size = 1622, normalized size = 3.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*((2*a^3 - 3*a*b^2)*d^2*f*x^2 - a*b^2*f*cos(d*x + c)^2 - 2*I*b^3*f*dilog
(cos(d*x + c) + I*sin(d*x + c)) + 2*I*b^3*f*dilog(cos(d*x + c) - I*sin(d*x
+ c)) - 2*I*b^3*f*dilog(-cos(d*x + c) + I*sin(d*x + c)) + 2*I*b^3*f*dilog(-
cos(d*x + c) - I*sin(d*x + c)) + 2*(2*a^3 - 3*a*b^2)*d^2*x*e - 2*I*(a^2*b -
 b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*I
*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x
 + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
 1) + 2*I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) -
 a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) - 2*I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) + 2*((a^2*b - b^3)*c*f - (a^2*b - b^3)*d*e)*sqrt(-(
a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) + 2*I*a) + 2*((a^2*b - b^3)*c*f - (a^2*b - b^3)*d*e)*sqrt(-(a^2
 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b
^2)/b^2) - 2*I*a) - 2*((a^2*b - b^3)*c*f - (a^2*b - b^3)*d*e)*sqrt(-(a^2 -
b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a) - 2*((a^2*b - b^3)*c*f - (a^2*b - b^3)*d*e)*sqrt(-(a^2 - b^
2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/
b^2) - 2*I*a) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^
2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 2*((a^2*b - b^3)*d*f*x + (a^2*b
 - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c)
 - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*(
(a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b) - 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sqrt(
-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 4*(a^2*b*d*f*x + a^2
*b*d*e)*cos(d*x + c) - 2*(b^3*d*f*x + b^3*d*e)*log(cos(d*x + c) + I*sin(d*x
 + c) + 1) - 2*(b^3*d*f*x + b^3*d*e)*log(cos(d*x + c) - I*sin(d*x + c) + 1)
 - 2*(b^3*c*f - b^3*d*e)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)
- 2*(b^3*c*f - b^3*d*e)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) +
 2*(b^3*d*f*x + b^3*c*f)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + 2*(b^3*d
*f*x + b^3*c*f)*log(-cos(d*x + c) - I*sin(d*x + c) + 1) - 2*(2*a^2*b*f + (a
*b^2*d*f*x + a*b^2*d*e)*cos(d*x + c))*sin(d*x + c))/(a*b^3*d^2)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)**3*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)
[Out] \text{Hanged}
```

# 3.336 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=124

$$\frac{(2a^2-3b^2)\,x}{2b^3}-\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{a\cos(c+dx)}{b^2d}-\frac{\cos(c+dx)\sin(}{2bd}$$

```
[Out] 1/2*(2*a^2-3*b^2)*x/b^3-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(
a^2-b^2)^(1/2))/a/b^3/d-arctanh(cos(d*x+c))/a/d+a*cos(d*x+c)/b^2/d-1/2*cos(
d*x+c)*sin(d*x+c)/b/d
```

**Rubi** [A]
time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2974, 3136, 2739, 632, 210, 3855}

$$-\frac{2(a^2-b^2)^{3/2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{x(2a^2-3b^2)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 - 3*b^2)*x)/(2*b^3) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (a*Cos[
c + d*x])/(b^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\int \frac{\csc(c+dx)\left(-2b^2-ab\sin(c+dx)-(2a^2-\right.}{a+b\sin(c+dx)}}{2b^2}$$

$$= \frac{(2a^2-3b^2)\,x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \csc(c+dx)}{a}$$

$$= \frac{(2a^2-3b^2)\,x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c}{2bd}$$

$$= \frac{(2a^2-3b^2)\,x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c}{2bd}$$

$$= \frac{(2a^2-3b^2)\,x}{2b^3} - \frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3d} - \frac{\tanh^{-1}(\cos(c+}{ad}$$

**Mathematica [A]**
time = 0.19, size = 143, normalized size = 1.15

$$-\frac{-4a^3c+6ab^2c-4a^3dx+6ab^2dx+8(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)-4a^2b\cos(c+dx)+4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+ab^2\sin(2(c+dx))}{4ab^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(-4*a^3*c + 6*a*b^2*c - 4*a^3*d*x + 6*a*b^2*d*x + 8*(a^2 - b^2)^(3/2)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 4*a^2*b*Cos[c + d*x] + 4
*b^3*Log[Cos[(c + d*x)/2]] - 4*b^3*Log[Sin[(c + d*x)/2]] + a*b^2*Sin[2*(c +
 d*x)])/(a*b^3*d)
```

**Maple [A]**
time = 0.28, size = 180, normalized size = 1.45

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\frac{2\left(\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(2a^2-3b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}+\frac{\left(-2a^4+4a^2b^2-2b^4\right)\arctan(}{ab^3\sqrt{a^2-}}}{d}$ |
| default | $\frac{\frac{\frac{2\left(\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(2a^2-3b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}+\frac{\left(-2a^4+4a^2b^2-2b^4\right)\arctan(}{ab^3\sqrt{a^2-}}}{d}$ |
| risch | $\frac{xa^2}{b^3}-\frac{3x}{2b}+\frac{a\,e^{i(dx+c)}}{2b^2d}+\frac{a\,e^{-i(dx+c)}}{2b^2d}+\frac{i\sqrt{a^2-b^2}\,a\ln\left(e^{i(dx+c)}-\frac{i\left(-a+\sqrt{a^2-b^2}\right)}{b}\right)}{d\,b^3}-\frac{i\sqrt{a^2-b^2}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^3*((1/2*b^2*tan(1/2*d*x+1/2*c)^3+a*b*tan(1/2*d*x+1/2*c)^2-1/2*b^2*
tan(1/2*d*x+1/2*c)+a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(2*a^2-3*b^2)*arctan
(tan(1/2*d*x+1/2*c)))+(-2*a^4+4*a^2*b^2-2*b^4)/a/b^3/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/a*ln(tan(1/2*d*x+1/2*c
)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**A**]
time = 0.46, size = 350, normalized size = 2.82

$$\left[-\frac{ab^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^{2}b\cos\left(dx+c\right)+b^{3}\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-b^{3}\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(2\,a^{3}-3\,ab^{2}\right)dx-\left(-a^{2}+b^{2}\right)^{\frac{3}{2}}\log\left(-\frac{\left(2\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}-2\,\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}}\right)}{2\,ab^{3}d},\ -\frac{ab^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^{2}b\cos\left(dx+c\right)+b^{3}\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-b^{3}\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(2\,a^{3}-3\,ab^{2}\right)dx-2\,\left(a^{2}-b^{2}\right)^{\frac{3}{2}}\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^{2}-b^{2}}\cos\left(dx+c\right)}\right)}{2\,ab^{3}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log(1/2
*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*b^2)
*d*x - (-a^2 + b^2)^(3/2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*
x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(
-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b^3
*d), -1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log
(1/2*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*
b^2)*d*x - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))))/(a*b^3*d)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 6.66, size = 183, normalized size = 1.48

$$\frac{\frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{\left(2\,a^{2}-3\,b^{2}\right)\left(dx+c\right)}{b^{3}}-\frac{4\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,ab^{3}}+\frac{2\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a + (2*a^2 - 3*b^2)*(d*x + c)/b^3 - 4
*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
```

```
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b^3) + 2*
(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/
2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

**Mupad [B]**
time = 6.64, size = 1320, normalized size = 10.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - sin(2*c + 2*d*x)/(4*b*d)
 - (3*atan((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a*b^2*c
os(c/2 + (d*x)/2))/(2*b^3*cos(c/2 + (d*x)/2) - 2*a^3*sin(c/2 + (d*x)/2) + 3
*a*b^2*sin(c/2 + (d*x)/2))))/(b*d) + (a*cos(c + d*x))/(b^2*d) + (2*a^2*atan
((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a*b^2*cos(c/2 + (
d*x)/2))/(2*b^3*cos(c/2 + (d*x)/2) - 2*a^3*sin(c/2 + (d*x)/2) + 3*a*b^2*sin
(c/2 + (d*x)/2))))/(b^3*d) + (atan((b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(3/2)*64i - a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*
b^4 + 3*a^4*b^2)^(1/2)*16i - a^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(3/2)*16i - a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(3/2)*42i + a^3*b^9*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2)*66i - a^5*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2)*176i + a^7*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
 + 3*a^4*b^2)^(1/2)*178i - a^9*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(1/2)*81i - a^2*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(3/2)*116i + a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(3/2)*72i + a^2*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*
b^4 + 3*a^4*b^2)^(1/2)*148i - a^4*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2)*460i + a^6*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2)*577i - a^8*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2)*368i + a^10*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3
*a^2*b^4 + 3*a^4*b^2)^(1/2)*120i + a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*
a^2*b^4 + 3*a^4*b^2)^(3/2)*32i + a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(3/2)*14i + a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2)*14i)/(64*b^15*sin(c/2 + (d*x)/2) + 32*a*b^14*cos(c/
2 + (d*x)/2) - 120*a^3*b^12*cos(c/2 + (d*x)/2) + 180*a^5*b^10*cos(c/2 + (d*
x)/2) - 137*a^7*b^8*cos(c/2 + (d*x)/2) + 54*a^9*b^6*cos(c/2 + (d*x)/2) - 9*
a^11*b^4*cos(c/2 + (d*x)/2) - 256*a^2*b^13*sin(c/2 + (d*x)/2) + 416*a^4*b^1
1*sin(c/2 + (d*x)/2) - 351*a^6*b^9*sin(c/2 + (d*x)/2) + 161*a^8*b^7*sin(c/2
 + (d*x)/2) - 37*a^10*b^5*sin(c/2 + (d*x)/2) + 3*a^12*b^3*sin(c/2 + (d*x)/2
)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*2i)/(a*b^3*d)
```

# 3.337 $\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=852

$$\frac{ib(e+fx)^4}{4a^2f}+\frac{i(a^2-b^2)(e+fx)^4}{4a^2bf}-\frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2}-\frac{(e+fx)^3\csc(c+dx)}{ad}-\frac{(a^2-b^2)(e+f}{}$$

```
[Out] -6*I*(a^2-b^2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/
d^4-6*I*(a^2-b^2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2
/b/d^4-6*f*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d^2-(f*x+e)^3*csc(d*x+c)/a/d
-b*(f*x+e)^3*ln(1-exp(2*I*(d*x+c)))/a^2/d-(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(
I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b/d-(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/d+1/4*I*(a^2-b^2)*(f*x+e)^4/a^2/b/f-3/4
*I*b*f^3*polylog(4,exp(2*I*(d*x+c)))/a^2/d^4+6*I*f^2*(f*x+e)*polylog(2,-exp
(I*(d*x+c)))/a/d^3+3*I*(a^2-b^2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(
a-(a^2-b^2)^(1/2)))/a^2/b/d^2+3/2*I*b*f*(f*x+e)^2*polylog(2,exp(2*I*(d*x+c)
))/a^2/d^2-6*f^3*polylog(3,-exp(I*(d*x+c)))/a/d^4+6*f^3*polylog(3,exp(I*(d*
x+c)))/a/d^4-3/2*b*f^2*(f*x+e)*polylog(3,exp(2*I*(d*x+c)))/a^2/d^3-6*(a^2-b
^2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b/d^3
-6*(a^2-b^2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/
a^2/b/d^3-6*I*f^2*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^3+3*I*(a^2-b^2)*f*(
f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/d^2+1/4*I*
b*(f*x+e)^4/a^2/f
```

**Rubi** [A]
time = 1.14, antiderivative size = 852, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 19, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.559$, Rules used = {4639, 4493, 3377, 2718, 4495, 4268, 2611, 2320, 6724, 4489, 3392, 32, 2715, 8, 3798, 2221, 6744, 4621, 4615}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((I/4)*b*(e + f*x)^4)/(a^2*f) + ((I/4)*(a^2 - b^2)*(e + f*x)^4)/(a^2*b*f) -
 (6*f*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d^2) - ((e + f*x)^3*Csc[c +
d*x])/(a*d) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(a^2*b*d) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d) - (b*(e + f*x)^3*Log[1 - E^((2*
I)*(c + d*x))])/(a^2*d) + ((6*I)*f^2*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))]
)/(a*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^3) + ((3
*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^
```

```
2 - b^2])])/(a^2*b*d^2) + ((3*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^2) + (((3*I)/2)*b*f*(e +
f*x)^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2*d^2) - (6*f^3*PolyLog[3, -E^(I
*(c + d*x))])/(a*d^4) + (6*f^3*PolyLog[3, E^(I*(c + d*x))])/(a*d^4) - (6*(a
^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(a^2*b*d^3) - (6*(a^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c +
d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^3) - (3*b*f^2*(e + f*x)*PolyLog[3,
E^((2*I)*(c + d*x))])/(2*a^2*d^3) - ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*
E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b*d^4) - ((6*I)*(a^2 - b^2)*f
^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^4) - (
((3*I)/4)*b*f^3*PolyLog[4, E^((2*I)*(c + d*x))])/(a^2*d^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot}{a+b\sin(c+dx)}}{a}$$

$$= -\frac{\int (e+fx)^3\cos(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\cot(c+dx)\csc(c+d}{a}$$

$$= -\frac{(e+fx)^3\csc(c+dx)}{ad} - \frac{(e+fx)^3\sin(c+dx)}{ad} + \frac{\int (e+fx)^3 c}{}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+}\right.}{ad^2}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3114 vs. 2(852) = 1704.
time = 42.15, size = 3114, normalized size = 3.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]

[Out] (3*e^2*f*Log[Tan[(c + d*x)/2]])/(a*d^2) + (6*e*f^2*((c + d*x)*(Log[1 - E^(I
*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(Pol
yLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^3) + (b*e*f
^2*Csc[c]*(2*d^2*x^2*(2*d*E^((2*I)*c)*x + (3*I)*(-1 + E^((2*I)*c))*Log[1 -
E^((2*I)*(c + d*x))]) + 6*d*(-1 + E^((2*I)*c))*x*PolyLog[2, E^((2*I)*(c + d
*x))] + (3*I)*(-1 + E^((2*I)*c))*PolyLog[3, E^((2*I)*(c + d*x))]))/(4*a^2*d
^3*E^(I*c)) - (6*f^3*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - I*d*
x*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + I*d*x*PolyLog[2, Cos[c + d*x
] + I*Sin[c + d*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - PolyLog[
3, Cos[c + d*x] + I*Sin[c + d*x]]))/(a*d^4) + (b*E^(I*c)*f^3*Csc[c]*(x^4 +
(-1 + E^((-2*I)*c))*x^4 + ((-1 + E^((2*I)*c))*(2*d^4*x^4 + (4*I)*d^3*x^3*Lo
g[1 - E^((2*I)*(c + d*x))] + 6*d^2*x^2*PolyLog[2, E^((2*I)*(c + d*x))] + (6
*I)*d*x*PolyLog[3, E^((2*I)*(c + d*x))] - 3*PolyLog[4, E^((2*I)*(c + d*x))]
))/(2*d^4*E^((2*I)*c))))/(4*a^2) + ((a^2 - b^2)*((4*I)*d^4*e^3*E^((2*I)*c)*
x + (6*I)*d^4*e^2*E^((2*I)*c)*f*x^2 + (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 + I*d
^4*E^((2*I)*c)*f^3*x^4 + (2*I)*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1
+ E^((2*I)*(c + d*x))))] - (2*I)*d^3*e^3*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c +
d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + d^3*e^3*Log[4*a^2*E^((2*I)*(c + d*
x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - d^3*e^3*E^((2*I)*c)*Log[4*a^2*E^(
(2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] + 6*d^3*e^2*f*x*Log[1
+ (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6
*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[
(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))
/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*E^((2*I)*c)*f^2*
x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)
*c)])] + 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-
a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c
+ d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e^2*f*x*Lo
g[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
 - 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + S
qrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*
x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*E^((2*I)*c)*
f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((
2*I)*c)])] + 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqr
t[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(
2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (6*I)*d^2*(-
1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c
) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e
+ f*x)^2*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2
)*E^((2*I)*c)]))] + 12*d*e*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c
) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*e*E^((2*I)*c)*f^2*PolyLog[3,
```

```
(I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 1
2*d*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^
2)*E^((2*I)*c)])] - 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)
))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*e*f^2*PolyLog[3,
-((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] -
12*d*e*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sq
rt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*
x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*E^((2*I)*c)*f^
3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^(
(2*I)*c)]))] + (12*I)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I
*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*E^((2*I)*c)*f^3*PolyLog[4, (I*b*
E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (12*I)
*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^
((2*I)*c)]))] - (12*I)*E^((2*I)*c)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(
I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/(2*a^2*b*d^4*(-1 + E^((2*
I)*c))) + ((-4*b*e^3 - 12*b*e^2*f*x - 12*b*e*f^2*x^2 - 4*b*f^3*x^3 - 4*a*d*
e^3*x*Cos[c] - 6*a*d*e^2*f*x^2*Cos[c] - 4*a*d*e*f^2*x^3*Cos[c] - a*d*f^3*x^
4*Cos[c])*Csc[c/2]*Sec[c/2])/(8*a*b*d) - (b*e^3*Csc[c]*(-(d*x*Cos[c]) + Log
[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a^2*d*(Cos[c]^2 + Sin[c]^2))
+ (Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(e^3*Sin[(d*x)/2]) - 3*e^2*f*x*Sin[(d*x)/2
] - 3*e*f^2*x^2*Sin[(d*x)/2] - f^3*x^3*Sin[(d*x)/2]))/(2*a*d) + (Csc[c/2]*C
sc[c/2 + (d*x)/2]*(e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*
Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2]))/(2*a*d) + (3*b*e^2*f*Csc[c]*Sec[c]*(d
^2*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi + 2*ArcTan[Tan[c]]) - Pi*Log[1 +
 E^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*Log[1 - E^((2*I)*(d*x + ArcTan[
Tan[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan[c]...
```

**Maple** [F]

time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cos(dx+c) \left(\cot^2(dx+c)\right)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3898 vs. $2(776) = 1552$.
time = 0.75, size = 3898, normalized size = 4.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a*b*d^3*f^3*x^3 + 6*a*b*d^3*f^2*x^2*e + 6*a*b*d^3*f*x*e^2 + 6*I*b^2
*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 6*I*b^2*f^3*p
olylog(4, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 6*I*b^2*f^3*polylog
(4, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 6*I*b^2*f^3*polylog(4, -
cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*a*b*d^3*e^3 - 6*I*(a^2 - b^
2)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 6*I*(a^2 - b^2)*
f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 6*I*(a^2 - b^2)*f^3
*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 6*I*(a^2 - b^2)*f^3*p
olylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 3*(-I*(a^2 - b^2)*d^2*f
^3*x^2 - 2*I*(a^2 - b^2)*d^2*f^2*x*e - I*(a^2 - b^2)*d^2*f*e^2)*dilog((I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 3*(-I*(a^2 - b^2)*d^2*f^3*x^2 -
2*I*(a^2 - b^2)*d^2*f^2*x*e - I*(a^2 - b^2)*d^2*f*e^2)*dilog((I*a*cos(d*x +
 c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b + 1)*sin(d*x + c) + 3*(I*(a^2 - b^2)*d^2*f^3*x^2 + 2*I*(a^2 -
 b^2)*d^2*f^2*x*e + I*(a^2 - b^2)*d^2*f*e^2)*dilog((-I*a*cos(d*x + c) - a*s
in(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1)*sin(d*x + c) + 3*(I*(a^2 - b^2)*d^2*f^3*x^2 + 2*I*(a^2 - b^2)*d^2
*f^2*x*e + I*(a^2 - b^2)*d^2*f*e^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x +
c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)
*sin(d*x + c) + 3*(-I*b^2*d^2*f^3*x^2 + 2*I*a*b*d*f^3*x - I*b^2*d^2*f*e^2 -
 2*I*(b^2*d^2*f^2*x - a*b*d*f^2)*e)*dilog(cos(d*x + c) + I*sin(d*x + c))*si
n(d*x + c) + 3*(I*b^2*d^2*f^3*x^2 - 2*I*a*b*d*f^3*x + I*b^2*d^2*f*e^2 + 2*I
*(b^2*d^2*f^2*x - a*b*d*f^2)*e)*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*
x + c) + 3*(I*b^2*d^2*f^3*x^2 + 2*I*a*b*d*f^3*x + I*b^2*d^2*f*e^2 + 2*I*(b^
2*d^2*f^2*x + a*b*d*f^2)*e)*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x +
```

```
 c) + 3*(-I*b^2*d^2*f^3*x^2 - 2*I*a*b*d*f^3*x - I*b^2*d^2*f*e^2 - 2*I*(b^2*
d^2*f^2*x + a*b*d*f^2)*e)*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c
) - ((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*
f*e^2 - (a^2 - b^2)*d^3*e^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - ((a^2 - b^2)*c^3*f^3 - 3*(
a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e^2 - (a^2 - b^2)*d^3*e^3)*l
og(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I
*a)*sin(d*x + c) - ((a^2 - b^2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^
2 - b^2)*c*d^2*f*e^2 - (a^2 - b^2)*d^3*e^3)*log(-2*b*cos(d*x + c) + 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - ((a^2 - b^
2)*c^3*f^3 - 3*(a^2 - b^2)*c^2*d*f^2*e + 3*(a^2 - b^2)*c*d^2*f*e^2 - (a^2 -
 b^2)*d^3*e^3)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) - 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*
c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x + (a^2 - b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2
)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*d*f^2)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*
x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)
*sin(d*x + c) + ((a^2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*c^3*f^3 + 3*((a^2 -
b^2)*d^3*f*x + (a^2 - b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2)*d^3*f^2*x^2 - (a^2
 - b^2)*c^2*d*f^2)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x
+ c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + ((a^
2 - b^2)*d^3*f^3*x^3 + (a^2 - b^2)*c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x + (a^2
- b^2)*c*d^2*f)*e^2 + 3*((a^2 - b^2)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*d*f^2)*e
)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + ((a^2 - b^2)*d^3*f^3*x^
3 + (a^2 - b^2)*c^3*f^3 + 3*((a^2 - b^2)*d^3*f*x + (a^2 - b^2)*c*d^2*f)*e^2
 + 3*((a^2 - b^2)*d^3*f^2*x^2 - (a^2 - b^2)*c^2*d*f^2)*e)*log(-(-I*a*cos(d*
x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) - b)/b)*sin(d*x + c) + (b^2*d^3*f^3*x^3 + 3*a*b*d^2*f^3*x^2 + b^2
*d^3*e^3 + 3*(b^2*d^3*f*x + a*b*d^2*f)*e^2 + 3*(b^2*d^3*f^2*x^2 + 2*a*b*d^2
*f^2*x)*e)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + (b^2*d^3*f
^3*x^3 + 3*a*b*d^2*f^3*x^2 + b^2*d^3*e^3 + 3*(b^2*d^3*f*x + a*b*d^2*f)*e^2
+ 3*(b^2*d^3*f^2*x^2 + 2*a*b*d^2*f^2*x)*e)*log(cos(d*x + c) - I*sin(d*x + c
) + 1)*sin(d*x + c) + (b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*f*e^2 + 3*(b^2*c^2
 + 2*a*b*c)*d*f^2*e - (b^2*c^3 + 3*a*b*c^2)*f^3)*log(-1/2*cos(d*x + c) + 1/
2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + (b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*f
*e^2 + 3*(b^2*c^2 + 2*a*b*c)*d*f^2*e - (b^2*c^3 + 3*a*b*c^2)*f^3)*log(-1/2*
cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*...
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.338 $\int \frac{(e+fx)^2 \cos(c+dx) \cot^2(c+dx)}{a+b \sin(c+dx)} \, dx$

**Optimal**. Leaf size=616

$$\frac{ib(e+fx)^3}{3a^2f}+\frac{i(a^2-b^2)(e+fx)^3}{3a^2bf}-\frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2}-\frac{(e+fx)^2\csc(c+dx)}{ad}-\frac{(a^2-b^2)(e+fx}{}$$

```
[Out] 1/3*I*b*(f*x+e)^3/a^2/f+1/3*I*(a^2-b^2)*(f*x+e)^3/a^2/b/f-4*f*(f*x+e)*arcta
nh(exp(I*(d*x+c)))/a/d^2-(f*x+e)^2*csc(d*x+c)/a/d-b*(f*x+e)^2*ln(1-exp(2*I*
(d*x+c)))/a^2/d-(a^2-b^2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1
/2)))/a^2/b/d-(a^2-b^2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2
)))/a^2/b/d+2*I*f^2*polylog(2,-exp(I*(d*x+c)))/a/d^3-2*I*f^2*polylog(2,exp(
I*(d*x+c)))/a/d^3+I*b*f*(f*x+e)*polylog(2,exp(2*I*(d*x+c)))/a^2/d^2+2*I*(a^
2-b^2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b/d^
2+2*I*(a^2-b^2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))
/a^2/b/d^2-1/2*b*f^2*polylog(3,exp(2*I*(d*x+c)))/a^2/d^3-2*(a^2-b^2)*f^2*po
lylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b/d^3-2*(a^2-b^2)*f^2*p
olylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/d^3
```

**Rubi** [A]
time = 0.90, antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4639, 4493, 3377, 2717, 4495, 4268, 2317, 2438, 4489, 3391, 3798, 2221, 2611, 2320, 6724, 4621, 4615}

$$-\frac{2f^2(a^2-b^2)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^3}-\frac{2f^2(a^2-b^2)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2bd^3}+\frac{2if(a^2-b^2)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^2}+\frac{2if(a^2-b^2)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2bd^2}-\frac{bf^2\text{PolyLog}(3,e^{2i(c+dx)})}{2a^2d^3}+\frac{ibf(e+fx)\text{PolyLog}(2,e^{2i(c+dx)})}{a^2d^2}+\frac{2if^2\text{PolyLog}(2,-e^{i(c+dx)})}{ad^3}-\frac{2if^2\text{PolyLog}(2,e^{i(c+dx)})}{ad^3}-\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd}-\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2bd}+\frac{i(a^2-b^2)(e+fx)^3}{3a^2bf}-\frac{b(e+fx)^2\log\left(1-e^{2i(c+dx)}\right)}{a^2d}+\frac{ib(e+fx)^3}{3a^2f}-\frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2}-\frac{(e+fx)^2\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((I/3)*b*(e + f*x)^3)/(a^2*f) + ((I/3)*(a^2 - b^2)*(e + f*x)^3)/(a^2*b*f) -
 (4*f*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d^2) - ((e + f*x)^2*Csc[c + d*
x])/(a*d) - ((a^2 - b^2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(a^2*b*d) - ((a^2 - b^2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d) - (b*(e + f*x)^2*Log[1 - E^((2*I)
*(c + d*x))])/(a^2*d) + ((2*I)*f^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^3) -
((2*I)*f^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^3) + ((2*I)*(a^2 - b^2)*f*(e +
 f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b*d^2)
+ ((2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(a^2*b*d^2) + (I*b*f*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x)
)])/(a^2*d^2) - (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(a^2*b*d^3) - (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^3) - (b*f^2*PolyLog[3, E^((2*I)*(c
 + d*x))])/(2*a^2*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot}{a+b\sin(c+dx)}}{a}$$

$$= -\frac{\int (e+fx)^2\cos(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\cot(c+dx)\csc(c+d}{a}$$

$$= -\frac{(e+fx)^2\csc(c+dx)}{ad} - \frac{(e+fx)^2\sin(c+dx)}{ad} + \frac{\int (e+fx)^2 c}{}$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+d}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+d}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+d}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+d}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+d}\right.}{ad^2}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1905 vs. 2(616) = 1232.
time = 22.02, size = 1905, normalized size = 3.09

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*e*f*Log[Tan[(c + d*x)/2]])/(a*d^2) + (2*f^2*((c + d*x)*(Log[1 - E^(I*(c
+ d*x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLog
[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^3) + (b*f^2*Csc
[c]*(2*d^2*x^2*(2*d*E^((2*I)*c)*x + (3*I)*(-1 + E^((2*I)*c))*Log[1 - E^((2*
I)*(c + d*x))]) + 6*d*(-1 + E^((2*I)*c))*x*PolyLog[2, E^((2*I)*(c + d*x))]
+ (3*I)*(-1 + E^((2*I)*c))*PolyLog[3, E^((2*I)*(c + d*x))]))/(12*a^2*d^3*E^
(I*c)) + ((a^2 - b^2)*((12*I)*d^3*e^2*E^((2*I)*c)*x + (12*I)*d^3*e*E^((2*I)
```

```
*c)*f*x^2 + (4*I)*d^3*E^((2*I)*c)*f^2*x^3 + (6*I)*d^2*e^2*ArcTan[(2*a*E^(I*
(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - (6*I)*d^2*e^2*E^((2*I)*c)*Arc
Tan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + 3*d^2*e^2*Log[4
*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - 3*d^2*e^2*E^
((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2]
 + 12*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b
^2)*E^((2*I)*c)])] - 12*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))
/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^2*f^2*x^2*Log[1 + (b
*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^2
*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^
2 + b^2)*E^((2*I)*c)])] + 12*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d^2*e*E^((2*I)*c)*f*x*Log[1
+ (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6
*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)
*E^((2*I)*c)])] - 6*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(
I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (12*I)*d*(-1 + E^((2*I)*c)
)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2
+ b^2)*E^((2*I)*c)])] + (12*I)*d*(-1 + E^((2*I)*c))*f*(e + f*x)*PolyLog[2,
-((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] +
12*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*
E^((2*I)*c)])] - 12*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E
^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*f^2*PolyLog[3, -((b*E^(I*(
2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*E^((2*I)
*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)
*E^((2*I)*c)]))]))/(6*a^2*b*d^3*(-1 + E^((2*I)*c))) + ((-3*b*e^2 - 6*b*e*f*
x - 3*b*f^2*x^2 - 3*a*d*e^2*x*Cos[c] - 3*a*d*e*f*x^2*Cos[c] - a*d*f^2*x^3*C
os[c])*Csc[c/2]*Sec[c/2])/(6*a*b*d) - (b*e^2*Csc[c]*(-(d*x*Cos[c]) + Log[Co
s[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a^2*d*(Cos[c]^2 + Sin[c]^2)) + (
Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(e^2*Sin[(d*x)/2]) - 2*e*f*x*Sin[(d*x)/2] - f
^2*x^2*Sin[(d*x)/2]))/(2*a*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]*(e^2*Sin[(d*x)
/2] + 2*e*f*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2]))/(2*a*d) + (b*e*f*Csc[c]
*Sec[c]*(d^2*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi + 2*ArcTan[Tan[c]]) -
Pi*Log[1 + E^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*Log[1 - E^((2*I)*(d*x
 + ArcTan[Tan[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan[c]]*Log[Sin[d*x + Ar
cTan[Tan[c]]]] + I*PolyLog[2, E^((2*I)*(d*x + ArcTan[Tan[c]]))])*Tan[c])/Sq
rt[1 + Tan[c]^2]))/(a^2*d^2*Sqrt[Sec[c]^2*(Cos[c]^2 + Sin[c]^2)])
```

**Maple [F]**
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \cos(dx+c)\left(\cot^2(dx+c)\right)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2541 vs. 2(559) = 1118.
time = 0.62, size = 2541, normalized size = 4.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a*b*d^2*f^2*x^2 + 4*a*b*d^2*f*x*e + 2*b^2*f^2*polylog(3, cos(d*x +
c) + I*sin(d*x + c))*sin(d*x + c) + 2*b^2*f^2*polylog(3, cos(d*x + c) - I*s
in(d*x + c))*sin(d*x + c) + 2*b^2*f^2*polylog(3, -cos(d*x + c) + I*sin(d*x
+ c))*sin(d*x + c) + 2*b^2*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c))*s
in(d*x + c) + 2*a*b*d^2*e^2 + 2*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x +
c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b)*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x + c) +
a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
)/b)*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*si
n(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*
x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin
(d*x + c) + 2*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*d*f*e)*dilog((I*a*cos
(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 2*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2
- b^2)*d*f*e)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 2*(I*(a
^2 - b^2)*d*f^2*x + I*(a^2 - b^2)*d*f*e)*dilog((-I*a*cos(d*x + c) - a*sin(d
*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
 + 1)*sin(d*x + c) + 2*(I*(a^2 - b^2)*d*f^2*x + I*(a^2 - b^2)*d*f*e)*dilog(
```

```
(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 2*(-I*b^2*d*f^2*x - I*b^2*
d*f*e + I*a*b*f^2)*dilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 2*(I
*b^2*d*f^2*x + I*b^2*d*f*e - I*a*b*f^2)*dilog(cos(d*x + c) - I*sin(d*x + c)
)*sin(d*x + c) + 2*(I*b^2*d*f^2*x + I*b^2*d*f*e + I*a*b*f^2)*dilog(-cos(d*x
 + c) + I*sin(d*x + c))*sin(d*x + c) + 2*(-I*b^2*d*f^2*x - I*b^2*d*f*e - I*
a*b*f^2)*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + ((a^2 - b^2)*
c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^2)*d^2*e^2)*log(2*b*cos(d*x + c)
 + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) +
((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^2)*d^2*e^2)*log(2*b
*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*si
n(d*x + c) + ((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (a^2 - b^2)*d^2
*e^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) + 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*c^2*f^2 - 2*(a^2 - b^2)*c*d*f*e + (
a^2 - b^2)*d^2*e^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-
(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d^2*f^2*x^2 - (a^2 -
b^2)*c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d*f)*e)*log(-(I*a*cos
(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b)*sin(d*x + c) + ((a^2 - b^2)*d^2*f^2*x^2 - (a^2 - b^2)*
c^2*f^2 + 2*((a^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d*f)*e)*log(-(I*a*cos(d*x
+ c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2) - b)/b)*sin(d*x + c) + ((a^2 - b^2)*d^2*f^2*x^2 - (a^2 - b^2)*c^2*f
^2 + 2*((a^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d*f)*e)*log(-(-I*a*cos(d*x + c)
 - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b)*sin(d*x + c) + ((a^2 - b^2)*d^2*f^2*x^2 - (a^2 - b^2)*c^2*f^2 +
 2*((a^2 - b^2)*d^2*f*x + (a^2 - b^2)*c*d*f)*e)*log(-(-I*a*cos(d*x + c) - a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b)*sin(d*x + c) + (b^2*d^2*f^2*x^2 + 2*a*b*d*f^2*x + b^2*d^2*e^2 + 2*(
b^2*d^2*f*x + a*b*d*f)*e)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x +
c) + (b^2*d^2*f^2*x^2 + 2*a*b*d*f^2*x + b^2*d^2*e^2 + 2*(b^2*d^2*f*x + a*b*
d*f)*e)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + (b^2*d^2*e^2
- 2*(b^2*c + a*b)*d*f*e + (b^2*c^2 + 2*a*b*c)*f^2)*log(-1/2*cos(d*x + c) +
1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + (b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*f
*e + (b^2*c^2 + 2*a*b*c)*f^2)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) +
1/2)*sin(d*x + c) + (b^2*d^2*f^2*x^2 - 2*a*b*d*f^2*x - (b^2*c^2 + 2*a*b*c)*
f^2 + 2*(b^2*d^2*f*x + b^2*c*d*f)*e)*log(-cos(d*x + c) + I*sin(d*x + c) + 1
)*sin(d*x + c) + (b^2*d^2*f^2*x^2 - 2*a*b*d*f^2*x - (b^2*c^2 + 2*a*b*c)*f^2
 + 2*(b^2*d^2*f*x + b^2*c*d*f)*e)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*s
in(d*x + c))/(a^2*b*d^3*sin(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.339 $\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=386

$$\frac{ib(e+fx)^2}{2a^2f}+\frac{i(a^2-b^2)(e+fx)^2}{2a^2bf}-\frac{f\tanh^{-1}(\cos(c+dx))}{ad^2}-\frac{(e+fx)\csc(c+dx)}{ad}-\frac{(a^2-b^2)(e+fx)\log\left(1-\right.}{a^2bd}$$

```
[Out] 1/2*I*b*(f*x+e)^2/a^2/f+1/2*I*(a^2-b^2)*(f*x+e)^2/a^2/b/f-f*arctanh(cos(d*x
+c))/a/d^2-(f*x+e)*csc(d*x+c)/a/d-b*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/a^2/d-(a
^2-b^2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b/d-(a^2-b
^2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/d+1/2*I*b*f*
polylog(2,exp(2*I*(d*x+c)))/a^2/d^2+I*(a^2-b^2)*f*polylog(2,I*b*exp(I*(d*x+
c))/(a-(a^2-b^2)^(1/2)))/a^2/b/d^2+I*(a^2-b^2)*f*polylog(2,I*b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/a^2/b/d^2
```

**Rubi** [A]
time = 0.51, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 15, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {4639, 4493, 3377, 2718, 4495, 3855, 4489, 2715, 8, 3798, 2221, 2317, 2438, 4621, 4615}

$$\frac{if(a^2-b^2)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^2}+\frac{if(a^2-b^2)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2bd^2}+\frac{ibf\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2a^2d^2}-\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd}-\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2bd}+\frac{i(a^2-b^2)(e+fx)^2}{2a^2bf}-\frac{b(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{a^2d}+\frac{ib(e+fx)^2}{2a^2f}-\frac{f\tanh^{-1}(\cos(c+dx))}{ad^2}-\frac{(e+fx)\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((I/2)*b*(e + f*x)^2)/(a^2*f) + ((I/2)*(a^2 - b^2)*(e + f*x)^2)/(a^2*b*f) -
 (f*ArcTanh[Cos[c + d*x]])/(a*d^2) - ((e + f*x)*Csc[c + d*x])/(a*d) - ((a^2
 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^
2*b*d) - ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(a^2*b*d) - (b*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a^2*d) +
 (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/
(a^2*b*d^2) + (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(a^2*b*d^2) + ((I/2)*b*f*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2
*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+}{a+b\sin(c+dx)}}{a}$$

$$= -\frac{\int (e+fx)\cos(c+dx)\,dx}{a} + \frac{\int (e+fx)\cot(c+dx)\csc(c+dx)}{a}$$

$$= -\frac{(e+fx)\csc(c+dx)}{ad} - \frac{(e+fx)\sin(c+dx)}{ad} + \frac{\int (e+fx)\cos(}{a}$$

$$= \frac{ib(e+fx)^2}{2a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^2}{2bf} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} -$$

$$= \frac{ib(e+fx)^2}{2a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^2}{2bf} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} -$$

$$= \frac{ib(e+fx)^2}{2a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^2}{2bf} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} -$$

$$= \frac{ib(e+fx)^2}{2a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^2}{2bf} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} -$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2314 vs. 2(386) = 772.
time = 13.63, size = 2314, normalized size = 5.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c + d*x)/2] - f*(c + d*x)*Cos[(c + d*x
)/2])*Csc[(c + d*x)/2])/(2*a*d^2) - (b*e*Log[Sin[c + d*x]])/(a^2*d) + (b*c*
f*Log[Sin[c + d*x]])/(a^2*d^2) + (f*Log[Tan[(c + d*x)/2]])/(a*d^2) - (b*f*(
(c + d*x)*Log[1 - E^((2*I)*(c + d*x))] - (I/2)*((c + d*x)^2 + PolyLog[2, E^
((2*I)*(c + d*x))])))/(a^2*d^2) + (Sec[(c + d*x)/2]*(-(d*e*Sin[(c + d*x)/2]
) + c*f*Sin[(c + d*x)/2] - f*(c + d*x)*Sin[(c + d*x)/2]))/(2*a*d^2) + ((f*(
c + d*x)^2 + (2*I)*d*e*Log[Sec[(c + d*x)/2]^2] - (2*I)*c*f*Log[Sec[(c + d*x
)/2]^2] - (2*I)*d*e*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])] + (2*I)*c*
f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])] - (4*I)*f*(c + d*x)*Log[(-2*
```

```
I)/(-I + Tan[(c + d*x)/2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt
[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + 2*f*Log[
1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(
I*a - b + Sqrt[-a^2 + b^2]))] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sq
rt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] - 2*f
*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]
)/(I*a + b + Sqrt[-a^2 + b^2])] + 4*f*PolyLog[2, -Cos[c + d*x] + I*Sin[c +
d*x]] + 2*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 +
 b^2]))] - 2*f*PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^
2 + b^2]))] + 2*f*PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^
2 + b^2])] - 2*f*PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a
^2 + b^2]))])*(-((e*Cos[c + d*x])/(a + b*Sin[c + d*x])) + (b^2*e*Cos[c + d*
x])/(a^2*(a + b*Sin[c + d*x])) + (c*f*Cos[c + d*x])/(d*(a + b*Sin[c + d*x])
) - (b^2*c*f*Cos[c + d*x])/(a^2*d*(a + b*Sin[c + d*x])) - (f*(c + d*x)*Cos[
c + d*x])/(d*(a + b*Sin[c + d*x])) + (b^2*f*(c + d*x)*Cos[c + d*x])/(a^2*d*
(a + b*Sin[c + d*x]))))/(d*(2*f*(c + d*x) - (4*I)*f*Log[(-2*I)/(-I + Tan[(c
 + d*x)/2])] - (4*f*Log[1 + Cos[c + d*x] - I*Sin[c + d*x]]*(I*Cos[c + d*x]
+ Sin[c + d*x]))/(-Cos[c + d*x] + I*Sin[c + d*x]) + (I*f*Log[1 - (a*(1 - I*
Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 -
 I*Tan[(c + d*x)/2]) - (I*f*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2
])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/
2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sq
rt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) + (I*f*Log[1
- (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x
)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(
c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(
c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a +
b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) + (2*I)
*d*e*Tan[(c + d*x)/2] - (2*I)*c*f*Tan[(c + d*x)/2] + ((2*I)*f*(c + d*x)*Sec
[(c + d*x)/2]^2)/(-I + Tan[(c + d*x)/2]) - (f*Log[1 - (a*(I + Tan[(c + d*x)
/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(I + Tan[(c + d*x)/
2]) + (I*a*f*Log[1 - (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^
2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[(
c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]
) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b
^2] + a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/
2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c +
 d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) -
 ((2*I)*d*e*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(c + d*x)/2]^2 + Sec[(c
+ d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/(a + b*Sin[c + d*x]) +
((2*I)*c*f*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(c + d*x)/2]^2 + Sec[(c +
 d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/(a + b*Sin[c + d*x])))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1731 vs. $2(351) = 702$.

time = 0.33, size = 1732, normalized size = 4.49

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1732 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/d/b*ln(exp(I*(d*x+c)))*e-1/d/b*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c
))-I*b)-2*I*(f*x+e)*exp(I*(d*x+c))/d/a/(exp(2*I*(d*x+c))-1)-I/b*e*x+1/2*I/b
*f*x^2+1/a^2*b^3/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2)
)/(I*a+(-a^2+b^2)^(1/2)))*c-I/a^2*b^3/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(
d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-I/a^2*b^3/d^2*f/(-a^2+b^2
)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-I/d
^2/b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+
b^2)^(1/2)))*a^2-I/d^2/b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2
)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2-2/d*b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*
x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-2/d^2*b*f/(-a^2+b^2)*ln((
I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-2/d*b*f/(-
a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))
*x-2/d^2*b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a
^2+b^2)^(1/2)))*c+1/d^2/b*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*
b)-2/d^2/b*f*c*ln(exp(I*(d*x+c)))-1/a^2/d*b*e*ln(exp(I*(d*x+c))-1)-1/a^2/d*
b*e*ln(exp(I*(d*x+c))+1)+1/a/d^2*f*ln(exp(I*(d*x+c))-1)-1/a/d^2*f*ln(exp(I*
(d*x+c))+1)+1/d/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(
I*a-(-a^2+b^2)^(1/2)))*a^2*x+1/d^2/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-
(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2*c+1/d/b*f/(-a^2+b^2)*ln((I*a+
b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*x+1/d^2/b*f/
(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)
))*a^2*c+1/a^2*b^3/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2)
)/(I*a+(-a^2+b^2)^(1/2)))*x+1/a^2*b^3/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c
))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/a^2*b^3/d^2*f/(-a^2+b^2)*l
n((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/a^2/d
*b*f*ln(exp(I*(d*x+c))+1)*x+1/a^2/d^2*b*f*c*ln(exp(I*(d*x+c))-1)-I/a^2/d^2*
b*f*dilog(exp(I*(d*x+c)))+I/a^2*b/d^2*f*dilog(exp(I*(d*x+c))+1)-1/a^2*b/d^2
*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+2*I*b/d^2*f/(-a^2+b^2)
*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+2*I*
b/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2
+b^2)^(1/2)))+2*I/b/d*c*f*x+I/b/d^2*c^2*f+1/a^2*b/d*e*ln(I*b*exp(2*I*(d*x+c
))-2*a*exp(I*(d*x+c))-I*b)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1432 vs. 2(349) = 698.
time = 0.63, size = 1432, normalized size = 3.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*a*b*d*f*x - I*b^2*f*dilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x +
c) + I*b^2*f*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + I*b^2*f*di
log(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - I*b^2*f*dilog(-cos(d*x +
 c) - I*sin(d*x + c))*sin(d*x + c) + 2*a*b*d*e - I*(a^2 - b^2)*f*dilog((I*a
*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - I*(a^2 - b^2)*f*dilog((I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1)*sin(d*x + c) + I*(a^2 - b^2)*f*dilog((-I*a*cos(d*x
+ c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2) - b)/b + 1)*sin(d*x + c) + I*(a^2 - b^2)*f*dilog((-I*a*cos(d*x + c)
 - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b + 1)*sin(d*x + c) - ((a^2 - b^2)*c*f - (a^2 - b^2)*d*e)*log(2*b*
cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin
(d*x + c) - ((a^2 - b^2)*c*f - (a^2 - b^2)*d*e)*log(2*b*cos(d*x + c) - 2*I*
b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - ((a^2 -
 b^2)*c*f - (a^2 - b^2)*d*e)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2
*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - ((a^2 - b^2)*c*f - (a^2 -
 b^2)*d*e)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) - 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-
(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + ((a^2 - b^2)*d*f*x + (a^2 - b^2
)*c*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + ((a^2 - b^2)*d*f*x
+ (a^2 - b^2)*c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + ((a^2
- b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) -
```

```
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x
+ c) + (b^2*d*f*x + b^2*d*e + a*b*f)*log(cos(d*x + c) + I*sin(d*x + c) + 1)
*sin(d*x + c) + (b^2*d*f*x + b^2*d*e + a*b*f)*log(cos(d*x + c) - I*sin(d*x
+ c) + 1)*sin(d*x + c) + (b^2*d*e - (b^2*c + a*b)*f)*log(-1/2*cos(d*x + c)
+ 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + (b^2*d*e - (b^2*c + a*b)*f)*log(
-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + (b^2*d*f*x + b
^2*c*f)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + (b^2*d*f*x +
 b^2*c*f)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c))/(a^2*b*d^2*
sin(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)*cot(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.340 $\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=60

$$-\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}$$

```
[Out] -csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d-(1-b^2/a^2)*ln(a+b*sin(d*x+c))/b/d
```

**Rubi** [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) - ((1 - b^2/a^2)*Log[
a + b*Sin[c + d*x]])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^2} - \frac{b^2}{a^2x} + \frac{-a^2+b^2}{a^2(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= -\frac{\csc(c+dx)}{ad} - \frac{b\log(\sin(c+dx))}{a^2 d} - \frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}$$

**Mathematica [A]**
time = 0.07, size = 54, normalized size = 0.90

$$\frac{-ab\csc(c+dx) - b^2\log(\sin(c+dx)) + (-a^2+b^2)\log(a+b\sin(c+dx))}{a^2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*b*Csc[c + d*x]) - b^2*Log[Sin[c + d*x]] + (-a^2 + b^2)*Log[a + b*Sin[c
 + d*x]])/(a^2*b*d)
```

**Maple [A]**
time = 0.13, size = 59, normalized size = 0.98

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{1}{a\sin(dx+c)} - \frac{b\ln(\sin(dx+c))}{a^2} + \frac{(-a^2+b^2)\ln(a+b\sin(dx+c))}{a^2b}}{d}$ |
| default | $\frac{-\frac{1}{a\sin(dx+c)} - \frac{b\ln(\sin(dx+c))}{a^2} + \frac{(-a^2+b^2)\ln(a+b\sin(dx+c))}{a^2b}}{d}$ |
| risch | $\frac{ix}{b} + \frac{2ic}{bd} - \frac{2ie^{i(dx+c)}}{da(e^{2i(dx+c)}-1)} - \frac{b\ln(e^{2i(dx+c)}-1)}{a^2d} - \frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{bd} + \frac{b\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx}{b}\right.}{a^2d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a/sin(d*x+c)-1/a^2*b*ln(sin(d*x+c))+(-a^2+b^2)/a^2/b*ln(a+b*sin(d*x
+c)))
```

**Maxima** [A]
time = 0.42, size = 57, normalized size = 0.95

$$-\frac{\frac{b\log(\sin(dx+c))}{a^2}+\frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{a^2b}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*log(sin(d*x + c))/a^2 + (a^2 - b^2)*log(b*sin(d*x + c) + a)/(a^2*b) + 1
/(a*sin(d*x + c)))/d
```

**Fricas** [A]
time = 0.37, size = 69, normalized size = 1.15

$$-\frac{b^2\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+(a^2-b^2)\log(b\sin(dx+c)+a)\sin(dx+c)+ab}{a^2bd\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b^2*log(1/2*sin(d*x + c))*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x + c) +
 a)*sin(d*x + c) + a*b)/(a^2*b*d*sin(d*x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [A]
time = 7.44, size = 72, normalized size = 1.20

$$-\frac{\frac{b\log(|\sin(dx+c)|)}{a^2}+\frac{(a^2-b^2)\log(|b\sin(dx+c)+a|)}{a^2b}-\frac{b\sin(dx+c)-a}{a^2\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(sin(d*x + c)))/a^2 + (a^2 - b^2)*log(abs(b*sin(d*x + c) + a))/(
a^2*b) - (b*sin(d*x + c) - a)/(a^2*sin(d*x + c)))/d
```

**<u>Mupad</u>** [B]
time = 4.81, size = 118, normalized size = 1.97

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,\left(\frac{b}{a^2}-\frac{1}{b}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{b\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(b/a^2 - 1/b))/d
- tan(c/2 + (d*x)/2)/(2*a*d) - cot(c/2 + (d*x)/2)/(2*a*d) + log(tan(c/2 + (
d*x)/2)^2 + 1)/(b*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d)
```

# 3.341 $\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=1144

$$-\frac{i(e+fx)^3}{ad}-\frac{(e+fx)^4}{4af}-\frac{(a^2-b^2)(e+fx)^4}{4ab^2f}+\frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d}+\frac{6bf^2(e+fx)\cos(c+dx)}{a^2d^3}+\frac{6(}{}$$

```
[Out] 6*(a^2-b^2)^(3/2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2
/b^2/d^4-6*(a^2-b^2)^(3/2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1
/2)))/a^2/b^2/d^4+6*b*f^2*(f*x+e)*polylog(3,-exp(I*(d*x+c)))/a^2/d^3-1/4*(f
*x+e)^4/a/f+2*b*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a^2/d+6*b*f^2*(f*x+e)*cos
(d*x+c)/a^2/d^3-(a^2-b^2)*(f*x+e)^3*cos(d*x+c)/a^2/b/d-6*(a^2-b^2)*f^3*sin(
d*x+c)/a^2/b/d^4-b*(f*x+e)^3*cos(d*x+c)/a^2/d-6*b*f^3*sin(d*x+c)/a^2/d^4-1/
4*(a^2-b^2)*(f*x+e)^4/a/b^2/f+I*(a^2-b^2)^(3/2)*(f*x+e)^3*ln(1-I*b*exp(I*(d
*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^2/d-3*(a^2-b^2)^(3/2)*f*(f*x+e)^2*polylog
(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d^2+3*(a^2-b^2)^(3/2)*f*
(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^2/d^2-I*(
a^2-b^2)^(3/2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b
^2/d+6*I*b*f^3*polylog(4,-exp(I*(d*x+c)))/a^2/d^4-3*I*b*f*(f*x+e)^2*polylog
(2,-exp(I*(d*x+c)))/a^2/d^2+3*f*(f*x+e)^2*ln(1-exp(2*I*(d*x+c)))/a/d^2+3*I*
b*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a^2/d^2-I*(f*x+e)^3/a/d-(f*x+e)^3*c
ot(d*x+c)/a/d+3/2*f^3*polylog(3,exp(2*I*(d*x+c)))/a/d^4-6*I*(a^2-b^2)^(3/2)
*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d^3+
6*I*(a^2-b^2)^(3/2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(
1/2)))/a^2/b^2/d^3+3*b*f*(f*x+e)^2*sin(d*x+c)/a^2/d^2-3*I*f^2*(f*x+e)*polyl
og(2,exp(2*I*(d*x+c)))/a/d^3-6*b*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a^2/
d^3-6*I*b*f^3*polylog(4,exp(I*(d*x+c)))/a^2/d^4+6*(a^2-b^2)*f^2*(f*x+e)*cos
(d*x+c)/a^2/b/d^3+3*(a^2-b^2)*f*(f*x+e)^2*sin(d*x+c)/a^2/b/d^2
```

**Rubi** [A]
time = 1.67, antiderivative size = 1144, normalized size of antiderivative = 1.00, number of steps used = 66, number of rules used = 20, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4639, 4493, 3392, 32, 3391, 3801, 3798, 2221, 2611, 2320, 6724, 4490, 3377, 2717, 2713, 4268, 6744, 4621, 3404, 2296}

---

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (e + f*x)^4/(4*a*f) - ((a^2 - b^2)*(e + f*x)^4)/
(4*a*b^2*f) + (2*b*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a^2*d) + (6*b*f^2
*(e + f*x)*Cos[c + d*x])/(a^2*d^3) + (6*(a^2 - b^2)*f^2*(e + f*x)*Cos[c + d
```

```
*x])/(a^2*b*d^3) - (b*(e + f*x)^3*Cos[c + d*x])/(a^2*d) - ((a^2 - b^2)*(e +
 f*x)^3*Cos[c + d*x])/(a^2*b*d) - ((e + f*x)^3*Cot[c + d*x])/(a*d) - (I*(a^
2 - b^2)^(3/2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(a^2*b^2*d) + (I*(a^2 - b^2)^(3/2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d) + (3*f*(e + f*x)^2*Log[1 - E^((
2*I)*(c + d*x))])/(a*d^2) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*
x))])/(a^2*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a^2*
d^2) - (3*(a^2 - b^2)^(3/2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/
(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) + (3*(a^2 - b^2)^(3/2)*f*(e + f*x)^2*
PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) - ((
3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^3) + (6*b*f^2*(e +
 f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[
3, E^(I*(c + d*x))])/(a^2*d^3) - ((6*I)*(a^2 - b^2)^(3/2)*f^2*(e + f*x)*Pol
yLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^3) + ((6*I
)*(a^2 - b^2)^(3/2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/(a^2*b^2*d^3) + (3*f^3*PolyLog[3, E^((2*I)*(c + d*x))])/(2*
a*d^4) + ((6*I)*b*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a^2*d^4) - ((6*I)*b*f^
3*PolyLog[4, E^(I*(c + d*x))])/(a^2*d^4) + (6*(a^2 - b^2)^(3/2)*f^3*PolyLog
[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^4) - (6*(a^2 -
 b^2)^(3/2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a
^2*b^2*d^4) - (6*b*f^3*Sin[c + d*x])/(a^2*d^4) - (6*(a^2 - b^2)*f^3*Sin[c +
 d*x])/(a^2*b*d^4) + (3*b*f*(e + f*x)^2*Sin[c + d*x])/(a^2*d^2) + (3*(a^2 -
 b^2)*f*(e + f*x)^2*Sin[c + d*x])/(a^2*b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos^2(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\,c}{a+b\sin(c+dx}}{a}$$

$$= -\frac{\int (e+fx)^3\cos^2(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\cot^2(c+dx)\,dx}{a} -$$

$$= -\frac{3f(e+fx)^2\cos^2(c+dx)}{4ad^2} - \frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{(e+fx}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{3(e+fx)^4}{8af} + \frac{3f^3\cos^2(c+dx)}{8ad^4} - \frac{(e+fx)^3\cot}{ad}$$

$$= \frac{3ef^2x}{4ad^2} + \frac{3f^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+f}{4b^2f}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4632 vs. $2(1144) = 2288$.
time = 40.58, size = 4632, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),
x]

[Out] -((b*e^3*Log[Tan[(c + d*x)/2]])/(a^2*d)) - (3*b*e^2*f*((c + d*x)*(Log[1 - E
^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(
PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a^2*d^2) - (
f^3*Csc[c]*(2*d^2*x^2*(2*d*E^((2*I)*c)*x + (3*I)*(-1 + E^((2*I)*c))*Log[1 -
 E^((2*I)*(c + d*x))]) + 6*d*(-1 + E^((2*I)*c))*x*PolyLog[2, E^((2*I)*(c +
d*x))] + (3*I)*(-1 + E^((2*I)*c))*PolyLog[3, E^((2*I)*(c + d*x))]))/(4*a*d^
4*E^(I*c)) + (6*b*e*f^2*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - I
*d*x*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + I*d*x*PolyLog[2, Cos[c +
d*x] + I*Sin[c + d*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - PolyL
og[3, Cos[c + d*x] + I*Sin[c + d*x]]))/(a^2*d^3) - (b*f^3*(-2*d^3*x^3*ArcTa
nh[Cos[c + d*x] + I*Sin[c + d*x]] + (3*I)*d^2*x^2*PolyLog[2, -Cos[c + d*x]
- I*Sin[c + d*x]] - (3*I)*d^2*x^2*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*x]]
 - 6*d*x*PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] + 6*d*x*PolyLog[3, Cos[
c + d*x] + I*Sin[c + d*x]] - (6*I)*PolyLog[4, -Cos[c + d*x] - I*Sin[c + d*x
]] + (6*I)*PolyLog[4, Cos[c + d*x] + I*Sin[c + d*x]]))/(a^2*d^4) + (3*e^2*f
*Csc[c]*(-(d*x*Cos[c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a
*d^2*(Cos[c]^2 + Sin[c]^2)) + (I*(a^2 - b^2)^(3/2)*((3*I)*Sqrt[a^2 - b^2]*d
^3*e^2*f*x*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sq
rt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + (
3*I)*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*(Cos[2*c + d*x] + I*Sin[2*c +
 d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c])]
*(Cos[c] + I*Sin[c]) + I*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*(Cos[2*c +
d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c
])^2] - a*Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^
2*PolyLog[2, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(
-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 3*Sq
rt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*(Cos[2*c + d*x] + I*Sin[2*c +
 d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c
])]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*(Co
s[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] +
 I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) + (6*I)*Sqrt[a^2 - b^2]*d*f
^3*x*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqr
t[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 6
*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(
I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c]
 + I*Sin[c]) + 6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*(Cos[2*c + d*x] + I*Sin[
2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*
Sin[c])]*(Cos[c] + I*Sin[c]) + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*(Co
s[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c
] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 3*Sqrt[a^2 - b^2]*d^
3*e*f^2*x^2*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c]
+ Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[
```

```
c]) + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c +
d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c]
)]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*(Cos[2*
c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] +
I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) + 6*Sqrt[a^2 - b^2]*d*f^3*
x*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[
(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*((-I)*Cos[c] + Sin[c]) - (
2*I)*d^3*e^3*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c + d*x]))/Sqrt[a^2 - b^
2]]*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]]))/(a^2*b^2*d^4*Sqrt[(-a^2 +
b^2)*(Cos[2*c] + I*Sin[2*c])]) + Csc[c]*Csc[c + d*x]*(Cos[c + d*x]/(16*a*b^
2*d^4) - ((I/16)*Sin[c + d*x])/(a*b^2*d^4))*((8*I)*b^2*d^3*e^3*Cos[c] + (24
*I)*b^2*d^3*e^2*f*x*Cos[c] + (24*I)*b^2*d^3*e*f^2*x^2*Cos[c] + (8*I)*b^2*d^
3*f^3*x^3*Cos[c] - 2*a*b*d^3*e^3*Cos[d*x] + (18*I)*a*b*d^2*e^2*f*Cos[d*x] +
 12*a*b*d*e*f^2*Cos[d*x] - (36*I)*a*b*f^3*Cos[d*x] - 6*a*b*d^3*e^2*f*x*Cos[
d*x] + (36*I)*a*b*d^2*e*f^2*x*Cos[d*x] + 12*a*b*d*f^3*x*Cos[d*x] - 6*a*b*d^
3*e*f^2*x^2*Cos[d*x] + (18*I)*a*b*d^2*f^3*x^2*Cos[d*x] - 2*a*b*d^3*f^3*x^3*
Cos[d*x] + 2*a*b*d^3*e^3*Cos[2*c + d*x] - (18*I)*a*b*d^2*e^2*f*Cos[2*c + d*
x] - 12*a*b*d*e*f^2*Cos[2*c + d*x] + (36*I)*a*b*f^3*Cos[2*c + d*x] + 6*a*b*
d^3*e^2*f*x*Cos[2*c + d*x] - (36*I)*a*b*d^2*e*f^2*x*Cos[2*c + d*x] - 12*a*b
*d*f^3*x*Cos[2*c + d*x] + 6*a*b*d^3*e*f^2*x^2*Cos[2*c + d*x] - (18*I)*a*b*d
^2*f^3*x^2*Cos[2*c + d*x] + 2*a*b*d^3*f^3*x^3*Cos[2*c + d*x] - (8*I)*b^2*d^
3*e^3*Cos[c + 2*d*x] - 4*a^2*d^4*e^3*x*Cos[c + 2*d*x] - (24*I)*b^2*d^3*e^2*
f*x*Cos[c + 2*d*x] - 6*a^2*d^4*e^2*f*x^2*Cos[c + 2*d*x] - (24*I)*b^2*d^3*e*
f^2*x^2*Cos[c + 2*d*x] - 4*a^2*d^4*e*f^2*x^3*Cos[c + 2*d*x] - (8*I)*b^2*d^3
*f^3*x^3*Cos[c + 2*d*x] - a^2*d^4*f^3*x^4*Cos[c...
```

**Maple [F]**
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\cos^2(dx+c)) (\cot^2(dx+c))}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4695 vs. $2(1052) = 2104$.
time = 0.96, size = 4695, normalized size = 4.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/4*(12*a^2*b*d^2*f^3*x^2 + 24*a^2*b*d^2*f^2*x*e - 12*I*b^3*f^3*polylog(4,
cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 12*I*b^3*f^3*polylog(4, cos(d
*x + c) - I*sin(d*x + c))*sin(d*x + c) - 12*I*b^3*f^3*polylog(4, -cos(d*x +
 c) + I*sin(d*x + c))*sin(d*x + c) + 12*I*b^3*f^3*polylog(4, -cos(d*x + c)
- I*sin(d*x + c))*sin(d*x + c) + 12*a^2*b*d^2*f*e^2 - 12*I*(a^2*b - b^3)*f^
3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c)
+ 12*I*(a^2*b - b^3)*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x +
c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b)*sin(d*x + c) + 12*I*(a^2*b - b^3)*f^3*sqrt(-(a^2 - b^2)/b^2)*poly
log(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 12*I*(a^2*b - b^3)*f^3*sqr
t(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 24
*a^2*b*f^3 - 6*(-I*(a^2*b - b^3)*d^2*f^3*x^2 - 2*I*(a^2*b - b^3)*d^2*f^2*x*
e - I*(a^2*b - b^3)*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x +
c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) - b)/b + 1)*sin(d*x + c) - 6*(I*(a^2*b - b^3)*d^2*f^3*x^2 + 2*I*(a^2*
b - b^3)*d^2*f^2*x*e + I*(a^2*b - b^3)*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*di
log((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 6*(I*(a^2*b - b^3)*d^2*
f^3*x^2 + 2*I*(a^2*b - b^3)*d^2*f^2*x*e + I*(a^2*b - b^3)*d^2*f*e^2)*sqrt(-
(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 6*(
-I*(a^2*b - b^3)*d^2*f^3*x^2 - 2*I*(a^2*b - b^3)*d^2*f^2*x*e - I*(a^2*b - b
^3)*d^2*f*e^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x
+ c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
1)*sin(d*x + c) - 2*((a^2*b - b^3)*c^3*f^3 - 3*(a^2*b - b^3)*c^2*d*f^2*e +
3*(a^2*b - b^3)*c*d^2*f*e^2 - (a^2*b - b^3)*d^3*e^3)*sqrt(-(a^2 - b^2)/b^2)
*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2
```

```
*I*a)*sin(d*x + c) - 2*((a^2*b - b^3)*c^3*f^3 - 3*(a^2*b - b^3)*c^2*d*f^2*e
 + 3*(a^2*b - b^3)*c*d^2*f*e^2 - (a^2*b - b^3)*d^3*e^3)*sqrt(-(a^2 - b^2)/b
^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
- 2*I*a)*sin(d*x + c) + 2*((a^2*b - b^3)*c^3*f^3 - 3*(a^2*b - b^3)*c^2*d*f^
2*e + 3*(a^2*b - b^3)*c*d^2*f*e^2 - (a^2*b - b^3)*d^3*e^3)*sqrt(-(a^2 - b^2
)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b
^2) + 2*I*a)*sin(d*x + c) + 2*((a^2*b - b^3)*c^3*f^3 - 3*(a^2*b - b^3)*c^2*
d*f^2*e + 3*(a^2*b - b^3)*c*d^2*f*e^2 - (a^2*b - b^3)*d^3*e^3)*sqrt(-(a^2 -
 b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) - 2*I*a)*sin(d*x + c) - 2*((a^2*b - b^3)*d^3*f^3*x^3 + (a^2*b - b^3
)*c^3*f^3 + 3*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*e^2 + 3*((a^2
*b - b^3)*d^3*f^2*x^2 - (a^2*b - b^3)*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*
log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + 2*((a^2*b - b^3)*d^3*f^3*x
^3 + (a^2*b - b^3)*c^3*f^3 + 3*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2
*f)*e^2 + 3*((a^2*b - b^3)*d^3*f^2*x^2 - (a^2*b - b^3)*c^2*d*f^2)*e)*sqrt(-
(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c)
+ I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) - 2*((a^2*b
 - b^3)*d^3*f^3*x^3 + (a^2*b - b^3)*c^3*f^3 + 3*((a^2*b - b^3)*d^3*f*x + (a
^2*b - b^3)*c*d^2*f)*e^2 + 3*((a^2*b - b^3)*d^3*f^2*x^2 - (a^2*b - b^3)*c^2
*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c)
+ (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*
x + c) + 2*((a^2*b - b^3)*d^3*f^3*x^3 + (a^2*b - b^3)*c^3*f^3 + 3*((a^2*b -
 b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*e^2 + 3*((a^2*b - b^3)*d^3*f^2*x^2 -
 (a^2*b - b^3)*c^2*d*f^2)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c)
 - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b)*sin(d*x + c) + 12*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*f^2*
e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) +
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c)
 - 12*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2
)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 12*((a^2*b - b^3)*d*f
^3*x + (a^2*b - b^3)*d*f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(
d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2))/b)*sin(d*x + c) - 12*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*
f^2*e)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x +
c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-...
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x)),
 x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.342 $\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=840

$$-\frac{i(e+fx)^2}{ad}-\frac{(e+fx)^3}{3af}-\frac{(a^2-b^2)(e+fx)^3}{3ab^2f}+\frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d}+\frac{2bf^2\cos(c+dx)}{a^2d^3}+\frac{2(a^2-b^2)}{}$$

```
[Out] 2*I*(a^2-b^2)^(3/2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a
^2/b^2/d^3-1/3*(f*x+e)^3/a/f-1/3*(a^2-b^2)*(f*x+e)^3/a/b^2/f+2*b*(f*x+e)^2*
arctanh(exp(I*(d*x+c)))/a^2/d+2*b*f^2*cos(d*x+c)/a^2/d^3+2*(a^2-b^2)*f^2*co
s(d*x+c)/a^2/b/d^3-b*(f*x+e)^2*cos(d*x+c)/a^2/d-(a^2-b^2)*(f*x+e)^2*cos(d*x
+c)/a^2/b/d-(f*x+e)^2*cot(d*x+c)/a/d+2*f*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/a/d
^2-I*f^2*polylog(2,exp(2*I*(d*x+c)))/a/d^3+I*(a^2-b^2)^(3/2)*(f*x+e)^2*ln(1
-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^2/d-I*(a^2-b^2)^(3/2)*(f*x+e
)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d+2*I*b*f*(f*x+e)*
polylog(2,exp(I*(d*x+c)))/a^2/d^2-2*I*b*f*(f*x+e)*polylog(2,-exp(I*(d*x+c))
)/a^2/d^2-2*(a^2-b^2)^(3/2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-
b^2)^(1/2)))/a^2/b^2/d^2+2*(a^2-b^2)^(3/2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d
*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^2/d^2+2*b*f^2*polylog(3,-exp(I*(d*x+c)))/
a^2/d^3-2*b*f^2*polylog(3,exp(I*(d*x+c)))/a^2/d^3-2*I*(a^2-b^2)^(3/2)*f^2*p
olylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d^3-I*(f*x+e)^2/a/
d+2*b*f*(f*x+e)*sin(d*x+c)/a^2/d^2+2*(a^2-b^2)*f*(f*x+e)*sin(d*x+c)/a^2/b/d
^2
```

**Rubi** [A]
time = 1.38, antiderivative size = 840, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 22, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {4639, 4493, 3392, 32, 2715, 8, 3801, 3798, 2221, 2317, 2438, 4490, 3391, 3377, 2718, 4268, 2611, 2320, 6724, 4621, 3404, 2296}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^2)/(a*d) - (e + f*x)^3/(3*a*f) - ((a^2 - b^2)*(e + f*x)^3)/
(3*a*b^2*f) + (2*b*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a^2*d) + (2*b*f^2
*Cos[c + d*x])/(a^2*d^3) + (2*(a^2 - b^2)*f^2*Cos[c + d*x])/(a^2*b*d^3) - (
b*(e + f*x)^2*Cos[c + d*x])/(a^2*d) - ((a^2 - b^2)*(e + f*x)^2*Cos[c + d*x]
)/(a^2*b*d) - ((e + f*x)^2*Cot[c + d*x])/(a*d) - (I*(a^2 - b^2)^(3/2)*(e +
f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d) +
(I*(a^2 - b^2)^(3/2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^
2 - b^2])])/(a^2*b^2*d) + (2*f*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a*d
```

```
^2) - ((2*I)*b*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + ((2*I)
*b*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - (2*(a^2 - b^2)^(3/2
)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2
*b^2*d^2) + (2*(a^2 - b^2)^(3/2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) - (I*f^2*PolyLog[2, E^((2*I)*(c +
d*x))])/(a*d^3) + (2*b*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a^2*d^3) - (2*b*f
^2*PolyLog[3, E^(I*(c + d*x))])/(a^2*d^3) - ((2*I)*(a^2 - b^2)^(3/2)*f^2*Po
lyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^3) + ((2*
I)*(a^2 - b^2)^(3/2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(a^2*b^2*d^3) + (2*b*f*(e + f*x)*Sin[c + d*x])/(a^2*d^2) + (2*(a^2 -
 b^2)*f*(e + f*x)*Sin[c + d*x])/(a^2*b*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
```

```
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos^2(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot}{a+b\sin(c+dx)}}{a}$$

$$= -\frac{\int (e+fx)^2\cos^2(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\cot^2(c+dx)\,dx}{a} - \frac{b}{}$$

$$= -\frac{f(e+fx)\cos^2(c+dx)}{2ad^2} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{(e+fx)^2\,c}{}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{2af} - \frac{(e+fx)^2\cot(c+dx)}{ad} + \frac{f^2\cos(c+}{}$$

$$= \frac{f^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e}{}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2}{}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2}{}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2}{}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2}{}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2}{}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2574 vs. 2(840) = 1680.
time = 21.13, size = 2574, normalized size = 3.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),
x]
```

```
[Out] -((b*e^2*Log[Tan[(c + d*x)/2]])/(a^2*d)) - (2*b*e*f*((c + d*x)*(Log[1 - E^(
I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(Po
lyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a^2*d^2) + (2*
b*f^2*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - I*d*x*PolyLog[2, -C
os[c + d*x] - I*Sin[c + d*x]] + I*d*x*PolyLog[2, Cos[c + d*x] + I*Sin[c + d
*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - PolyLog[3, Cos[c + d*x]
 + I*Sin[c + d*x]]))/(a^2*d^3) + (2*e*f*Csc[c]*(-(d*x*Cos[c]) + Log[Cos[d*x
]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*d^2*(Cos[c]^2 + Sin[c]^2)) + (I*(a^
2 - b^2)^(3/2)*(2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*(Cos[2*c +
d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c
])^2] - a*Sin[c]))]*(Cos[c] + I*Sin[c]) - 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*P
olyLog[2, (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a
^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[c])]*(Cos[c] + I*Sin[c]) - I*(-2*S
qrt[a^2 - b^2]*f^2*PolyLog[3, -((b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*
a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] - a*Sin[c]))]*(Cos[c] +
 I*Sin[c]) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*(Cos[2*c + d*x] + I*Sin[2*
c + d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Si
n[c])]*(Cos[c] + I*Sin[c]) + d^2*(Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(-Log[1 +
 (b*(Cos[2*c + d*x] + I*Sin[2*c + d*x]))/(I*a*Cos[c] + Sqrt[(-a^2 + b^2)*(C
os[c] + I*Sin[c])^2] - a*Sin[c])] + Log[1 - (b*(Cos[2*c + d*x] + I*Sin[2*c
+ d*x]))/((-I)*a*Cos[c] + Sqrt[(-a^2 + b^2)*(Cos[c] + I*Sin[c])^2] + a*Sin[
c])])*(Cos[c] + I*Sin[c]) + 2*e^2*ArcTan[(b*Cos[c + d*x] + I*(a + b*Sin[c +
 d*x]))/Sqrt[a^2 - b^2]]*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]))))/(a^
2*b^2*d^3*Sqrt[(-a^2 + b^2)*(Cos[2*c] + I*Sin[2*c])]) + Csc[c]*Csc[c + d*x]
*(Cos[c + d*x]/(24*a*b^2*d^3) - ((I/24)*Sin[c + d*x])/(a*b^2*d^3))*((12*I)*
b^2*d^2*e^2*Cos[c] + (24*I)*b^2*d^2*e*f*x*Cos[c] + (12*I)*b^2*d^2*f^2*x^2*C
os[c] - 3*a*b*d^2*e^2*Cos[d*x] + (18*I)*a*b*d*e*f*Cos[d*x] + 6*a*b*f^2*Cos[
d*x] - 6*a*b*d^2*e*f*x*Cos[d*x] + (18*I)*a*b*d*f^2*x*Cos[d*x] - 3*a*b*d^2*f
^2*x^2*Cos[d*x] + 3*a*b*d^2*e^2*Cos[2*c + d*x] - (18*I)*a*b*d*e*f*Cos[2*c +
 d*x] - 6*a*b*f^2*Cos[2*c + d*x] + 6*a*b*d^2*e*f*x*Cos[2*c + d*x] - (18*I)*
a*b*d*f^2*x*Cos[2*c + d*x] + 3*a*b*d^2*f^2*x^2*Cos[2*c + d*x] - (12*I)*b^2*
d^2*e^2*Cos[c + 2*d*x] - 6*a^2*d^3*e^2*x*Cos[c + 2*d*x] - (24*I)*b^2*d^2*e*
f*x*Cos[c + 2*d*x] - 6*a^2*d^3*e*f*x^2*Cos[c + 2*d*x] - (12*I)*b^2*d^2*f^2*
x^2*Cos[c + 2*d*x] - 2*a^2*d^3*f^2*x^3*Cos[c + 2*d*x] + 6*a^2*d^3*e^2*x*Cos
[3*c + 2*d*x] + 6*a^2*d^3*e*f*x^2*Cos[3*c + 2*d*x] + 2*a^2*d^3*f^2*x^3*Cos[
3*c + 2*d*x] - 3*a*b*d^2*e^2*Cos[2*c + 3*d*x] - (6*I)*a*b*d*e*f*Cos[2*c + 3
*d*x] + 6*a*b*f^2*Cos[2*c + 3*d*x] - 6*a*b*d^2*e*f*x*Cos[2*c + 3*d*x] - (6*
I)*a*b*d*f^2*x*Cos[2*c + 3*d*x] - 3*a*b*d^2*f^2*x^2*Cos[2*c + 3*d*x] + 3*a*
b*d^2*e^2*Cos[4*c + 3*d*x] + (6*I)*a*b*d*e*f*Cos[4*c + 3*d*x] - 6*a*b*f^2*C
os[4*c + 3*d*x] + 6*a*b*d^2*e*f*x*Cos[4*c + 3*d*x] + (6*I)*a*b*d*f^2*x*Cos[
4*c + 3*d*x] + 3*a*b*d^2*f^2*x^2*Cos[4*c + 3*d*x] - 12*b^2*d^2*e^2*Sin[c] -
 (12*I)*a^2*d^3*e^2*x*Sin[c] - 24*b^2*d^2*e*f*x*Sin[c] - (12*I)*a^2*d^3*e*f
*x^2*Sin[c] - 12*b^2*d^2*f^2*x^2*Sin[c] - (4*I)*a^2*d^3*f^2*x^3*Sin[c] + (3
*I)*a*b*d^2*e^2*Sin[d*x] - 6*a*b*d*e*f*Sin[d*x] - (6*I)*a*b*f^2*Sin[d*x] +
(6*I)*a*b*d^2*e*f*x*Sin[d*x] - 6*a*b*d*f^2*x*Sin[d*x] + (3*I)*a*b*d^2*f^2*x
```

```
^2*Sin[d*x] - (3*I)*a*b*d^2*e^2*Sin[2*c + d*x] + 6*a*b*d*e*f*Sin[2*c + d*x]
 + (6*I)*a*b*f^2*Sin[2*c + d*x] - (6*I)*a*b*d^2*e*f*x*Sin[2*c + d*x] + 6*a*
b*d*f^2*x*Sin[2*c + d*x] - (3*I)*a*b*d^2*f^2*x^2*Sin[2*c + d*x] + 12*b^2*d^
2*e^2*Sin[c + 2*d*x] - (6*I)*a^2*d^3*e^2*x*Sin[c + 2*d*x] + 24*b^2*d^2*e*f*
x*Sin[c + 2*d*x] - (6*I)*a^2*d^3*e*f*x^2*Sin[c + 2*d*x] + 12*b^2*d^2*f^2*x^
2*Sin[c + 2*d*x] - (2*I)*a^2*d^3*f^2*x^3*Sin[c + 2*d*x] + (6*I)*a^2*d^3*e^2
*x*Sin[3*c + 2*d*x] + (6*I)*a^2*d^3*e*f*x^2*Sin[3*c + 2*d*x] + (2*I)*a^2*d^
3*f^2*x^3*Sin[3*c + 2*d*x] - (3*I)*a*b*d^2*e^2*Sin[2*c + 3*d*x] + 6*a*b*d*e
*f*Sin[2*c + 3*d*x] + (6*I)*a*b*f^2*Sin[2*c + 3*d*x] - (6*I)*a*b*d^2*e*f*x*
Sin[2*c + 3*d*x] + 6*a*b*d*f^2*x*Sin[2*c + 3*d*x] - (3*I)*a*b*d^2*f^2*x^2*S
in[2*c + 3*d*x] + (3*I)*a*b*d^2*e^2*Sin[4*c + 3*d*x] - 6*a*b*d*e*f*Sin[4*c
+ 3*d*x] - (6*I)*a*b*f^2*Sin[4*c + 3*d*x] + (6*I)*a*b*d^2*e*f*x*Sin[4*c + 3
*d*x] - 6*a*b*d*f^2*x*Sin[4*c + 3*d*x] + (3*I)*a*b*d^2*f^2*x^2*Sin[4*c + 3*
d*x]) - (f^2*Csc[c]*Sec[c]*(d^2*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi + 2
*ArcTan[Tan[c]]) - Pi*Log[1 + E^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*Lo
g[1 - E^((2*I)*(d*x + ArcTan[Tan[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan[c
]]*Log[Sin[d*x + ArcTan[Tan[c]]]] + I*PolyLog[2, E^((2*I)*(d*x + ArcTan[Tan
[c]]))])*Tan[c])/Sqrt[1 + Tan[c]^2]))/(a*d^3*Sqrt[Sec[c]^2*(Cos[c]^2 + Sin[
c]^2)])
```

**Maple [F]**
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 (\cos^2(dx+c)) (\cot^2(dx+c))}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3089 vs. $2(769) = 1538$.
time = 0.73, size = 3089, normalized size = 3.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/6*(12*a^2*b*d*f^2*x - 6*b^3*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c))
*sin(d*x + c) - 6*b^3*f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c))*sin(d*x
 + c) + 6*b^3*f^2*polylog(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) +
 6*b^3*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 12*a^2
*b*d*f*e + 6*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(
d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2))/b)*sin(d*x + c) - 6*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*p
olylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 6*(a^2*b - b^3)*f^2*sqrt
(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos
(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 6*(
a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*
sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b)*sin(d*x + c) - 6*(-I*(a^2*b - b^3)*d*f^2*x - I*(a^2*b - b^3)*d*f*e)*sqrt
(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x +
c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 6*
(I*(a^2*b - b^3)*d*f^2*x + I*(a^2*b - b^3)*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*di
log((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 6*(I*(a^2*b - b^3)*d*f^
2*x + I*(a^2*b - b^3)*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c
) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1)*sin(d*x + c) - 6*(-I*(a^2*b - b^3)*d*f^2*x - I*(a^2*b - b^
3)*d*f*e)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c)
- (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*si
n(d*x + c) + 3*((a^2*b - b^3)*c^2*f^2 - 2*(a^2*b - b^3)*c*d*f*e + (a^2*b -
b^3)*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + 3*((a^2*b - b^3)*c
^2*f^2 - 2*(a^2*b - b^3)*c*d*f*e + (a^2*b - b^3)*d^2*e^2)*sqrt(-(a^2 - b^2)
/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2
) - 2*I*a)*sin(d*x + c) - 3*((a^2*b - b^3)*c^2*f^2 - 2*(a^2*b - b^3)*c*d*f*
e + (a^2*b - b^3)*d^2*e^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2
*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 3*((
a^2*b - b^3)*c^2*f^2 - 2*(a^2*b - b^3)*c*d*f*e + (a^2*b - b^3)*d^2*e^2)*sqr
t(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-
(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 3*((a^2*b - b^3)*d^2*f^2*x^2 - (a^
```

```
2*b - b^3)*c^2*f^2 + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*c*d*f)*e)*sqr
t(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x +
c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + 3*((a^
2*b - b^3)*d^2*f^2*x^2 - (a^2*b - b^3)*c^2*f^2 + 2*((a^2*b - b^3)*d^2*f*x +
 (a^2*b - b^3)*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*
sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
 b)/b)*sin(d*x + c) - 3*((a^2*b - b^3)*d^2*f^2*x^2 - (a^2*b - b^3)*c^2*f^2
+ 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*c*d*f)*e)*sqrt(-(a^2 - b^2)/b^2)
*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + 3*((a^2*b - b^3)*d^2*f^2
*x^2 - (a^2*b - b^3)*c^2*f^2 + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*c*d
*f)*e)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x +
c) - 12*(a^2*b*d*f^2*x + a^2*b*d*f*e)*cos(d*x + c)^2 - 6*(-I*b^3*d*f^2*x -
I*b^3*d*f*e + I*a*b^2*f^2)*dilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c
) - 6*(I*b^3*d*f^2*x + I*b^3*d*f*e - I*a*b^2*f^2)*dilog(cos(d*x + c) - I*si
n(d*x + c))*sin(d*x + c) - 6*(-I*b^3*d*f^2*x - I*b^3*d*f*e - I*a*b^2*f^2)*d
ilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 6*(I*b^3*d*f^2*x + I*b^
3*d*f*e + I*a*b^2*f^2)*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) +
 3*(b^3*d^2*f^2*x^2 + 2*a*b^2*d*f^2*x + b^3*d^2*e^2 + 2*(b^3*d^2*f*x + a*b^
2*d*f)*e)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + 3*(b^3*d^2*
f^2*x^2 + 2*a*b^2*d*f^2*x + b^3*d^2*e^2 + 2*(b^3*d^2*f*x + a*b^2*d*f)*e)*lo
g(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) - 3*(b^3*d^2*e^2 - 2*(b^3
*c + a*b^2)*d*f*e + (b^3*c^2 + 2*a*b^2*c)*f^2)*log(-1/2*cos(d*x + c) + 1/2*
I*sin(d*x + c) + 1/2)*sin(d*x + c) - 3*(b^3*d^2*e^2 - 2*(b^3*c + a*b^2)*d*f
*e + (b^3*c^2 + 2*a*b^2*c)*f^2)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c)
+ 1/2)*sin(d*x + c) - 3*(b^3*d^2*f^2*x^2 - 2*a*b^2*d*f^2*x - (b^3*c^2 + 2*a
*b^2*c)*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*log(-cos(d*x + c) + I*sin(d*x
+ c) + 1)*sin(d*x + c) - 3*(b^3*d^2*f^2*x^2 - 2*a*b^2*d*f^2*x - (b^3*c^2 +
2*a*b^2*c)*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*e)*log(-cos(d*x + c) - I*sin(d
*x + c) + 1)*sin(d*x + c) - 6*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*f*x*e + a*b^
2*d^2*e^2)*cos(d*x + c) - 2*(a^3*d^3*f^2*x^3 + ...
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x)),
 x)
```

**Giac** [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.343 $\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=517

$$-\frac{ex}{a}+\frac{\left(1-\frac{a^2}{b^2}\right)ex}{a}-\frac{fx^2}{2a}+\frac{\left(1-\frac{a^2}{b^2}\right)fx^2}{2a}+\frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d}-\frac{b(e+fx)\cos(c+dx)}{a^2d}-\frac{(a^2-b^2)\,(e$$

```
[Out] -e*x/a+(1-a^2/b^2)*e*x/a-1/2*f*x^2/a+1/2*(1-a^2/b^2)*f*x^2/a+2*b*(f*x+e)*ar
ctanh(exp(I*(d*x+c)))/a^2/d-b*(f*x+e)*cos(d*x+c)/a^2/d-(a^2-b^2)*(f*x+e)*co
s(d*x+c)/a^2/b/d-(f*x+e)*cot(d*x+c)/a/d+f*ln(sin(d*x+c))/a/d^2+I*b*f*polylo
g(2,exp(I*(d*x+c)))/a^2/d^2-I*(a^2-b^2)^(3/2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c
))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d-I*b*f*polylog(2,-exp(I*(d*x+c)))/a^2/d^2+
I*(a^2-b^2)^(3/2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/
b^2/d-(a^2-b^2)^(3/2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a
^2/b^2/d^2+(a^2-b^2)^(3/2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2
)))/a^2/b^2/d^2+b*f*sin(d*x+c)/a^2/d^2+(a^2-b^2)*f*sin(d*x+c)/a^2/b/d^2
```

**Rubi** [A]
time = 0.71, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 16, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4639, 4493, 3391, 3801, 3556, 4490, 2713, 3377, 2717, 4268, 2317, 2438, 4621, 3404, 2296, 2221}

$$-\frac{f(a^2-b^2)^{3/2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^2d^2}+\frac{f(a^2-b^2)^{3/2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2b^2d^2}-\frac{ibf\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}+\frac{ibf\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2}+\frac{f(a^2-b^2)\sin(c+dx)}{a^2bd^2}-\frac{i(a^2-b^2)^{3/2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{i(a^2-b^2)^{3/2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2b^2d}-\frac{(a^2-b^2)(e+fx)\cos(c+dx)}{a^2bd}+\frac{ex\left(1-\frac{a^2}{b^2}\right)}{a}+\frac{fx^2\left(1-\frac{a^2}{b^2}\right)}{2a}+\frac{bf\sin(c+dx)}{a^2d^2}-\frac{b(e+fx)\cos(c+dx)}{a^2d}+\frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d}+\frac{f\log(\sin(c+dx))}{ad^2}-\frac{(e+fx)\cot(c+dx)}{ad}-\frac{ex}{a}-\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((e*x)/a) + ((1 - a^2/b^2)*e*x)/a - (f*x^2)/(2*a) + ((1 - a^2/b^2)*f*x^2)/
(2*a) + (2*b*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a^2*d) - (b*(e + f*x)*Cos
[c + d*x])/(a^2*d) - ((a^2 - b^2)*(e + f*x)*Cos[c + d*x])/(a^2*b*d) - ((e +
 f*x)*Cot[c + d*x])/(a*d) - (I*(a^2 - b^2)^(3/2)*(e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d) + (I*(a^2 - b^2)^(3/2)*(e
 + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d) +
 (f*Log[Sin[c + d*x]])/(a*d^2) - (I*b*f*PolyLog[2, -E^(I*(c + d*x))])/(a^2*
d^2) + (I*b*f*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - ((a^2 - b^2)^(3/2)*f
*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) + (
(a^2 - b^2)^(3/2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(a^2*b^2*d^2) + (b*f*Sin[c + d*x])/(a^2*d^2) + ((a^2 - b^2)*f*Sin[c + d*x
])/(a^2*b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos^2(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+}{a+b\sin(c+dx)}}{a}$$

$$= -\frac{\int (e+fx)\cos^2(c+dx)\,dx}{a} + \frac{\int (e+fx)\cot^2(c+dx)\,dx}{a} - \frac{b\int(}{}$$

$$= -\frac{f\cos^2(c+dx)}{4ad^2} - \frac{(e+fx)\cot(c+dx)}{ad} - \frac{(e+fx)\cos(c+dx)\,\mathrm{s}}{2ad}$$

$$= -\frac{3ex}{2a} - \frac{3fx^2}{4a} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{f\log(\sin(c+dx))}{ad^2} + \int($$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh}{a^2d}$$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh}{a^2d}$$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh}{a^2d}$$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh}{a^2d}$$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh}{a^2d}$$

**Mathematica** [A]
time = 9.26, size = 1019, normalized size = 1.97

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(a*(c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(b^2*d^2) - ((d*e - c*f +
f*(c + d*x))*Cos[c + d*x])/(b*d^2) + ((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c
 + d*x)/2] - f*(c + d*x)*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(2*a*d^2) + (f
*Log[Sin[c + d*x]])/(a*d^2) - (b*e*Log[Tan[(c + d*x)/2]])/(a^2*d) + (b*c*f*
Log[Tan[(c + d*x)/2]])/(a^2*d^2) - (b*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))
] - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2
```

```
, E^(I*(c + d*x))])))/(a^2*d^2) + ((a^2 - b^2)^2*(d*e + d*f*x)*((2*(d*e - c
*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*
f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/
2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2
]))/(a + I*(b + Sqrt[-a^2 + b^2]))]]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Ta
n[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b +
Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sq
rt[-a^2 + b^2]))]]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Lo
g[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]
))] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))
/Sqrt[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 +
b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a +
I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]]))/Sqrt[-a^2 + b^2]))
/(a^2*b^2*d^2*(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*
Tan[(c + d*x)/2]])) + (Sec[(c + d*x)/2]*(d*e*Sin[(c + d*x)/2] - c*f*Sin[(c
+ d*x)/2] + f*(c + d*x)*Sin[(c + d*x)/2]))/(2*a*d^2) + (f*Sin[c + d*x])/(b*
d^2)
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1889 vs. 2(475) = 950.
time = 1.55, size = 1890, normalized size = 3.66

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1890 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*I*a^2/b^2/d^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)
/(-a^2+b^2)^(1/2))-1/2*(d*x*f+I*f+d*e)/b/d^2*exp(I*(d*x+c))-1/2*(d*x*f-I*f+
d*e)/b/d^2*exp(-I*(d*x+c))-2/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-
(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-2/d^2*f/(-a^2+b^2)^(1/2)*ln((I*
a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+2/d*f/(-a^2+
b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)
))*x+2/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I
*a+(-a^2+b^2)^(1/2)))*c-2/d^2/a*f*ln(exp(I*(d*x+c)))-2*I*(f*x+e)/d/a/(exp(2
*I*(d*x+c))-1)+1/a^2/d*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^
2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/a^2/d^2*b^2*f/(-a^2+b^2)^(1/2)*ln
((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-2*I/a^2/
d^2*b^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^
2)^(1/2))+4*I/d^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a
)/(-a^2+b^2)^(1/2))-1/a^2/d*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))
+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/a^2/d^2*b^2*f/(-a^2+b^2)^(1/
2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/a
```

```
^2/d^2*b^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))
/(I*a+(-a^2+b^2)^(1/2)))+2*I/a^2/d*b^2*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b
*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-I/a^2/d^2*b^2*f/(-a^2+b^2)^(1/2)*dil
og((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-1/a^2/d*
b*e*ln(exp(I*(d*x+c))-1)+1/a^2/d*b*e*ln(exp(I*(d*x+c))+1)+1/a/d^2*f*ln(exp(
I*(d*x+c))-1)+1/a/d^2*f*ln(exp(I*(d*x+c))+1)-a*e*x/b^2+1/a^2/d*b*f*ln(exp(I
*(d*x+c))+1)*x+1/a^2/d^2*b*f*c*ln(exp(I*(d*x+c))-1)-I/a^2/d^2*b*f*dilog(exp
(I*(d*x+c))+1)-I/a^2/d^2*b*f*dilog(exp(I*(d*x+c)))-1/2*a*f*x^2/b^2-4*I/d*e/
(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+2*
I/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a
-(-a^2+b^2)^(1/2)))-2*I/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+
(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+a^2/b^2/d*f/(-a^2+b^2)^(1/2)*ln((
I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+a^2/b^2/d^
2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+
b^2)^(1/2)))*c-a^2/b^2/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+
b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*ln((I*
a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I*a^2/b^2/d^
2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a
^2+b^2)^(1/2)))+2*I*a^2/b^2/d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d
*x+c))-2*a)/(-a^2+b^2)^(1/2))-I*a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b
*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1762 vs. $2(470) = 940$.
time = 0.69, size = 1762, normalized size = 3.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a^2*b*f*cos(d*x + c)^2 - I*b^3*f*dilog(cos(d*x + c) + I*sin(d*x + c
))*sin(d*x + c) + I*b^3*f*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c)
 - I*b^3*f*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + I*b^3*f*dil
og(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - I*(a^2*b - b^3)*f*sqrt(-(
a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c)
+ I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + I*(a^
2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c
) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*
sin(d*x + c) + I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x
 + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) - b)/b + 1)*sin(d*x + c) - I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2
)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 2*a^2*b*f + ((a^2*
b - b^3)*c*f - (a^2*b - b^3)*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x +
c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c)
+ ((a^2*b - b^3)*c*f - (a^2*b - b^3)*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*co
s(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d
*x + c) - ((a^2*b - b^3)*c*f - (a^2*b - b^3)*d*e)*sqrt(-(a^2 - b^2)/b^2)*lo
g(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I
*a)*sin(d*x + c) - ((a^2*b - b^3)*c*f - (a^2*b - b^3)*d*e)*sqrt(-(a^2 - b^2
)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b
^2) - 2*I*a)*sin(d*x + c) + ((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sqrt(
-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c)
 + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) - ((a^2*b
- b^3)*d*f*x + (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x
+ c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2) - b)/b)*sin(d*x + c) + ((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sq
rt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) - ((a^
2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos
(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b)*sin(d*x + c) - (b^3*d*f*x + b^3*d*e + a*b^2*f)*log(cos
(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) - (b^3*d*f*x + b^3*d*e + a*b^2
*f)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + (b^3*d*e - (b^3*c
 + a*b^2)*f)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c)
 + (b^3*d*e - (b^3*c + a*b^2)*f)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c)
 + 1/2)*sin(d*x + c) + (b^3*d*f*x + b^3*c*f)*log(-cos(d*x + c) + I*sin(d*x
+ c) + 1)*sin(d*x + c) + (b^3*d*f*x + b^3*c*f)*log(-cos(d*x + c) - I*sin(d*
x + c) + 1)*sin(d*x + c) + 2*(a*b^2*d*f*x + a*b^2*d*e)*cos(d*x + c) + (a^3*
d^2*f*x^2 + 2*a^3*d^2*x*e + 2*(a^2*b*d*f*x + a^2*b*d*e)*cos(d*x + c))*sin(d
*x + c))/(a^2*b^2*d^2*sin(d*x + c))
```

**Sympy** [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Integral((e + f*x)*cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
giac")
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
[Out] \text{Hanged}
```

# 3.344 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=104

$$-\frac{ax}{b^2}+\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{bd}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -a*x/b^2+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))
/a^2/b^2/d+b*arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/b/d-cot(d*x+c)/a/d
```

**Rubi** [A]
time = 0.17, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2973, 3136, 2739, 632, 210, 3855}

$$\frac{2(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{ax}{b^2}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^2*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cos[c + d*x]/(
b*d) - Cot[c + d*x]/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)}{bd}-\frac{\cot(c+dx)}{ad}-\frac{\int \frac{\csc(c+dx)\left(b^2+2ab\sin(c+dx)+a^2\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{ab}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}-\frac{\cot(c+dx)}{ad}-\frac{b\int\csc(c+dx)\,dx}{a^2}+\frac{\left(a^2-b^2\right)^2\int \frac{}{a+b}}{a^2b^2}$$

$$= -\frac{ax}{b^2}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{bd}-\frac{\cot(c+dx)}{ad}+\frac{\left(2\left(a^2-b^2\right)\right.}{}$$

$$= -\frac{ax}{b^2}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{bd}-\frac{\cot(c+dx)}{ad}-\frac{\left(4\left(a^2-b^2\right)\right.}{}$$

$$= -\frac{ax}{b^2}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-$$

## Mathematica [A]

time = 0.53, size = 146, normalized size = 1.40

$$-\frac{2a^3c+2a^3dx-4(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+2a^2b\cos(c+dx)+ab^2\cot\left(\frac{1}{2}(c+dx)\right)-2b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-ab^2\tan\left(\frac{1}{2}(c+dx)\right)}{2a^2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(2*a^3*c + 2*a^3*d*x - 4*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]] + 2*a^2*b*Cos[c + d*x] + a*b^2*Cot[(c + d*x)/2] - 2*b
^3*Log[Cos[(c + d*x)/2]] + 2*b^3*Log[Sin[(c + d*x)/2]] - a*b^2*Tan[(c + d*x
)/2])/(a^2*b^2*d)
```

## Maple [A]

time = 0.23, size = 155, normalized size = 1.49

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{2\left(\frac{b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}+\frac{\left(4a^4-8a^2b^2+4b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2b^2\sqrt{a^2-b^2}}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{d}$ |
| default | $\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{2\left(\frac{b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}+\frac{\left(4a^4-8a^2b^2+4b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2b^2\sqrt{a^2-b^2}}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{d}$ |
| risch | $-\frac{ax}{b^2}-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}-\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^2d}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^2d}-\frac{\sqrt{-a^2+b^2}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tan(1/2*d*x+1/2*c)-2/b^2*(b/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(ta
n(1/2*d*x+1/2*c)))+1/2*(4*a^4-8*a^2*b^2+4*b^4)/a^2/b^2/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/2/a/tan(1/2*d*x+1/2*
c)-1/a^2*b*ln(tan(1/2*d*x+1/2*c)))
```

## Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [A]
time = 0.47, size = 396, normalized size = 3.81

$$\left[\frac{b^{3} \log \left(\frac{1}{2} \cos \left(dx + c\right) + \frac{1}{2}\right) \sin \left(dx + c\right) - b^{3} \log \left(-\frac{1}{2} \cos \left(dx + c\right) + \frac{1}{2}\right) \sin \left(dx + c\right) - 2 \, a b^{2} \cos \left(dx + c\right) - \left(a^{2} - b^{2}\right) \sqrt{-a^{2} + b^{2}} \log \left(\frac{\left(2 \, a^{2} - b^{2}\right) \cos\left(dx + c\right)^{2} - 2 \, a b \sin\left(dx + c\right) - a^{2} - b^{2} + 2 \, \left(a \cos\left(dx + c\right) \sin\left(dx + c\right) + b \cos\left(dx + c\right)\right) \sqrt{-a^{2} + b^{2}}}{b^{2} \cos\left(dx + c\right)^{2} - 2 \, a b \sin\left(dx + c\right) - a^{2} - b^{2}}\right) \sin \left(dx + c\right) - 2 \, \left(a^{3} dx + a^{2} b \cos \left(dx + c\right)\right) \sin \left(dx + c\right)}{2 \, a^{2} b^{2} d \sin \left(dx + c\right)}, \frac{b^{3} \log \left(\frac{1}{2} \cos \left(dx + c\right) + \frac{1}{2}\right) \sin \left(dx + c\right) - b^{3} \log \left(-\frac{1}{2} \cos \left(dx + c\right) + \frac{1}{2}\right) \sin \left(dx + c\right) - 2 \, a b^{2} \cos \left(dx + c\right) - 2 \, \left(a^{2} - b^{2}\right)^{\frac{3}{2}} \arctan \left(-\frac{a \sin\left(dx + c\right) + b}{\sqrt{a^{2} - b^{2}} \cos\left(dx + c\right)}\right) \sin \left(dx + c\right) - 2 \, \left(a^{3} dx + a^{2} b \cos \left(dx + c\right)\right) \sin \left(dx + c\right)}{2 \, a^{2} b^{2} d \sin \left(dx + c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 2*a*b^2*cos(d*x + c) - (a^2 - b^2)*sqrt(-a^2 + b^
2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 2*(a^3*d*x + a^2
*b*cos(d*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c)), 1/2*(b^3*log(1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 2*a*b^2*cos(d*x + c) - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) +
 b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 2*(a^3*d*x + a^2*b*cos(d
*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c))]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c + dx\right)\cot^{2}\left(c + dx\right)}{a + b\sin\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(99) = 198.
time = 5.61, size = 221, normalized size = 2.12

$$-\frac{\frac{6\,(dx+c)a}{b^{2}} + \frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}} - \frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{12\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,a^{2}b^{2}} - \frac{2\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,ab}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^{2}b}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a/b^2 + 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/
2*d*x + 1/2*c)/a - 12*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi +
```

```
1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a
^2 - b^2)*a^2*b^2) - (2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/
2*c)^2 - 12*a^2*tan(1/2*d*x + 1/2*c) + 2*b^2*tan(1/2*d*x + 1/2*c) - 3*a*b)/
((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^2*b))/d
```

**Mupad [B]**
time = 6.11, size = 1167, normalized size = 11.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (atan((16*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)
- 4*a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 4*a
^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 12*a^3*b^
3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a^5*b^7*co
s(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 4*a^7*b^5*cos(
c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 6*a^9*b^3*cos(c/
2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 29*a^2*b^4*sin(c/2
 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 18*a^4*b^2*sin(c/2
+ (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a^2*b^10*sin(c/2 + (
d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 4*a^4*b^8*sin(c/2 + (d*
x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 22*a^6*b^6*sin(c/2 + (d*x
)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 32*a^8*b^4*sin(c/2 + (d*x)
/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 18*a^10*b^2*sin(c/2 + (d*x)
/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*a*b^5*cos(c/2 + (d*x)/2)*
(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 5*a^5*b*cos(c/2 + (d*x)/2)*(b^6
 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^11*b*cos(c/2 + (d*x)/2)*(b^6 -
a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^15*sin(c/2 + (d*x)/2)*16i + a*b^14*c
os(c/2 + (d*x)/2)*8i - a^14*b*sin(c/2 + (d*x)/2)*3i - a^3*b^12*cos(c/2 + (d
*x)/2)*48i + a^5*b^10*cos(c/2 + (d*x)/2)*123i - a^7*b^8*cos(c/2 + (d*x)/2)*
167i + a^9*b^6*cos(c/2 + (d*x)/2)*126i - a^11*b^4*cos(c/2 + (d*x)/2)*51i +
a^13*b^2*cos(c/2 + (d*x)/2)*9i - a^2*b^13*sin(c/2 + (d*x)/2)*100i + a^4*b^1
1*sin(c/2 + (d*x)/2)*269i - a^6*b^9*sin(c/2 + (d*x)/2)*390i + a^8*b^7*sin(c
/2 + (d*x)/2)*323i - a^10*b^5*sin(c/2 + (d*x)/2)*151i + a^12*b^3*sin(c/2 +
(d*x)/2)*36i))*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^2*b^2*d) - (b*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d) - sin(2*c + 2*d*x)/(2*b*d*sin(c +
 d*x)) - (2*a*atan((a^3*cos(c/2 + (d*x)/2) + b^3*sin(c/2 + (d*x)/2))/(b^3*c
os(c/2 + (d*x)/2) - a^3*sin(c/2 + (d*x)/2))))/(b^2*d) - cot(c + d*x)/(a*d)
```

# 3.345 $\int \frac{(e+fx)^3\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=1432

$$\frac{3bf^3x}{8a^2d^3}+\frac{3(a^2-b^2)\,f^3x}{8a^2bd^3}-\frac{b(e+fx)^3}{4a^2d}-\frac{(a^2-b^2)\,(e+fx)^3}{4a^2bd}+\frac{ib(e+fx)^4}{4a^2f}-\frac{i(a^2-b^2)^2\,(e+fx)^4}{4a^2b^3f}-\frac{6f(e+fx)^2\,t}{}$$

```
[Out] -b*(f*x+e)^3*ln(1-exp(2*I*(d*x+c)))/a^2/d-3*(a^2-b^2)*f*(f*x+e)^2*cos(d*x+c
)/a/b^2/d^2+6*(a^2-b^2)*f^2*(f*x+e)*sin(d*x+c)/a/b^2/d^3-3/8*(a^2-b^2)*f^3*
cos(d*x+c)*sin(d*x+c)/a^2/b/d^4+3/4*b*f*(f*x+e)^2*cos(d*x+c)*sin(d*x+c)/a^2
/d^2-3/4*(a^2-b^2)*f^2*(f*x+e)*sin(d*x+c)^2/a^2/b/d^3+(a^2-b^2)^2*(f*x+e)^3
*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^3/d+(a^2-b^2)^2*(f*x+e)
^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^3/d+6*(a^2-b^2)^2*f^2
*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^3/d^3+6*(a
^2-b^2)^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2
/b^3/d^3+6*I*(a^2-b^2)^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2
)))/a^2/b^3/d^4+6*I*(a^2-b^2)^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^
2)^(1/2)))/a^2/b^3/d^4-3*f*(f*x+e)^2*cos(d*x+c)/a/d^2+6*f^2*(f*x+e)*sin(d*x
+c)/a/d^3-(f*x+e)^3*csc(d*x+c)/a/d-6*f^3*polylog(3,-exp(I*(d*x+c)))/a/d^4+6
*f^3*polylog(3,exp(I*(d*x+c)))/a/d^4+1/4*I*b*(f*x+e)^4/a^2/f-3/2*b*f^2*(f*x
+e)*polylog(3,exp(2*I*(d*x+c)))/a^2/d^3-6*I*f^2*(f*x+e)*polylog(2,exp(I*(d*
x+c)))/a/d^3-3/4*I*b*f^3*polylog(4,exp(2*I*(d*x+c)))/a^2/d^4+6*f^3*cos(d*x+
c)/a/d^4+3/2*I*b*f*(f*x+e)^2*polylog(2,exp(2*I*(d*x+c)))/a^2/d^2-1/4*(a^2-b
^2)*(f*x+e)^3/a^2/b/d+1/2*b*(f*x+e)^3*sin(d*x+c)^2/a^2/d+3/8*b*f^3*x/a^2/d^
3-6*f*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d^2-1/4*b*(f*x+e)^3/a^2/d-(f*x+e)
^3*sin(d*x+c)/a/d-3*I*(a^2-b^2)^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/
(a-(a^2-b^2)^(1/2)))/a^2/b^3/d^2-3*I*(a^2-b^2)^2*f*(f*x+e)^2*polylog(2,I*b*
exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^3/d^2+3/4*(a^2-b^2)*f*(f*x+e)^2*c
os(d*x+c)*sin(d*x+c)/a^2/b/d^2+3/8*(a^2-b^2)*f^3*x/a^2/b/d^3+6*(a^2-b^2)*f^
3*cos(d*x+c)/a/b^2/d^4-(a^2-b^2)*(f*x+e)^3*sin(d*x+c)/a/b^2/d-3/8*b*f^3*cos
(d*x+c)*sin(d*x+c)/a^2/d^4-3/4*b*f^2*(f*x+e)*sin(d*x+c)^2/a^2/d^3+1/2*(a^2-
b^2)*(f*x+e)^3*sin(d*x+c)^2/a^2/b/d-1/4*I*(a^2-b^2)^2*(f*x+e)^4/a^2/b^3/f+6
*I*f^2*(f*x+e)*polylog(2,-exp(I*(d*x+c)))/a/d^3
```

**Rubi** [A]
time = 1.90, antiderivative size = 1432, normalized size of antiderivative = 1.00, number of steps used = 85, number of rules used = 21, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {4639, 4493, 3392, 3377, 2718, 3391, 4495, 4268, 2611, 2320, 6724, 4490, 32, 2715, 8, 4489, 3798, 2221, 6744, 4621, 4615}

---

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*f^3*x)/(8*a^2*d^3) + (3*(a^2 - b^2)*f^3*x)/(8*a^2*b*d^3) - (b*(e + f*x
)^3)/(4*a^2*d) - ((a^2 - b^2)*(e + f*x)^3)/(4*a^2*b*d) + ((I/4)*b*(e + f*x)
^4)/(a^2*f) - ((I/4)*(a^2 - b^2)^2*(e + f*x)^4)/(a^2*b^3*f) - (6*f*(e + f*x
)^2*ArcTanh[E^(I*(c + d*x))])/(a*d^2) + (6*f^3*Cos[c + d*x])/(a*d^4) + (6*(
a^2 - b^2)*f^3*Cos[c + d*x])/(a*b^2*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x])/(
a*d^2) - (3*(a^2 - b^2)*f*(e + f*x)^2*Cos[c + d*x])/(a*b^2*d^2) - ((e + f*x
)^3*Csc[c + d*x])/(a*d) + ((a^2 - b^2)^2*(e + f*x)^3*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d) + ((a^2 - b^2)^2*(e + f*x)^3*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d) - (b*(e + f
*x)^3*Log[1 - E^((2*I)*(c + d*x))])/(a^2*d) + ((6*I)*f^2*(e + f*x)*PolyLog[
2, -E^(I*(c + d*x))])/(a*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, E^(I*(c + d
*x))])/(a*d^3) - ((3*I)*(a^2 - b^2)^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^2) - ((3*I)*(a^2 - b^2)^2*f*(e
 + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3
*d^2) + (((3*I)/2)*b*f*(e + f*x)^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2*d^
2) - (6*f^3*PolyLog[3, -E^(I*(c + d*x))])/(a*d^4) + (6*f^3*PolyLog[3, E^(I*
(c + d*x))])/(a*d^4) + (6*(a^2 - b^2)^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^3) + (6*(a^2 - b^2)^2*f^2*(e
 + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d
^3) - (3*b*f^2*(e + f*x)*PolyLog[3, E^((2*I)*(c + d*x))])/(2*a^2*d^3) + ((6
*I)*(a^2 - b^2)^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]
)])/(a^2*b^3*d^4) + ((6*I)*(a^2 - b^2)^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d^4) - (((3*I)/4)*b*f^3*PolyLog[4, E^((
2*I)*(c + d*x))])/(a^2*d^4) + (6*f^2*(e + f*x)*Sin[c + d*x])/(a*d^3) + (6*(
a^2 - b^2)*f^2*(e + f*x)*Sin[c + d*x])/(a*b^2*d^3) - ((e + f*x)^3*Sin[c + d
*x])/(a*d) - ((a^2 - b^2)*(e + f*x)^3*Sin[c + d*x])/(a*b^2*d) - (3*b*f^3*Co
s[c + d*x]*Sin[c + d*x])/(8*a^2*d^4) - (3*(a^2 - b^2)*f^3*Cos[c + d*x]*Sin[
c + d*x])/(8*a^2*b*d^4) + (3*b*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*
a^2*d^2) + (3*(a^2 - b^2)*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*b
*d^2) - (3*b*f^2*(e + f*x)*Sin[c + d*x]^2)/(4*a^2*d^3) - (3*(a^2 - b^2)*f^2
*(e + f*x)*Sin[c + d*x]^2)/(4*a^2*b*d^3) + (b*(e + f*x)^3*Sin[c + d*x]^2)/(
2*a^2*d) + ((a^2 - b^2)*(e + f*x)^3*Sin[c + d*x]^2)/(2*a^2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos^3(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx}}{a}$$

$$= -\frac{\int (e+fx)^3\cos^3(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\cos(c+dx)\cot^2(c}{a}$$

$$= -\frac{f(e+fx)^2\cos^3(c+dx)}{3ad^2} - \frac{(e+fx)^3\cos^2(c+dx)\sin(c+dx}{3ad}$$

$$= \frac{2f^3\cos^3(c+dx)}{27ad^4} - \frac{(e+fx)^3\csc(c+dx)}{ad} - \frac{5(e+fx)^3\sin(c+}{3ad}$$

$$= \frac{ib(e+fx)^4}{4a^2f} - \frac{i(a^2-b^2)^2(e+fx)^4}{4a^2b^3f} - \frac{6f(e+fx)^2\tanh^{-1}(e^{i(}}{ad^2}$$

$$= \frac{ib(e+fx)^4}{4a^2f} - \frac{i(a^2-b^2)^2(e+fx)^4}{4a^2b^3f} - \frac{6f(e+fx)^2\tanh^{-1}(e^{i(}}{ad^2}$$

$$= -\frac{b(e+fx)^3}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{4bd} + \frac{ib(e+fx)^4}{4a^2f} - \frac{i(a^2-b^2}{4a}$$

$$= \frac{3bf^3x}{8a^2d^3} + \frac{3\left(1-\frac{b^2}{a^2}\right)f^3x}{8bd^3} - \frac{b(e+fx)^3}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{4bd} +$$

$$= \frac{3bf^3x}{8a^2d^3} + \frac{3\left(1-\frac{b^2}{a^2}\right)f^3x}{8bd^3} - \frac{b(e+fx)^3}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{4bd} +$$

$$= \frac{3bf^3x}{8a^2d^3} + \frac{3\left(1-\frac{b^2}{a^2}\right)f^3x}{8bd^3} - \frac{b(e+fx)^3}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{4bd} +$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4084 vs. $2(1432) = 2864$.
time = 30.10, size = 4084, normalized size = 2.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),
x]
```

```
[Out] ((-e^3 - 3*e^2*f*x - 3*e*f^2*x^2 - f^3*x^3)*Csc[c + d*x])/(a*d) + (3*e^2*f*
Log[Tan[(c + d*x)/2]])/(a*d^2) + (6*e*f^2*((c + d*x)*(Log[1 - E^(I*(c + d*x
))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLog[2, -
E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^3) + (b*e*f^2*Csc[c]
*(2*d^2*x^2*(2*d*E^((2*I)*c)*x + (3*I)*(-1 + E^((2*I)*c))*Log[1 - E^((2*I)*
(c + d*x))]) + 6*d*(-1 + E^((2*I)*c))*x*PolyLog[2, E^((2*I)*(c + d*x))] + (
3*I)*(-1 + E^((2*I)*c))*PolyLog[3, E^((2*I)*(c + d*x))]))/(4*a^2*d^3*E^(I*c
)) - (6*f^3*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - I*d*x*PolyLog
[2, -Cos[c + d*x] - I*Sin[c + d*x]] + I*d*x*PolyLog[2, Cos[c + d*x] + I*Sin
[c + d*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - PolyLog[3, Cos[c
+ d*x] + I*Sin[c + d*x]]))/(a*d^4) + (b*E^(I*c)*f^3*Csc[c]*(x^4 + (-1 + E^(
(-2*I)*c))*x^4 + ((-1 + E^((2*I)*c))*(2*d^4*x^4 + (4*I)*d^3*x^3*Log[1 - E^(
(2*I)*(c + d*x))] + 6*d^2*x^2*PolyLog[2, E^((2*I)*(c + d*x))] + (6*I)*d*x*P
olyLog[3, E^((2*I)*(c + d*x))] - 3*PolyLog[4, E^((2*I)*(c + d*x))]))/(2*d^4
*E^((2*I)*c))))/(4*a^2) + ((a^2 - b^2)^2*((-4*I)*d^4*e^3*E^((2*I)*c)*x - (6
*I)*d^4*e^2*E^((2*I)*c)*f*x^2 - (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 - I*d^4*E^(
(2*I)*c)*f^3*x^4 - (2*I)*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((
2*I)*(c + d*x))))] + (2*I)*d^3*e^3*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))
/(b*(-1 + E^((2*I)*(c + d*x))))] - d^3*e^3*Log[4*a^2*E^((2*I)*(c + d*x)) +
b^2*(-1 + E^((2*I)*(c + d*x)))^2] + d^3*e^3*E^((2*I)*c)*Log[4*a^2*E^((2*I)*
(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - 6*d^3*e^2*f*x*Log[1 + (b*E
^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e
^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2
+ b^2)*E^((2*I)*c)])] - 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*
E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*c)*f^2*x^2*Lo
g[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
 - 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 +
b^2)*E^((2*I)*c)])] + 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)
))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*f*x*Log[1 +
(b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d
^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-
a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(
I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*c)*f^2*x^
2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c
)])] - 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^
2 + b^2)*E^((2*I)*c)])] + 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c +
d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (6*I)*d^2*(-1 + E^
((2*I)*c))*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*
Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)
^2*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((
2*I)*c)]))] - 12*d*e*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*
Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E
^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*f^
3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^(
(2*I)*c)])] + 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*
```

```
E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*e*f^2*PolyLog[3, -((b*E
^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*e
*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a
^2 + b^2)*E^((2*I)*c)]))] - 12*d*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(
I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*E^((2*I)*c)*f^3*x*Po
lyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*
c)]))] - (12*I)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[
(-a^2 + b^2)*E^((2*I)*c)])] + (12*I)*E^((2*I)*c)*f^3*PolyLog[4, (I*b*E^(I*(
2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*f^3*P
olyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)
*c)]))] + (12*I)*E^((2*I)*c)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^
(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/(2*a^2*b^3*d^4*(-1 + E^((2*I)*c
))) - (b*e^3*Csc[c]*(-(d*x*Cos[c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]
*Sin[c]))/(a^2*d*(Cos[c]^2 + Sin[c]^2)) - (I*(-a^2 + 2*b^2)*e^3*x*(1 + Cos[
2*c] + I*Sin[2*c]))/(b^3*(-1 + Cos[2*c] + I*Sin[2*c])) - (((3*I)/2)*(-a^2 +
 2*b^2)*e^2*f*x^2*(1 + Cos[2*c] + I*Sin[2*c]))/(b^3*(-1 + Cos[2*c] + I*Sin[
2*c])) - (I*(-a^2 + 2*b^2)*e*f^2*x^3*(1 + Cos[2*c] + I*Sin[2*c]))/(b^3*(-1
+ Cos[2*c] + I*Sin[2*c])) - ((I/4)*(-a^2 + 2*b^2)*f^3*x^4*(1 + Cos[2*c] + I
*Sin[2*c]))/(b^3*(-1 + Cos[2*c] + I*Sin[2*c])) + (((-1/2*I)*a*f^3*x^3*Cos[c
])/(b^2*d) - (a*f^3*x^3*Sin[c])/(2*b^2*d) + ((-I)*d^3*e^3 - 3*d^2*e^2*f + (
6*I)*d*e*f^2 + 6*f^3)*((a*Cos[c])/(2*b^2*d^4) - ((I/2)*a*Sin[c])/(b^2*d^4))
 + (a*d^2*e^2*f - (2*I)*a*d*e*f^2 - 2*a*f^3)*((...
```

**Maple** [**F**]
time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{3} \left(\cos^{3}\left(dx+c\right)\right) \left(\cot^{2}\left(dx+c\right)\right)}{a+b\sin\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4903 vs. $2(1336) = 2672$.
time = 1.03, size = 4903, normalized size = 3.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -1/8*(8*(a^3*b + a*b^3)*d^3*f^3*x^3 - 48*a^3*b*d*f^3*x + 24*I*b^4*f^3*polyl
og(4, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 24*I*b^4*f^3*polylog(4,
 cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 24*I*b^4*f^3*polylog(4, -cos
(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 24*I*b^4*f^3*polylog(4, -cos(d*x
 + c) - I*sin(d*x + c))*sin(d*x + c) + 24*(a^3*b + a*b^3)*d^3*f*x*e^2 + 24*
I*(a^4 - 2*a^2*b^2 + b^4)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c
) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x
+ c) + 24*I*(a^4 - 2*a^2*b^2 + b^4)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*s
in(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
)*sin(d*x + c) - 24*I*(a^4 - 2*a^2*b^2 + b^4)*f^3*polylog(4, -(-I*a*cos(d*x
 + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2))/b)*sin(d*x + c) - 24*I*(a^4 - 2*a^2*b^2 + b^4)*f^3*polylog(4, -(-
I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 8*(a^3*b + a*b^3)*d^3*e^3 + 3*(2*a^2
*b^2*d^2*f^3*x^2 + 4*a^2*b^2*d^2*f^2*x*e + 2*a^2*b^2*d^2*f*e^2 - a^2*b^2*f^
3)*cos(d*x + c)^3 - 8*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*f*x*e^2 - 6*a^3*b*d*
f^3*x + a^3*b*d^3*e^3 + 3*(a^3*b*d^3*f^2*x^2 - 2*a^3*b*d*f^2)*e)*cos(d*x +
c)^2 + 12*(I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*I*(a^4 - 2*a^2*b^2 + b
^4)*d^2*f^2*x*e + I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f*e^2)*dilog((I*a*cos(d*x +
 c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b + 1)*sin(d*x + c) + 12*(I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f^3*x^2
 + 2*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x*e + I*(a^4 - 2*a^2*b^2 + b^4)*d^2*
f*e^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 12*(-I*(a^4 -
2*a^2*b^2 + b^4)*d^2*f^3*x^2 - 2*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x*e - I*
(a^4 - 2*a^2*b^2 + b^4)*d^2*f*e^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c
) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*
sin(d*x + c) + 12*(-I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f^3*x^2 - 2*I*(a^4 - 2*a^
2*b^2 + b^4)*d^2*f^2*x*e - I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f*e^2)*dilog((-I*a
*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 12*(-I*b^4*d^2*f^3*x^2 + 2*I*a*
b^3*d*f^3*x - I*b^4*d^2*f*e^2 - 2*I*(b^4*d^2*f^2*x - a*b^3*d*f^2)*e)*dilog(
cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 12*(I*b^4*d^2*f^3*x^2 - 2*I*a
*b^3*d*f^3*x + I*b^4*d^2*f*e^2 + 2*I*(b^4*d^2*f^2*x - a*b^3*d*f^2)*e)*dilog
(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 12*(I*b^4*d^2*f^3*x^2 + 2*I*
```

```
a*b^3*d*f^3*x + I*b^4*d^2*f*e^2 + 2*I*(b^4*d^2*f^2*x + a*b^3*d*f^2)*e)*dilo
g(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 12*(-I*b^4*d^2*f^3*x^2 - 2
*I*a*b^3*d*f^3*x - I*b^4*d^2*f*e^2 - 2*I*(b^4*d^2*f^2*x + a*b^3*d*f^2)*e)*d
ilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 4*((a^4 - 2*a^2*b^2 + b
^4)*c^3*f^3 - 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*f^2*e + 3*(a^4 - 2*a^2*b^2 +
b^4)*c*d^2*f*e^2 - (a^4 - 2*a^2*b^2 + b^4)*d^3*e^3)*log(2*b*cos(d*x + c) +
2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + 4*(
(a^4 - 2*a^2*b^2 + b^4)*c^3*f^3 - 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*f^2*e + 3
*(a^4 - 2*a^2*b^2 + b^4)*c*d^2*f*e^2 - (a^4 - 2*a^2*b^2 + b^4)*d^3*e^3)*log
(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a
)*sin(d*x + c) + 4*((a^4 - 2*a^2*b^2 + b^4)*c^3*f^3 - 3*(a^4 - 2*a^2*b^2 +
b^4)*c^2*d*f^2*e + 3*(a^4 - 2*a^2*b^2 + b^4)*c*d^2*f*e^2 - (a^4 - 2*a^2*b^2
 + b^4)*d^3*e^3)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^
2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + 4*((a^4 - 2*a^2*b^2 + b^4)*c^3*f^3 -
3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*f^2*e + 3*(a^4 - 2*a^2*b^2 + b^4)*c*d^2*f*e
^2 - (a^4 - 2*a^2*b^2 + b^4)*d^3*e^3)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 4*((a^4 - 2*a^2*
b^2 + b^4)*d^3*f^3*x^3 + (a^4 - 2*a^2*b^2 + b^4)*c^3*f^3 + 3*((a^4 - 2*a^2*
b^2 + b^4)*d^3*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*d^2*f)*e^2 + 3*((a^4 - 2*a^2
*b^2 + b^4)*d^3*f^2*x^2 - (a^4 - 2*a^2*b^2 + b^4)*c^2*d*f^2)*e)*log(-(I*a*c
os(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) - b)/b)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*f^3*x^
3 + (a^4 - 2*a^2*b^2 + b^4)*c^3*f^3 + 3*((a^4 - 2*a^2*b^2 + b^4)*d^3*f*x +
(a^4 - 2*a^2*b^2 + b^4)*c*d^2*f)*e^2 + 3*((a^4 - 2*a^2*b^2 + b^4)*d^3*f^2*x
^2 - (a^4 - 2*a^2*b^2 + b^4)*c^2*d*f^2)*e)*log(-(I*a*cos(d*x + c) - a*sin(d
*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + (a^4 - 2*a^2*b^2
+ b^4)*c^3*f^3 + 3*((a^4 - 2*a^2*b^2 + b^4)*d^3*f*x + (a^4 - 2*a^2*b^2 + b^
4)*c*d^2*f)*e^2 + 3*((a^4 - 2*a^2*b^2 + b^4)*d^3*f^2*x^2 - (a^4 - 2*a^2*b^2
 + b^4)*c^2*d*f^2)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) - 4*(
(a^4 - 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + (a^4 - 2*...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.346 $\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=1051

$$-\frac{befx}{2a^2d}-\frac{(a^2-b^2)\,efx}{2a^2bd}-\frac{bf^2x^2}{4a^2d}-\frac{(a^2-b^2)\,f^2x^2}{4a^2bd}+\frac{ib(e+fx)^3}{3a^2f}-\frac{i(a^2-b^2)^2\,(e+fx)^3}{3a^2b^3f}-\frac{4f(e+fx)\tanh^{-1}\left(e\right.}{ad^2}$$

```
[Out] -b*(f*x+e)^2*ln(1-exp(2*I*(d*x+c)))/a^2/d-1/2*(a^2-b^2)*e*f*x/a^2/b/d-2*(a^
2-b^2)*f*(f*x+e)*cos(d*x+c)/a/b^2/d^2+1/2*b*f*(f*x+e)*cos(d*x+c)*sin(d*x+c)
/a^2/d^2+2*(a^2-b^2)^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2))
)/a^2/b^3/d^3+2*(a^2-b^2)^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(
1/2)))/a^2/b^3/d^3+(a^2-b^2)^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/a^2/b^3/d+(a^2-b^2)^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-
b^2)^(1/2)))/a^2/b^3/d+2*(a^2-b^2)*f^2*sin(d*x+c)/a/b^2/d^3-(a^2-b^2)*(f*x+
e)^2*sin(d*x+c)/a/b^2/d-1/4*(a^2-b^2)*f^2*sin(d*x+c)^2/a^2/b/d^3+1/2*(a^2-b
^2)*(f*x+e)^2*sin(d*x+c)^2/a^2/b/d-1/3*I*(a^2-b^2)^2*(f*x+e)^3/a^2/b^3/f-2*
f*(f*x+e)*cos(d*x+c)/a/d^2-(f*x+e)^2*csc(d*x+c)/a/d+1/3*I*b*(f*x+e)^3/a^2/f
+2*f^2*sin(d*x+c)/a/d^3-1/4*b*f^2*x^2/a^2/d-1/4*b*f^2*sin(d*x+c)^2/a^2/d^3+
1/2*b*(f*x+e)^2*sin(d*x+c)^2/a^2/d+2*I*f^2*polylog(2,-exp(I*(d*x+c)))/a/d^3
-4*f*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d^2-1/2*b*f^2*polylog(3,exp(2*I*(d*x
+c)))/a^2/d^3-2*I*f^2*polylog(2,exp(I*(d*x+c)))/a/d^3-(f*x+e)^2*sin(d*x+c)/
a/d-2*I*(a^2-b^2)^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/
2)))/a^2/b^3/d^2-2*I*(a^2-b^2)^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+
(a^2-b^2)^(1/2)))/a^2/b^3/d^2+1/2*(a^2-b^2)*f*(f*x+e)*cos(d*x+c)*sin(d*x+c)
/a^2/b/d^2-1/2*b*e*f*x/a^2/d-1/4*(a^2-b^2)*f^2*x^2/a^2/b/d+I*b*f*(f*x+e)*po
lylog(2,exp(2*I*(d*x+c)))/a^2/d^2
```

**Rubi** [A]
time = 1.40, antiderivative size = 1051, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 20, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4639, 4493, 3392, 3377, 2717, 2713, 4495, 4268, 2317, 2438, 4490, 3391, 4489, 3798, 2221, 2611, 2320, 6724, 4621, 4615}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(b*e*f*x)/(a^2*d) - ((a^2 - b^2)*e*f*x)/(2*a^2*b*d) - (b*f^2*x^2)/(4*a
^2*d) - ((a^2 - b^2)*f^2*x^2)/(4*a^2*b*d) + ((I/3)*b*(e + f*x)^3)/(a^2*f) -
 ((I/3)*(a^2 - b^2)^2*(e + f*x)^3)/(a^2*b^3*f) - (4*f*(e + f*x)*ArcTanh[E^(
I*(c + d*x))])/(a*d^2) - (2*f*(e + f*x)*Cos[c + d*x])/(a*d^2) - (2*(a^2 - b
^2)*f*(e + f*x)*Cos[c + d*x])/(a*b^2*d^2) - ((e + f*x)^2*Csc[c + d*x])/(a*d
```

```
) + ((a^2 - b^2)^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(a^2*b^3*d) + ((a^2 - b^2)^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d
*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d) - (b*(e + f*x)^2*Log[1 - E^((2*I)
*(c + d*x))])/(a^2*d) + ((2*I)*f^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^3) -
((2*I)*f^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^3) - ((2*I)*(a^2 - b^2)^2*f*(e
 + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d
^2) - ((2*I)*(a^2 - b^2)^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(a^2*b^3*d^2) + (I*b*f*(e + f*x)*PolyLog[2, E^((2*I)*(
c + d*x))])/(a^2*d^2) + (2*(a^2 - b^2)^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^3) + (2*(a^2 - b^2)^2*f^2*PolyLog[3,
(I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d^3) - (b*f^2*PolyLo
g[3, E^((2*I)*(c + d*x))])/(2*a^2*d^3) + (2*f^2*Sin[c + d*x])/(a*d^3) + (2*
(a^2 - b^2)*f^2*Sin[c + d*x])/(a*b^2*d^3) - ((e + f*x)^2*Sin[c + d*x])/(a*d
) - ((a^2 - b^2)*(e + f*x)^2*Sin[c + d*x])/(a*b^2*d) + (b*f*(e + f*x)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a^2*d^2) + ((a^2 - b^2)*f*(e + f*x)*Cos[c + d*x]*S
in[c + d*x])/(2*a^2*b*d^2) - (b*f^2*Sin[c + d*x]^2)/(4*a^2*d^3) - ((a^2 - b
^2)*f^2*Sin[c + d*x]^2)/(4*a^2*b*d^3) + (b*(e + f*x)^2*Sin[c + d*x]^2)/(2*a
^2*d) + ((a^2 - b^2)*(e + f*x)^2*Sin[c + d*x]^2)/(2*a^2*b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos^3(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot}{a+b\sin(c+dx)}}{a}$$

$$= -\frac{\int (e+fx)^2\cos^3(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\cos(c+dx)\cot^2(c+}{a}$$

$$= -\frac{2f(e+fx)\cos^3(c+dx)}{9ad^2} - \frac{(e+fx)^2\cos^2(c+dx)\sin(c+dx)}{3ad}$$

$$= -\frac{(e+fx)^2\csc(c+dx)}{ad} - \frac{5(e+fx)^2\sin(c+dx)}{3ad} + \frac{2\int (e+fx)}{}$$

$$= \frac{ib(e+fx)^3}{3a^2f} - \frac{i(a^2-b^2)^2(e+fx)^3}{3a^2b^3f} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+}\right.}{ad^2}$$

$$= \frac{ib(e+fx)^3}{3a^2f} - \frac{i(a^2-b^2)^2(e+fx)^3}{3a^2b^3f} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+}\right.}{ad^2}$$

$$= -\frac{befx}{2a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)efx}{2bd} - \frac{bf^2x^2}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)f^2x^2}{4bd} + \frac{ib(e+fx)}{3a^2f}$$

$$= -\frac{befx}{2a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)efx}{2bd} - \frac{bf^2x^2}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)f^2x^2}{4bd} + \frac{ib(e+fx)}{3a^2f}$$

$$= -\frac{befx}{2a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)efx}{2bd} - \frac{bf^2x^2}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)f^2x^2}{4bd} + \frac{ib(e+fx)}{3a^2f}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5228 vs. $2(1051) = 2102$.
time = 10.38, size = 5228, normalized size = 4.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),
x]
```

```
[Out] Result too large to show
```

**Maple** [**F**]

time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2\left(\cos^3(dx+c)\right)\left(\cot^2(dx+c)\right)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3146 vs. 2(978) = 1956.
time = 0.77, size = 3146, normalized size = 2.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -1/8*(8*b^4*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 8*
b^4*f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 8*b^4*f^2*
polylog(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 8*b^4*f^2*polylog
(3, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 8*(a^3*b + a*b^3)*d^2*f^
2*x^2 - 16*a^3*b*f^2 + 16*(a^3*b + a*b^3)*d^2*f*x*e - 8*(a^4 - 2*a^2*b^2 +
b^4)*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 8*(a^4 - 2*a^2
*b^2 + b^4)*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 8*(a^4
- 2*a^2*b^2 + b^4)*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) -
 8*(a^4 - 2*a^2*b^2 + b^4)*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x +
 c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*
```

```
x + c) + 4*(a^2*b^2*d*f^2*x + a^2*b^2*d*f*e)*cos(d*x + c)^3 + 8*(a^3*b + a*
b^3)*d^2*e^2 - 8*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*f*x*e + a^3*b*d^2*e^2 - 2
*a^3*b*f^2)*cos(d*x + c)^2 + 8*(I*(a^4 - 2*a^2*b^2 + b^4)*d*f^2*x + I*(a^4
- 2*a^2*b^2 + b^4)*d*f*e)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos
(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x +
c) + 8*(I*(a^4 - 2*a^2*b^2 + b^4)*d*f^2*x + I*(a^4 - 2*a^2*b^2 + b^4)*d*f*e
)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 8*(-I*(a^4 - 2*a^2*
b^2 + b^4)*d*f^2*x - I*(a^4 - 2*a^2*b^2 + b^4)*d*f*e)*dilog((-I*a*cos(d*x +
 c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b + 1)*sin(d*x + c) + 8*(-I*(a^4 - 2*a^2*b^2 + b^4)*d*f^2*x - I
*(a^4 - 2*a^2*b^2 + b^4)*d*f*e)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) -
 (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin
(d*x + c) + 8*(-I*b^4*d*f^2*x - I*b^4*d*f*e + I*a*b^3*f^2)*dilog(cos(d*x +
c) + I*sin(d*x + c))*sin(d*x + c) + 8*(I*b^4*d*f^2*x + I*b^4*d*f*e - I*a*b^
3*f^2)*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 8*(I*b^4*d*f^2*x
 + I*b^4*d*f*e + I*a*b^3*f^2)*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x
 + c) + 8*(-I*b^4*d*f^2*x - I*b^4*d*f*e - I*a*b^3*f^2)*dilog(-cos(d*x + c)
- I*sin(d*x + c))*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*c^2*f^2 - 2*(a^
4 - 2*a^2*b^2 + b^4)*c*d*f*e + (a^4 - 2*a^2*b^2 + b^4)*d^2*e^2)*log(2*b*cos
(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*
x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*c^2*f^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*c*d
*f*e + (a^4 - 2*a^2*b^2 + b^4)*d^2*e^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*
x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 4*((a^4 - 2*a^2
*b^2 + b^4)*c^2*f^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*c*d*f*e + (a^4 - 2*a^2*b^2
+ b^4)*d^2*e^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*c^2*f^2 - 2
*(a^4 - 2*a^2*b^2 + b^4)*c*d*f*e + (a^4 - 2*a^2*b^2 + b^4)*d^2*e^2)*log(-2*
b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*s
in(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x^2 - (a^4 - 2*a^2*b^2 + b
^4)*c^2*f^2 + 2*((a^4 - 2*a^2*b^2 + b^4)*d^2*f*x + (a^4 - 2*a^2*b^2 + b^4)*
c*d*f)*e)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) - 4*((a^4 - 2*a^2*
b^2 + b^4)*d^2*f^2*x^2 - (a^4 - 2*a^2*b^2 + b^4)*c^2*f^2 + 2*((a^4 - 2*a^2*
b^2 + b^4)*d^2*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*d*f)*e)*log(-(I*a*cos(d*x +
c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) - b)/b)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x^2 - (a^4
- 2*a^2*b^2 + b^4)*c^2*f^2 + 2*((a^4 - 2*a^2*b^2 + b^4)*d^2*f*x + (a^4 - 2*
a^2*b^2 + b^4)*c*d*f)*e)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) -
4*((a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x^2 - (a^4 - 2*a^2*b^2 + b^4)*c^2*f^2 +
2*((a^4 - 2*a^2*b^2 + b^4)*d^2*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*d*f)*e)*log(
-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) + 4*(b^4*d^2*f^2*x^2 + 2*a*b^3*
d*f^2*x + b^4*d^2*e^2 + 2*(b^4*d^2*f*x + a*b^3*d*f)*e)*log(cos(d*x + c) + I
```

```
*sin(d*x + c) + 1)*sin(d*x + c) + 4*(b^4*d^2*f^2*x^2 + 2*a*b^3*d*f^2*x + b^
4*d^2*e^2 + 2*(b^4*d^2*f*x + a*b^3*d*f)*e)*log(cos(d*x + c) - I*sin(d*x + c
) + 1)*sin(d*x + c) + 4*(b^4*d^2*e^2 - 2*(b^4*c + a*b^3)*d*f*e + (b^4*c^2 +
 2*a*b^3*c)*f^2)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)*sin(d*x
+ c) + 4*(b^4*d^2*e^2 - 2*(b^4*c + a*b^3)*d*f*e + (b^4*c^2 + 2*a*b^3*c)*f^2
)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 4*(b^4*d
^2*f^2*x^2 - 2*a*b^3*d*f^2*x - (b^4*c^2 + 2*a*b^3*c)*f^2 + 2*(b^4*d^2*f*x +
 b^4*c*d*f)*e)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + 4*(b^
4*d^2*f^2*x^2 - 2*a*b^3*d*f^2*x - (b^4*c^2 + 2*...
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cos(c + d*x)**3*cot(c + d*x)**2/(a + b*sin(c + d*x)),
 x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

# 3.347 $\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=641

$$-\frac{bfx}{4a^2d}-\frac{(a^2-b^2)\,fx}{4a^2bd}+\frac{ib(e+fx)^2}{2a^2f}-\frac{i(a^2-b^2)^2\,(e+fx)^2}{2a^2b^3f}-\frac{f\tanh^{-1}(\cos(c+dx))}{ad^2}-\frac{f\cos(c+dx)}{ad^2}-\frac{(a^2-b^2)}{}$$

```
[Out] -1/4*b*f*x/a^2/d-1/4*(a^2-b^2)*f*x/a^2/b/d-1/2*I*(a^2-b^2)^2*(f*x+e)^2/a^2/
b^3/f+1/2*I*b*(f*x+e)^2/a^2/f-f*arctanh(cos(d*x+c))/a/d^2-f*cos(d*x+c)/a/d^
2-(a^2-b^2)*f*cos(d*x+c)/a/b^2/d^2-(f*x+e)*csc(d*x+c)/a/d-b*(f*x+e)*ln(1-ex
p(2*I*(d*x+c)))/a^2/d+(a^2-b^2)^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b
^2)^(1/2)))/a^2/b^3/d+(a^2-b^2)^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b
^2)^(1/2)))/a^2/b^3/d-I*(a^2-b^2)^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-
b^2)^(1/2)))/a^2/b^3/d^2+1/2*I*b*f*polylog(2,exp(2*I*(d*x+c)))/a^2/d^2-I*(a
^2-b^2)^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^3/d^2-(
f*x+e)*sin(d*x+c)/a/d-(a^2-b^2)*(f*x+e)*sin(d*x+c)/a/b^2/d+1/4*b*f*cos(d*x+
c)*sin(d*x+c)/a^2/d^2+1/4*(a^2-b^2)*f*cos(d*x+c)*sin(d*x+c)/a^2/b/d^2+1/2*b
*(f*x+e)*sin(d*x+c)^2/a^2/d+1/2*(a^2-b^2)*(f*x+e)*sin(d*x+c)^2/a^2/b/d
```

**Rubi** [A]
time = 0.79, antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4639, 4493, 3391, 3377, 2718, 4495, 3855, 4490, 2715, 8, 4489, 3798, 2221, 2317, 2438, 4621, 4615}

$$-\frac{if(a^2-b^2)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^3d^2}-\frac{if(a^2-b^2)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2b^3d^2}+\frac{ibf\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2a^2d^2}-\frac{f(a^2-b^2)\cos(c+dx)}{ab^2d^2}+\frac{f(a^2-b^2)\sin(c+dx)\cos(c+dx)}{4a^2bd^2}+\frac{(a^2-b^2)\,(e+fx)\sin^2(c+dx)}{2a^2bd}-\frac{(a^2-b^2)\,(e+fx)\sin(c+dx)}{ab^2d}-\frac{fx(a^2-b^2)}{4a^2bd}+\frac{(a^2-b^2)^2\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^3d}+\frac{(a^2-b^2)^2\,(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2b^3d}-\frac{i(a^2-b^2)^2\,(e+fx)^2}{2a^2b^3f}+\frac{bf\sin(c+dx)\cos(c+dx)}{4a^2d^2}-\frac{b(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{a^2d}+\frac{b(e+fx)\sin^2(c+dx)}{2a^2d}-\frac{bfx}{4a^2d}+\frac{ib(e+fx)^2}{2a^2f}-\frac{f\cos(c+dx)}{ad^2}-\frac{f\tanh^{-1}(\cos(c+dx))}{ad^2}-\frac{(e+fx)\sin(c+dx)}{ad}-\frac{(e+fx)\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(b*f*x)/(a^2*d) - ((a^2 - b^2)*f*x)/(4*a^2*b*d) + ((I/2)*b*(e + f*x)^2
)/(a^2*f) - ((I/2)*(a^2 - b^2)^2*(e + f*x)^2)/(a^2*b^3*f) - (f*ArcTanh[Cos[
c + d*x]])/(a*d^2) - (f*Cos[c + d*x])/(a*d^2) - ((a^2 - b^2)*f*Cos[c + d*x]
)/(a*b^2*d^2) - ((e + f*x)*Csc[c + d*x])/(a*d) + ((a^2 - b^2)^2*(e + f*x)*L
og[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d) + ((a^2 -
b^2)^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2
*b^3*d) - (b*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a^2*d) - (I*(a^2 - b^
2)^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^
2) - (I*(a^2 - b^2)^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])])/(a^2*b^3*d^2) + ((I/2)*b*f*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2*d^2)
 - ((e + f*x)*Sin[c + d*x])/(a*d) - ((a^2 - b^2)*(e + f*x)*Sin[c + d*x])/(a
*b^2*d) + (b*f*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*d^2) + ((a^2 - b^2)*f*Cos[
c + d*x]*Sin[c + d*x])/(4*a^2*b*d^2) + (b*(e + f*x)*Sin[c + d*x]^2)/(2*a^2*
d) + ((a^2 - b^2)*(e + f*x)*Sin[c + d*x]^2)/(2*a^2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
```

```
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4621

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Cos[c + d*x]^(n
 - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4639

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos^3(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+}{a+b\sin(c+dx)}}{a}$$

$$= -\frac{\int (e+fx)\cos^3(c+dx)\,dx}{a} + \frac{\int (e+fx)\cos(c+dx)\cot^2(c+dx}{a}$$

$$= -\frac{f\cos^3(c+dx)}{9ad^2} - \frac{(e+fx)\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{2\int (e+j}{}$$

$$= -\frac{(e+fx)\csc(c+dx)}{ad} - \frac{5(e+fx)\sin(c+dx)}{3ad} + \frac{2\int (e+fx)\,\mathrm{co}}{3a}$$

$$= \frac{ib(e+fx)^2}{2a^2f} - \frac{i(a^2-b^2)^2(e+fx)^2}{2a^2b^3f} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} -$$

$$= \frac{ib(e+fx)^2}{2a^2f} - \frac{i(a^2-b^2)^2(e+fx)^2}{2a^2b^3f} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} -$$

$$= -\frac{bfx}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)fx}{4bd} + \frac{ib(e+fx)^2}{2a^2f} - \frac{i(a^2-b^2)^2(e+fx)^2}{2a^2b^3f} -$$

$$= -\frac{bfx}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)fx}{4bd} + \frac{ib(e+fx)^2}{2a^2f} - \frac{i(a^2-b^2)^2(e+fx)^2}{2a^2b^3f} -$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2504 vs. $2(641) = 1282$.
time = 13.81, size = 2504, normalized size = 3.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*f*Cos[c + d*x])/(b^2*d^2)) - ((d*e - c*f + f*(c + d*x))*Cos[2*(c + d*x
)])/(4*b*d^2) + ((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c + d*x)/2] - f*(c + d
*x)*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(2*a*d^2) - (b*e*Log[Sin[c + d*x]])
/(a^2*d) + (b*c*f*Log[Sin[c + d*x]])/(a^2*d^2) + (f*Log[Tan[(c + d*x)/2]])/
(a*d^2) - (b*f*((c + d*x)*Log[1 - E^((2*I)*(c + d*x))] - (I/2)*((c + d*x)^2
 + PolyLog[2, E^((2*I)*(c + d*x))])))/(a^2*d^2) + (Sec[(c + d*x)/2]*(-(d*e*
Sin[(c + d*x)/2]) + c*f*Sin[(c + d*x)/2] - f*(c + d*x)*Sin[(c + d*x)/2]))/(
```

2*a*d^2) - (a*(d*e - c*f + f*(c + d*x))*Sin[c + d*x])/(b^2*d^2) + (f*Sin[2*(c + d*x)])/(8*b*d^2) + ((f*(c + d*x)^2 + (2*I)*d*e*Log[Sec[(c + d*x)/2]^2] - (2*I)*c*f*Log[Sec[(c + d*x)/2]^2] - (2*I)*d*e*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])] + (2*I)*c*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])] - (4*I)*f*(c + d*x)*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + 4*f*PolyLog[2, -Cos[c + d*x] + I*Sin[c + d*x]] + 2*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))] - 2*f*PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))] + 2*f*PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])] - 2*f*PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]])*((-2*e*Cos[c + d*x])/(a + b*Sin[c + d*x]) + (a^2*e*Cos[c + d*x])/(b^2*(a + b*Sin[c + d*x])) + (b^2*e*Cos[c + d*x])/(a^2*(a + b*Sin[c + d*x])) + (2*c*f*Cos[c + d*x])/(d*(a + b*Sin[c + d*x])) - (a^2*c*f*Cos[c + d*x])/(b^2*d*(a + b*Sin[c + d*x])) - (b^2*c*f*Cos[c + d*x])/(a^2*d*(a + b*Sin[c + d*x])) - (2*f*(c + d*x)*Cos[c + d*x])/(d*(a + b*Sin[c + d*x])) + (a^2*f*(c + d*x)*Cos[c + d*x])/(b^2*d*(a + b*Sin[c + d*x])) + (b^2*f*(c + d*x)*Cos[c + d*x])/(a^2*d*(a + b*Sin[c + d*x]))))/(d*(2*f*(c + d*x) - (4*I)*f*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - (4*f*Log[1 + Cos[c + d*x] - I*Sin[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/(-Cos[c + d*x] + I*Sin[c + d*x]) + (I*f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) + (I*f*Log[1 - (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) + (2*I)*d*e*Tan[(c + d*x)/2] - (2*I)*c*f*Tan[(c + d*x)/2] + ((2*I)*f*(c + d*x)*Sec[(c + d*x)/2]^2)/(-I + Tan[(c + d*x)/2]) - (f*Log[1 - (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(I + Tan[(c + d*x)/2]) + (I*a*f*Log[1 - (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - ((2*I)*d*e*Cos[(c + d*x)/

```
2]^2*(b*Cos[c + d*x]*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c +
 d*x])*Tan[(c + d*x)/2]))/(a + b*Sin[c + d*x]) + ((2*I)*c*f*Cos[(c + d*x)/2
]^2*(b*Cos[c + d*x]*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c +
d*x])*Tan[(c + d*x)/2]))/(a + b*Sin[c + d*x])))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2484 vs. $2(594) = 1188$.
time = 3.30, size = 2485, normalized size = 3.88

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2485 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/d/b^3*a^2*e*ln(exp(I*(d*x+c)))+1/d/b^3*a^2*e*ln(I*b*exp(2*I*(d*x+c))-2*a
*exp(I*(d*x+c))-I*b)+3*I*b/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a
^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+3*I*b/d^2*f/(-a^2+b^2)*dilog((I*a+b*
exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+4*I/b/d*c*f*x-I/d^
2/b^3*a^2*f*c^2-1/16*(2*d*x*f+I*f+2*d*e)/b/d^2*exp(2*I*(d*x+c))-1/16*(2*d*x
*f-I*f+2*d*e)/b/d^2*exp(-2*I*(d*x+c))+4/d/b*ln(exp(I*(d*x+c)))*e-2/d/b*e*ln
(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-2*I*(f*x+e)*exp(I*(d*x+c))/d/
a/(exp(2*I*(d*x+c))-1)-1/d/b^3*a^4*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-
a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-1/d^2/b^3*a^4*f/(-a^2+b^2)*ln((I*
a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/d/b^3*a^4*
f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/
2)))*x-1/d^2/b^3*a^4*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2)
)/(I*a+(-a^2+b^2)^(1/2)))*c-1/2*I/b^3*a^2*f*x^2+1/a^2*b^3/d^2*f/(-a^2+b^2)*
ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-I/a^2*
b^3/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a
^2+b^2)^(1/2)))-I/a^2*b^3/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^
2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-3/d*b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d
*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-3/d^2*b*f/(-a^2+b^2)*ln(
(I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-3/d*b*f/(
-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2))
)*x-3/d^2*b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-
a^2+b^2)^(1/2)))*c+2/d^2/b*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I
*b)-4/d^2/b*f*c*ln(exp(I*(d*x+c)))+1/2*I*a*(d*x*f+I*f+d*e)/b^2/d^2*exp(I*(d
*x+c))-1/a^2/d*b*e*ln(exp(I*(d*x+c))-1)-1/a^2/d*b*e*ln(exp(I*(d*x+c))+1)+1/
a/d^2*f*ln(exp(I*(d*x+c))-1)-1/a/d^2*f*ln(exp(I*(d*x+c))+1)+3/d/b*f/(-a^2+b
^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2*
x+3/d^2/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^
2+b^2)^(1/2)))*a^2*c+3/d/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)
^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*x+3/d^2/b*f/(-a^2+b^2)*ln((I*a+b*exp(I*
```

```
(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*c+1/a^2*b^3/d*f/(-a^
2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x
+1/a^2*b^3/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(
-a^2+b^2)^(1/2)))*x+1/a^2*b^3/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a
^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/a^2/d*b*f*ln(exp(I*(d*x+c))+1)*x
+1/a^2/d^2*b*f*c*ln(exp(I*(d*x+c))-1)-I/a^2/d^2*b*f*dilog(exp(I*(d*x+c)))+I
/b^3*a^2*e*x+I/a^2*b/d^2*f*dilog(exp(I*(d*x+c))+1)-1/a^2*b/d^2*f*c*ln(I*b*e
xp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-1/d^2/b^3*a^2*f*c*ln(I*b*exp(2*I*(d
*x+c))-2*a*exp(I*(d*x+c))-I*b)+2/d^2/b^3*a^2*f*c*ln(exp(I*(d*x+c)))+1/a^2*b
/d*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+2*I/b/d^2*c^2*f+I/b*f*
x^2-2*I/b*e*x-1/2*I*a*(d*x*f-I*f+d*e)/b^2/d^2*exp(-I*(d*x+c))+I*a^4/b^3/d^2
*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)
^(1/2)))+I*a^4/b^3/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^
(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2*I*a^2/b^3/d*c*f*x-3*I/d^2/b*f/(-a^2+b^2)*d
ilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2-3*
I/d^2/b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a
^2+b^2)^(1/2)))*a^2
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1719 vs. 2(592) = 1184.
time = 0.69, size = 1719, normalized size = 2.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/4*(a^2*b^2*f*cos(d*x + c)^3 - 2*I*b^4*f*dilog(cos(d*x + c) + I*sin(d*x +
 c))*sin(d*x + c) + 2*I*b^4*f*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x
+ c) + 2*I*b^4*f*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 2*I*b
^4*f*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - a^2*b^2*f*cos(d*x
 + c) + 4*(a^3*b + a*b^3)*d*f*x + 2*I*(a^4 - 2*a^2*b^2 + b^4)*f*dilog((I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
```

```
a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 2*I*(a^4 - 2*a^2*b^2 + b^4)*f*di
log((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 2*I*(a^4 - 2*a^2*b^2 +
b^4)*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 2*I*(a^4 - 2*
a^2*b^2 + b^4)*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 4*(
a^3*b*d*f*x + a^3*b*d*e)*cos(d*x + c)^2 + 4*(a^3*b + a*b^3)*d*e + 2*((a^4 -
 2*a^2*b^2 + b^4)*c*f - (a^4 - 2*a^2*b^2 + b^4)*d*e)*log(2*b*cos(d*x + c) +
 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + 2*
((a^4 - 2*a^2*b^2 + b^4)*c*f - (a^4 - 2*a^2*b^2 + b^4)*d*e)*log(2*b*cos(d*x
 + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x +
c) + 2*((a^4 - 2*a^2*b^2 + b^4)*c*f - (a^4 - 2*a^2*b^2 + b^4)*d*e)*log(-2*b
*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*si
n(d*x + c) + 2*((a^4 - 2*a^2*b^2 + b^4)*c*f - (a^4 - 2*a^2*b^2 + b^4)*d*e)*
log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2
*I*a)*sin(d*x + c) - 2*((a^4 - 2*a^2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 +
b^4)*c*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) - 2*((a^4 - 2*a^2*
b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(-(I*a*cos(d*x + c) - a*
sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
 b)/b)*sin(d*x + c) - 2*((a^4 - 2*a^2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 +
 b^4)*c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b)*sin(d*x + c) - 2*((a^4 - 2*a^
2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(-(-I*a*cos(d*x + c) -
 a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b)*sin(d*x + c) + 2*(b^4*d*f*x + b^4*d*e + a*b^3*f)*log(cos(d*x + c)
 + I*sin(d*x + c) + 1)*sin(d*x + c) + 2*(b^4*d*f*x + b^4*d*e + a*b^3*f)*log
(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + 2*(b^4*d*e - (b^4*c + a*
b^3)*f)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 2*
(b^4*d*e - (b^4*c + a*b^3)*f)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) +
1/2)*sin(d*x + c) + 2*(b^4*d*f*x + b^4*c*f)*log(-cos(d*x + c) + I*sin(d*x +
 c) + 1)*sin(d*x + c) + 2*(b^4*d*f*x + b^4*c*f)*log(-cos(d*x + c) - I*sin(d
*x + c) + 1)*sin(d*x + c) - (a^2*b^2*d*f*x - 4*a^3*b*f*cos(d*x + c) + a^2*b
^2*d*e - 2*(a^2*b^2*d*f*x + a^2*b^2*d*e)*cos(d*x + c)^2)*sin(d*x + c))/(a^2
*b^3*d^2*sin(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)**3*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
giac")
[Out] integrate((f*x + e)*cos(d*x + c)^3*cot(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
[Out] \text{Hanged}
```

# 3.348 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=96

$$-\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d+(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^2/b
^3/d-a*sin(d*x+c)/b^2/d+1/2*sin(d*x+c)^2/b/d
```

**Rubi** [A]
time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{a\sin(c+dx)}{b^2d}-\frac{\csc(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])/(a^2*b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c + d*x]^
2/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{b^4}{ax^2}-\frac{b^4}{a^2x}+x+\frac{(a^2-b^2)^2}{a^2(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2 d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-$$

**Mathematica** [A]
time = 0.13, size = 86, normalized size = 0.90

$$\frac{-\frac{2\csc(c+dx)}{a}-\frac{2b\log(\sin(c+dx))}{a^2}+\frac{2(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3}-\frac{2a\sin(c+dx)}{b^2}+\frac{\sin^2(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*Csc[c + d*x])/a - (2*b*Log[Sin[c + d*x]])/a^2 + (2*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(a^2*b^3) - (2*a*Sin[c + d*x])/b^2 + Sin[c + d*x]^2/b)/
(2*d)
```

**Maple** [A]
time = 0.15, size = 90, normalized size = 0.94

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{-\frac{(\sin^2(dx+c))b}{2}+a\sin(dx+c)}{b^2}-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}+\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx+c))}{b^3a^2}}{d}$ |
| default | $\frac{-\frac{-\frac{(\sin^2(dx+c))b}{2}+a\sin(dx+c)}{b^2}-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}+\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx+c))}{b^3a^2}}{d}$ |
| risch | $-\frac{ix\,a^2}{b^3}+\frac{2ix}{b}-\frac{e^{2i(dx+c)}}{8bd}+\frac{ia\,e^{i(dx+c)}}{2b^2d}-\frac{ia\,e^{-i(dx+c)}}{2b^2d}-\frac{e^{-2i(dx+c)}}{8bd}-\frac{2ia^2c}{b^3d}+\frac{4ic}{bd}-\frac{2ie^{i(dx+c)}}{da(e^{2i(dx+c)}-1)}-$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2*(-1/2*sin(d*x+c)^2*b+a*sin(d*x+c))-1/a/sin(d*x+c)-1/a^2*b*ln(si
n(d*x+c))+1/b^3*(a^4-2*a^2*b^2+b^4)/a^2*ln(a+b*sin(d*x+c)))
```

**Maxima** [A]
time = 0.29, size = 91, normalized size = 0.95

$$-\frac{\frac{2\,b\log\left(\sin\left(dx+c\right)\right)}{a^2}-\frac{b\sin\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)}{b^2}+\frac{2}{a\sin\left(dx+c\right)}-\frac{2\,\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(sin(d*x + c))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2
 + 2/(a*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/(
a^2*b^3))/d
```

**Fricas** [A]
time = 0.43, size = 133, normalized size = 1.39

$$\frac{4\,a^3b\cos\left(dx+c\right)^2-4\,b^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-4\,a^3b-4\,ab^3+4\,\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)-\left(2\,a^2b^2\cos\left(dx+c\right)^2-a^2b^2\right)\sin\left(dx+c\right)}{4\,a^2b^3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*b*cos(d*x + c)^2 - 4*b^4*log(1/2*sin(d*x + c))*sin(d*x + c) - 4*
a^3*b - 4*a*b^3 + 4*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)*sin(d*x
 + c) - (2*a^2*b^2*cos(d*x + c)^2 - a^2*b^2)*sin(d*x + c))/(a^2*b^3*d*sin(d
*x + c))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 4.71, size = 105, normalized size = 1.09

$$-\frac{\frac{2\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^2}-\frac{b\sin\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)}{b^2}-\frac{2\,\left(b\sin\left(dx+c\right)-a\right)}{a^2\sin\left(dx+c\right)}-\frac{2\,\left(a^4-2\,a^2b^2+b^4\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(sin(d*x + c)))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c)
)/b^2 - 2*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^
4)*log(abs(b*sin(d*x + c) + a))/(a^2*b^3))/d
```

**Mupad** [B]
time = 5.05, size = 233, normalized size = 2.43

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,{\left(a^2-b^2\right)}^2}{a^2\,b^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(a^2-2\,b^2\right)}{b^3\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+b^2\right)}{b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(4\,a^2+b^2\right)}{b^2}-\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{b}+1}{d\,\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^2)/(a
^2*b^3*d) - tan(c/2 + (d*x)/2)/(2*a*d) - (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^
2 - 2*b^2))/(b^3*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d) - ((2*tan(c/2 + (
d*x)/2)^2*(2*a^2 + b^2))/b^2 + (tan(c/2 + (d*x)/2)^4*(4*a^2 + b^2))/b^2 - (
4*a*tan(c/2 + (d*x)/2)^3)/b + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 +
 (d*x)/2)^3 + 2*a*tan(c/2 + (d*x)/2)^5))
```

# Chapter 4

# Appendix

## Local contents

## 4.1 Download section

The following zip files contain the raw integrals used in this test.

**Mathematica format** Mathematica_syntax.zip

**Maple and Mupad format** Maple_syntax.zip

**Sympy format** SYMPY_syntax.zip

**Sage math format** SAGE_syntax.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

### 4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
       if m:
          print ("func ", func , " is elementary_function")
       else:
          print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
       print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
       print ("m=",m)
       if m:
          print ("func ", func ," is special_function")
       else:
          print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```